THE POSITIVE GEOMETRY OF MASSIVE AMPLITUDES AND BEYOND

AIDAN HERDERSCHEE
UNIVERSITY OF MICHIGAN: LCTP

Quad-Fundamental ϕ^3 Theory

Key Points:

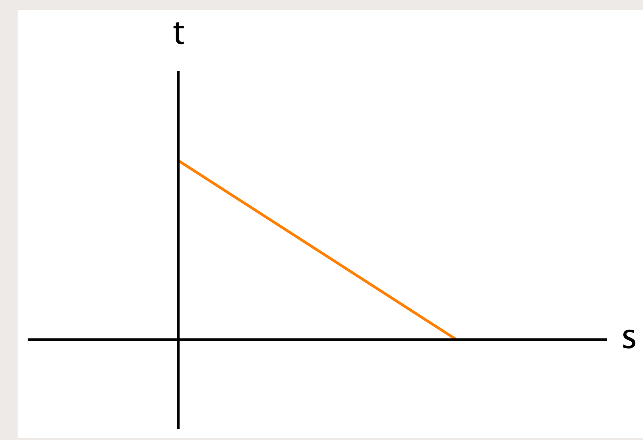
1. Analogous to the Coulomb branch of N=4 SYM.
2. Can kinematically be interpreted as a dimensional reduction of bi-adjoint theory.
3. The positive geometry is also the associahedron
4. In planar variable coordinates, the boundaries are simply shifted by an amount proportional to the mass of the state.

$$M_I = \frac{1}{2}\sum_{i,j\in I} \triangle_{i,j} \text{ for all } I, \qquad \sum_{j=1; j\neq i}^{n} \triangle_{i,j} = -2m_i^2 \text{ for all } i$$

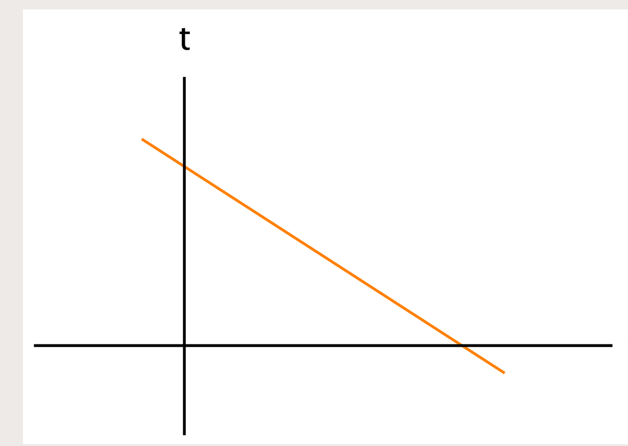
$$\mathcal{L}_{int} = \mathrm{Tr}[[T^a, T^b]T^c]\mathrm{Tr}[[\tilde{T}^{a'}, \tilde{T}^{b'}]\tilde{T}^{c'}]\phi^{aa'}\phi^{bb'}\phi^{cc'}$$

$$\langle \phi^{A,A'}_{B,B'} \rangle = (\oplus v_k \delta^{i_k}_{j_k}) \otimes (\oplus v'_{k'} \delta^{i'_{k'}}_{j'_{k'}})$$

$$U(N) \times U(\tilde{N}) \to \Pi_k U(N_k) \times \Pi_{k'} U(\tilde{N}_{k'})$$



Massless $\mathcal{A}_4$



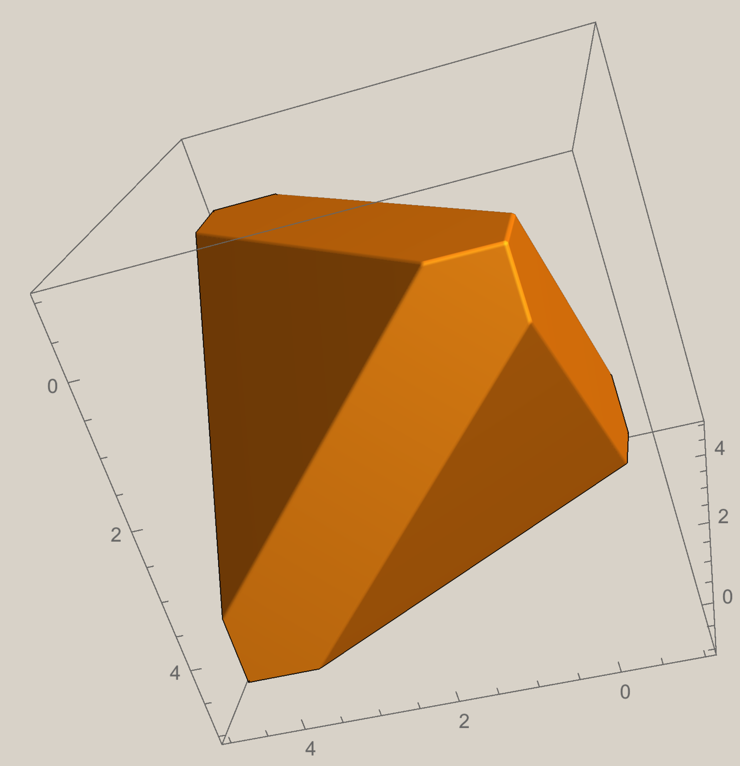
Massive $\mathcal{A}_4$

Bi-Fundamental ϕ^3 Theory

Example: $m[\psi_a \tilde{\psi}_a \psi_b \tilde{\psi}_b \psi_c \tilde{\psi}_c]$

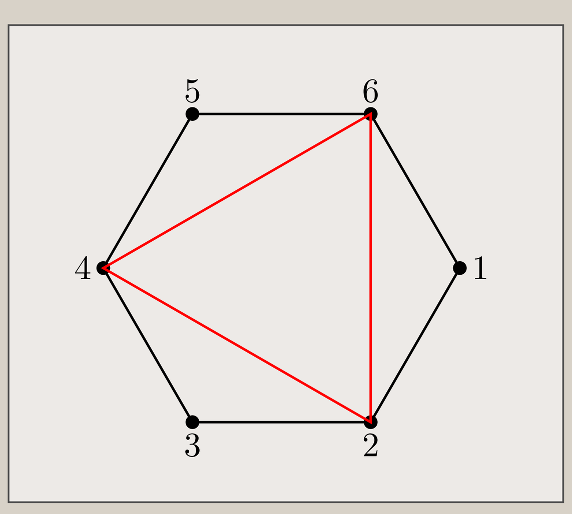
$$\mathcal{L}_{int} \ni f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} + (T^a)^i_j (\tilde{T}^{a'})^{i'}_{j'}\phi^{aa'}\psi^{jj'}_s \psi^s_{ii'}$$

$$H_6^{BF}: \begin{aligned} c_{2,5} &= X_{2,5} + X_{1,3} - X_{1,5} - X_{3,5} \\ c_{1,4} &= X_{1,4} + X_{3,5} - X_{1,3} - X_{1,5} \\ c_{3,6} &= X_{3,6} + X_{1,5} - X_{1,3} - X_{3,5} \end{aligned} \quad + \quad \triangle : X_{i,j} > 0 \quad \rightarrow$$



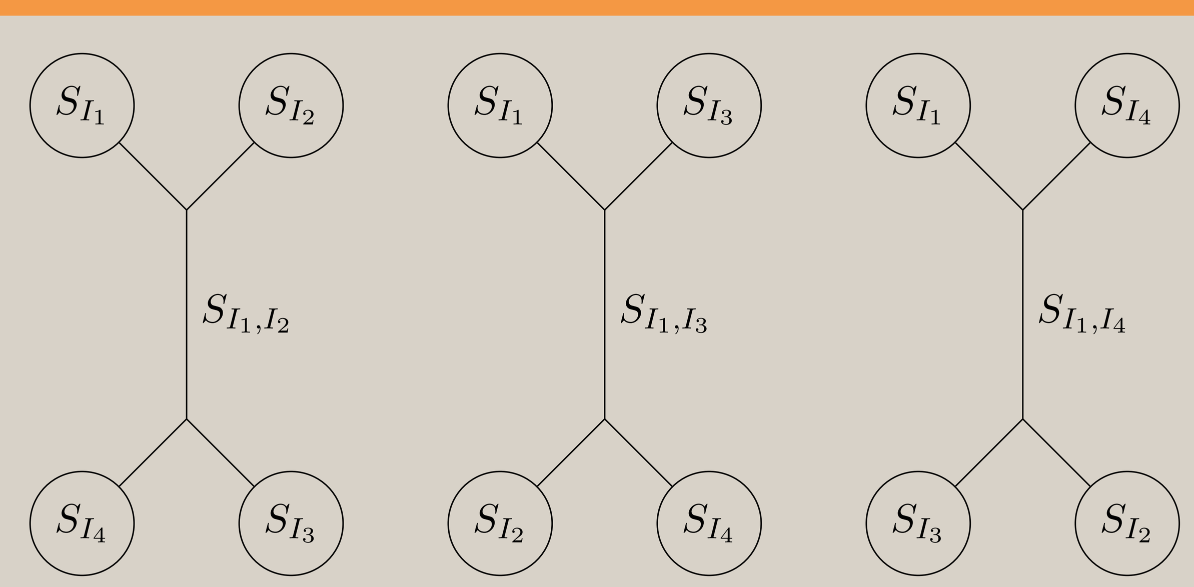
Key Points:

1. Amplitudes can be imbued with more exotic flavor and color structure than bi-adjoint amplitudes
2. Boundaries appear at infinity in the positive geometry



$$H_6^{BF} \cap \triangle : \quad \Omega = \frac{1}{X_{1,3}X_{3,5}X_{1,5}} + \frac{1}{X_{1,3}X_{3,5}X_{3,6}} + \frac{1}{X_{1,3}X_{1,4}X_{1,5}} + \frac{1}{X_{2,5}X_{3,5}X_{1,5}}$$

Color-Kinematics Duality



Key Points:

1. Small kinematic space encodes the color and flavor structure of the theory.
   1.1. Planar variables forbidden by flavor or fundamental charge conservation are set to constants.
   1.2. The 7-term identity only applies to a sub-set of 4-point sub-graphs
2. The mass generation mechanism must be equivalent to a dimensional reduction procedure.

Towards the Massive Amplituhedron

$$X_I \to X_I + M_I$$
$$\Rightarrow$$
$$|i\rangle[i| \to |i^1\rangle[i_1| + |i^2\rangle[i_2|$$

Key Points:

1. Conjecture: A projection through the Positive Lagrangian Grassmannian, with an additional restriction, into Gr(n,n+2)
2. Future work:
   2.1. Confirming the exact structure of the massive Amplituhedron
   2.2. Understanding how band structure emerges from the massive Amplituhedron.