Scattering Amplitudes on the Celestial Sphere

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Based on work with Dhritiman Nandan, Anastasia Volovich, and Michael Zlotnikov: 1711.08435 and 1904.10940

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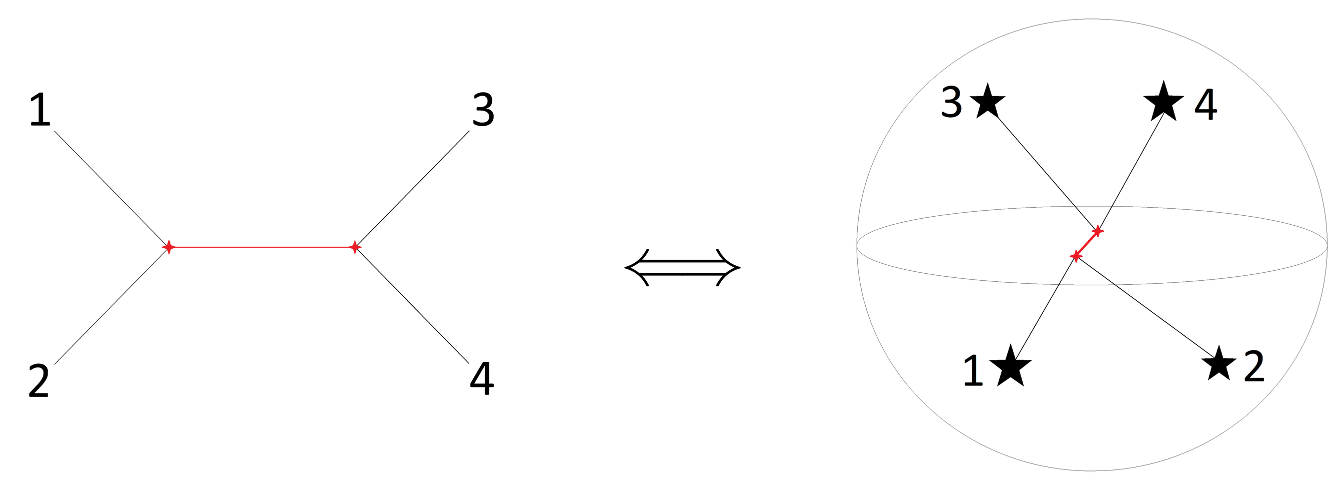
Introduction

The study of mathematical properties of scattering amplitudes have yielded fruitful results from various directions of research. On this poster, we present results from the study of scattering amplitudes, recast in the basis of conformal primary wavefunctions by Pasterski, Strominger, and Shao (PSS) [1, 2] in $4D$ Minkowski space. The PSS basis makes $SL(2,\mathbb{C})$ Lorentz transformations manifest and takes scattering amplitudes to conformal correlation functions on the celestial sphere at null infinity.



Conformal Primary Wavefunctions

The PSS basis for massless particles [2] is given by a Mellin transform, in the particle energy, of the usual plane-wave basis that we compute amplitudes in. For scalars, the basis is given by

$$\varphi^{\pm}_{\Delta}(X^\mu, z, \bar z) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i k\cdot X}\,,$$

where the $-$ $(+)$ indicates an incoming (outgoing) particle in a scattering process. To ensure orthogonality and completeness of the basis, we require $\Delta = 1 + i\mathbb{R}$.

Massless Scattering Amplitudes on the Celestial Sphere

We parameterize null momenta in terms of coordinates on the celestial sphere $(z,\bar z)$ with $\epsilon = -1$ $(+1)$ for incoming (outgoing) particles:

$$k^\mu = \epsilon\omega q^\mu\,, \quad q^\mu \equiv (1+|z|^2, z+\bar z, -i(z-\bar z), 1-|z|^2)\,,$$

which allows us to translate between spinor-helicity expressions and celestial sphere coordinates

$$[ij] = 2\sqrt{\omega_i\omega_j}\bar z_{ij}\,, \qquad \langle ij\rangle = -2\epsilon_i\epsilon_j\sqrt{\omega_i\omega_j} z_{ij}\,,$$

with $z_{ij} = z_i - z_j$ and $\bar z_{ij} = \bar z_i - \bar z_j$. Starting with a scattering amplitude of massless external particles, with helicities ℓ_i, we can implement the PSS basis,

$$\tilde{\mathcal{A}}_{J_1\cdots J_n}(\Delta_j, z_j, \bar z_j) = \prod_{j=1}^n \int_0^\infty d\omega_j\, \omega_j^{\Delta_j-1}\, \mathcal{A}_{\ell_1\cdots\ell_n}(\omega_j, z_j, \bar z_j)\,,$$

where $\tilde{\mathcal{A}}$ transforms as a correlator of spin $J_i = \ell_i$ conformal primaries of dimension Δ_i on the celestial sphere under $SL(2,\mathbb{C})$, $z' = \frac{az+b}{cz+d}$,

$$\tilde{\mathcal{A}}_{J_1\cdots J_n}\left(\Delta_j, z'_j, \bar z'_j\right) = \prod_{k=1}^n \left[(cz_k+d)^{\Delta_k+J_k}(\bar c\bar z_k+\bar d)^{\Delta_k-J_k}\right] \tilde{\mathcal{A}}_{J_1\cdots J_n}(\Delta_j, z_j, \bar z_j)\,.$$

We call these objects *celestial amplitudes.*

Yang-Mills Amplitudes on the Celestial Sphere

Using the PSS basis, we compute the celestial amplitudes for the n-point MHV scattering amplitude in Yang-Mills theory, which takes the form of an Aomoto-Gelfand hypergeometric function [3],

$$\hat\varphi^{(n-4,n)}(\alpha_i,\beta_j;\gamma;x_{i,j}) = \int\limits_{\substack{u_j\geq 0\\ 1-\sum_j u_j\geq 0}} \prod_{j=6}^n du_j\, u_j^{\alpha_j-1}\left(1-\sum_{i=6}^n u_i\right)^{\gamma-\sum_{i=6}^n\alpha_i-1} \prod_{j=2}^5\left(1-\sum_{i=6}^n x_{i,j}u_i\right)^{-\beta_j}\,,$$

with coordinates living on $Gr(n-4,n)$. N^{k}MHV amplitudes are found to be more complicated Gelfand A-hypergeometric functions.

Soft Limits

The analog of soft limits for celestial amplitudes is taking conformal dimensions to unity: $\Delta \to 1$. This limit is closely tied to the usual soft limit in the plane-wave basis

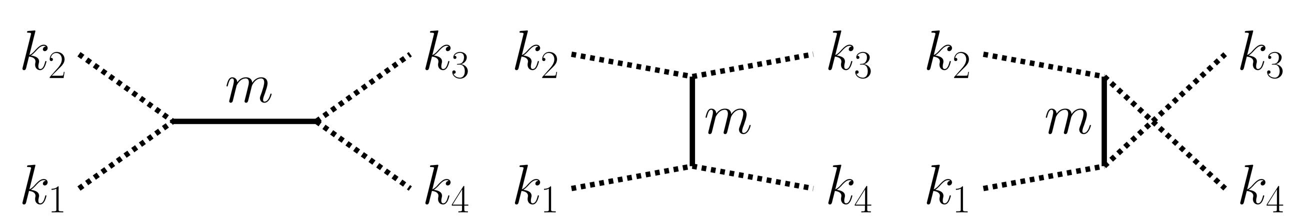
$$\lim_{\Delta_k\to 1} \tilde{\mathcal{A}}_{J_1\cdots J_n}(z_j,\bar z_j) = \frac{1}{\Delta_k - 1}\prod_{j=1, j\neq k}^n \int_0^\infty d\omega_j\, \omega_j^{\Delta_j-1} \times \int_0^\infty d\omega_k\, 2\,\delta(\omega_k)\,\omega_k\, \mathcal{A}_{\ell_1\cdots\ell_n}(\omega_j; z_j, \bar z_j)\,.$$

Using the soft behavior of color-ordered amplitudes in Yang-Mills theory, we find [4]

$$\lim_{\Delta_k\to 1}\tilde{\mathcal{A}}_{J_1\cdots J_n}(z_j,\bar z_j) = \begin{cases} \frac{1}{\Delta_k-1}\frac{z_{k-1\,k+1}}{z_{k-1\,k}z_{k\,k+1}}\tilde{\mathcal{A}}_{n-1}, & J=+1\,,\\ \frac{1}{\Delta_k-1}\frac{\bar z_{k-1\,k+1}}{\bar z_{k-1\,k}\bar z_{k\,k+1}}\tilde{\mathcal{A}}_{n-1}, & J=-1\,. \end{cases}$$

Conformal Partial Wave Decomposition: Scalars

We consider the tree-level four-point amplitude in scalar ϕ^3 theory, with massless external particles and a massive propagator.



We compute the conformally invariant part of the four-point celestial amplitude in the s-channel (1 and 2 incoming, 3 and 4 outgoing) [4]

$$\tilde A_4^{12\leftrightarrow 34}(z) \sim \delta(i\bar z - iz)\, z^2\,(z-1)^{h_{12}-h_{34}}\left(e^{\pi i\alpha} + \left(\frac{z}{z-1}\right)^\alpha + z^\alpha\right)\,,$$

where cross ratios are $z = \frac{z_{12}z_{34}}{z_{13}z_{24}}$ and $\bar z = \frac{\bar z_{12}\bar z_{34}}{\bar z_{13}\bar z_{24}}$, and

$$\alpha = \sum_{i=1}^4 h_i - 2\,, \quad \text{with} \quad h_i = \frac{\Delta_i + J_i}{2}, \quad \bar h_i = \frac{\Delta_i - J_i}{2}\,.$$

The expansion in terms of conformal partial waves is given by [5]

$$\tilde A_4(z,\bar z) = i\sum_{J=0}^{\infty}{}' \int_{\mathcal{C}} d\Delta\, \Psi^{h,\bar h}_{h_i,\bar h_i}(z,\bar z)\frac{(1-2h)(2\bar h-1)}{(2\pi)^2}\left\langle \tilde A_4(z,\bar z), \Psi^{h,\bar h}_{h_i,\bar h_i}(z,\bar z)\right\rangle\,,$$

where the inner product is defined by

$$\langle G(z,\bar z), F(z,\bar z)\rangle \equiv \int \frac{dz d\bar z}{(z\bar z)^2} G(z,\bar z)\overline{F(z,\bar z)}\,.$$

Doing this for the s-channel diagram above, and obtain [4]

$$\tilde A_{4,s}^{12\leftrightarrow 34}(z) \sim \frac{1}{2}\int_{\mathcal{C}}\frac{d\Delta}{4\pi^2}\frac{\Gamma\left(1-\frac{\Delta}{2}-h_{12}\right)\Gamma\left(\frac{\Delta}{2}-h_{12}\right)\Gamma\left(1-\frac{\Delta}{2}-h_{34}\right)\Gamma\left(\frac{\Delta}{2}-h_{34}\right)}{\Gamma(1-\Delta)\Gamma(\Delta-1)}\Psi^{\Delta}_{h_i}(z,\bar z)\,.$$

Closing the contour on either side, we find either two sequences of poles and two sequences of shadow poles from the ratio of Gamma functions in the integrand. From further study, one might extract OPE coefficients from these decompositions.

References

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