All-Multiplicity One-Loop Amplitudes in Born-Infeld Electrodynamics from Generalized Unitarity

Henriette Elvang, Marios Hadjiantonis, **Callum R. T. Jones** and Shruti Paranjape

Leinweber Center for Theoretical Physics, University of Michigan

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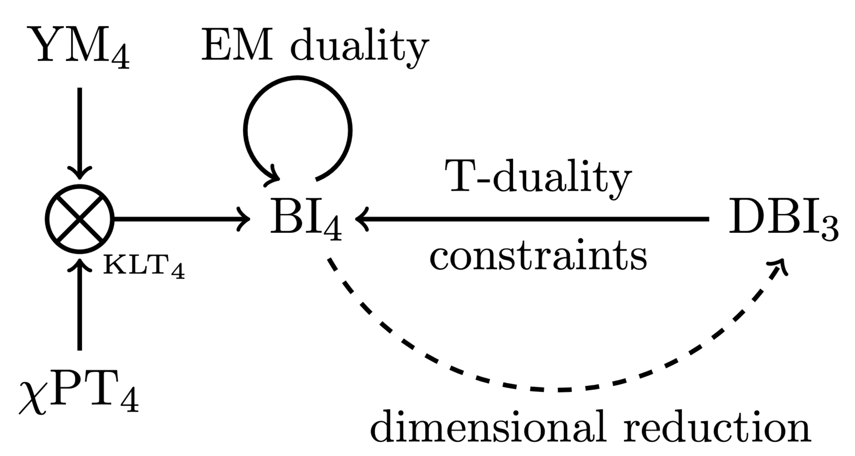
Classical Born-Infeld$_4$ Electrodynamics



Statement of Problem

Electromagnetic duality (helicity conservation) is a symmetry of the classical equations of motion, **not** the action:

$$\mathbf{S}_{\mathrm{BI}_4} = -\Lambda^4 \int \mathrm{d}^4x \left[\sqrt{-\det\left(g_{\mu\nu} + \frac{1}{\Lambda^2}F_{\mu\nu}\right)} - 1\right].$$

Is it a symmetry of the quantum theory? Is helicity conserved at one-loop? Is duality violation removable with finite local counterterms?

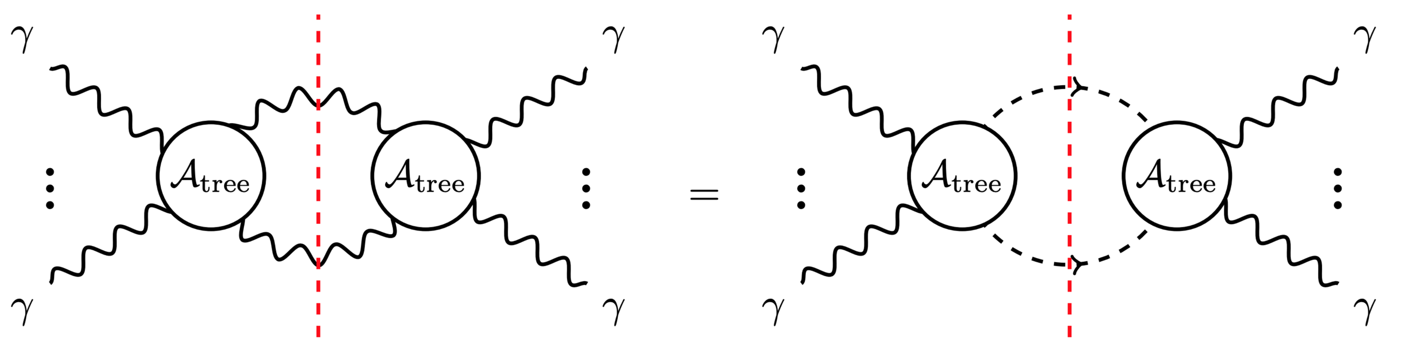
Strategy: Explicitly calculate one-loop amplitudes in duality violating self-dual (all-plus) and next-to-self-dual (one-minus) helicity sectors:

$$\mathcal{A}_n^{\text{(1-loop) BI}_4}\left(1_\gamma^+, 2_\gamma^+, ..., n_\gamma^\pm\right)$$

Supersymmetric Decomposition

Born-Infeld$_4$ is a consistent truncation of $\mathcal{N} \geq 1$ Super Born-Infeld$_4$:

$$\mathcal{A}_n^{\text{(1-loop) BI}_4} = \underbrace{-\mathcal{A}_n^{\text{(1-loop) }\mathcal{N}=2\text{ BI}_4} + 2\mathcal{A}_n^{\text{(1-loop) }\mathcal{N}=1\text{ BI}_4}}_{=0 \text{ in (N)SD sector}} + \mathcal{A}_n^{\text{(1-loop) [scalar]}}$$



d-Dimensional massless vector cuts $\Longleftrightarrow$ 4-Dimensional **massive** scalar cuts.

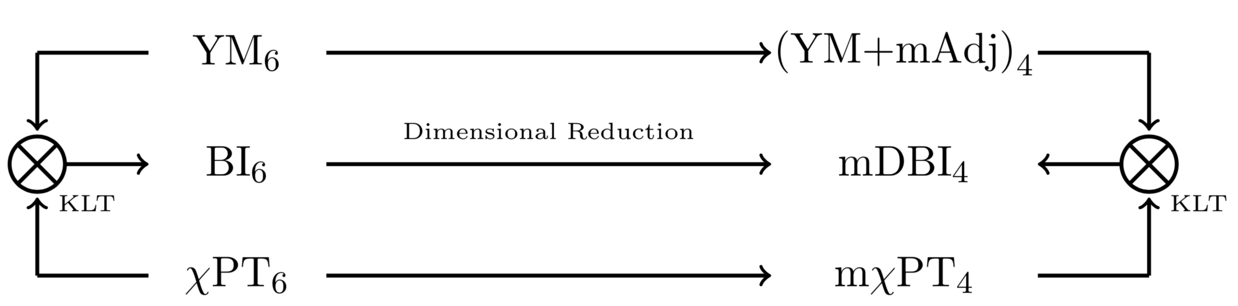
mDBI$_4$ $\equiv$ Born-Infeld$_4$ + Massive Scalar

Problem: Which massive scalar model gives the correct d-dimensional cuts of BI$_4$?

$$\mathbf{S}_{\mathrm{mDBI}_4} = \mathbf{S}_{\mathrm{BI}} + \int \mathrm{d}^4x \left[|\partial\phi|^2 + \mu^2|\phi|^2 + \frac{c_0}{\Lambda^4}F^+F^-|\partial\phi|^2 + \mu^2\sum_{n=2}^{\infty}\frac{c_{2n}}{\Lambda^{4n-4}}|\phi|^2(F^+)^{2n-2} + \text{h.c.}\right]$$

Claim: Given by dimensional reduction from 6d Born-Infeld:
$\implies c_n = 0$ if $n > 4$.

Method 1: Massive Double Copy



YM + mAdj$_4$: Determined by massless limit:

$$\mathcal{A}_n^{\text{YM+mAdj}_4}\left[1_\phi, 2_g, ..., (n-1)_g, n_{\bar\phi}\right] \xrightarrow{\mu^2=0} \mathcal{A}_n^{\mathcal{N}=2\text{ SYM}_4}\left[1_\phi, 2_g, ..., (n-1)_g, n_{\bar\phi}\right].$$

mχPT$_4$: Determined by d-dimensional Adler zero:

$$\mathcal{A}_n^{\chi\mathrm{PT}_d}[1, 2, ..., i, ..., n-1, n] \to \mathcal{O}(p_i), \quad \text{as} \quad p_i \to 0.$$

Method 2: Dimensional Reduction and T-Duality

	1	2	3	4	5
$\vec{p}_{1,n}$	x	x		x	x
$\vec{\epsilon}_{1,n}$				x	x
$\vec{p}_{2,3,...,n-2}$	x	x			
$\vec{\epsilon}_{2,3,...,n-2}$	x	x			
$\vec{p}_{n-1}$	x	x			
$\vec{\epsilon}_{n-1}$			x		

$|\gamma^\top(\vec{p}_{n-1})\rangle$ w/ $x_3 \sim x_3 + 2\pi R$, $R \to 0$

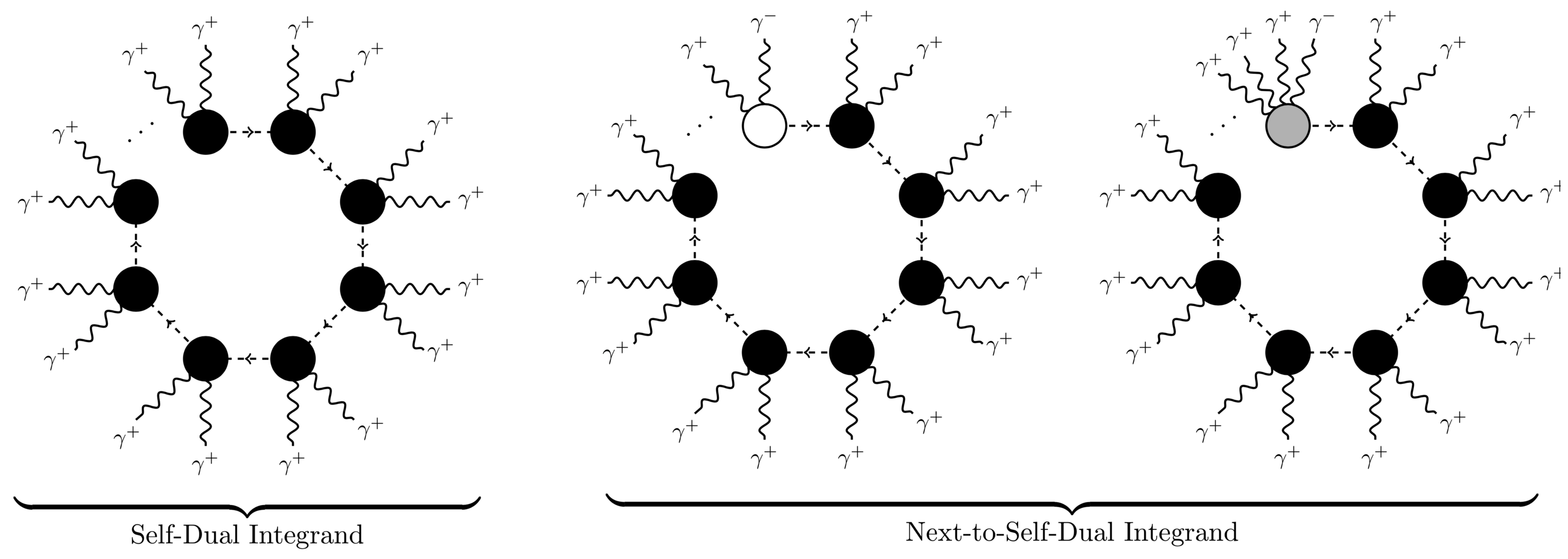
$\xleftrightarrow{\text{T-dual}}$

$|\Phi(\vec{p}_{n-1})\rangle = \dfrac{ISO(5,1)}{ISO(3,1)}$ Goldstone

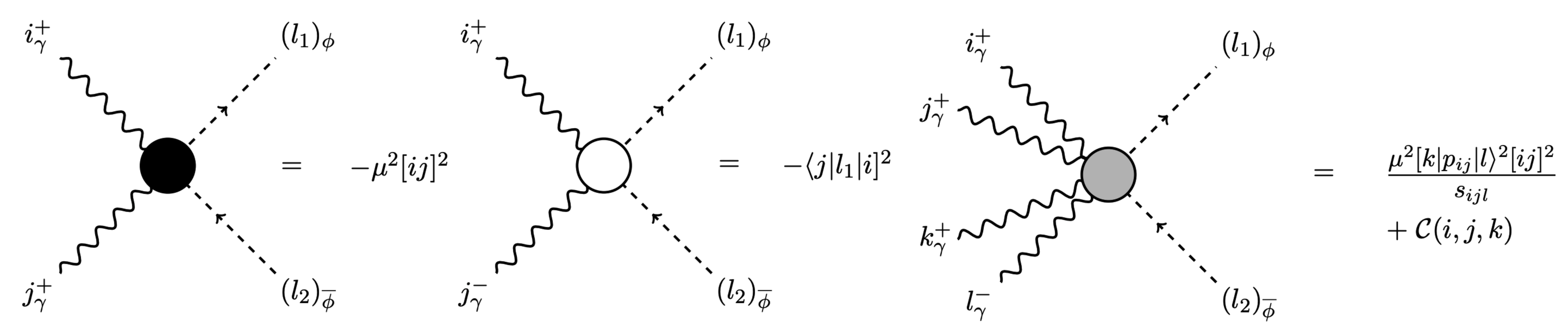
$$\mathcal{A}_n^{\mathrm{mDBI}_4}\left(1_\phi, 2_\gamma^+, ..., (n-1)_\gamma^+, n_{\bar\phi}^-\right) - \mathcal{A}_n^{\mathrm{mDBI}_4}\left(1_\phi, 2_\gamma^+, ..., (n-1)_\gamma^-, n_{\bar\phi}^-\right) \xrightarrow{3d+\text{soft}} \mathcal{O}\left(p_{n-1}^2\right)$$

Result: Numerical agreement between T-duality and double-copy up to $n = 8$.

Constructing Integrands



Unexpected Simplification: $c_n = 0$ for $n > 4 \implies$ finitely many "vertex rules":



Self-Dual Amplitudes

$$\mathcal{A}_{2n}^{\text{(1-loop) BI}_4}\left(1_\gamma^+, 2_\gamma^+, \ldots, 2n_\gamma^+\right) = \frac{i}{32\pi^2}\left(-\frac{1}{2}\right)^{n-1}\frac{1}{n(n+1)(n+2)(n+3)} \times \left[[12]^2[34]^2\ldots[2n-1,2n]^2\left(\sum_{i<j}^n\sum_{k<l}^n a_{ijkl}\left(\sum_{m=2i+1}^{2j}p_m\right)^2\left(\sum_{m=2k+1}^{2l}p_m\right)^2\right) + \mathcal{P}(2,3,\ldots,2n)\right] + \mathcal{O}(\epsilon),$$

with

$$a_{ijkl} = \begin{cases} 1 & \text{if all } i,j,k,l \text{ are different} \\ 2 & \text{if exactly 2 of } i,j,k,l \text{ are identical} \\ 4 & \text{if } i=k \text{ and } j=l \end{cases}.$$

Next-to-Self-Dual Amplitudes

$$\mathcal{A}_{2n}^{\text{(1-loop) BI}_4}\left(1_\gamma^+, 2_\gamma^+, \ldots, (2n-1)_\gamma^+, 2n_\gamma^-\right) = \frac{-i}{16\pi^2}\frac{1}{(n+2)!}\left(-\frac{1}{2}\right)^{n-1}[12]^2\ldots[2n-3\;2n-2]^2\left[\frac{1}{(n-1)(n+3)} \times \sum_{i<j}^n\left(\sum_{m=2i+1}^{2j}p_m\right)^2\left[\sum_{k<l}^n 2\,a_{ijkl}\left(\sum_{m=1}^{2k}[2n-1|p_m|2n\rangle\right)\left(\sum_{m=1}^{2l}[2n-1|p_m|2n\rangle\right) + \sum_{k=1}^n b_{ijk}\left(\sum_{m=1}^{2k}[2n-1|p_m|2n\rangle\right)^2\right] - \frac{1}{2}(n-2)!\frac{[2n-1|p_{2n-2}+p_{2n-3}|2n\rangle^2}{s_{2n,2n-2,2n-3}} \times \left[\sum_{i<j}^{n-2}\sum_{k<l}^{n-2}a_{ijkl}\left(\sum_{m=2i+1}^{2j}p_m\right)^2\left(\sum_{m=2k+1}^{2l}p_m\right)^2 + 4\sum_{i\leq j}^{n-2}\left(\sum_{m=1}^{2i}p_m\right)^2\left(\sum_{m=1}^{2j}p_m\right)^2 + 2\sum_{i=1}^{n-2}\sum_{k<l}^{n-2}a_{i(n-1)kl}\left(\sum_{m=1}^{2i}p_m\right)^2\left(\sum_{m=2k+1}^{2l}p_m\right)^2\right]\right] + \mathcal{P}(1,2,\ldots,2n-1) + \mathcal{O}(\epsilon),$$

with

$$b_{ijk} = \begin{cases} 2 & \text{if } i \neq k \text{ and } j \neq k \\ 6 & \text{if } i = k \text{ or } j = k \end{cases}.$$