The Full Colour Two-Loop 5-pt All-Plus Helicity Gluon Amplitude Using a 4-D Unitarity Approach

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Abstract

We present the current progress in the computation of the full colour two-loop 5-pt all-plus helicity gluon amplitude using a 4-dimensional unitarity [1] approach. The calculation follows closely to the work presented in [2], but extends to include all colour contributions. Motivated by the discovery of compact expressions for the subleading in colour one-loop amplitudes, we use a combination of 4-d unitarity, complex recursion [3,4] and augmented recursion [2,5] to calculate the full amplitude in U(N) Yang-Mills. We recover the leading in colour result and show some interesting results that have been produced thus far.

Introduction

There has been a great deal of progress in the calculation of two-loop gluon amplitudes in recent years drawing from various methods: analytic techniques using an alternative colour decomposition, IBP relations, finite field methods and numerical unitarity [6,7].

The full colour 5-pt all-plus amplitude has been calculated in the above.

We propose an alternative approach, to use simple 4-d unitarity techniques but to extend the calculation to include the full colour algebra associated with the partial amplitudes. In this way, simple analytic expressions can be used to generate the amplitudes to all orders in the number of colours.

U(N) Yang-Mills

We consider the calculation of the full colour amplitude in U(N) Yang-Mills. This has the advantage of utilising decoupling identities to verify the amplitudes we obtain, whilst simplifying the colour algebra.

The normalisation of the U(N) generators are taken to be, $\mathrm{tr}\left(T^a T^b\right) = \delta^{ab}$, such that the colour decomposition of the two loop amplitude is the following [8],

$$\begin{aligned}\mathcal{A}_5^{(2)}\left(a^+,b^+,c^+,d^+,e^+\right) =& N_c^2\,\mathrm{tr}(abcde)\,A_{5;1}^{(2)}\left(a^+,b^+,c^+,d^+,e^+\right)\\ &+N_c\,\mathrm{tr}(a)\mathrm{tr}(bcde)\,A_{5;2}^{(2)}\left(a^+;b^+,c^+,d^+,e^+\right)\\ &+N_c;\mathrm{tr}(ab)\mathrm{tr}(cde);\,A_{5;3}^{(2)}\left(a^+,b^+;c^+,d^+,e^+\right)\\ &+\mathrm{tr}(abcde)\,A_{5;B}^{(2)}\left(a^+;b^+,c^+,d^+,e^+\right)\\ &+\mathrm{tr}(a)\mathrm{tr}(b)\mathrm{tr}(cde)\,A_{5;B;1,1}^{(2)}\left(a^+;b^+;c^+,d^+,e^+\right)\\ &+\mathrm{tr}(a)\mathrm{tr}(bc)\mathrm{tr}(de)\,A_{5;B;1,2}^{(2)}\left(a^+;b^+,c^+;d^+,e^+\right).\end{aligned}$$

We can treat the one-loop all-plus amplitude as a vertex since it is a rational function to order epsilon and the 4-d cuts of the amplitude vanish. Consequently, the cuts of the two-loop amplitude appear as cuts of one-loop integral functions so we may apply one-loop unitarity techniques to the full amplitude.

Cut Constructible Pieces

Beginning with the familiar decomposition of a one-loop amplitude into box, triangle and bubble integral functions,

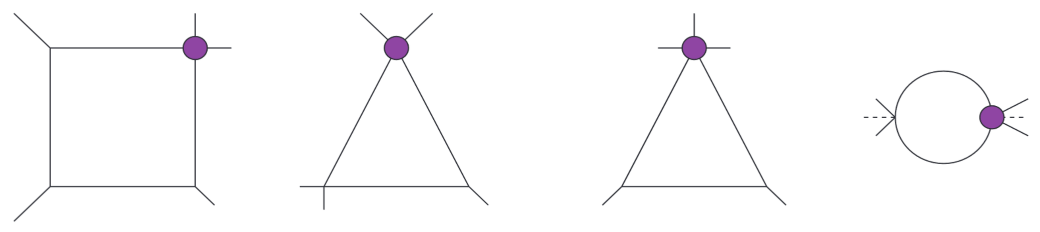

Figure 1: Diagrams that contribute to the two-loop 5-pt all-plus amplitude, where the purple vertices are the one-loop all-plus amplitude.

we use generalised unitarity to compute the coefficients of the integral functions.

In the full colour case however, the cut integral is composed of a product of the full colour amplitudes instead of just the partial amplitudes.

$$d = \int \mathrm{dLIPS}\, \mathcal{A}_4^{(1)}(1^+,2^+,l_1^+,-l_4^-)\, \mathcal{A}_3^{(0)}(-l_1^-,3^+,l_2^+)\, \mathcal{A}_3^{(0)}(-l_2^-,4^+,l_3^-)\, \mathcal{A}_3^{(0)}(-l_3^+,5^+,l_4^-)$$

Subleading in colour partial amplitudes are required for this calculation. These have been previously obtained from the leading partial amplitude through decoupling equations like,

$$\sum_{\sigma\in \mathbf{Z}_{n-1}} A_{4;1}^{(1)}(\sigma(1),\sigma(2),\cdots,\sigma(n-1),n) + A_{4;2}^{(1)}(n;1,2,\cdots,n-1) = 0$$

Here we present alternative, compact expressions for the subleading partial amplitudes at one-loop level.

$$A_{n;2}^{(1)}(1^+;2^+,3^+,...,n^+) = -i\sum_{i<j}\frac{[1|ij|1]}{\langle 23\rangle\langle 34\rangle\cdots\langle n2\rangle},$$

$$A_{n;3}^{(1)}(1^+,2^+,\cdots,r^+;(r+1)^+,...,n^+) = -2i\frac{t_{1\cdots r}\,t_{r+1\cdots n}}{\langle 12\rangle\cdots\langle r1\rangle\langle (r+1)(r+2)\rangle\cdots\langle n(r+1)\rangle}$$

The expressions have the correct spinor weight, mass dimension and satisfy decoupling identities between the trace structures at one-loop.

Using canonical forms to solve the cut integrals, including the colour algebra, and summing over all independent diagrams returns all trace structures in the full two-loop colour decomposition.

Interestingly, we find that the coefficient of the trace structures, $\mathrm{tr}(a)\mathrm{tr}(bc)\mathrm{tr}(de)$, vanishes.

The cut constructible parts of the amplitude satisfy decoupling identities at two-loop and the leading in colour piece has been reconstructed. These act as a strong check that the full colour cut constructible piece has been computed correctly.

Infra-Red Singularities

Theorems for IR divergences are well established for leading and subleading in colour for two loop amplitudes [9].

$$|\mathcal{M}_n^{(1)}(\mu;\{p\})\rangle = \mathbf{I}^{(1)}(\mu;\{p\})\,|\mathcal{M}_n^{(0)}(\mu;\{p\})\rangle + |\mathcal{M}_n^{(1),\,\mathrm{fin}}(\mu;\{p\})\rangle$$

$$\mathbf{I}^{(1)}(\epsilon,\mu;\{p\}) = \frac{1}{2}\frac{e^{-\epsilon\psi(1)}}{\Gamma(1-\epsilon)}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\mathbf{T}_i\cdot\mathbf{T}_j\left[\frac{1}{\epsilon^2}+\frac{\gamma_i}{\mathbf{T}_i^2}\frac{1}{\epsilon}\right]\left(\frac{\mu^2\,e^{-i\lambda_{ij}\pi}}{2p_i\cdot p_j}\right)^{\epsilon}$$

The leading IR divergence for 5-pt was verified in [10] and the subleading in colour in [6,7], showing agreement with [9].

We find agreement to all orders in the colour sum for the coefficient of ϵ^{-2}, following an alternative definition of the IR divergences [11],

$$\begin{aligned}&\mathcal{A}_n^{(2),(i,j),\mathrm{ren}}(1,2,...,n)\big|_{\mathrm{div.}} \equiv\\ &-g^2 c_\Gamma \tilde{f}^{b_i a_i c} f^{c a_j b_j}\left[\frac{1}{\epsilon^2}(-s_{ij})^{-\epsilon}+\frac{11}{6\epsilon}\right]\mathcal{A}_n^{(1)}(1^{a_1},2^{a_2},...,i^{b_i},...,j^{b_j},...n^{a_n}).\end{aligned}$$

Rational Piece

The rational terms are computed using complex recurison. Using a Risager shift [12],

$$\lambda_{\hat{c}} = \lambda_c + z[de]\lambda_\eta,\ \lambda_{\hat{d}} = \lambda_d + z[ec]\lambda_\eta,\ \lambda_{\hat{e}} = \lambda_e + z[cd]\lambda_\eta.$$

simple and double poles are excited, BCFW recursion [3] is used to reconstruct the rational piece.

Loop amplitudes may contain double poles. The leading double pole arises from the structure in Figure 2, and can be calculated using factorisation theorems and recursion [3].

Double poles are only present in the leading in colour amplitude. The single pole terms arise from a non-factorising structure (Figure 3) and can be calculated using Augmented Recursion [2,3,5].

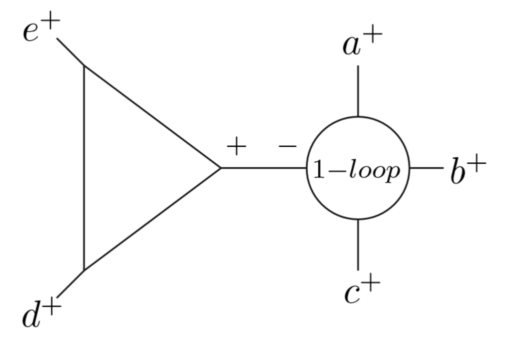

Figure 2: Factorising structure that gives the double pole.

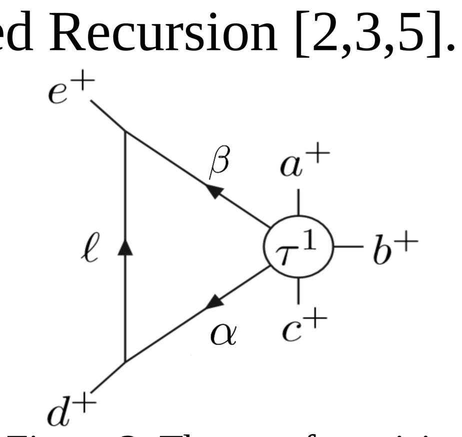

Figure 3: The non-factorising structure from which the full rational piece is calculated .

Axial gauge techniques [13] are used to construct an off-shell current from, $\mathcal{A}_5^{(1)}\left(\alpha^\mp,\beta^\pm,a^+,b^+,c^+\right)$.

The current must:

1) reproduce the amplitude in the limit $\alpha^2,\beta^2\to 0$.
2) reproducing the leading poles as, $s_{\alpha\beta}\to 0$.

We capture the singular structure of the rational piece by taking the amplitude off shell and satisfying the above conditions.

Integrating this full colour current, denoted $\tau_5^{(1)}$, as a one-loop insertion (Figure 3),

$$\int \mathrm{d^D}l\,\frac{[d|l|q\rangle[e|l|q\rangle}{\langle dq\rangle\langle eq\rangle l^2\alpha^2\beta^2}\frac{\langle\beta q\rangle^2}{\langle\alpha q\rangle^2}\tau_5^{(1)}\left(\alpha^-,\beta^+,a^+,b^+,c^+\right),$$

captures the correct pole structure, allowing the residues to be extracted.

By simply using the full colour one-loop single-minus amplitude to generate the current, the full colour rational piece can be computed. We find that the coefficient of the trace structure $\mathrm{tr}(a)\mathrm{tr}(bc)\mathrm{tr}(de)$, vanishes here as well.

We also remark that at subleading in colour, the rational piece contains no double poles, which can be shown by considering decoupling identities.

This result extends to the case of n-gluons.

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