The Rise of SMEFT On-shell Amplitudes

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Introduction and motivation

We present a map between the tree-level Standard Model Effective Field Theory (SMEFT) in the Warsaw basis and massive on-shell amplitudes. As a first step, we focus on the electroweak sector without fermions. We describe the Feynman rules for a particular choice of input scheme and compare them with the 3-point massive amplitudes in the broken phase. Thereby we fix an on-shell basis which allows us to study scattering amplitudes with recursion relations. We hope to open up new avenues of exploration to a complete formulation of massive EFTs in the on-shell language.

SMEFT and Massive Amplitude Formalism

Massive On-shell: usual formalism for massless and SU(2) gives us the little-group action [Arkani-Hamed et. al (2017)]:

$$\mathbf{p}_{\alpha\dot\beta} = \chi^I_\alpha \epsilon_{IJ} \tilde\chi^J_{\dot\beta} \equiv \epsilon_{IJ} |\mathbf{p}\rangle^I_\alpha \, \mathbf{p}|^J_{\dot\beta}$$

Holomorphic SMEFT: purely bosonic holomorphic SMEFT operators

$$\mathcal{L} \supset c_X \mathcal{O}_X + c_{\tilde X} \mathcal{O}_{\tilde X} = c^+_X \mathcal{O}^+_X + c^-_X \mathcal{O}^-_X$$

where $c^\pm_X = c_X \pm i c_{\tilde X}$ and $X^\pm_{\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} \mp i \tilde X_{\mu\nu}\right)$.

Bosonic operators			
$\mathcal{O}^\pm_W$	$\epsilon^{ijk} W^{i,\pm}_{\mu\nu} W^{j,\pm}_{\nu\rho} W^{k,\pm}_{\rho\mu}$	$\mathcal{O}_H$	$(H^\dagger H)^3$
$\mathcal{O}^\pm_{HB}$	$H^\dagger H B^\pm_{\mu\nu} B^\pm_{\mu\nu}$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}^\pm_{HW}$	$H^\dagger H W^\pm_{\mu\nu} W^\pm_{\mu\nu}$	$\mathcal{O}_{HD}$	$\lvert H^\dagger D_\mu H\rvert^2$
$\mathcal{O}^\pm_{HWB}$	$H^\dagger \sigma^i H W^{i,\pm}_{\mu\nu} B^\pm_{\mu\nu}$		

Massive 3-point Amplitudes

Higgs couplings

$$\mathcal{M}(\mathbf{1}_h \mathbf{2}_h \mathbf{3}_h) = v\,\mathcal{G}_{hhh} + \frac{v^3}{\Lambda^2}\mathcal{C}_{hhh}$$

$$\mathcal{M}(\mathbf{1}_h 2^+_\gamma 3^+_\gamma) = \frac{v}{\Lambda^2}\mathcal{C}^+_{h\gamma\gamma}\,[23]^2, \quad \mathcal{M}(\mathbf{1}_h \mathbf{2}^{J_{1,2}}_Z 3^+_\gamma) = \frac{v}{\Lambda^2}\mathcal{C}^+_{hZ\gamma}[\mathbf{2}3]^2$$

$$\mathcal{M}(\mathbf{1}_h \mathbf{2}^{I_{1,2}}_V \mathbf{3}^{J_{1,2}}_{\bar V}) = \left(\frac{1}{v}\mathcal{G}_{hV\bar V} + \frac{v}{\Lambda^2}\mathcal{C}_{hV\bar V}\right)\langle \mathbf{23}\rangle[\mathbf{23}] + \frac{v}{\Lambda^2}\mathcal{C}^-_{hV\bar V}\langle\mathbf{23}\rangle^2 + \frac{v}{\Lambda^2}\mathcal{C}^+_{hV\bar V}[\mathbf{23}]^2$$

Gauge couplings

$$\mathcal{M}(\mathbf{1}^{I_{1,2}}_W \mathbf{2}^{J_{1,2}}_{\bar W} 3^-_\gamma) = \left(\frac{1}{v\,m_W}\mathcal{G}_{W\bar W\gamma} + \frac{v}{m_W\Lambda^2}\mathcal{C}_{W\bar W\gamma}\right) x^{-1}[\mathbf{12}]^2 + \frac{v\,\mathcal{C}^-_{W\bar W\gamma}}{m_W\Lambda^2}[\mathbf{12}]\langle\mathbf{2}3\rangle\langle 3\mathbf{1}\rangle + \frac{\mathcal{C}'^-_{W\bar W\gamma}}{\Lambda^2}\langle\mathbf{12}\rangle\langle\mathbf{2}3\rangle\langle 3\mathbf{1}\rangle\,.$$

SM Map

$\mathcal{G}_i$	$\mathcal{G}_{hhh}$	$\mathcal{G}_{hV\bar V}$	$\mathcal{G}_{W\bar W Z}$	$\mathcal{G}_{W\bar W\gamma}$
SM	$-3\lambda_h$	-2	$-2\sqrt{2}\,c_\theta$	$-2\sqrt{2}\,s_\theta$

Dimension Six Map

- $\mathcal{C}$'s represent dim-6.
- $\mathcal{C}^\pm$ **only** receives $\mathcal{O}^\pm$contributions.
- Non-holomorphic coefficients only in SM-like structures.
- $\mathcal{C}$'s relations inherits the SM group structures

$\mathcal{C}_i$	Warsaw Basis
$\mathcal{C}_{hhh}$	$3\lambda_h\delta\tilde v + 6c_H - 9\lambda_h c_{H\Box} + 9/4\lambda_h c_{HD}$
$\mathcal{C}_{W\bar W Z}$	$-4\sqrt{2}\,c_\theta\left[c_{HD}/4 + c_{HWB}\,s_\theta c_\theta - \delta\tilde v\right]$
$\mathcal{C}_{W\bar W\gamma}$	$-\sqrt{2}\,c_\theta/t_\theta\left[c_{HD}/4 + c_{HWB}s_\theta c_\theta + \delta\tilde v\right]$
$\mathcal{C}_{hZZ}$	$-2\left[c_{H\Box} + c_{HD}/4 - \delta\tilde v\right]$
$\mathcal{C}_{hW\bar W}$	$-2\left[c_{H\Box} - c_{HD}/4 - \delta\tilde v\right]$
$\mathcal{C}^\pm_{h\gamma\gamma}$	$-2\left[s^2_\theta\,c^\pm_{HW} - s_\theta\,c_\theta\,c^\pm_{HWB} + c^2_\theta\,c^\pm_{HB}\right]$
$\mathcal{C}^\pm_{hZ\gamma}$	$-\left[(s^2_\theta - c^2_\theta)\,c^\pm_{HWB} + 2s_\theta\,c_\theta\,(c^\pm_{HW} - c^\pm_{HB})\right]$
$\mathcal{C}^\pm_{hZZ}$	$-2\left[c^2_\theta\,c^\pm_{HW} + s^2_\theta\,c^\pm_{HB} + c_\theta\,s_\theta\,c^\pm_{HWB}\right]$
$\mathcal{C}^\pm_{hW\bar W}$	$-2\,c^\pm_{HW}$
$\mathcal{C}^\pm_{W\bar W Z}$	$c^\pm_{HWB}\sqrt{2}\,s_\theta$
$\mathcal{C}^\pm_{W\bar W\gamma}$	$c^\pm_{HWB}\sqrt{2}\,c_\theta$
$\mathcal{C}'^\pm_{W\bar W Z}$	$c^\pm_W\,3\sqrt{2}\,c_\theta$
$\mathcal{C}'^\pm_{W\bar W\gamma}$	$c^\pm_W\,3\sqrt{2}\,s_\theta$

Four-point Amplitudes

- Recursion relations can fail for EFT → not fully fixed by factorization
- Factorizable → BCFW and/or consistent factorization channels

$W\bar W\gamma\gamma$

We calculate the $\mathcal{M}^{\rm SM}(\mathbf{1}_W\mathbf{2}_{\bar W}3^-_\gamma 4^+_\gamma)$ Amplitude, under a $[3^-, 4^+\rangle$- shift.

$$\mathcal{M}(\mathbf{1}_W\mathbf{2}_{\bar W}3^-_\gamma 4^+_\gamma) = -\frac{(\mathcal{G}_{W\bar W\gamma}\,m_W/v)^2}{(t-m^2_W)(u-m^2_W)}\Big(\langle 3\,\mathbf{1}\rangle[4\,\mathbf{2}] + \langle 3\,\mathbf{2}\rangle[4\,\mathbf{1}]\Big)^2,$$

- Consistent factorization and a naive large-z behaviour of z^{-2}.
- SM Map → $\left(\mathcal{G}_{W\overline{W}\gamma}m_W/v\right)^2 = 2e^2$.

Calculating $\mathcal{M}^{\rm BSM}(\mathbf{1}_W\mathbf{2}_{\bar W}3^-_\gamma 4^+_\gamma)$ and keeping just $\mathcal{O}(\Lambda^{-2})$ terms leads to

$$\mathcal{M}^{\rm BSM,-}(\mathbf{1}_W\mathbf{2}_{\bar W}3^-_\gamma 4^+_\gamma) = \left(\frac{\mathcal{G}_{W\bar W\gamma}}{\Lambda^2}\right)\frac{1}{(t-m^2_W)\,(u-m^2_W)}\times \Big\{\mathcal{C}^-_{W\bar W\gamma}m_W\Big(\langle 3\,\mathbf{1}\rangle[4\,\mathbf{2}] + \langle 3\,\mathbf{2}\rangle[4\,\mathbf{1}]\Big)\langle 3\,\mathbf{1}\rangle\langle 3\,\mathbf{2}\rangle[4\,3] + \mathcal{C}'^-_{W\bar W\gamma}\langle\mathbf{1}\,\mathbf{2}\rangle\langle 3\,\mathbf{1}\rangle\langle 3\,\mathbf{2}\rangle\langle 3|p_{\mathbf{2}}|4][4\,3]\Big\}\,,$$

- For the non-minimal coupling in the vertex $\hat{\mathcal{M}}(\mathbf{1}_W 3^-_\gamma \boldsymbol{q}_{\bar W})$ we obtain the same result with $\langle...\rangle \leftrightarrow [...]$ and $3 \leftrightarrow 4$.

$W\bar W hh$

- Bad large-z behaviour doing a BCFW shift due to the pure scalar amplitude with longitudinal W's and the Higgs.
- Reconstruct from the residues and requiring a well-behaved UV.

$$\mathcal{M}(\mathbf{1}_W\mathbf{2}_{\bar W}\mathbf{3}_h\mathbf{4}_h) = \frac{R_t}{t-m^2_W} + \frac{R_u}{u-m^2_W} + \frac{R_s}{s-m^2_h} + \mathcal{G}_{W\bar W hh}\langle\mathbf{12}\rangle[\mathbf{12}]\,,$$

via gluing $R_t = \mathcal{M}(\mathbf{1}_W\boldsymbol{q}_{\bar W}\mathbf{3}_h)\mathcal{M}(-\boldsymbol{q}_W\mathbf{2}_{\bar W}\mathbf{4}_h)|_{t\to m^2_W}$ as similar for R_s and R_u.

$$\mathcal{M}(\mathbf{1}_W\mathbf{2}_{\bar W}\mathbf{3}_h\mathbf{4}_h) = \left[-\frac{(\mathcal{G}_{W\bar W h})^2 m^2_W(2m^2_h - s)}{v^2(t-m^2_W)(u-m^2_W)} - \frac{(\mathcal{G}_{W\bar W h})(\mathcal{G}_{hhh})}{(s-m^2_h)} + \mathcal{G}_{W\bar W hh}\right]\langle\mathbf{12}\rangle[\mathbf{12}]\,.$$

In the HE limit, $\mathcal{M}_{\rm HE} \to (\mathcal{G}_{W\bar W h})^2 s/(2v^2) + \mathcal{G}_{W\bar W hh}s$ and by unitarity

$$\mathcal{G}_{W\bar W hh} = -\frac{1}{2}\frac{(\mathcal{G}_{W\bar W h})^2}{v^2} = -\frac{2}{v^2}\,,$$

- For SMEFT amplitude, the SM-like part $\mathcal{G}_i \to \mathcal{C}_i$.
- Cannot use the HE constraint for the 4-point contact interaction $\mathcal{C}_{W\bar W hh}$

Future directions?

- General method to obtain n-point massive amplitudes with higher dim ops
- Everything purely from on-shell: 4-point test, unitarity, soft/collinear limits
- On-shell map for HEFT Lagrangian and its difference to SMEFT looking to multi boson legs processes.
- Symmetry breaking understanding via on-shell methods.

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