LEINWEBER CENTER FOR THEORETICAL PHYSICS
UNIVERSITY OF MICHIGAN

Born Infeld Theory Beyond Leading Order

Henriette Elvang, Marios Hadjiantonis, Callum R. T. Jones, *Shruti Paranjape*
University of Michigan, Ann Arbor

Born-Infeld Theory at Tree-Level

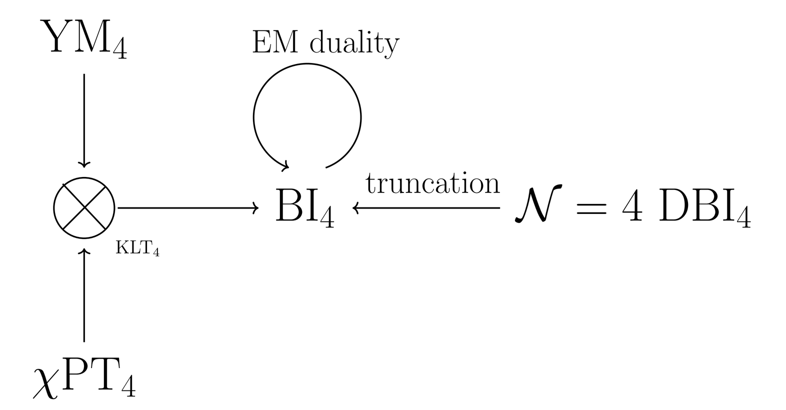
Born-Infeld (BI) theory is a vector EFT which arose as:

- A model of nonlinear electrodynamics.
- A low energy EFT of gauge fields on D-branes.

$$S_{\rm BI} = -16\Lambda^4 \int d^4x \left[\sqrt{-\det\left(g_{\mu\nu} + \frac{1}{16\Lambda^4}F_{\mu\nu}\right)} - 1\right],$$

where Λ is the brane tension and serves as the cutoff of the Born-Infeld EFT.

Two possible definitions of BI theory at leading order



Duality-Violating Amplitudes at One-Loop

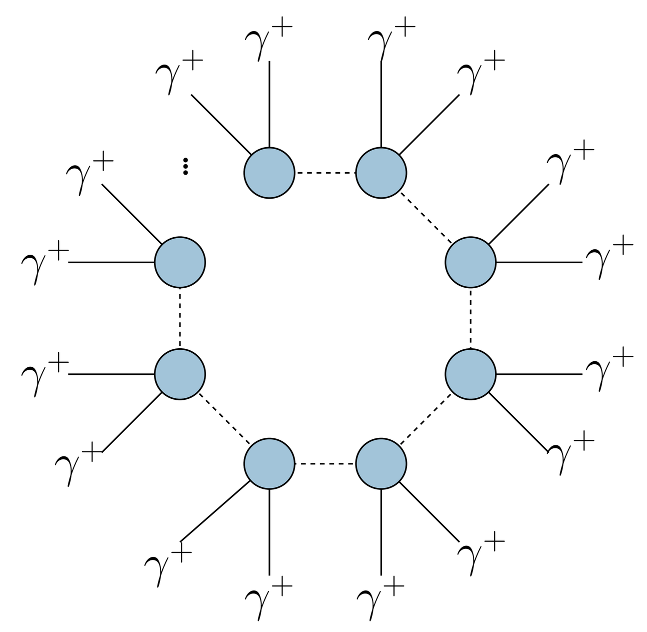
Generalized unitarity

→ d-dimensional unitarity

Absence of $4d$ cuts

→ Integrands built of mDBI$_4$ trees ← BCJ Double Copy

SUSY decomposition



On-shell rules: In the self-dual (SD) and next-to-self-dual (NSD) sectors,

$\mathcal{A}_n\left(1^+_\gamma \dots (n-1)^+_\gamma n^+_\gamma\right) = \text{local}$

$\mathcal{A}_n\left(1^+_\gamma \dots (n-1)^+_\gamma n^-_\gamma\right) = \text{factorizing} + \text{local}$

$\Rightarrow$ local counterterms must be added to preserve EM duality for each n.

Double Copy at One-Loop

Tree-level double copy tells us that

$$\frac{g^4}{f_\pi^4} \to \frac{1}{\Lambda^8}$$

where g = YM coupling and f_π = cutoff scale of χPT.

$$i\mathcal{A}_4^{\rm YM} = \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}}\left[\frac{c_{1234}^{(\rm box)}n_{1234}^{(\rm box)}(l)}{l^2(l-p_2)^2(l-p_2-p_3)^2(l+p_1)^2} + \text{other boxes}\right]$$

YM Numerator for $\mathcal{A}_4(\mathbf{1^+2^+3^+4^+})$	$n_{1234}^{(\rm box)}(l) = 2g^4\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}(l_{-2\epsilon}^2)^2$

d-dimensional unitarity can be used to find the BCJ representation of χPT at one-loop.

BCJ Representation of χPT integrand	$\mathcal{I}_4^{\chi\rm PT}[1_{a_1}2_{a_2}3_{a_3}4_{a_4}] = \frac{2}{f_\pi^4}c_{1234}^{(\rm box)}\frac{(p_1\cdot l)(-p_{12}\cdot l)+\frac{1}{16}s_{12}^2}{(l-\frac{1}{2}p_{12})^2(l+\frac{1}{2}p_{12})^2} + \frac{2}{f_\pi^4}c_{1243}^{(\rm box)}\frac{(p_1\cdot l)(p_{12}\cdot l)+\frac{1}{16}s_{12}^2}{(l-\frac{1}{2}p_{12})^2(l+\frac{1}{2}p_{12})^2}$

Putting these together, BI is given by

$$c_{1234}^{(\rm box)}, c_{1243}^{(\rm box)} \to 2g^4\frac{[12]^2[34]^2}{s_{12}^2}(l_{-2\epsilon}^2)^2$$

Using dimension shifting to perform the rational integrals,

$$\mathcal{A}_4^{\text{1-loop BI}}(1^+2^+3^+4^+) = [12]^2[34]^2\int\frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}}\frac{(l_{-2\epsilon}^2)^2}{(l-\frac{1}{2}p_{12})^2(l+\frac{1}{2}p_{12})^2} + \text{other cuts}$$
$$= -\frac{i}{960\pi^2}\left([12]^2[34]^2s_{12}^2 + [13]^2[24]^2s_{13}^2 + [14]^2[23]^2s_{14}^2\right)$$

BCJ Compatible Higher Derivative Corrections to YM and χPT

Features of YM Ansatze: At each derivative order, find the most general on-shell element of $\mathcal{A}_4[1234]$ with:

- Little group scaling
- BCJ and KK relations:

$$\mathcal{A}_4[1234] = \frac{t}{u}\mathcal{A}_4[1243] = \frac{t}{s}\mathcal{A}_4[1324] = \frac{t}{u}\mathcal{A}_4[1342] = \frac{t}{s}\mathcal{A}_4[1423] = \mathcal{A}_4[1432]$$

- Locality: $\mathcal{A}_4[1234]$ cannot have t-pole
- Unitarity: On s and u pole, $\mathcal{A}_4[1234]$ must factorize into 3-point amplitudes of particles in the spectrum

$\mathcal{A}_4^{\rm YM}[\mathbf{1^+2^+3^+4^+}]$	$=\frac{\tilde{c}_2}{\Lambda^2}\frac{[12]^2[34]^2s+[13]^2[24]^2t+[14]^2[23]^2u}{su} + \frac{\tilde{c}_6}{\Lambda^6}t\left([12]^2[34]^2+[13]^2[24]^2+[14]^2[23]^2\right)+\mathcal{O}(\Lambda^{-8})$
$\mathcal{A}_4^{\rm YM}[\mathbf{1^+2^+3^+4^-}]$	$=\frac{1}{\Lambda^2}\frac{[12]^2[3\vert p_1\vert 4\rangle^2}{su}\left(\tilde{b}_0+\frac{\tilde{b}_6}{\Lambda^6}stu+\mathcal{O}(\Lambda^{-8})\right)$
$\mathcal{A}_4^{\rm YM}[\mathbf{1^+2^+3^-4^-}]$	$=\frac{[12]^2\langle 34\rangle^2}{su}\left(\tilde{a}_0+\frac{\tilde{a}_4}{\Lambda^4}tu+\frac{\tilde{a}_6}{\Lambda^6}stu+\mathcal{O}(\Lambda^{-8})\right)$

Features of χPT Ansatz: At each derivative order, find the most general on-shell element of $\mathcal{A}_4[1234]$ with:

- Little group scaling
- Locality + Unitarity: $\mathcal{A}_4[1234]$ cannot have any poles since there are no 3-point amplitudes
- Soft behaviour: Goldstone scalars have Adler zeros

$\mathbf{A_4^{\chi PT}[1234]}$	$=\frac{1}{\Lambda^2}t\left(1+\frac{c_4}{\Lambda^4}(s^2+t^2+u^2)+\frac{c_6}{\Lambda^6}stu+\mathcal{O}(\Lambda^{-8})\right).$

BI Theory as a Double Copy

$\mathcal{A}_4(1^+2^+3^-4^-)$ can be calculated in mutliple ways,

$$\mathcal{A}_4^{\text{1-loop BI}}(1^+2^+3^-4^-) = \frac{1}{\epsilon}\frac{1}{16\pi}[12]^2\langle 34\rangle^2\left(\frac{1}{2}s_{12}^2+\frac{1}{5}s_{13}^2+\frac{1}{5}s_{14}^2\right)$$

Compare this to double copy counterterms,

$\mathcal{A}_4^{\rm BI}(\mathbf{1^+2^+3^-4^-})$	$=-\frac{[12]^2\langle 34\rangle^2}{\Lambda^4}\left(\tilde{a}_0+\frac{1}{\Lambda^4}\left(2\tilde{a}_0c_4s^2+(\tilde{a}_4-2\tilde{a}_0c_4)\,tu\right) + \frac{1}{\Lambda^6}(\tilde{a}_6+\tilde{a}_0c_6)\,stu+\mathcal{O}(\Lambda^{-8})\right)$

$\Rightarrow$ double copy generates necessary infinite counterterm.
$\Rightarrow$ true even with $\mathcal{N}=4$ supersymmetry.

Double Copy in Duality-Violating Sectors

$\mathcal{A}_4^{\rm BI}(\mathbf{1^+2^+3^+4^-})$	$=-\frac{[12]^2[3\vert p_1\vert 4\rangle^2}{\Lambda^6}\left(\tilde{b}_0+\frac{\tilde{b}_0c_4}{\Lambda^4}(s^2+t^2+u^2)+\frac{\tilde{b}_0c_6+\tilde{b}_6}{\Lambda^6}stu+\mathcal{O}(\Lambda^{-8})\right)$
$\mathcal{A}_4^{\rm BI}(\mathbf{1^+2^+3^+4^+})$	$=-\frac{\tilde{c}_2}{\Lambda^6}\left([12]^2[34]^2s+[13]^2[24]^2t+[14]^2[23]^2u\right) - \frac{\tilde{c}_6+3\tilde{c}_2c_4}{\Lambda^{10}}stu\left([12]^2[34]^2+[13]^2[24]^2+[14]^2[23]^2\right)+\mathcal{O}(\Lambda^{-12})$

$\Rightarrow$ double copy fails to generate counterterms necessary for preservation of EM duality

Results

- Counterterms are needed to preserve EM duality at each n
- 4-point counterterms are not generated by the double copy construction of BI theory at 1-loop
- With $\mathcal{N}=4$ supersymmetry, no tension
- Proposal: the counterterms that restore the duality symmetry in the supersymmetric theory are exactly those generated by the double-copy

Partial work presented in `arXiv:1906.05321`, with more to come in `arXiv:19xx.xxxx`.