A family of two-loop non-planar master integrals for Higgs + jet production with full heavy-quark mass dependence.

Poster author: Martijn Hidding

Based on work by: R. Bonciani[a,b], V. Del Duca[c,d], H. Frellesvig[e,f], J.M. Henn[g], M. Hidding[h], L. Maestri[g], F. Moriello[c], G. Salvatori[c,j,k], V.A. Smirnov[l]

[a] Sapienza - Università di Roma, [b] INFN Sezione di Roma, [c] ETH Zurich, Institut fur theoretische Physik, [d] INFN Laboratori Nazionali di Frascati, [e] Dipartimento di Fisica e Astronomia, Università di Padova, [f] INFN Sezione di Padova, [g] Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, [h] School of Mathematics, Trinity College Dublin, [j] Dipartimento di Fisica, Università degli Studi di Milano, [k] NFN, Sezione di Milano, [l] Skobeltsyn Inst. of Nuclear Physics of Moscow State University,

Introduction

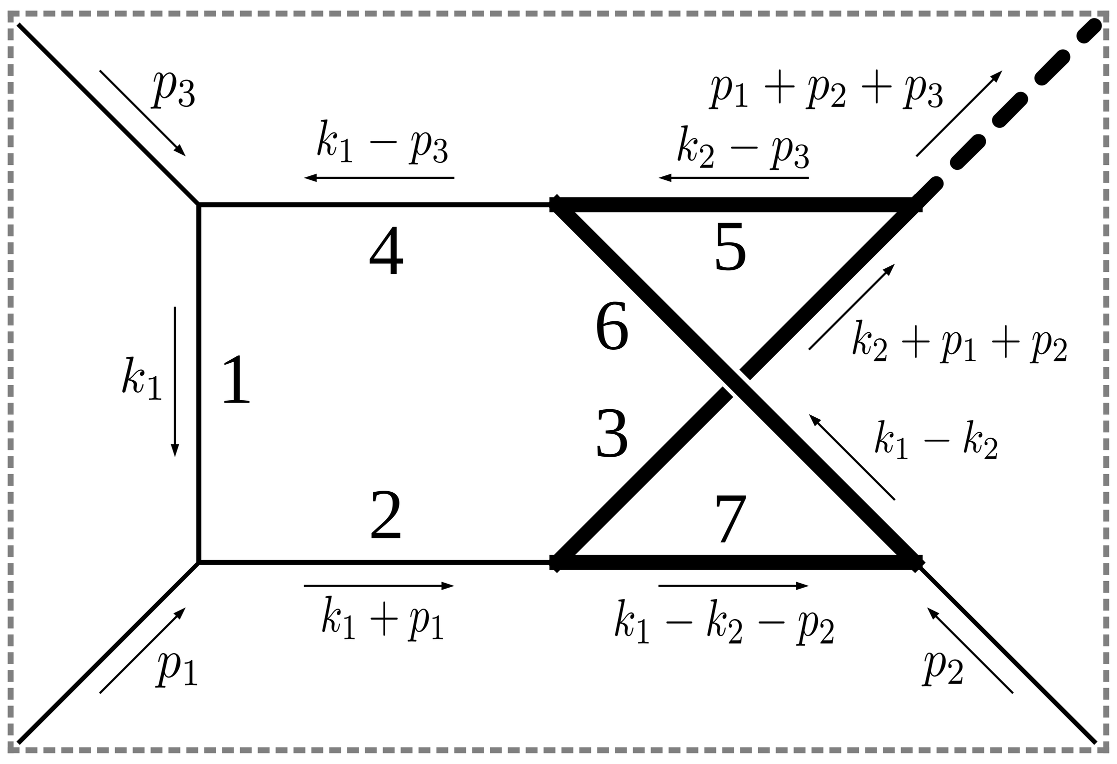
After the discovery of the Higgs boson at the Large Hadron Collider (LHC) at CERN, the LHC physics program has been centered around measuring the properties, couplings and quantum numbers of the Higgs boson, and looking for footprints of New Physics (NP) effects. An observable which may be useful in this respect is the production of a **Higgs boson in association with a jet**. In this work, we compute analytically one of two families of **non-planar master integrals** which contribute to the **two-loop** $gg \to gH$ amplitudes, with **full heavy-quark mass dependence**. This family is given by:

$$\iint \frac{d^dk_1 d^dk_2}{(i\pi^{d/2})^2} \frac{P_8^{-a_8} P_9^{-a_9}}{P_1^{a_1}P_2^{a_2}P_3^{a_3}P_4^{a_4}P_5^{a_5}P_6^{a_6}P_7^{a_7}},$$

Only P_1-P_7 are actual propagators, and therefore a_8 and a_9 are restricted to non-positive numbers. The kinematics is such that $p_1^2 = p_2^2 = p_3^2 = 0$, and we parametrize it by:

$$s = (p_1+p_2)^2,\quad t = (p_1+p_3)^2,\quad u = (p_2+p_3)^2,\quad m_H^2 = p_4^2.$$

$$P_1 = -k_1^2, \qquad P_2 = -(k_1+p_1)^2,$$
$$P_3 = m_t^2 - (k_2+p_1+p_2)^2, \qquad P_4 = -(k_1-p_3)^2,$$
$$P_5 = m_t^2 - (k_2-p_3)^2, \qquad P_6 = m_t^2 - (k_1-k_2)^2,$$
$$P_7 = m_t^2 - (k_1-k_2-p_2)^2, \qquad P_8 = m_t^2 - k_2^2,$$
$$P_9 = m_t^2 - (k_1-k_2-p_1-p_2)^2.$$



System of differential equations and canonical basis

For a complete set of master integrals $\vec{M}$ one may write down a closed form system of **first order linear differential equations**:

$$\frac{\partial}{\partial s_i}\vec{M} = \mathrm{A}_i \vec{M},$$

where $\{s_i\}$ denotes the set of external scales, and where the matrix A_i consists of rational functions of the external scales. Our family has **73 independent master integrals**, of which **65 are polylogarithmic**.

Performing a change of basis $M = \mathrm{T}\vec{B}$, where T is a matrix of rational or algebraic functions, we may put the **polylogarithmic sectors** in a **canonical form** [1].

In the canonical form ϵ is factored out, and all equations are combined into a single total differential:

$$d\vec{B}_{\text{poly}} = \epsilon d\tilde{\mathrm{A}} \vec{B}_{\text{poly}},$$

The entries of the matrix $\tilde{\mathrm{A}}$ are $\mathbb{Q}$-linear combinations of logarithms of rational or algebraic functions which are called **letters**. The formal solution of this system of differential equations is given by:

$$\vec{B}_{\text{poly}} = \mathbb{P}\exp\left[\epsilon \int_\gamma d\tilde{\mathrm{A}}\right]\vec{B}_{\text{bdry}},$$

where $\vec{B}_{\text{bdry}}$ denotes a boundary point, which we take to be the heavy mass limit.

Order-by-order in ϵ the result can be expressed in terms of iterated integrals:

$$\vec{B}_{\text{poly}} = \vec{B}_{\text{bdry}} + \sum_{k\geq 1}\epsilon^k \sum_{j=1}^{k}\int_0^1 \gamma^*(d\tilde{\mathrm{A}})(t_1) \times \int_0^{t_1}\gamma^*(d\tilde{\mathrm{A}})(t_2)\ldots \times \int_0^{t_{j-1}}\gamma^*(d\tilde{\mathrm{A}})(t_j)\,\vec{B}_{\text{bdry}}^{(k-j)}.$$

where $\gamma: [0,1] \to \mathbb{C}^4$ is some path in the phase space of (s,t,m^2,p_4^2). The $\tilde{\mathrm{A}}$-matrix can be expressed in terms of an alphabet of **69 linearly independent letters**, many of which are algebraic. The alphabet contains **12 independent square roots**, which can't be simultaneously rationalized, and therefore it is not clear how to write down the iterated integrals in terms of classical polylogarithms. We explore 2 ways of performing the integrations:

- Analytic **integration** up to **weight 2** in terms of **logarithms and dilogarithms**, with 1-fold integrals for weight 3 and 4. We consider these results in a region $\mathcal{R}$, which is the subregion of the Euclidean region where all square roots in the canonical basis are positive.
- **Expanding** the $\tilde{\mathrm{A}}$-matrix **as a series**, and "analytically" integrating the series up to a desired order. **This method may be used for the elliptic sectors** by considering the full system of differential equations.

Boundary terms in heavy mass limit

We may use **expansions by regions** [2] to compute the boundary terms in the heavy mass limit. This limit may be divergent for some integrals, so we parametrize along a line $\gamma: x \mapsto (xs, xt, xp_4^2, m^2)$, with $x \to 0$. As an illustrative example consider the computation of the boundary term B_{73}:

$$B_{73} = t\epsilon^4\left(\mathrm{I}_{\bar{1}_7,-2,0} + \frac{4s\mathrm{I}_{\bar{1}_7,-1,-1}}{2s+t-p_4^2} + \mathrm{I}_{\bar{1}_7,0,-2}\right) - \frac{t\epsilon^4\left(-4s-t+p_4^2\right)}{4}\left(\mathrm{I}_{\bar{1}_7,-1,0} + \mathrm{I}_{\bar{1}_7,0,-1}\right).$$

In the asymptotic limit one finds that:

$$\mathrm{I}_{\bar{1}_7,\sigma_1,\sigma_2} = \mathrm{I}^{(1)}_{\bar{1}_7,\sigma_1,\sigma_2} + x^{-\epsilon-1}\mathrm{I}^{(2)}_{\bar{1}_7,\sigma_1,\sigma_2},\quad \lim_{x\to 0} B_{73} \sim \epsilon^4 x^{-\epsilon}\left(-\frac{4st\mathrm{I}^{(2),(x=0)}_{\bar{1},-1,-1}}{p_4^2-2s-t} + t\left(\mathrm{I}^{(2),(x=0)}_{\bar{1},-2,0} + \mathrm{I}^{(2),(x=0)}_{\bar{1},0,-2}\right)\right).$$

Using a nice integration sequence, relying on particular choices of Cheng-Wu transforms, the above integrals may be integrated in closed form:

$$\mathrm{I}^{(2),(x=0)}_{\bar{1},-2,0} = \mathrm{I}^{(2),(x=0)}_{\bar{1},-1,-1} = \mathrm{I}^{(2),(x=0)}_{\bar{1},0,-2} = \frac{4\pi e^{2\gamma\epsilon}\epsilon^3 m^{-2\epsilon}(-t)^{-\epsilon}\cot(\pi\epsilon)\Gamma(2\epsilon)(p_4^2-4s-t)}{p_4^2-2s-t},$$

which yields $\lim_{x\to 0} B_{73} \sim -4\pi e^{2\gamma\epsilon}\epsilon^3\frac{(p_4^2-4s-t)}{(p_4^2-2s-t)}(m^2)^{-\epsilon}(-t)^{-\epsilon}\Gamma(2\epsilon)\cot(\pi\epsilon)$.

Ansatz for the symbol

We write each weight 2 polylogarithmic integral B_i in terms of an ansatz $B^{(2)}_{\text{ans},i} = c_i + \sum c_{ij}A_j$, with $c_i \propto \pi^2$, $c_{ij} \in \mathbb{Q}$ and $\{A_j\} = \left\{\mathrm{Li}_2(\pm l_r l_s), \mathrm{Li}_2\left(\pm\frac{l_r}{l_s}\right), \mathrm{Li}_2\left(\pm\frac{1}{l_r l_s}\right), \log(\pm l_r)\log(\pm l_s)\right\}$ in the spirit of [3]. The absence of terms $i\pi\log(\ldots)$ is guaranteed by working in region $\mathcal{R}$, where the canonical basis is real-valued, and by choosing real-valued basis functions in the ansatz. The coefficients c_{ij} may then be fixed by equating the symbol of the ansatz with $\mathcal{S}(\vec{B}^{(2)}) = \vec{B}^{(0),\mathsf{T}} \cdot (\tilde{\mathrm{A}}^{\mathsf{T}})^{\otimes 2}$. The coefficients c_i can be determined by taking the limit to a boundary point where the solution is known. For example:

$$B_{38} = \epsilon^3 r_2 r_3 r_{11}\mathrm{I}_{1,0,2,1,0,1,1,0,0} = -4\mathrm{Li}_2\left(-l_{33}^{-1}\right) + 2\mathrm{Li}_2\left(-l_{25}l_{33}^{-1}\right) + 2\mathrm{Li}_2\left(-l_{26}l_{33}^{-1}\right) + \ldots$$

The weight 3 and 4 results may be written as a 1-fold integrals over the weight 2 expressions.

Expansions for elliptic sectors

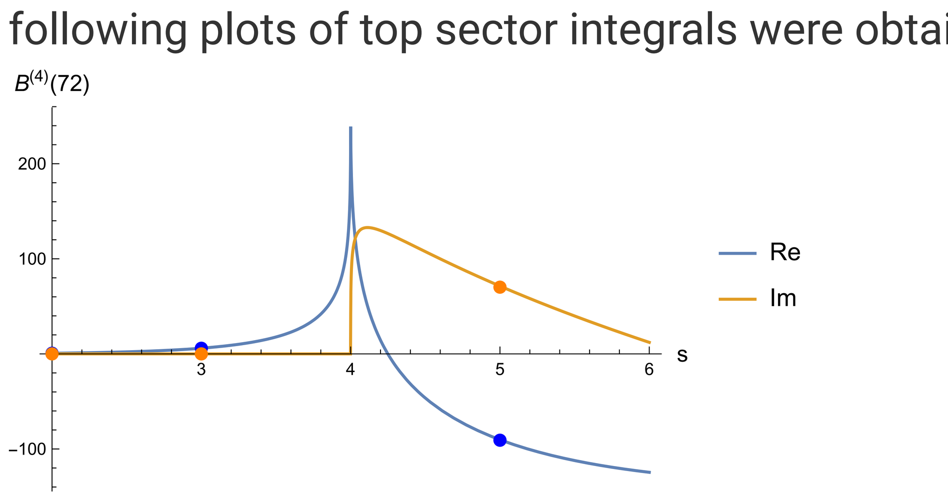
We may solve the elliptic sectors by considering a **(generalized) series expansion** of the differential equations. We follow the strategy of [4], which is based on the following steps:

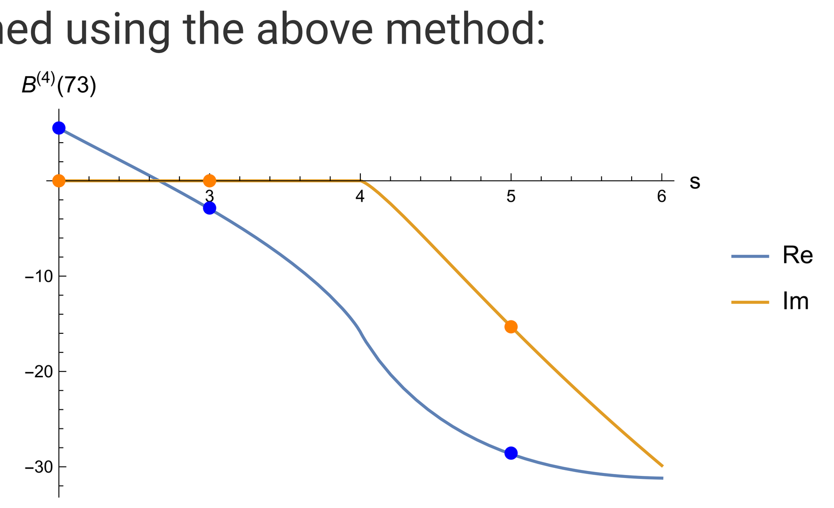
1. Find an analytical expression $\vec{B}(x_0)$ for the integrals at a given kinematical point x_0.
2. Write down the differential equations $\partial_t\vec{B}(x) = \mathrm{A}(\epsilon,t)\vec{B}(x)$ for the integrals along a line (or any other path) $\gamma(t)$ joining the original boundary point x_0 to any other point x_1 of interest.
3. Write the solution of the differential equations using series expansion methods, such as Frobenius' method, or Euler's method.
4. Obtain the values $\vec{B}(x_1)$

By a convenient choice of basis the diagonal blocks in the elliptic sectors can be reduced to 2x2, so that at most one has to solve second order differential equations, e.g. using Frobenius' method. All integrals to be performed are of the type:

$$\int t^q \log(t)^n\, dt = (-1)^n(1+q)^{-(n+1)}\Gamma(1+n, -(1+q)\log(t)) \quad q\in\mathbb{Q},\ n\in\mathbb{N},$$

whose series expansion involves again only terms of the form $t^q\log(t)^n$. The **analytic continuation** along physical singularities can furthermore be done by performing the analytic continuation of the logarithms according to the Feynman prescription. The following plots of top sector integrals were obtained using the above method:





where the dots are numerical evaluations performed using FIESTA.

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