

# Point Cloud Strategies for Boosted Tops

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Massachusetts Institute of Technology  
Center for Theoretical Physics

[Boosted Objects for New Physics Searches](#)

Fermilab, Illinois – 11/13/2018

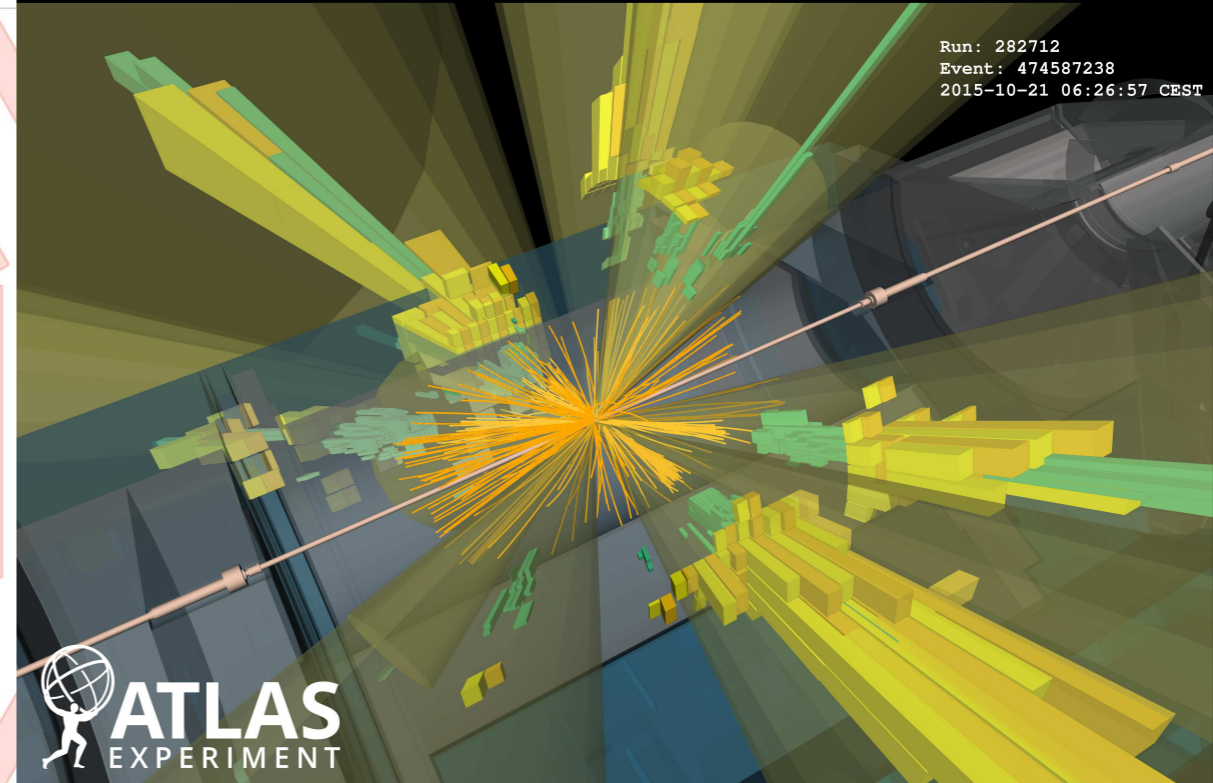
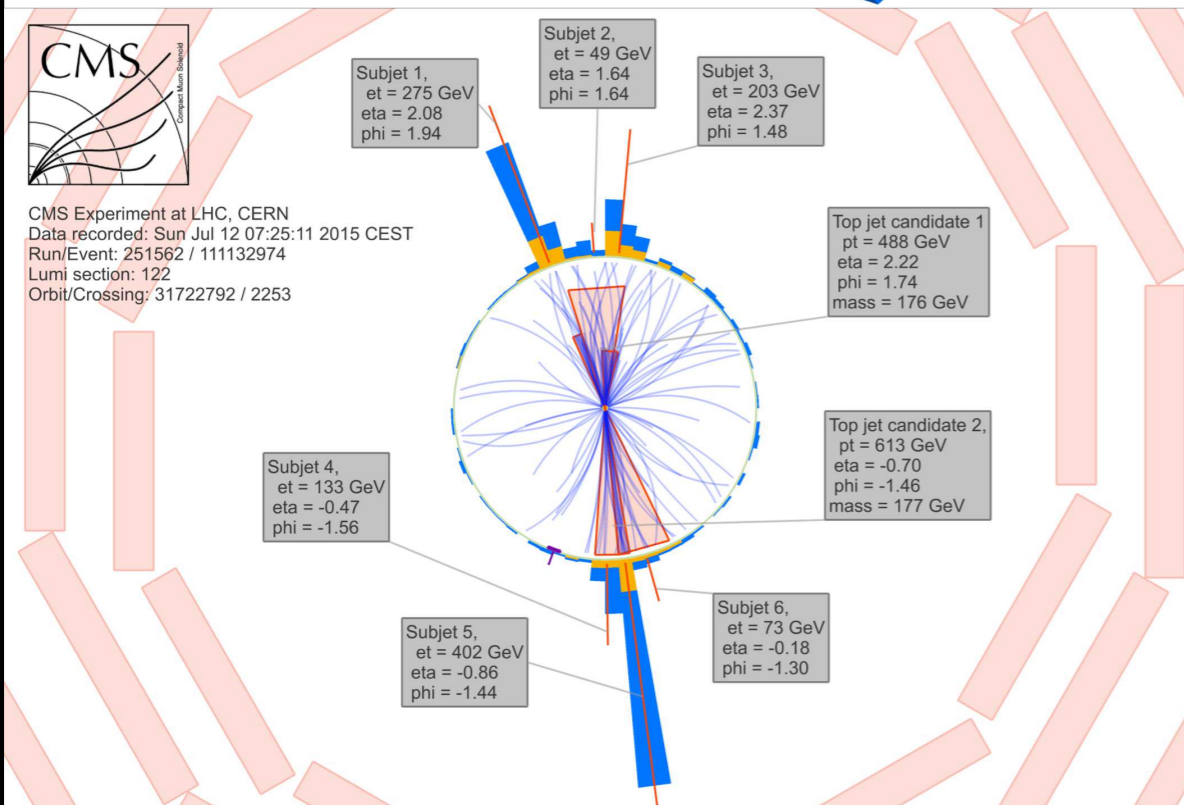
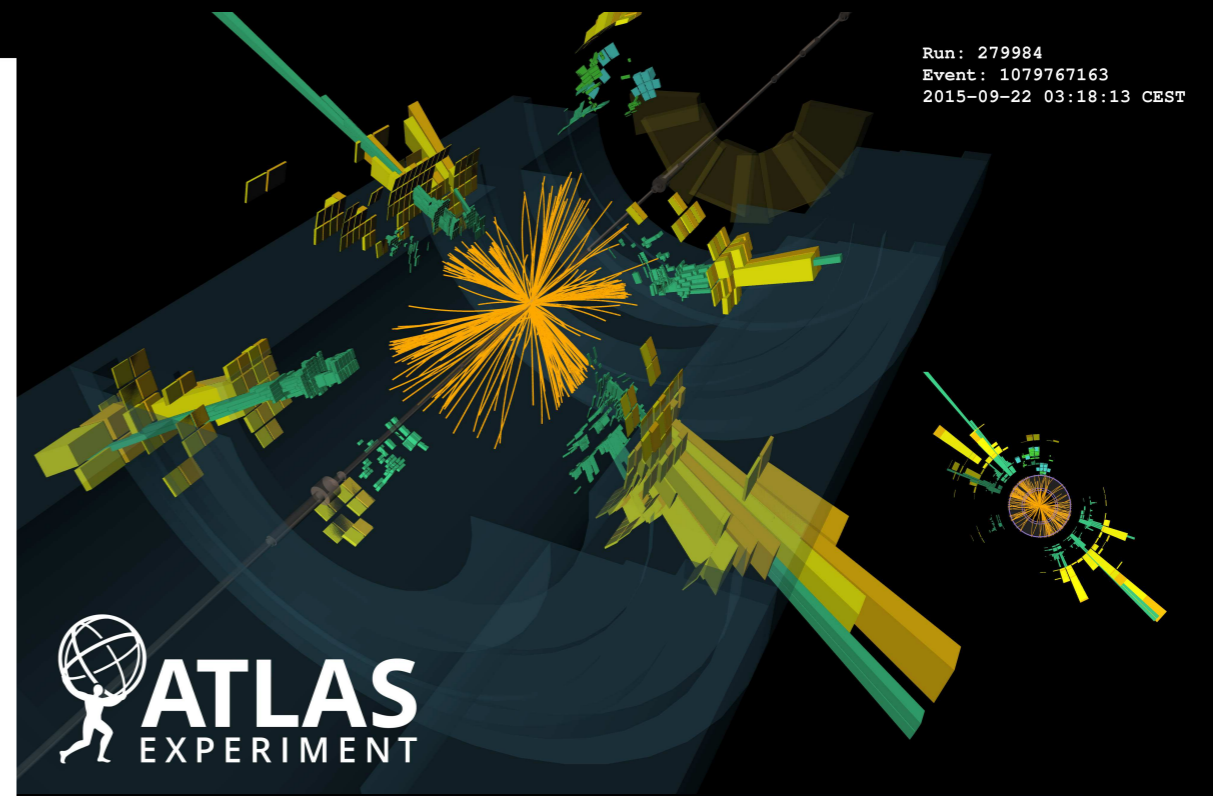
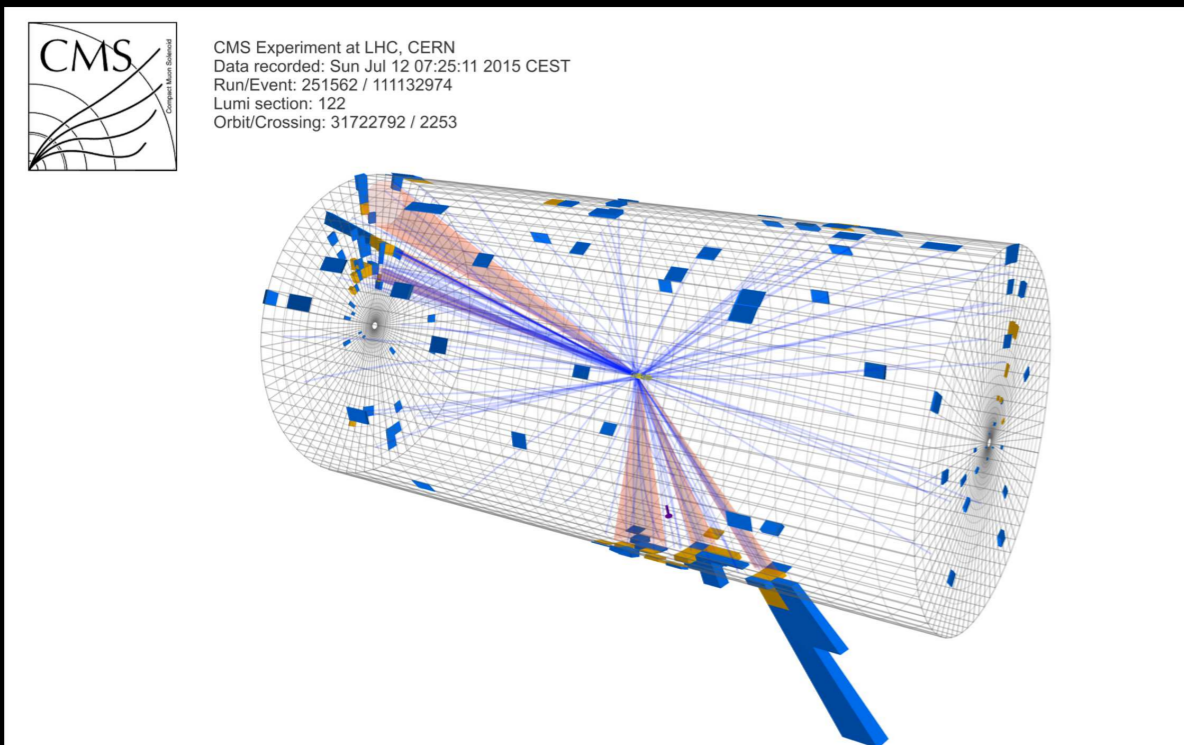
Based on work with Eric Metodiev and Jesse Thaler

[1712.07124](#)

[1810.05165](#)

<https://energyflow.network>

# Boosted Event Topologies at the LHC



# Why Boosted Tops?

Many models of new physics contain boosted Standard Model final states

e.g.  $Z' \rightarrow t\bar{t}$ , cascade decays, various SUSY scenarios

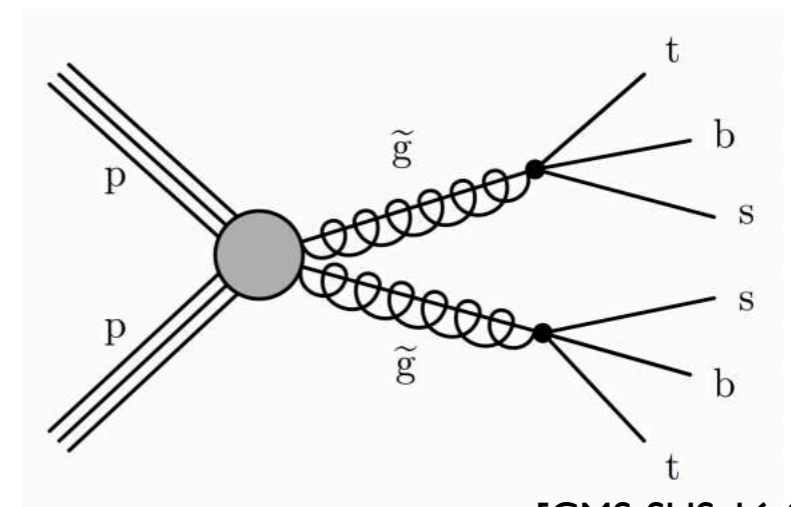
Boosted tops provide a way of testing and benchmarking multi-prong substructure techniques

Modern boosted top tagging is extremely effective!

Current CMS default – AK8 PUPPI jets, b tagged subjets, Soft

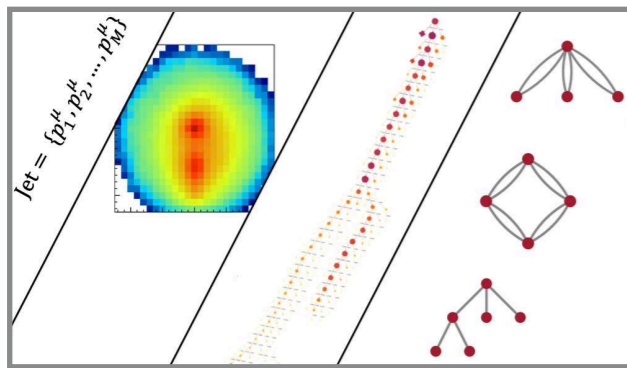
Drop mass cuts,  $\tau_{32}$  cut

Goal of this talk: *Demonstrate alternative, bottom up approaches to top tagging that go back to the basics and attempt to harness the power of the ML revolution*

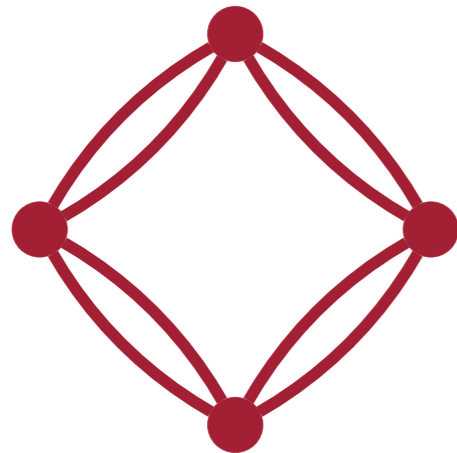


[CMS-SUS-16-040]

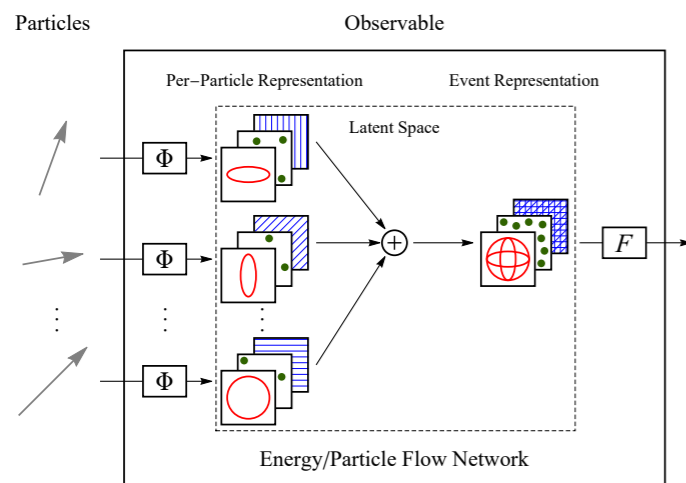
[CMS-B2G-17-017, [1810.05905](#)]



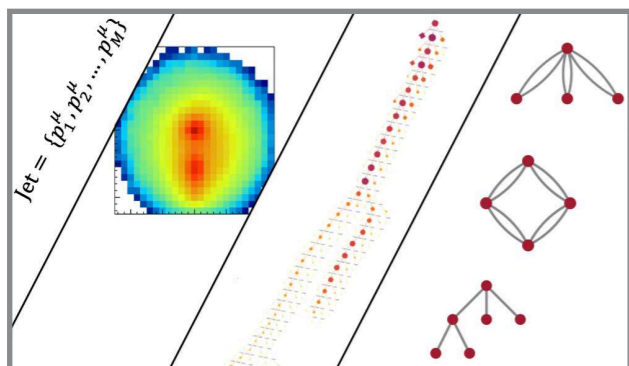
# Jets as Point Clouds



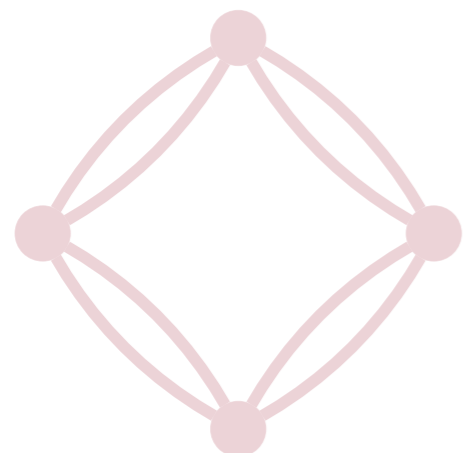
# Energy Flow Polynomials



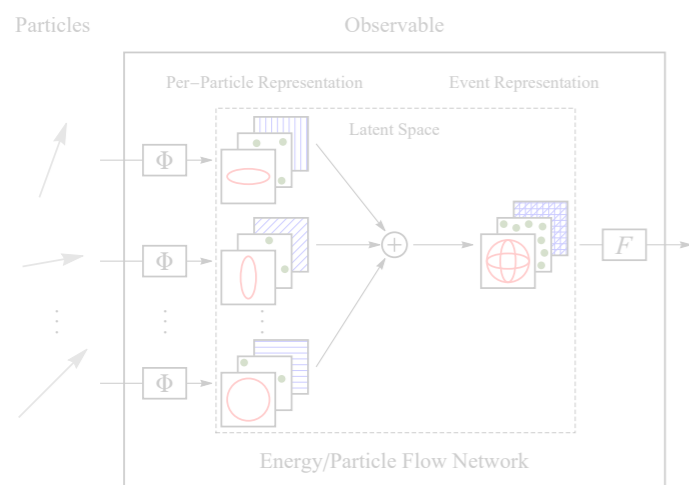
# Energy Flow Networks



# Jets as Point Clouds



# Energy Flow Polynomials



# Energy Flow Networks

# What is a Jet?

An **unordered**, **variable length** collection of particles

Due to quantum-mechanical indistinguishability  
Due to probabilistic nature of jet formation

$$J(\{p_1^\mu, \dots, p_M^\mu\}) = J(\{p_{\pi(1)}^\mu, \dots, p_{\pi(M)}^\mu\}), \quad \underbrace{M \geq 1}_{\text{Multiplicity}}, \quad \underbrace{\forall \pi \in S_M}_{\text{Permutations}}$$

$p_i^\mu$  represents *all* the particle properties:

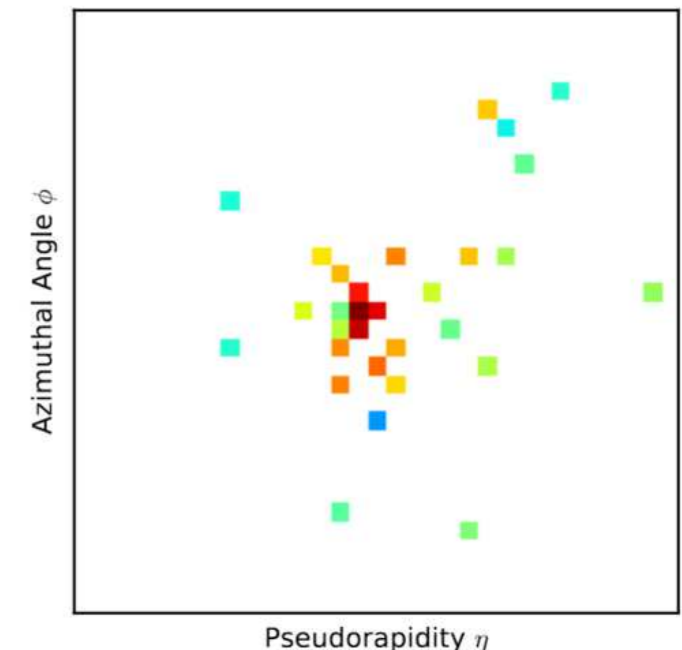
- Four-momentum –  $(E, p_x, p_y, p_z)_i^\mu$
- Other quantum numbers (e.g. particle id, charge)
- Experimental information (e.g. vertex info, quality criteria, PUPPI weights)

Contrast with jet images

$d$  dimensional particles,  $N \times N$  pixels  
 $dN^2$  jet image inputs,  $dM$  point cloud inputs

Particles are the medium in which theory and experiment meet

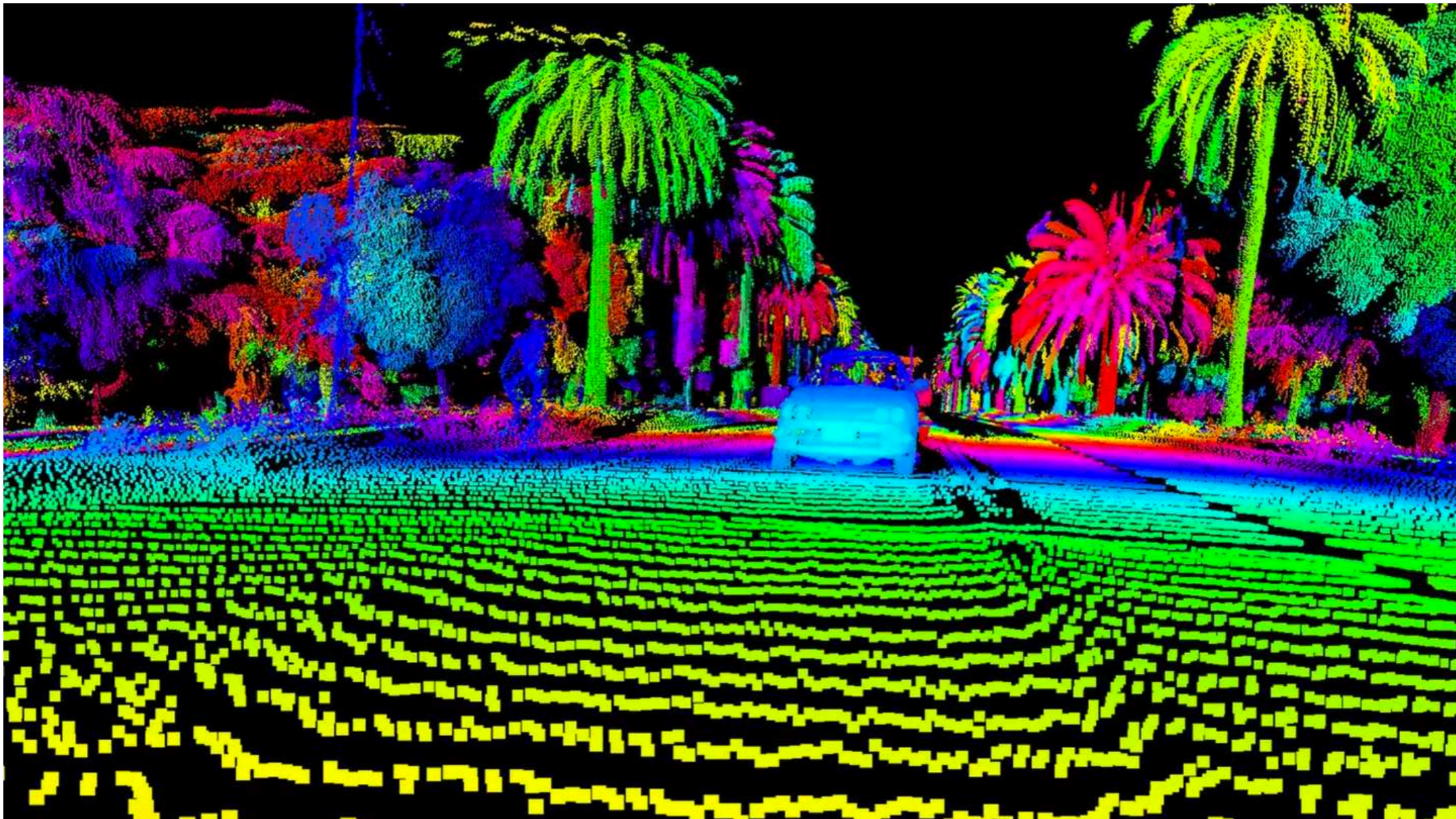
Success of CMS Particle Flow validates particles as fundamental objects in particle physics



# Point Clouds

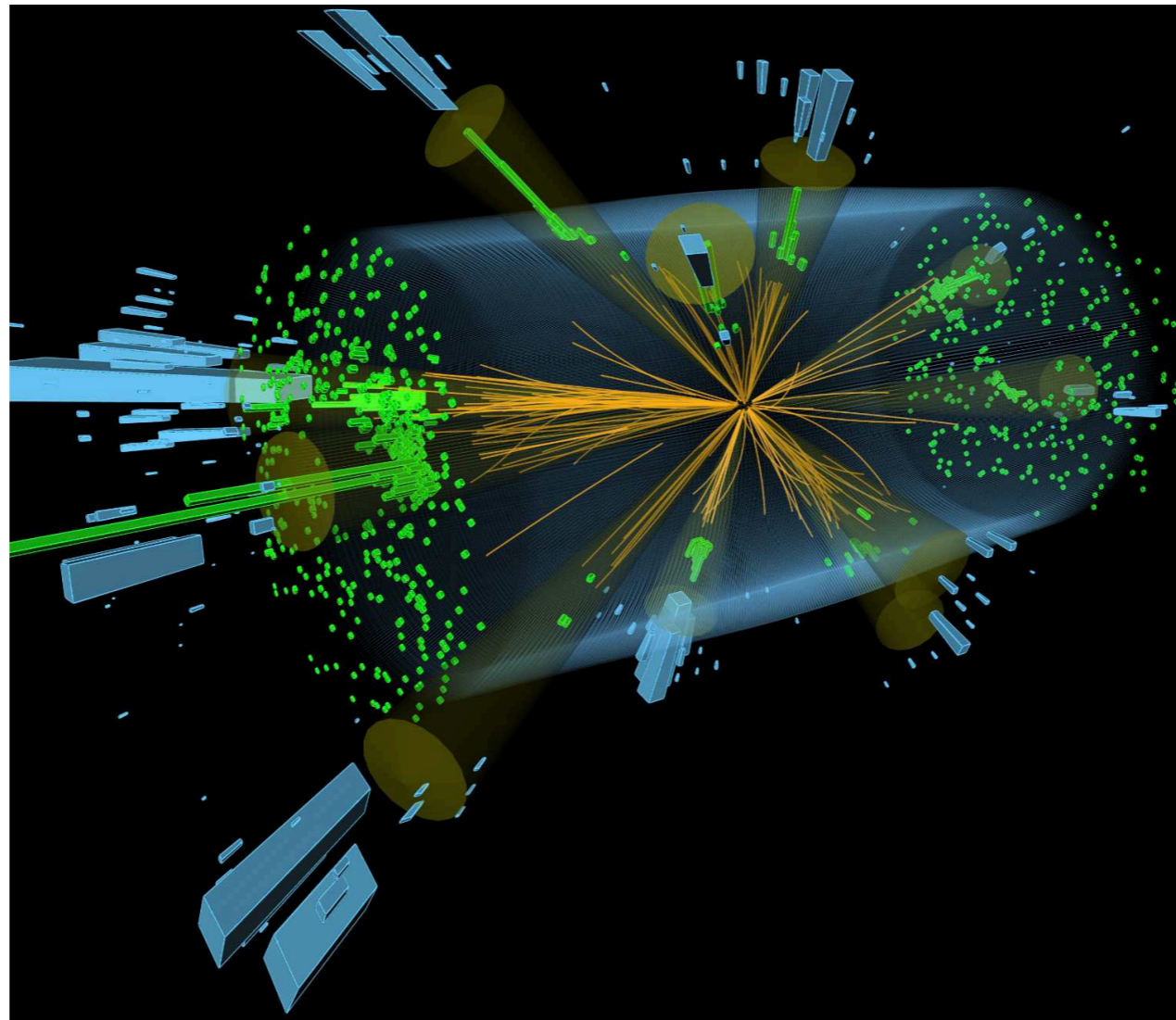
Point cloud: "A set of data points in space" –Wikipedia

LIDAR data from self-driving  
car sensor



# Particle Collision Events as Point Clouds

Point cloud: "A set of data points in space" –Wikipedia



Multi-jet event at CMS



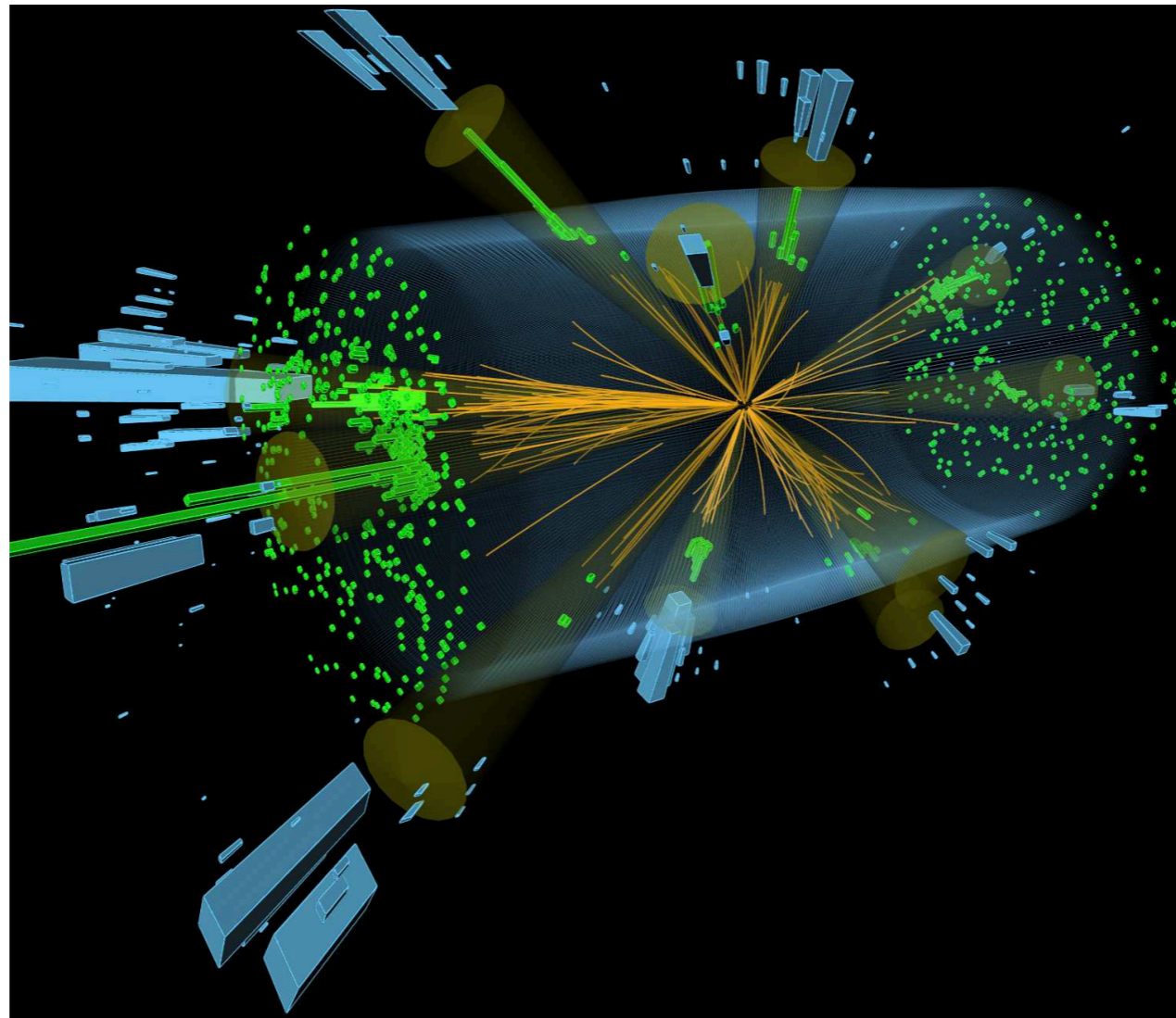
# Particle Collision Events as Point Clouds

Point cloud: "A set of data points in space" –Wikipedia

Jet/event

Particles

Feature space



Multi-jet event at CMS

# Processing Point Clouds

*Methods for processing point clouds/jets should respect the appropriate symmetries*

**Variable constituent multiplicity** requires at least one of:

Preprocessing to another representation (jet images,  $N$ -subjettiness, etc.)

Truncation to an (arbitrary) fixed size

Recurrent NN structure

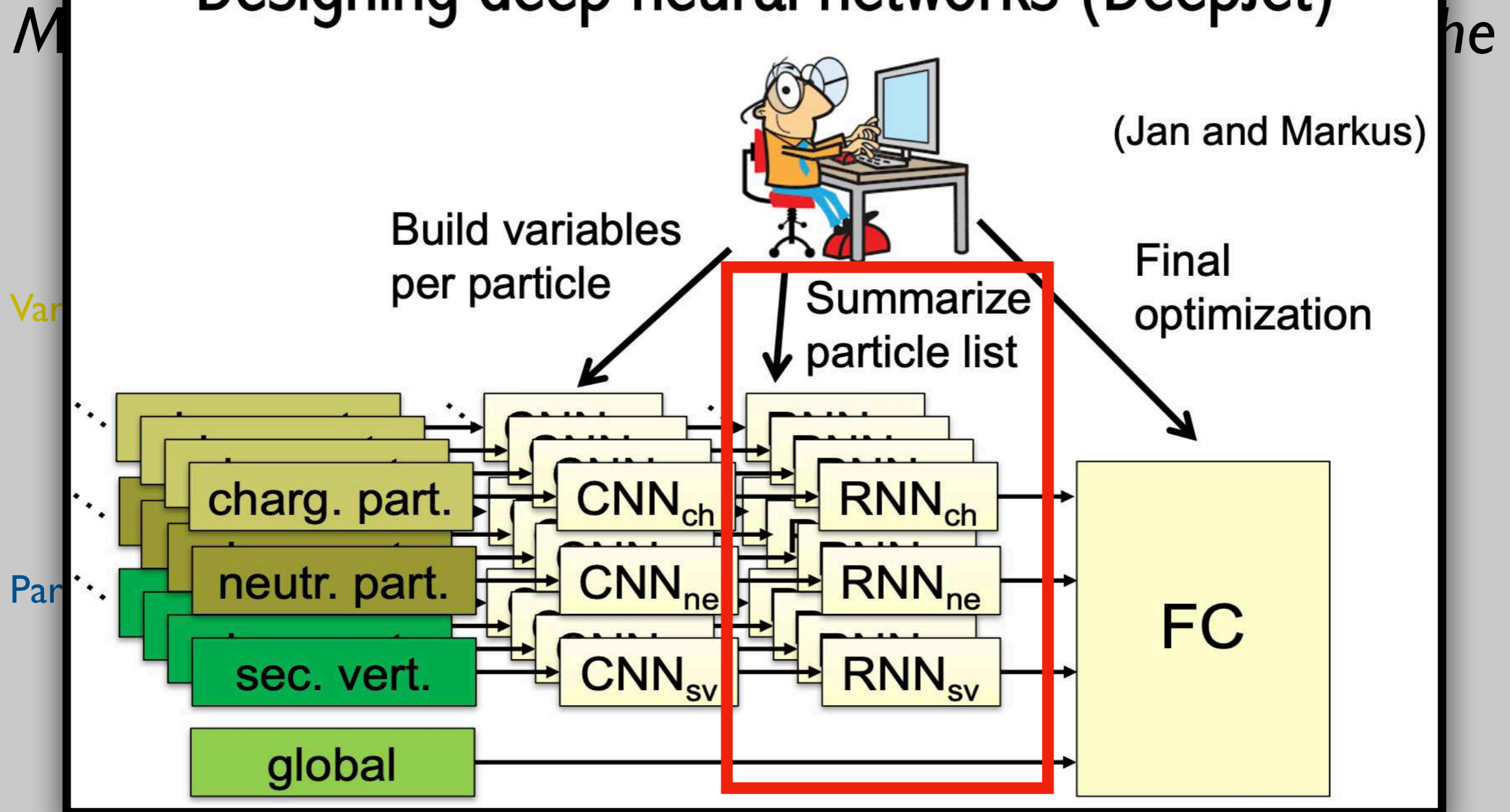
**Particle permutation symmetry** requires:

Permutation symmetric observables

Permutation symmetric architectures

# Processing Point Clouds

## Designing deep neural networks (DeepJet)



Slide from [Markus Stoye's talk](#)

# Jet Representations ↔ Analysis Tools

## Two key choices when analyzing jets

How to represent the jet



How to analyze that representation

- Single expert observable
- A few expert observables
- Many expert observables

**Fixed Processing**

- Jet images
- $N$ -subjettiness basis
- Energy flow polynomials

- Threshold cut
- Multidimensional likelihood
- Boosted decision tree (BDT), shallow neural network (NN)
- Convolutional NN (CNN)
- Dense neural network (DNN)
- Linear classification

- List of particles
- Clustering tree
- Set of particles

**Flexible Processing**

- Recurrent NN (RNN)
- Recursive NN
- Energy flow network

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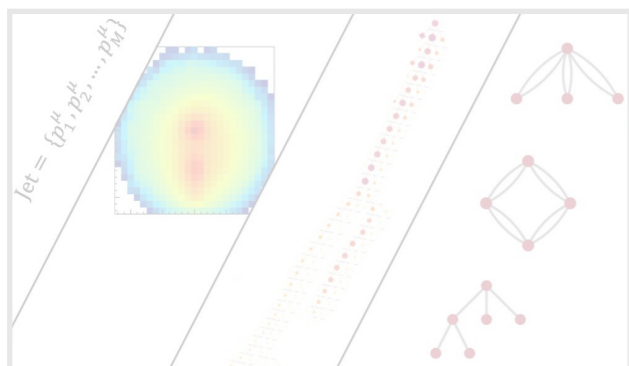
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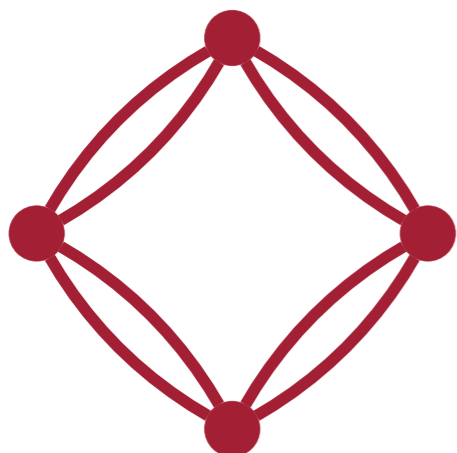
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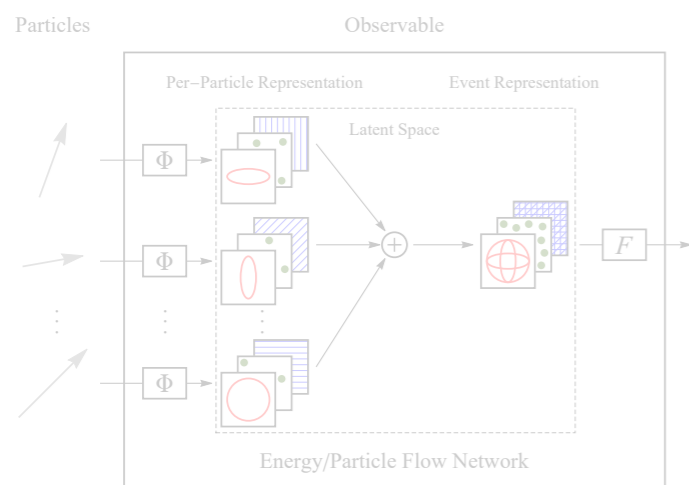


# Jets as Point Clouds



# Energy Flow Polynomials

*Fixed point cloud processing*

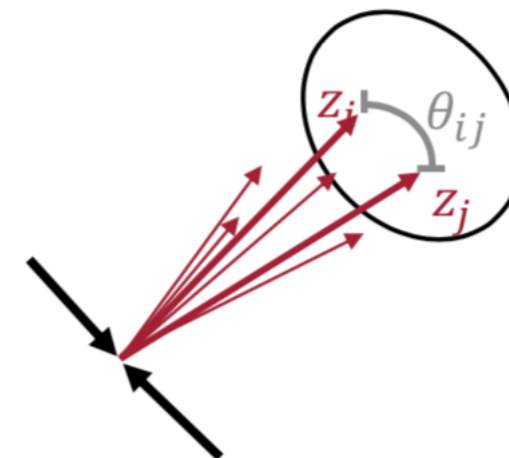


# Energy Flow Networks

# Energy Flow Polynomials (EFPs)

[PTK, Metodiev, Thaler, [1712.07124](#)]

$$\text{EFP}_G = \underbrace{\sum_{i_1=1}^M \cdots \sum_{i_N=1}^M}_{\text{Correlator of}} \underbrace{z_{i_1} \cdots z_{i_N}}_{\text{Energies}} \underbrace{\prod_{(k,l) \in G} \theta_{i_k i_l}}_{\text{and Angles}}$$



Generalizes many well-known and studied classes of energy correlators observables

A family of energy correlators with angular structures determined by multigraphs

$$= \sum_{i_1=1}^M \sum_{i_2=1}^M \sum_{i_3=1}^M \sum_{i_4=1}^M \sum_{i_5=1}^M z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5} \theta_{i_1 i_2} \theta_{i_2 i_3} \theta_{i_1 i_3} \theta_{i_1 i_4} \theta_{i_1 i_5} \theta_{i_4 i_5}^2$$

Multigraph correspondence



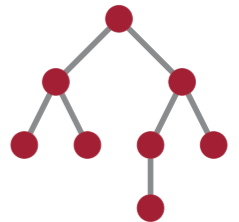
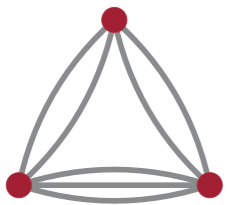
**Energy and Angle Measure**

Hadronic :  $z_i = \frac{p_{Ti}}{\sum_j p_{Tj}}$ ,  $\theta_{ij} = (\Delta y_{ij}^2 + \Delta \phi_{ij}^2)^{\beta/2}$

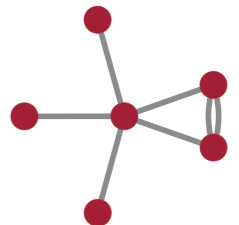
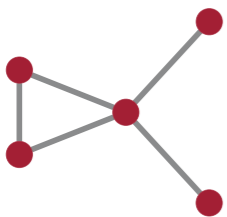
# Linear Basis of IRC-Safe Observables

One can show via the Stone-Weierstrass approximation theorem that any IRC-safe observable is a linear combination of EFPs

$$\mathcal{S} \simeq \sum_{G \in \mathcal{G}} s_G \text{EFP}_G, \quad \mathcal{G} \text{ a set of multigraphs}$$



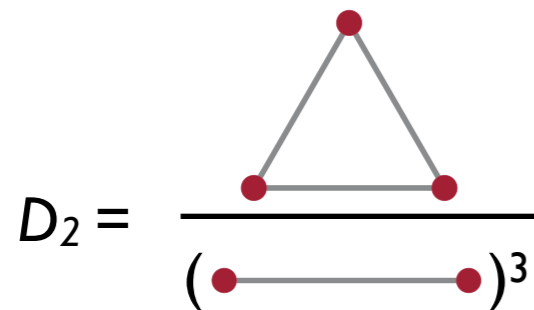
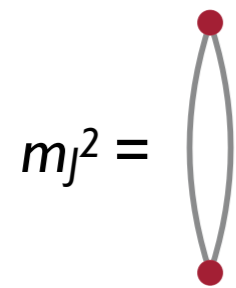
*Multivariate combinations of EFPs only require linear methods to achieve full generality*



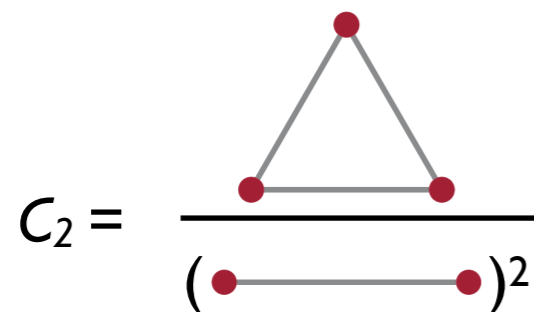
Strategy: Learn coefficients  $s_G$  via linear regression or classification



# Familiar Observables as EFPs



[Larkoski, Mout, Neill, 2014]



[Larkoski, Salam, Thaler, 2013]

Energy correlation functions are complete graphs

Even angularities are exact linear combinations of EFPs

## EFPs organized by degree $d$ – number of edges

Degree	Connected Multigraphs
$d = 1$	
$d = 2$	
$d = 3$	
$d = 4$	
$d = 5$	

# Computation Complexity of EFPs – Variable Elimination

Naive computation complexity of an energy correlator is  $\mathcal{O}(M^N)$

For  $\sim 100$  particles this becomes intractable for  $N > 4$

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⇒ EnergyCorrelator fjcontrib package gives up in this case

```
// if N > 5, then throw error
if (_N > 5) {
    throw Error("EnergyCorrelator is only hard coded for N = 0,1,2,3,4,5");
}
```

# Computation Complexity of EFPs – Variable Elimination

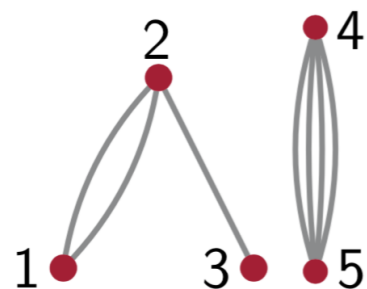
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Variable elimination (VE) algorithm can speedup EFPs by finding efficient elimination ordering



$$= \left( \sum_{i_1=1}^M \sum_{i_2=1}^M \sum_{i_3=1}^M z_{i_1} z_{i_2} z_{i_3} \theta_{i_1 i_2}^2 \theta_{i_2 i_3} \right) \left( \sum_{i_4=1}^M \sum_{i_5=1}^M z_{i_4} z_{i_5} \theta_{i_4 i_5}^4 \right)$$

Disconnected is product of connected



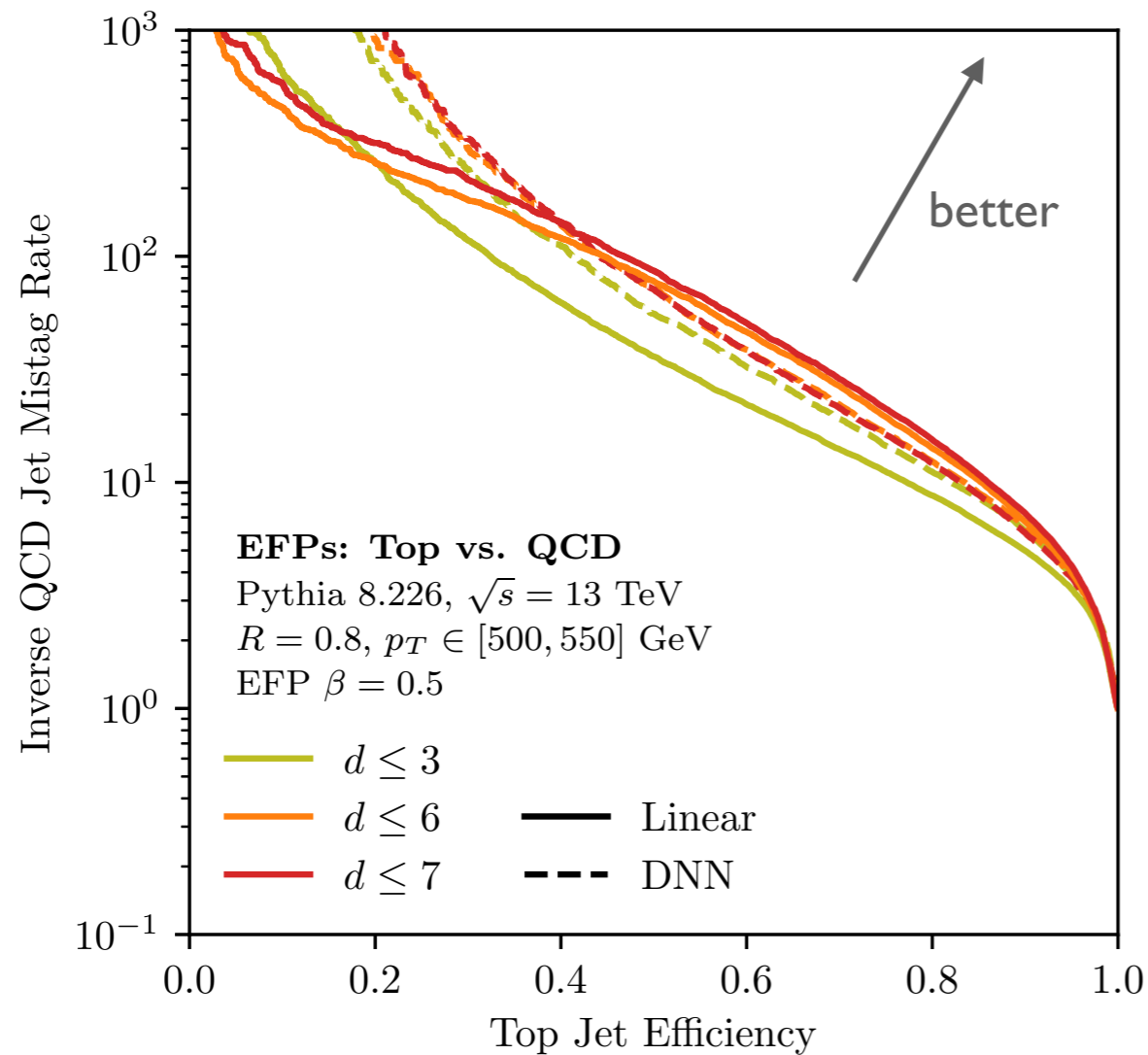
$$= \underbrace{\sum_{i_1=1}^M \sum_{i_2=1}^M \sum_{i_3=1}^M \sum_{i_4=1}^M \sum_{i_5=1}^M \sum_{i_6=1}^M \sum_{i_7=1}^M \sum_{i_8=1}^M z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5} z_{i_6} z_{i_7} z_{i_8} \prod_{j=2}^7 \theta_{i_1 i_j}}_{\mathcal{O}(M^8)}$$

$$= \underbrace{\sum_{i_1=1}^M z_{i_1} \left( \sum_{i_2=1}^M z_{i_2} \theta_{i_1 i_2} \right)^7}_{\mathcal{O}(M^2)}$$

Clever parentheses placement corresponds to good elimination ordering

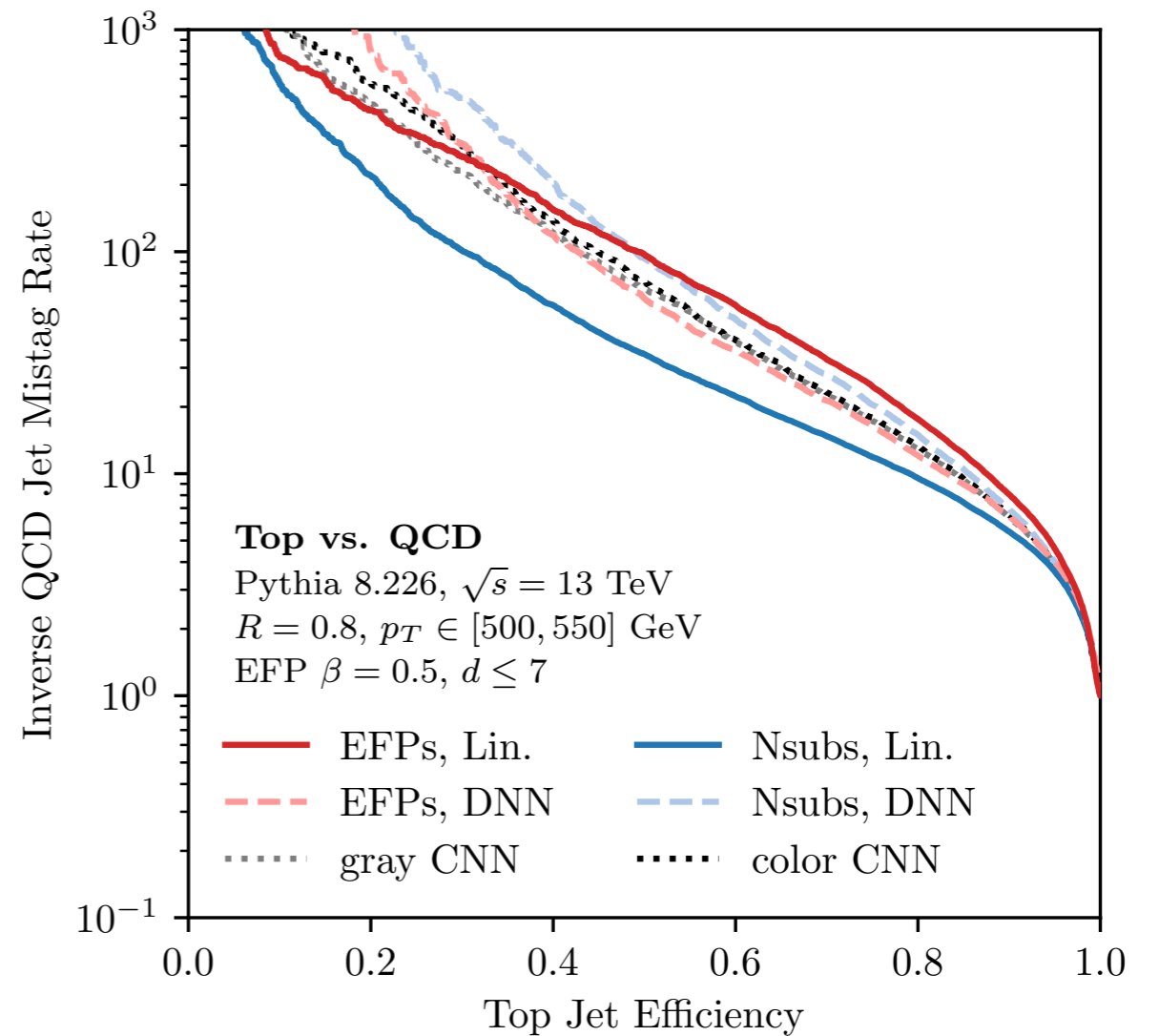
All tree graphs become  $\mathcal{O}(M^2)$

# EFPs for Boosted Tops



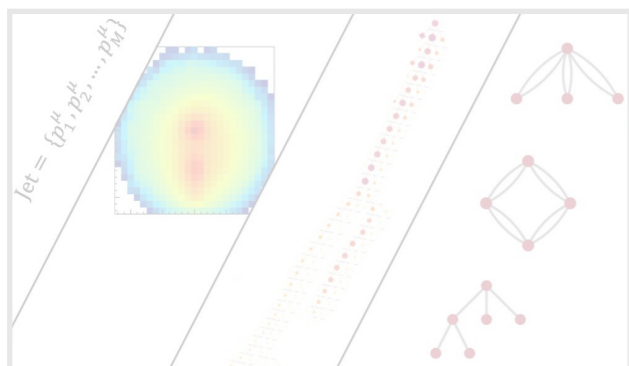
Saturation observed with more EFPs

DNN gets there faster but linear suffices

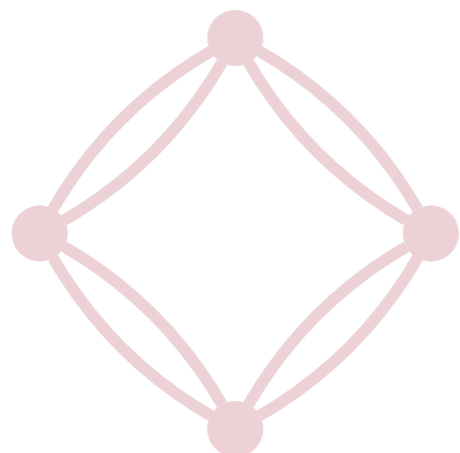


Linear EFPs excel at high efficiency

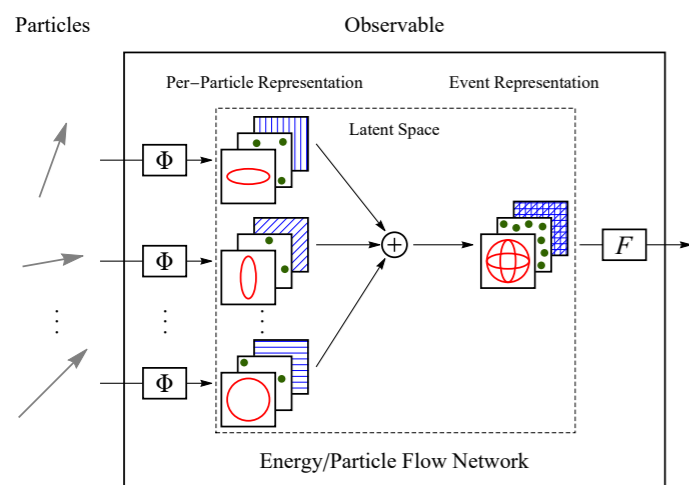
[de Oliviera, Kagan, Mackey, Nachman, Schwartzman, 2015]  
 [PTK, Metodiev, Schwartz, 2016]  
 [Datta, Larkoski, 2017]



# Jets as Point Clouds



# Energy Flow Polynomials



# Energy Flow Networks

*Flexible/learnable point cloud processing*

(EFNs for Q/G talk on Thursday @ ML4Jets!)

# Symmetric Function Parametrization

A general permutation-symmetric function is *additive* in a latent space

*Deep Sets*: Namespace for additive symmetric function parametrization

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## Deep Sets

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[\[1703.06114\]](#)

Manzil Zaheer<sup>1,2</sup>, Satwik Kottur<sup>1</sup>, Siamak Ravanbakhsh<sup>1</sup>,  
Barnabás Póczos<sup>1</sup>, Ruslan Salakhutdinov<sup>1</sup>, Alexander J Smola<sup>1,2</sup>  
<sup>1</sup> Carnegie Mellon University    <sup>2</sup> Amazon Web Services

**Deep Sets Theorem [63]**. *Let  $\mathfrak{X} \subset \mathbb{R}^d$  be compact,  $X \subset 2^{\mathfrak{X}}$  be the space of sets with bounded cardinality of elements in  $\mathfrak{X}$ , and  $Y \subset \mathbb{R}$  be a bounded interval. Consider a continuous function  $f : X \rightarrow Y$  that is invariant under permutations of its inputs, i.e.  $f(x_1, \dots, x_M) = f(x_{\pi(1)}, \dots, x_{\pi(M)})$  for all  $x_i \in \mathfrak{X}$  and  $\pi \in S_M$ . Then there exists a sufficiently large integer  $\ell$  and continuous functions  $\Phi : \mathfrak{X} \rightarrow \mathbb{R}^\ell$ ,  $F : \mathbb{R}^\ell \rightarrow Y$  such that the following holds to an arbitrarily good approximation:<sup>1</sup>*

$$f(\{x_1, \dots, x_M\}) = F\left(\sum_{i=1}^M \Phi(x_i)\right). \quad (2.1)$$

# Symmetric Function Parametrization

A general permutation-symmetric function is *additive* in a latent space

Deep Sets: Namespace for additive symmetric function parametrization

## Deep Sets

[1703.06114]

Manzil Zaheer<sup>1,2</sup>, Satwik Kottur<sup>1</sup>, Siamak Ravanbakhsh<sup>1</sup>,  
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Feature space

Variable length

Permutation invariance

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$$f(\{x_1, \dots, x_M\}) = F \left( \sum_{i=1}^M \Phi(x_i) \right). \quad (2.1)$$

Latent space

General parametrization for a function of sets



# Deep Sets for Particle Jets

[PTK, Metodiev, Thaler, [1810.05165](#)]

*Particle Flow Network (PFN)*

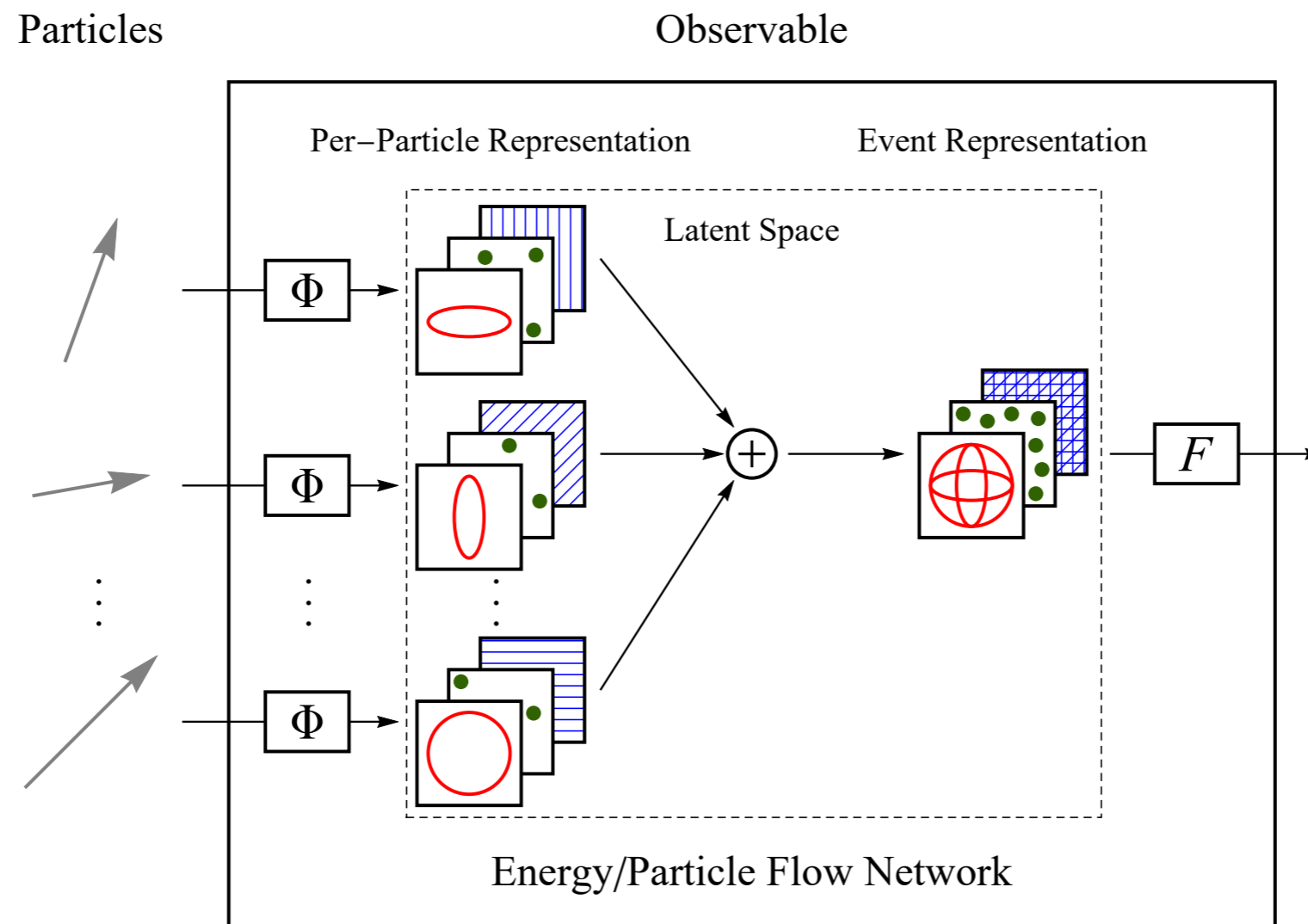
$$\text{PFN}(\{p_1^\mu, \dots, p_M^\mu\}) = F \left( \sum_{i=1}^M \Phi(p_i^\mu) \right)$$

Fully general latent space

*Energy Flow Network (EFN)*

$$\text{EFN}(\{p_1^\mu, \dots, p_M^\mu\}) = F \left( \sum_{i=1}^M z_i \Phi(\hat{p}_i) \right)$$

IRC-safe latent space

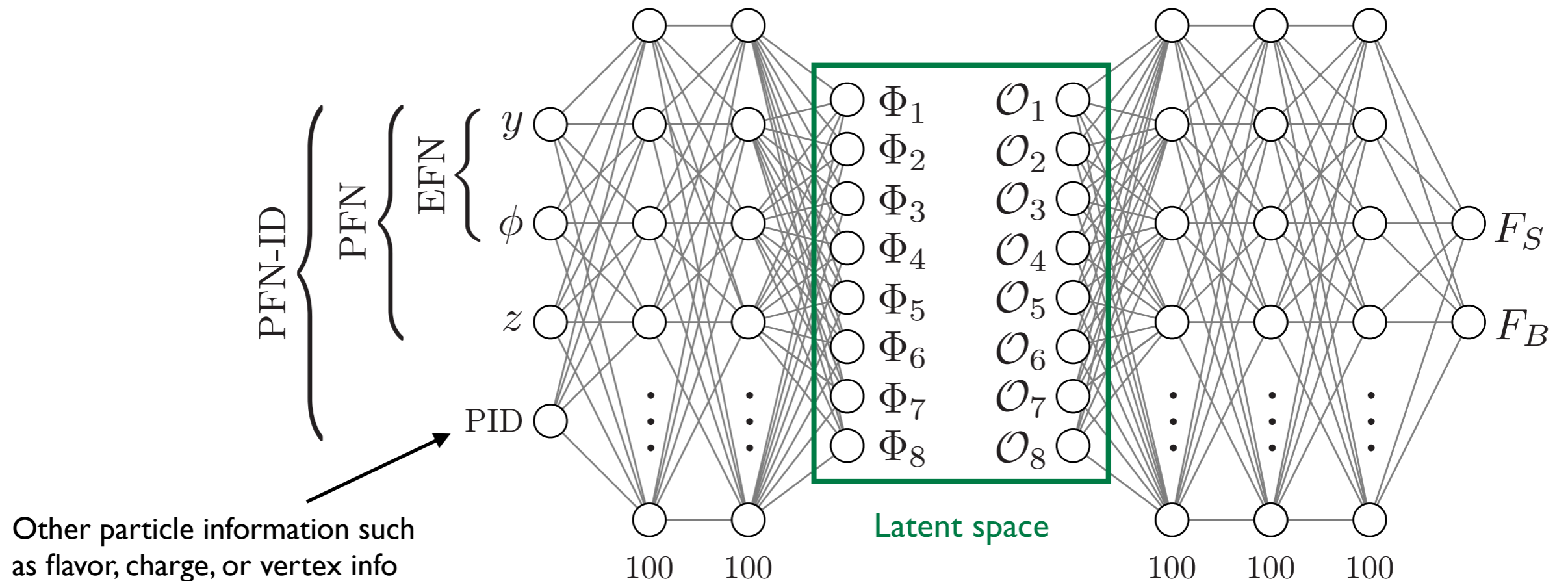


# Approximating $\Phi$ and $F$ with Neural Networks

Employ neural networks as arbitrary function approximators

Use fully-connected networks for simplicity

Default sizes –  $\Phi$ : (100, 100,  $\ell$ ),  $F$ : (100, 100, 100)



$$\text{PFN} : \mathcal{O}_a = \sum_{i=1}^M \Phi_a(z_i, y_i, \phi_i, [\text{PID}_i])$$

$$\text{EFN} : \mathcal{O}_a = \sum_{i=1}^M z_i \Phi_a(y_i, \phi_i)$$

# Top Jet Samples and Other Methods

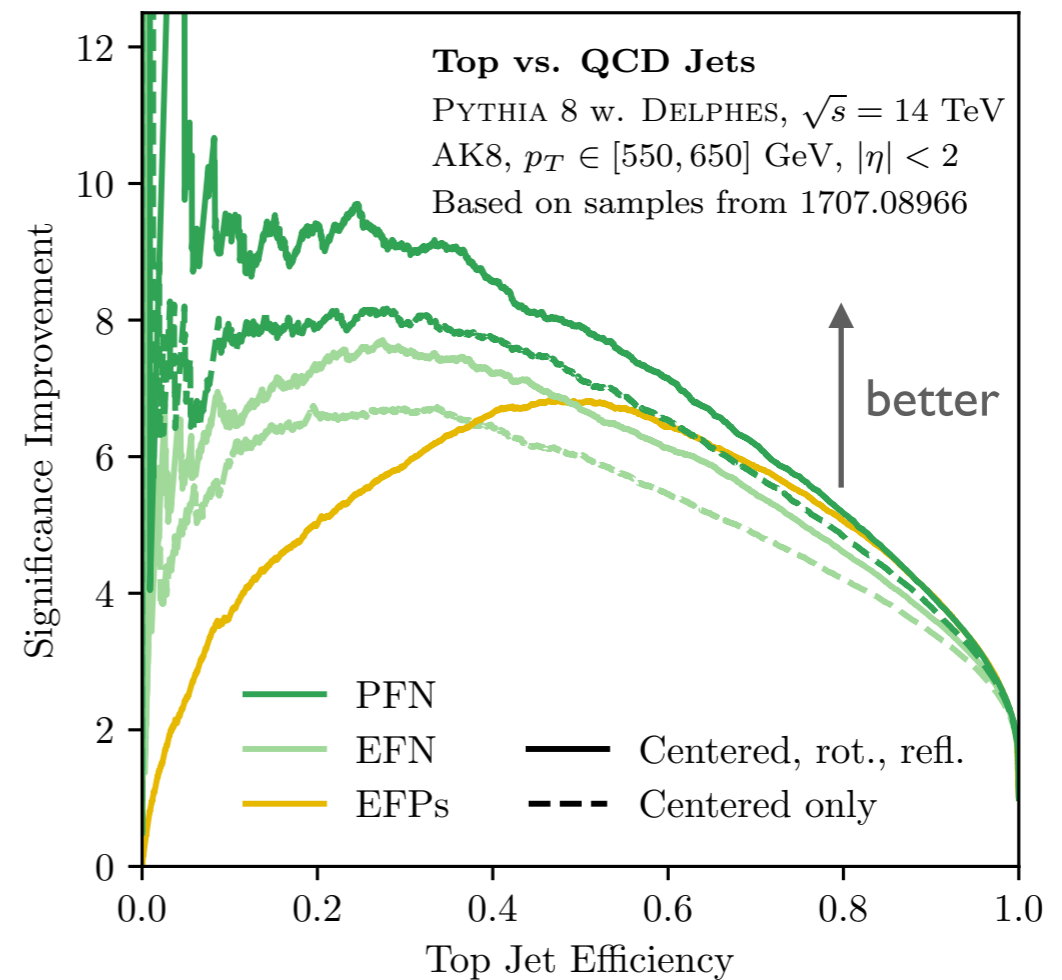
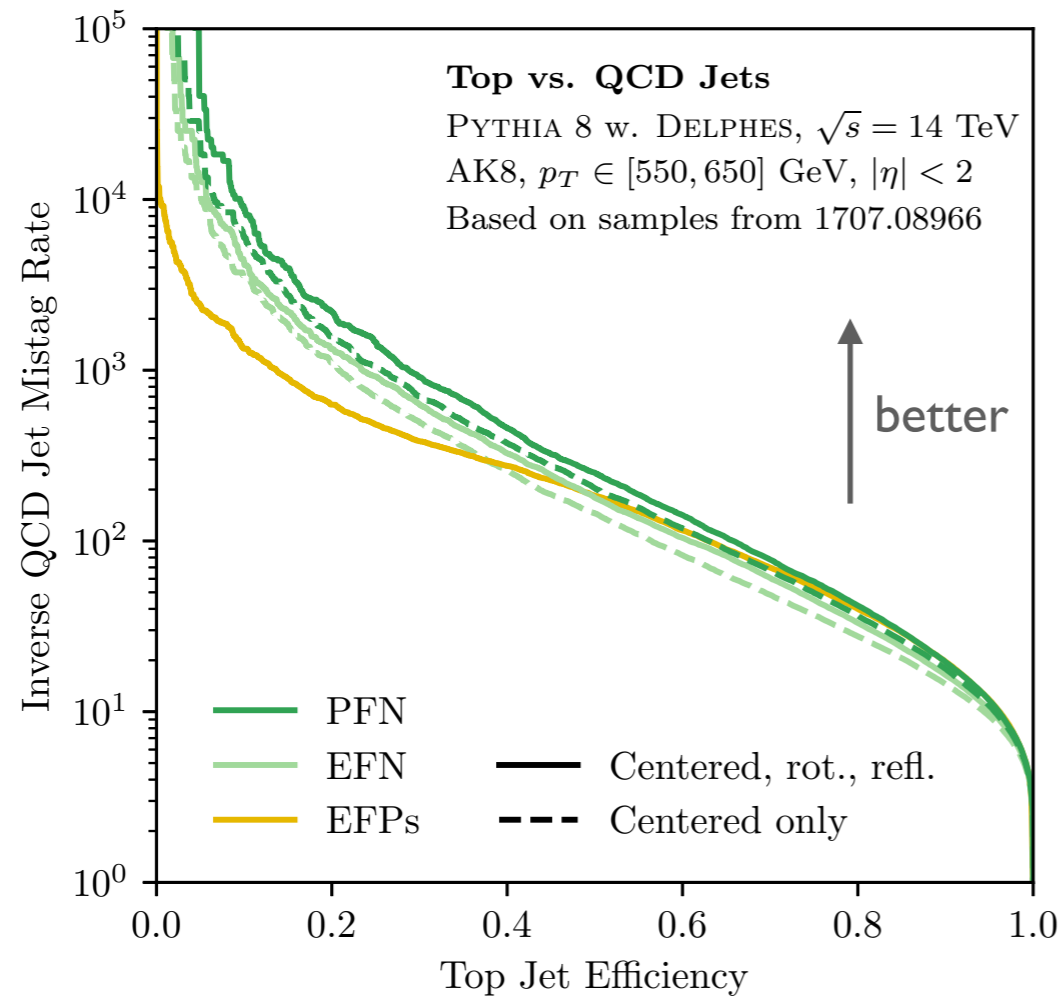
[Butter, Kasieczka, Plehn, Russell, 2017]

Common top and QCD dijet samples for standardized benchmarking

$p_T \in [550, 650]$  GeV, AK8 jets, fully-merged, Delphes simulation, 2m jets total

Approach	AUC	Acc.	1/eB @ (eS=0.3)	Contact	Comments
LoLa	0.979	0.928		G. Kasieczka S. Leiss	Preliminary number, based on LoLa
LBN	0.981	0.931	863	M. Rieger	Preliminary number
CNN	0.981	0.93	780	D. Shih	Model from (1803.00107)
P-CNN (1D CNN)	0.980	0.930	782	H. Qu, L. Gouskos	Preliminary, use kinematic info only
6-body N-sub. (+mass and pT) NN	0.979	0.922	856	K. Nordstrom	Based on 1807.04769
8-body N-sub. (+mass and pT) NN	0.980	0.928	795	K. Nordstrom	Based on 1807.04769
Linear EFPs	0.980	0.932	380	PTK, E. Metodiev	$d \leq 7$ , $\chi \leq 3$ EFPs with FLD. Based on 1712.07124
Particle Flow Network (PFN)	0.982	0.932	888	PTK, E. Metodiev	Median over ten trainings. Based on Table 5 in 1810.05165
Energy Flow Network (EFN)	0.979	0.927	619	PTK, E. Metodiev	Median over ten trainings. Based on Table 5 in 1810.05165

# Classification Performance

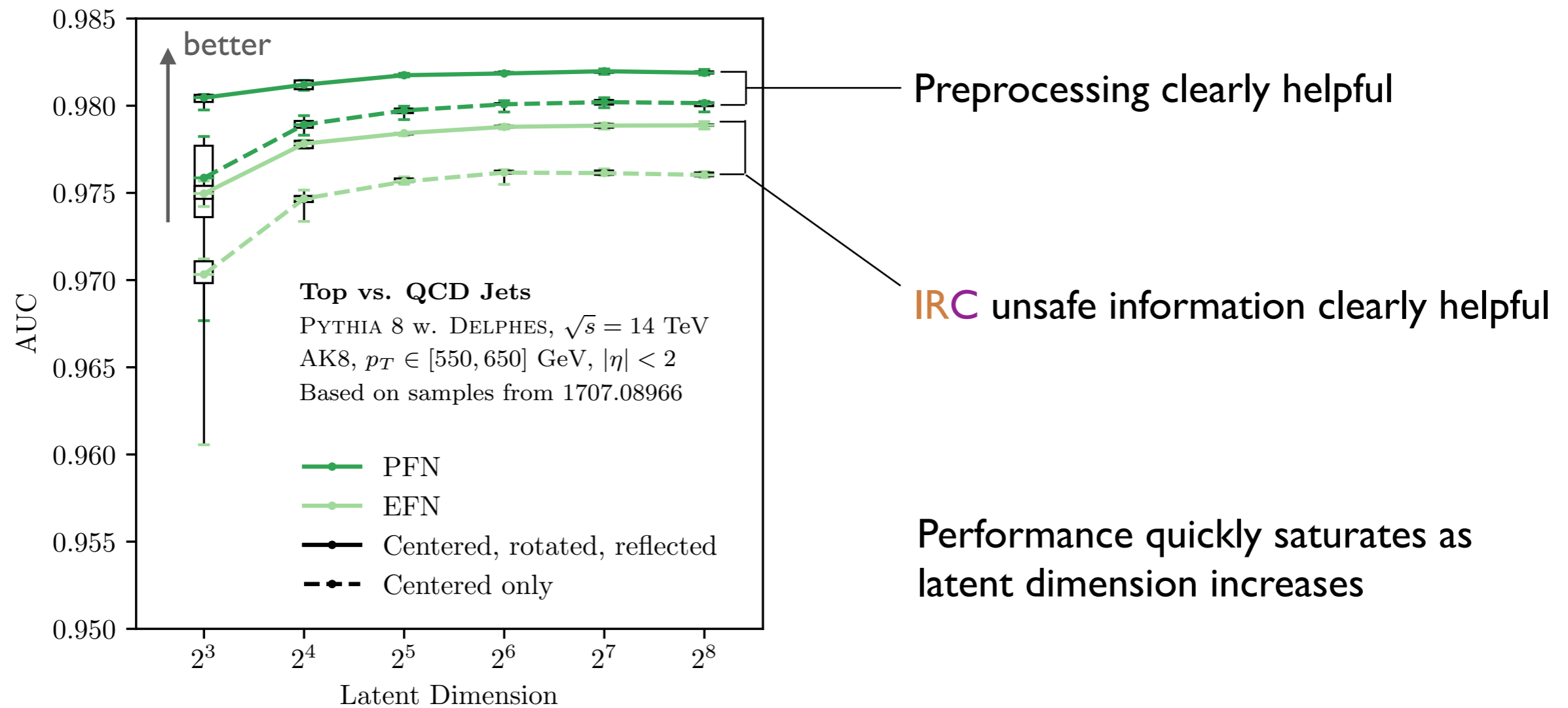


Latent space dimension  $\ell = 256$

EFN/PFN rotation and reflection preprocessing helpful

EFPs are comparable to EFN and even better at high signal efficiency

# EFN Latent Dimension Sweep

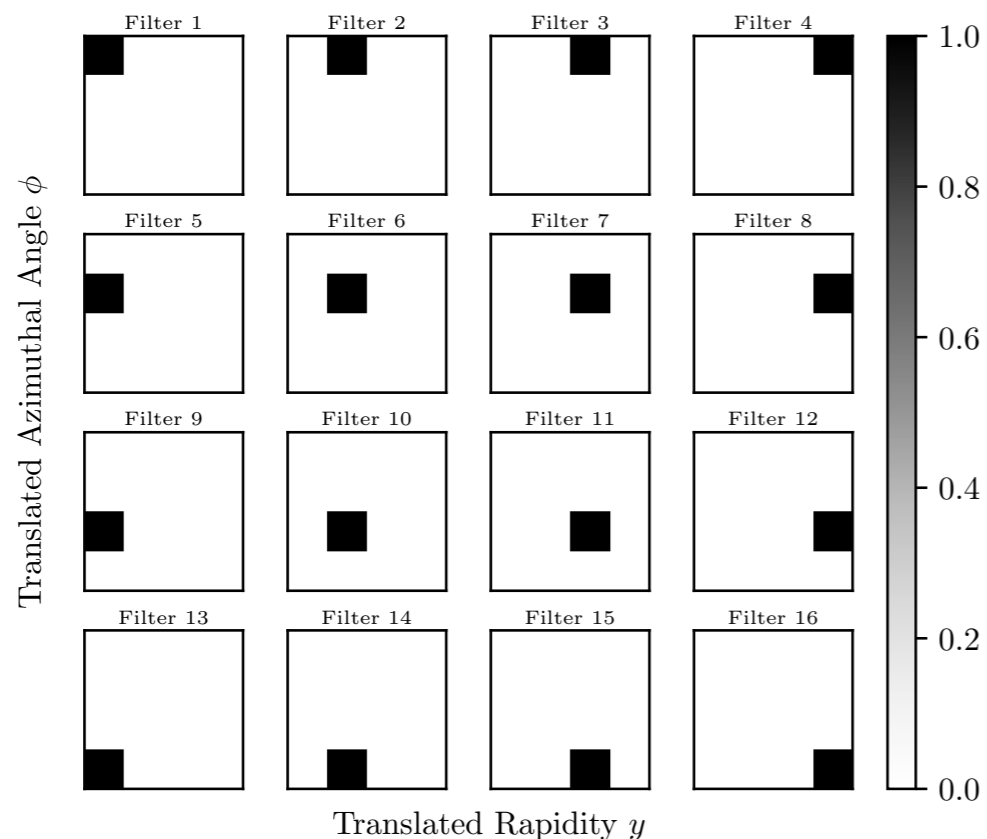


# Energy Flow Network Visualization

EFN observables are two-dimensional geometric functions

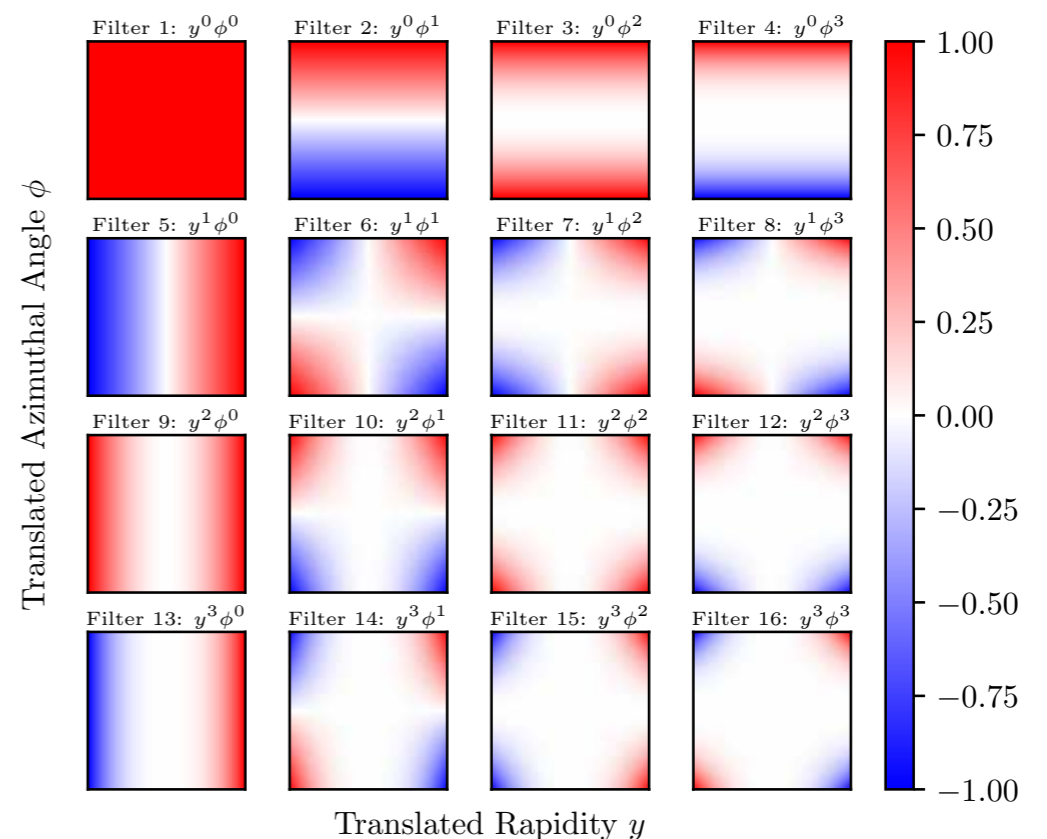
Visualize EFN observables as *filters* in the translated rapidity-azimuth plane

Jet images as EFN filters



[Cogan, Kagan, Strauss, Schwartzman, 2014]  
[de Oliveira, Kagan, Mackey, Nachman, Schwartzman, 2015]

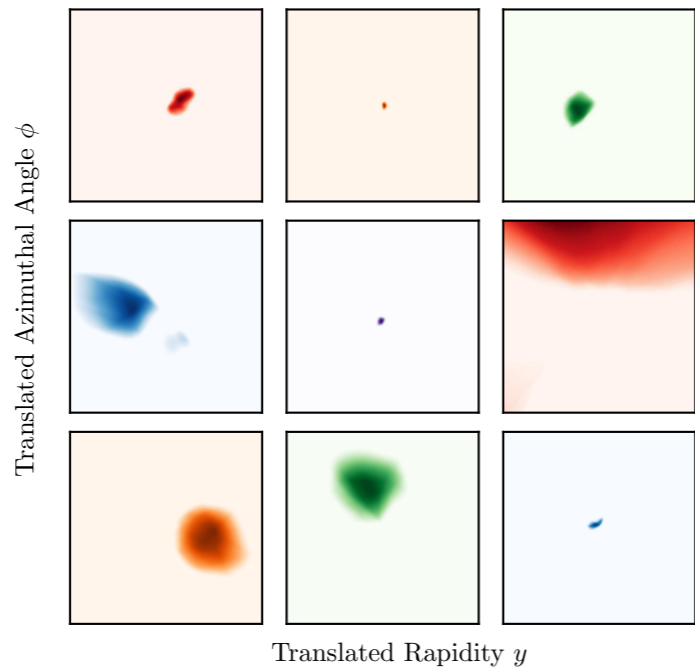
Moments as EFN filters



[Donoghue, Low, Pi, 1979]  
[Gur-Ari, Papucci, Perez, 2011]

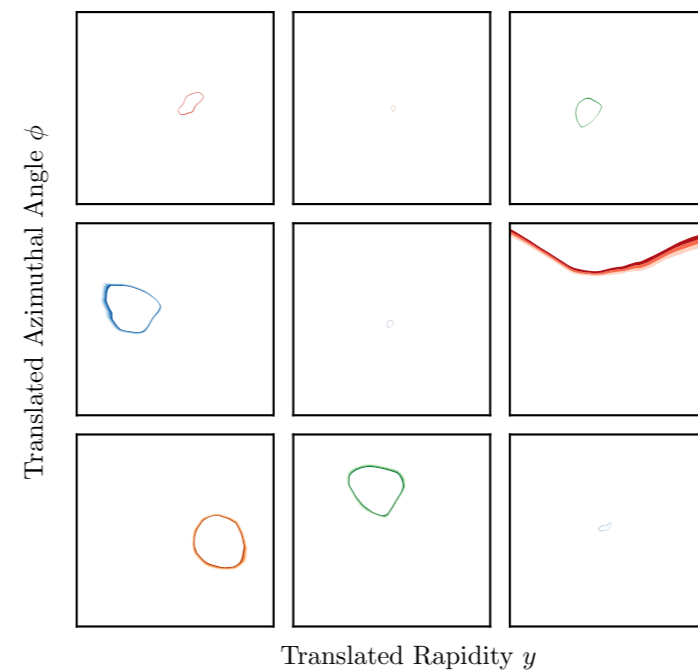
# Energy Flow Network Visualization

EFN of  
Visualiz

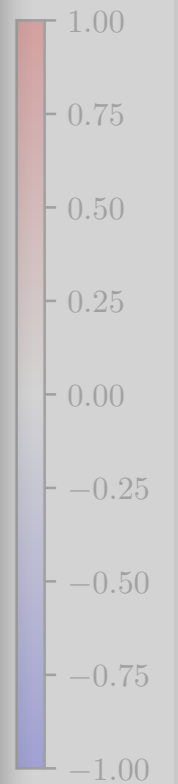
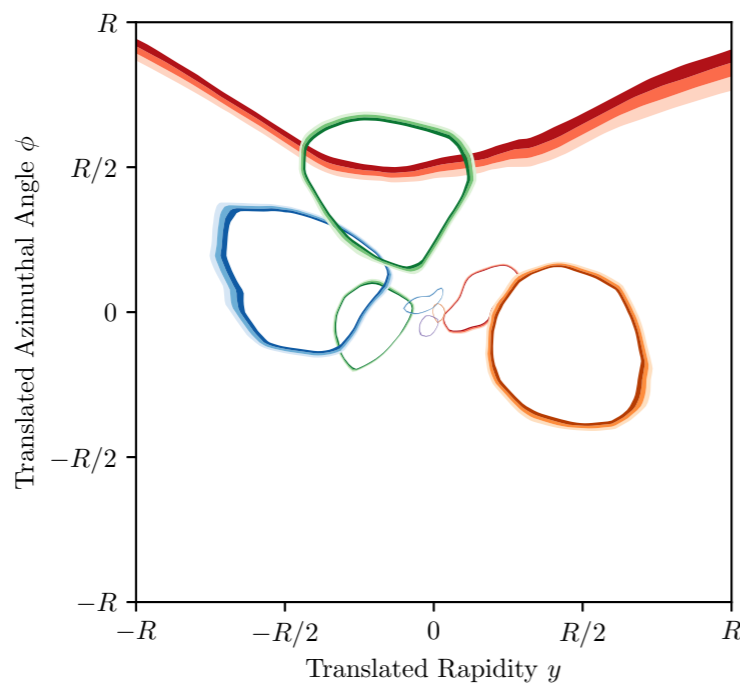


*Simultaneous visualization strategy*

Contour



Overlay

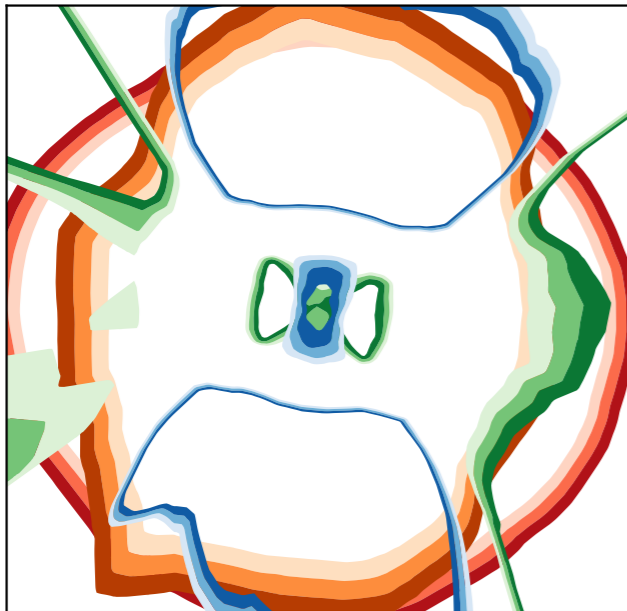


[Cogan, Ka  
[de Oliveira, Kagan, Flückiger, Flückiger, Schmittmann, 2015]

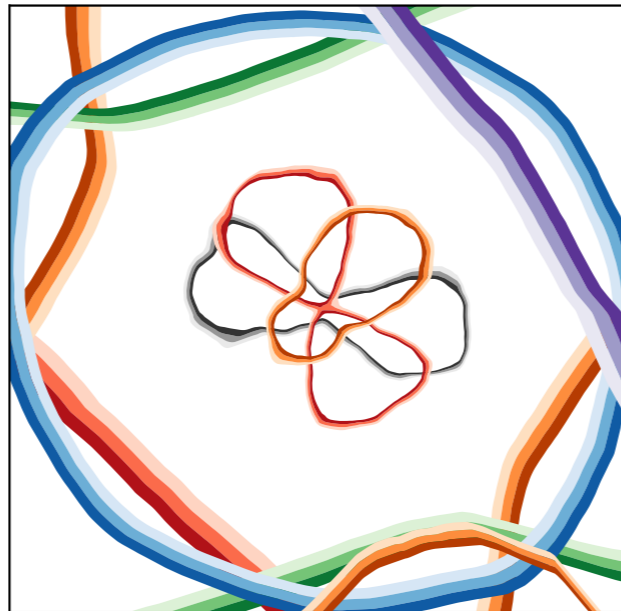
[Cogan, Pi, 1979]  
[Cogan, Raposo, Perez, 2011]

# Visualizing EFN Filters

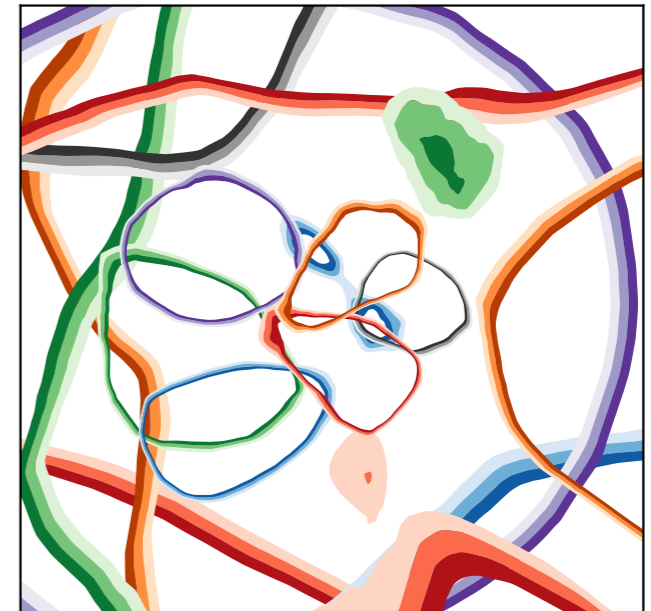
Without rotation/reflection preprocessing



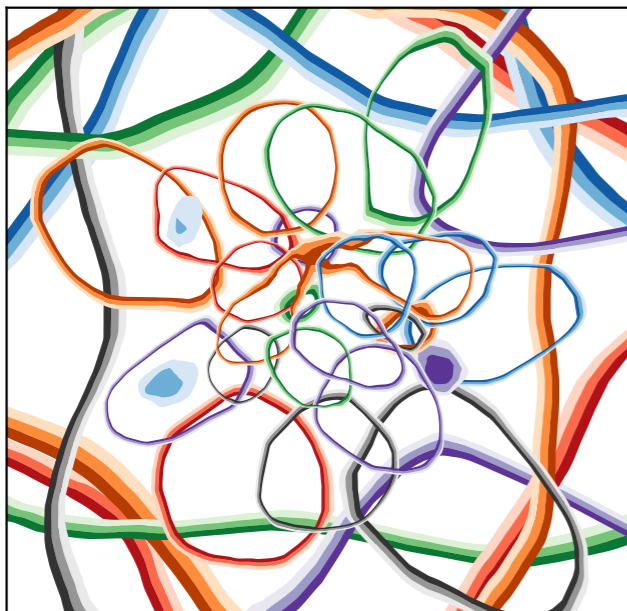
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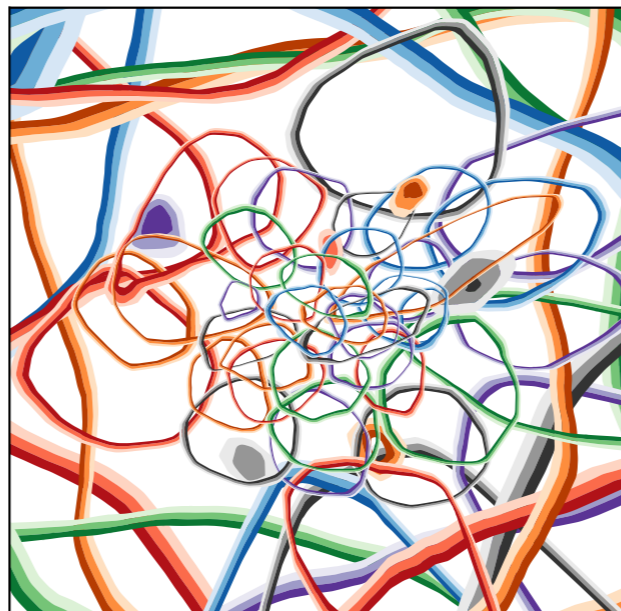
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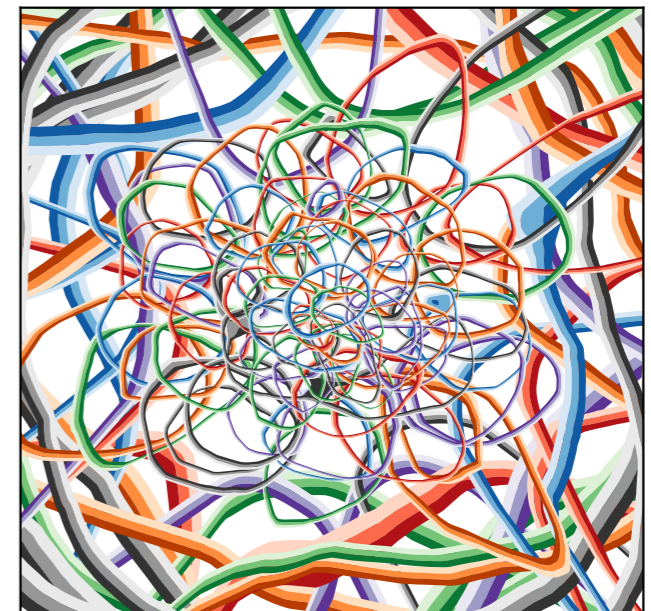
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$\ell = 32$



$\ell = 64$

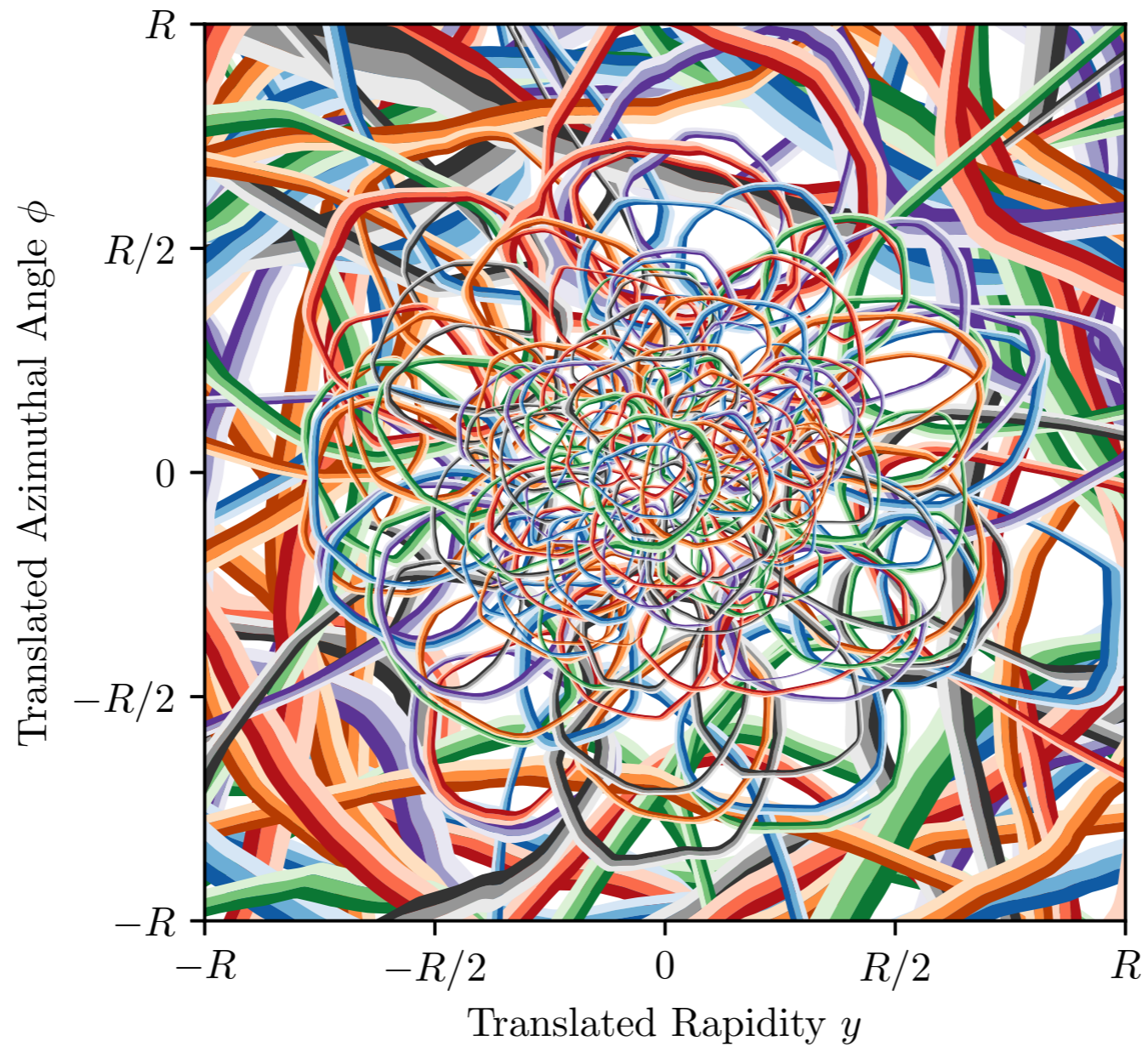


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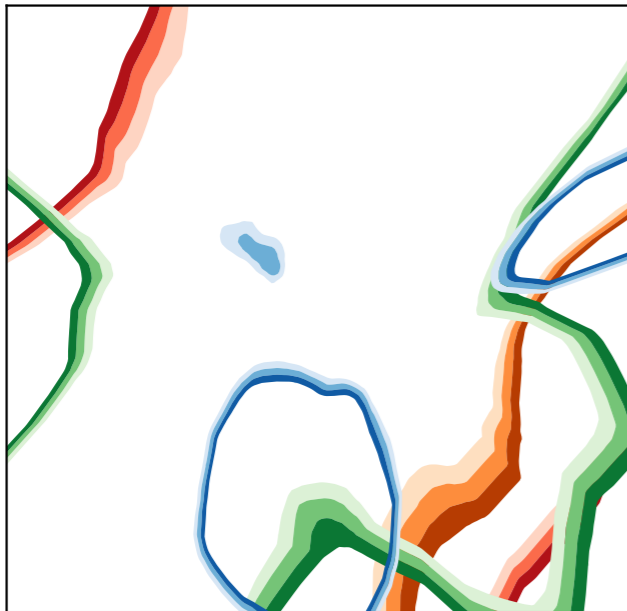
# Visualizing EFN Filters

Without rotation/reflection preprocessing

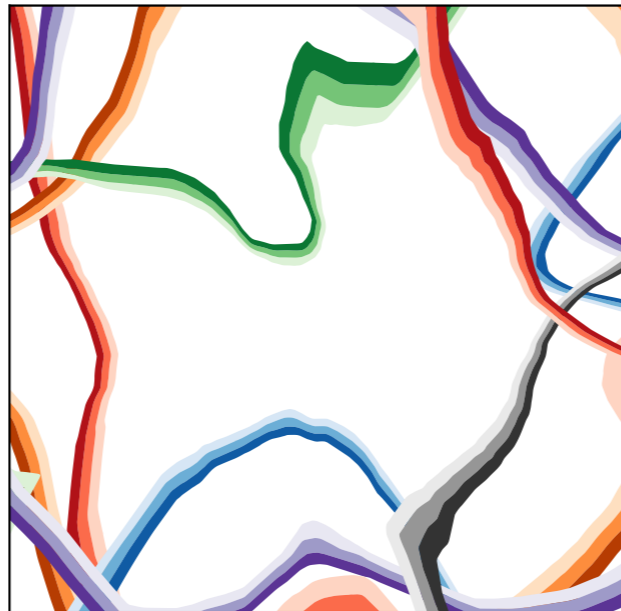


# Visualizing EFN Filters

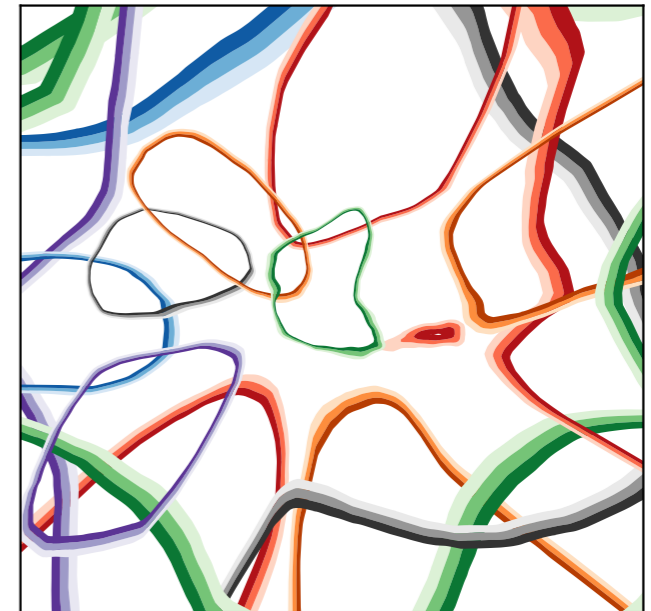
With rotation/reflection preprocessing



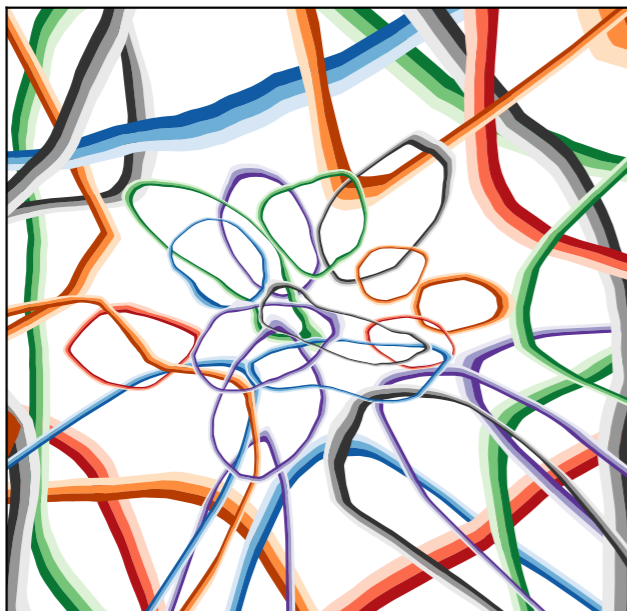
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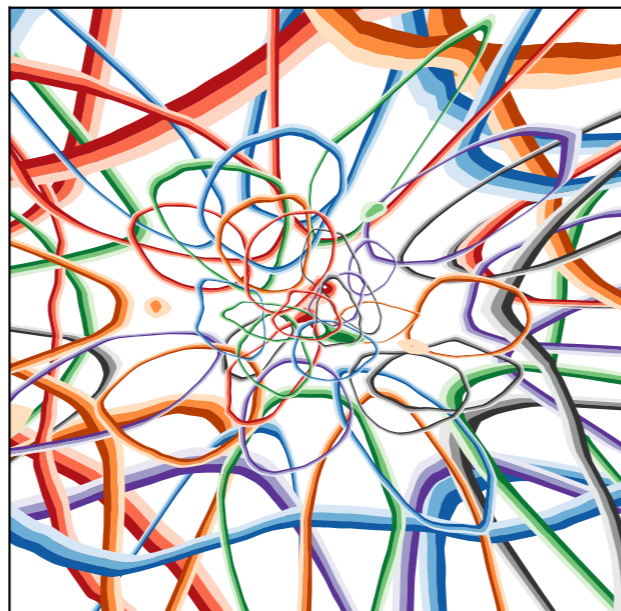
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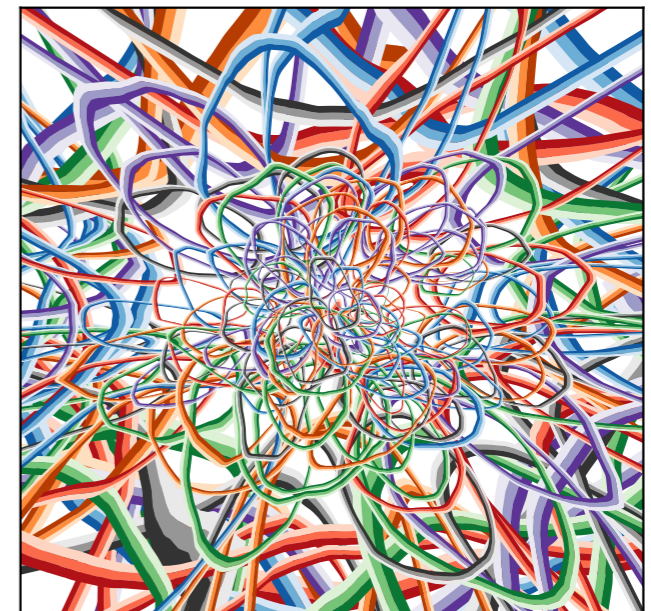
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$\ell = 32$



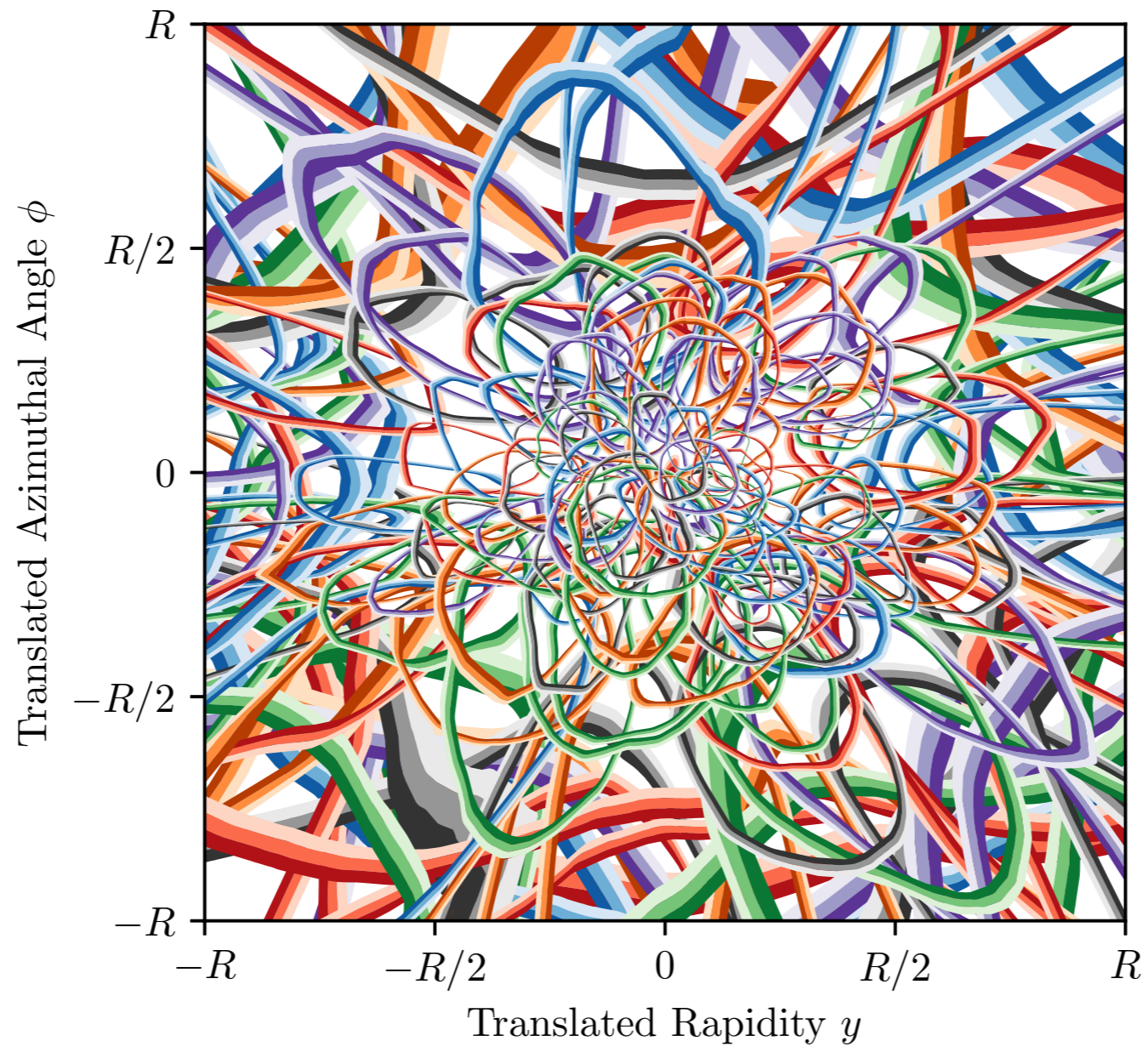
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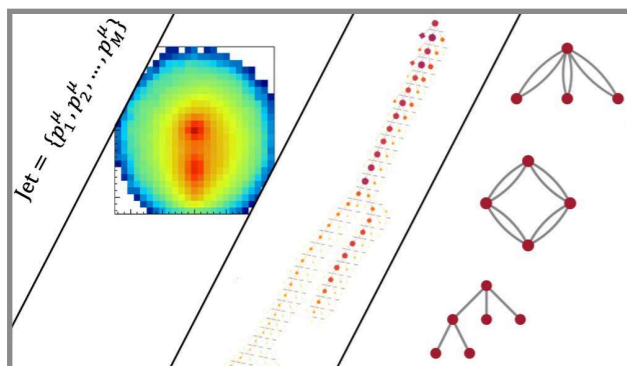


$\ell = 128$

# Visualizing EFN Filters

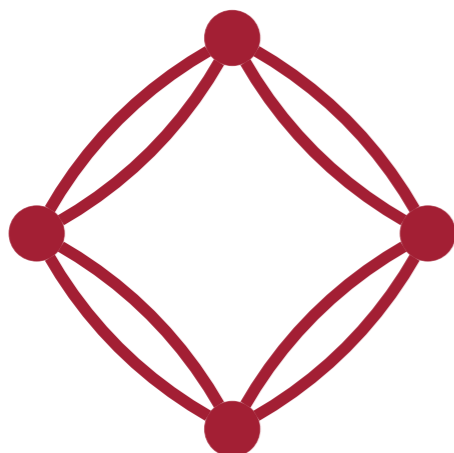
Without rotation/reflection preprocessing





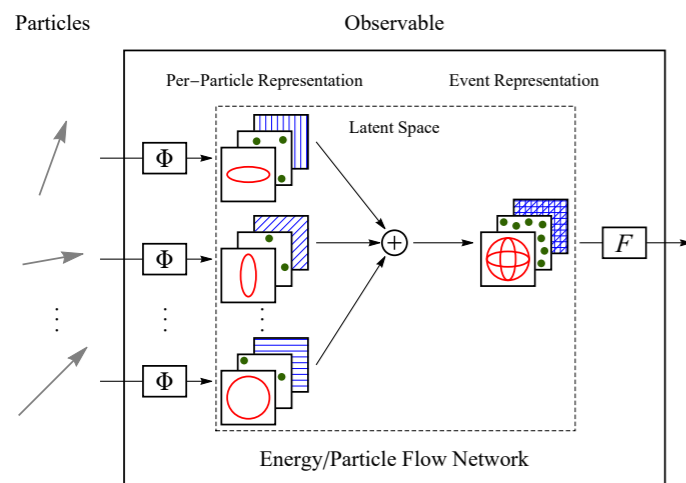
## Jets as Point Clouds

*Jets have the same symmetries as point clouds  
Respecting symmetries key for maximal performance*



## Energy Flow Polynomials

*Linear basis of IRC-safe observables  
Incredibly simple architecture competes with modern ML*



## Energy Flow Networks

*Excellent performance, fascinating visualizations via IRC safety  
(EFNs for Q/G talk on Thursday @ ML4Jets!)*

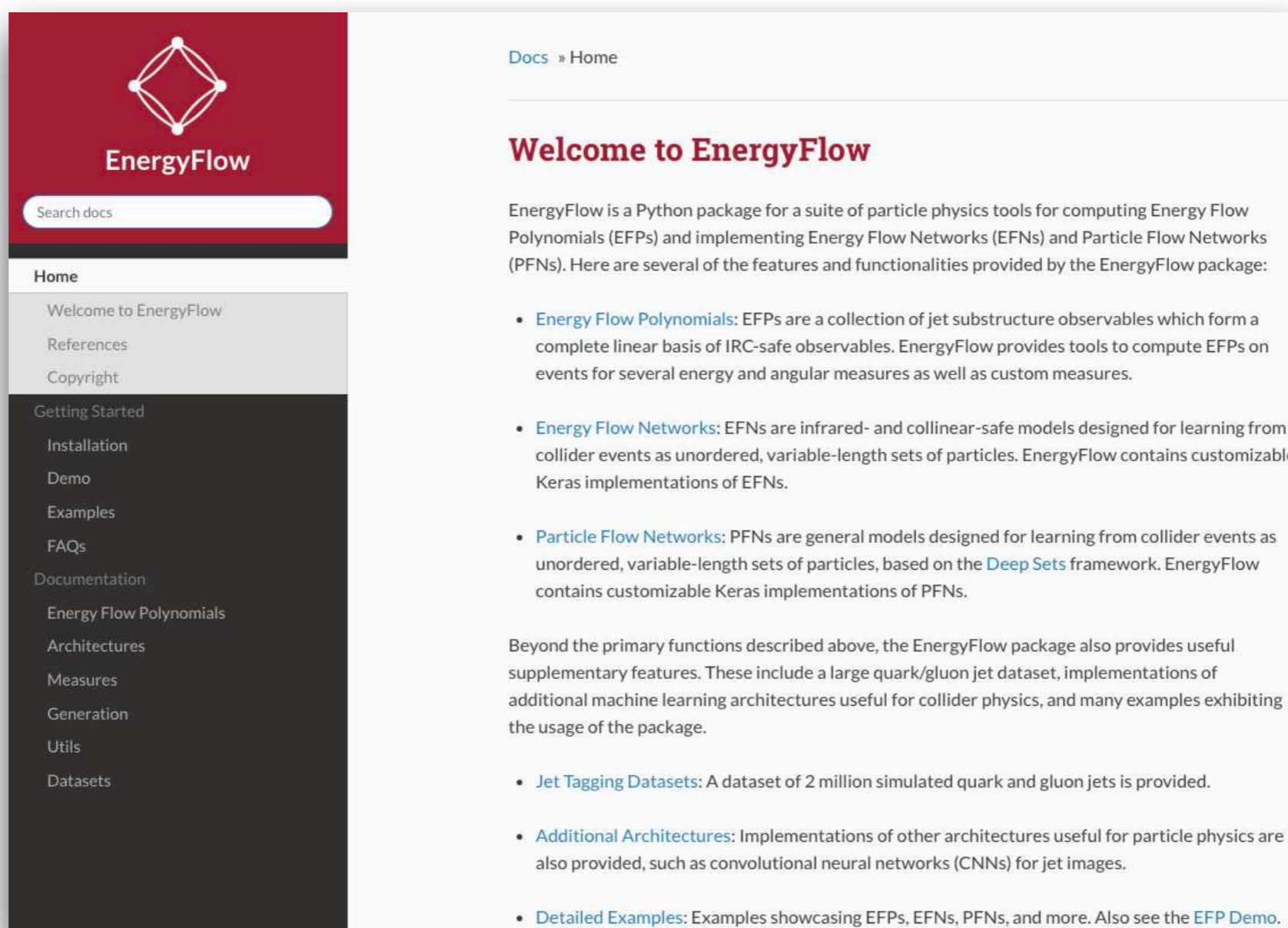
# EnergyFlow Python Package

Implements variable elimination for efficient EFP computation

Contains EFN and PFN implementations in Keras

CNN, DNN architectures included for easy model comparison

Several detailed examples demonstrating how to train models and make visualizations



Docs » Home

## Welcome to EnergyFlow

EnergyFlow is a Python package for a suite of particle physics tools for computing Energy Flow Polynomials (EFPs) and implementing Energy Flow Networks (EFNs) and Particle Flow Networks (PFNs). Here are several of the features and functionalities provided by the EnergyFlow package:

- **Energy Flow Polynomials:** EFPs are a collection of jet substructure observables which form a complete linear basis of IRC-safe observables. EnergyFlow provides tools to compute EFPs on events for several energy and angular measures as well as custom measures.
- **Energy Flow Networks:** EFNs are infrared- and collinear-safe models designed for learning from collider events as unordered, variable-length sets of particles. EnergyFlow contains customizable Keras implementations of EFNs.
- **Particle Flow Networks:** PFNs are general models designed for learning from collider events as unordered, variable-length sets of particles, based on the [Deep Sets](#) framework. EnergyFlow contains customizable Keras implementations of PFNs.

Beyond the primary functions described above, the EnergyFlow package also provides useful supplementary features. These include a large quark/gluon jet dataset, implementations of additional machine learning architectures useful for collider physics, and many examples exhibiting the usage of the package.

- **Jet Tagging Datasets:** A dataset of 2 million simulated quark and gluon jets is provided.
- **Additional Architectures:** Implementations of other architectures useful for particle physics are also provided, such as convolutional neural networks (CNNs) for jet images.
- **Detailed Examples:** Examples showcasing EFPs, EFNs, PFNs, and more. Also see the [EFP Demo](#).

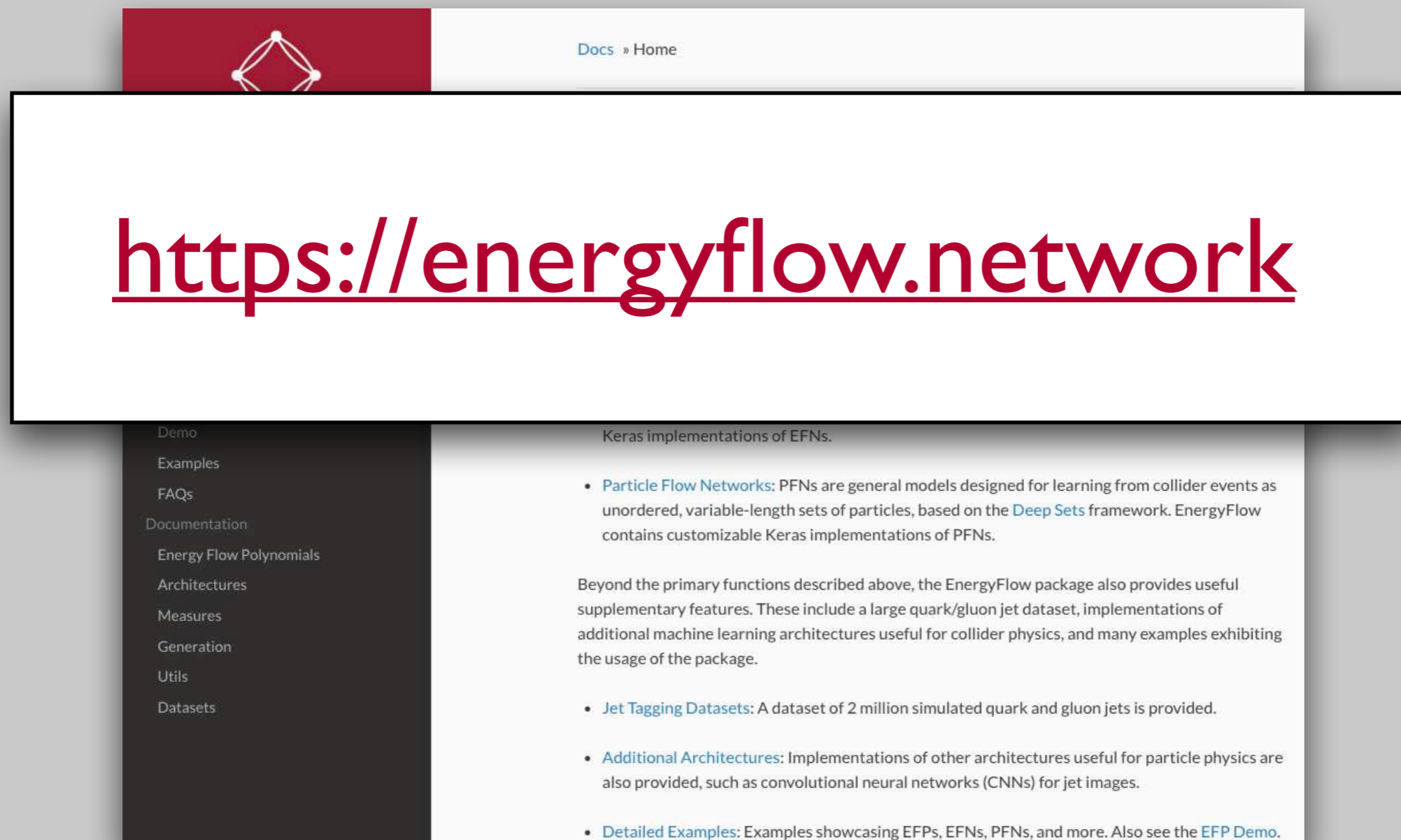
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CNN, DNN architectures included for easy model comparison

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The screenshot shows the EnergyFlow website documentation page. At the top left is a red header with a white network diagram logo. To its right is a navigation link "Docs » Home". The main content area features the URL <https://energyflow.network> in large red text. On the left side, there is a dark sidebar with a list of navigation items: Demo, Examples, FAQs, Documentation, Energy Flow Polynomials, Architectures, Measures, Generation, Utils, and Datasets. The main content area below the URL contains the text "Keras implementations of EFNs." followed by a bulleted list of features:

- **Particle Flow Networks:** PFNs are general models designed for learning from collider events as unordered, variable-length sets of particles, based on the [Deep Sets](#) framework. EnergyFlow contains customizable Keras implementations of PFNs.

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**Thank You!**

# Classification Performance

