W Reconstruction in semi/full-leptonic Vector Boson Scattering

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New techniques in particle reconstruction for VBS

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Formula Derivation for a process with one ν

$$m_{\mathrm{W}}^2 = (p_\mu + p_\nu)^2 \;\; \textit{ultra relativistic limit} \; \xrightarrow{m \to 0} \; 2 p_\mu p_\nu;$$

- Let's solve for the longitudinal component of the neutrino $p_{\nu L}$;

$$\underbrace{\left(p_{lL}^2 - E_l^2\right)}_{a} p_{\nu L}^2 +$$

$$\underbrace{\left(m_{\mathrm{W}}^2 p_{lL} + 2 p_{lL} \vec{p}_{lT} \vec{p}_{\nu T}\right)}_{b} p_{\nu L} +$$

$$\underbrace{\frac{m_{\mathrm{W}}^4}{4} + (\vec{p}_{lT} \vec{p}_{\nu T})^2 + m_{\mathrm{W}}^2 \vec{p}_{lT} \vec{p}_{\nu T} - E_l^2 \vec{p}_{\nu T}^{\,2})}_{c} = 0;$$

$$\boxed{p_{\nu L_{1,2}} = \frac{-b \pm \sqrt{\Delta}}{2a}} \quad \text{where} \quad \Delta = b^2 - 4ac$$

As a second order parametric equation, Δ determines the number of solution and their nature.

$m_W \Rightarrow$ **fixed value** $(80.385$ GeV$)$

- if $\Delta > 0 \Rightarrow$ 2 solutions $(+/-)$
- if $\Delta < 0$, from the formula:

we have two working options, we choose the first one

$$\Delta(p_L) \begin{cases} \text{put } \Delta = 0 \\ m_W = m_{WT} \Rightarrow \quad \text{correct } m_W \text{ with transverse mass} \end{cases}$$

VBS Semileptonic W boson reconstruction

PHANTOM PARAMETERS for the production

- semi-leptonic: $pp \to jjjj\mu^+\nu_\mu$
- full-leptonic: $pp \to jj\mu^+\nu_\mu e^+\nu_e$
- Parton level events
- MC generator: `Phantom`
- events generated with `NNPDF30_nnlo_as_0118`
- CALCULATION TYPE: α_e^6
- SCALE CHOICE: (invariant mass of the 2 central jets and of 2 leptons)/$\sqrt{2}$

Kinematical cuts:

- $p_\mathrm{T}^\ell > 20$ GeV
- $|\eta^\ell| < 3$
- $p_\mathrm{T}^{min} > 30$ GeV
- $|\eta_j| < 5.4$
- $p_\mathrm{T}^\mathrm{miss} > 20$ GeV
- $m_{jj} > 500$ GeV
- $\Delta R_{j\ell} > 0.3$

VBS Semileptonic channel

Selection criteria

- Selection 1: $p_{\nu L} > 50$ GeV
- Selection 2: $p_{\nu L} a/b > -0.5 \; \rightarrow$ choosing larger* solution
- Selection 3: $\vec{p}_\nu \cdot \vec{p}_W < 5000$ GeV^2
- Selection 4: $\vec{p}_\nu \cdot \vec{p}_W a/b < 25$ GeV
- Combined:
  - Selection of solutions with $p_{\nu L} < 50$ GeV
  - If both solutions lie above 50 GeV, Selection 3 is applied
  - solution with lower value of the $|\vec{p}_\nu \cdot \vec{p}_W a/b|$ is taken.

$^* p_{\nu L} a/b = -\frac{1}{2} \pm \frac{\sqrt{\Delta}}{2b}$

Selection efficiencies

- **Efficiencies**: count number of "correct solutions"
  - correct solution: the one which lies closest to the truth
  - wrong solution: the one which lies further from to the truth
- Solutions with negative discriminant are not taken into account

Fixed m_W (on-shell)

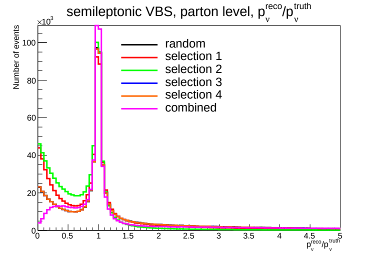
- Random: 49.9 $\%$
- Selection 1: 56.3 $\%$
- Selection 2: 61.3 $\%$
- Selection 3: 46.0 $\%$
- Selection 4: 52.6 $\%$
- Combined selection 44.6 $\%$

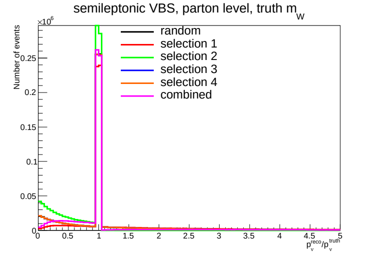
Truth m_W (off-shell)

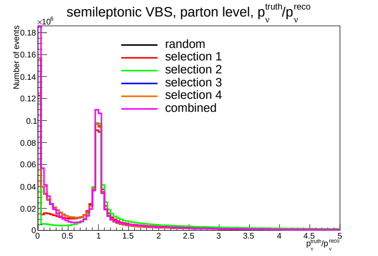
- Random: 50.0 $\%$
- Selection 1: 53.7 $\%$
- Selection 2: 58.4 $\%$
- Selection 3: 48.1 $\%$
- Selection 4: 51.4 $\%$
- Combined selection 46.4 $\%$

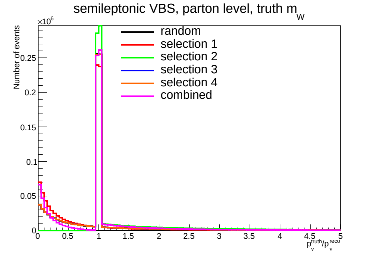
It turns out that efficiencies is not always a good measure for selection performance

Selection criteria







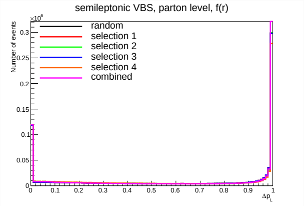


- Fixed m_W
- Truth m_W

Ljubljana variable

In order to visualize the whole range of relative error we introduced a variable

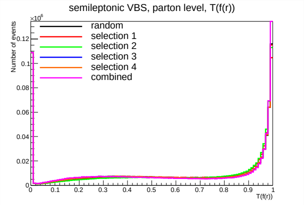
$$f(r) = \frac{2}{\frac{1}{r} + r}; \text{ where } r = \frac{p_L^{reco}}{p_L^{truth}}$$



Properties:

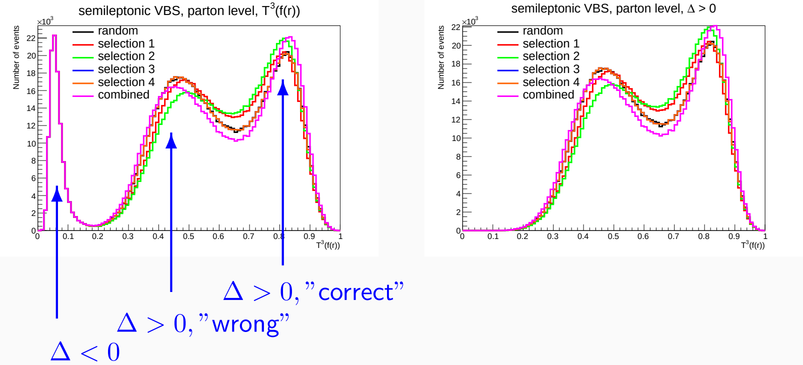
- wrong solutions: $f(r) \to 0$
- correct solutions: $f(r) \to 1$

In order to enhance separation between selection criterion we need a phase space transformation T:



- $T(x) = \text{arcsin}(2x - 1)/\pi + \frac{1}{2};$ where $x = f(r)$

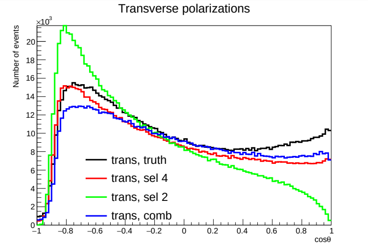
Ljubljana variable

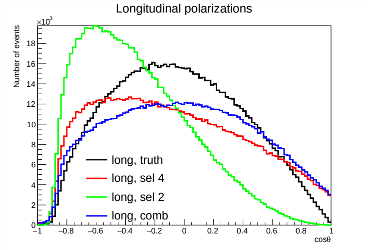


- first peak are the events with negative discriminant
- second peak are wrong reconstructed events with positive discriminant
- third peak are correctly reconstructed events

semileptonic angular distributions

cosθ distribution of the charged lepton in the W reference frame.





Combined selection criterion gives angular distribution closer to truth.

VBS Fully leptonic channel

Reco of neutrino momentum in $pp \to jje^+\nu_e\mu^+\nu_\mu$

- 8 unknown parameters (2 x neutrino four momentum)
- 6 equations:
  - $\vec{p}_{\mathrm{T}}^{\nu_\mu} + \vec{p}_{\mathrm{T}}^{\nu_e} = \vec{p}_{\mathrm{T}}^{\mathrm{miss}}$ (2x)
  - $(p^\ell + p^\nu)^2 = m_W^2$ (2x)
  - $p_\nu^2 = 0$ (2x)
- Remaining 2 equations:
  1. Setting some parameters to fixed values - for example: $M_{WW}^2 = (p_e + p_{\nu_e} + p_\mu + p_{\nu_\mu})^2$ and $M_{\nu\nu}^2 = (p_{\nu_e} + p_{\nu_\mu})^2$, M_{WW} and $M_{\nu\nu}$ are fixed numbers.[1]
  2. Using of $MT2$-Assisted On-Shell (MAOS) quantites, i.e. minimization of the transverse masses of the lepton-neutrino pairs.[1]
  3. Other ideas ??

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[1] arXiv:hep-ph/0603011, Higgs spin analysis in Collins-Soper frame using opening angles of different-flavour final state leptons

[1] arXiv:0908.0079

2. Reco with MAOS quantities - Equations

- MAOS estimations $\vec{p}_{\mathsf{T}}^{\nu_e\prime}$ and $\vec{p}_{\mathsf{T}}^{\nu_\mu\prime}$ for neutrinos transverse momentums can be obtained by minimizing the function $f(\vec{p}_1, \vec{p}_2) = \max\{M_{\mathsf{T}}^{W_1}, M_{\mathsf{T}}^{W_2}\}$, constrained by a bond $\vec{p}_1 + \vec{p}_2 = \vec{p}_{\mathsf{T}}^{\mathsf{miss}}$, where

$$M_{\mathsf{T}}^{W_1} = 2(|\vec{p}_{\mathsf{T}}^{\mu}||\vec{p}_1| - \vec{p}_{\mathsf{T}}^{\mu} \cdot \vec{p}_1), \qquad M_{\mathsf{T}}^{W_2} = 2(|\vec{p}_{\mathsf{T}}^{e}||\vec{p}_2| - \vec{p}_{\mathsf{T}}^{e} \cdot \vec{p}_2)$$

- Minimum of the function f defines quantity M_{T2}:

$$M_{\mathsf{T2}} \equiv \min_{\vec{p}_1 + \vec{p}_2 = \vec{p}_{\mathsf{T}}^{\mathsf{miss}}} f(\vec{p}_1, \vec{p}_2) = f|_{\vec{p}_{\mathsf{T}}^{\nu_e\prime}, \vec{p}_{\mathsf{T}}^{\nu_\mu\prime}} \tag{1}$$

- p_L are then determined from the m_W constraints
- Solution of the problem (1), under assumption $p_{\mathsf{T}}^{WW} \sim 0$ (approximate solution):

$$\vec{p}_{\mathsf{T}}^{\nu_e\prime} = -\vec{p}_{\mathsf{T}}^{\mu} \qquad \vec{p}_{\mathsf{T}}^{\nu_\mu\prime} = -\vec{p}_{\mathsf{T}}^{e}$$

2. Reco with MAOS quantities - Equations

Exact solution:

- $\min\left[\max\{M_{\mathsf{T}}^{W_1}, M_{\mathsf{T}}^{W_2}\}\right]$ can always lie only on the intersection of $M_{\mathsf{T}}^{W_1}$ and $M_{\mathsf{T}}^{W_2}$. $\Rightarrow$ Additional bond: $M_{\mathsf{T}}^{W_1} = M_{\mathsf{T}}^{W_2}$
  - It follows that:

$$\Rightarrow \quad 2(|\vec{p}_{\mathsf{T}}^{\mu}||\vec{p}_1| - \vec{p}_{\mathsf{T}}^{\mu} \cdot \vec{p}_1) = 2(|\vec{p}_{\mathsf{T}}^{e}||\vec{p}_2| - \vec{p}_{\mathsf{T}}^{e} \cdot (\vec{p}_{\mathsf{T}}^{\text{miss}} - \vec{p}_1))$$

$$|\vec{p}_{\mathsf{T}}^{\mu}||\vec{p}_1| - \vec{p}_{\mathsf{T}}^{\ell\ell} \cdot \vec{p}_1 + \vec{p}_{\mathsf{T}}^{e} \cdot \vec{p}_{\mathsf{T}}^{\text{miss}} = |\vec{p}_{\mathsf{T}}^{e}|\sqrt{|\vec{p}_{\mathsf{T}}^{\text{miss}}|^2 - 2\vec{p}_{\mathsf{T}}^{\text{miss}} \cdot \vec{p}_1 + |\vec{p}_1|^2}$$

$$(|\vec{p}_{\mathsf{T}}^{\mu}||\vec{p}_1| - |\vec{p}_{\mathsf{T}}^{\ell\ell}||\vec{p}_1|\cos\varphi + \vec{p}_{\mathsf{T}}^{e} \cdot \vec{p}_{\mathsf{T}}^{\text{miss}})^2 =$$
$$|\vec{p}_{\mathsf{T}}^{e}|^2|\vec{p}_{\mathsf{T}}^{\text{miss}}|^2 - 2|\vec{p}_{\mathsf{T}}^{e}|^2|\vec{p}_{\mathsf{T}}^{\text{miss}}||\vec{p}_1|\cos(\varphi + \varphi_0) + |\vec{p}_{\mathsf{T}}^{e}|^2|\vec{p}_1|^2$$

  - φ_0 - angle between $\vec{p}_{\mathsf{T}}^{\text{miss}}$ and $\vec{p}_{\mathsf{T}}^{\ell\ell}$
    $\rightarrow$ Parameter of the equation: $\varphi_0 = \arccos\left(\dfrac{\vec{p}_{\mathsf{T}}^{\ell\ell} \cdot \vec{p}_{\mathsf{T}}^{\text{miss}}}{|\vec{p}_{\mathsf{T}}^{\ell\ell}||\vec{p}_{\mathsf{T}}^{\text{miss}}|}\right)$
  - φ - angle between $\vec{p}_{\mathsf{T}}^{\ell\ell}$ and $\vec{p}_1$;

2. Reco with MAOS quantities - Equations

$$\underbrace{\left(|\vec{p}_{\mathsf{T}}^{\mu}|^2 + |\vec{p}_{\mathsf{T}}^{\ell\ell}|^2\cos^2\varphi - 2|\vec{p}_{\mathsf{T}}^{\mu}||\vec{p}_{\mathsf{T}}^{\ell\ell}|\cos\varphi - |\vec{p}_{\mathsf{T}}^{e}|^2\right)}_{f(\varphi)}|\vec{p}_1|^2 \quad +$$

$$\underbrace{\left(2(|\vec{p}_{\mathsf{T}}^{\mu}| - |\vec{p}_{\mathsf{T}}^{\ell\ell}|\cos\varphi)\vec{p}_{\mathsf{T}}^{e}\cdot\vec{p}_{\mathsf{T}}^{\mathsf{miss}} + 2|\vec{p}_{\mathsf{T}}^{e}|^2|\vec{p}_{\mathsf{T}}^{\mathsf{miss}}|\cos(\varphi+\varphi_0)\right)}_{g(\varphi)}|\vec{p}_1| \quad +$$

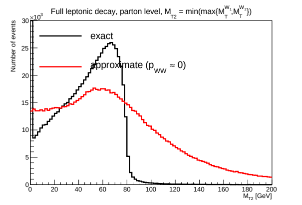
$$\underbrace{(\vec{p}_{\mathsf{T}}^{e}\cdot\vec{p}_{\mathsf{T}}^{\mathsf{miss}})^2 - |\vec{p}_{\mathsf{T}}^{\mathsf{miss}}|^2|\vec{p}_{\mathsf{T}}^{e}|^2}_{c} \quad = \quad 0$$

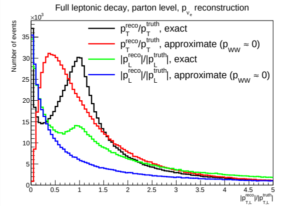
- Equation of the intersection curve in parametric form - x-axis of coordinate system coinciding with the $\vec{p}_{\mathsf{T}}^{\ell\ell}$ direction;

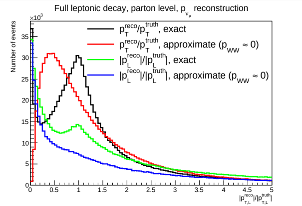
$$|\vec{p}_1| = \frac{-g(\varphi) \pm \sqrt{g(\varphi)^2 - 4cf(\varphi)}}{2f(\varphi)}, \qquad \vec{p}_2 = \vec{p}_{\mathsf{T}}^{\mathsf{miss}} - \vec{p}_1$$

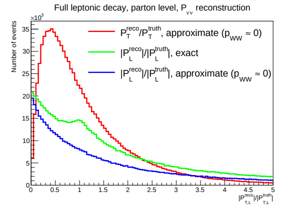
- Minimum of M_{T2} on the intersection curve can be found numerically;
- The following plots are produced by evaluating M_{T2} in 2000 points;

Results - random solution choice





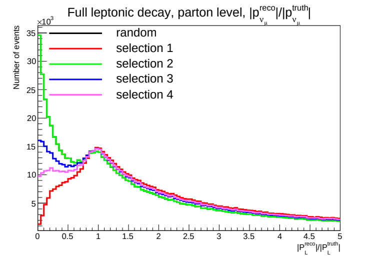


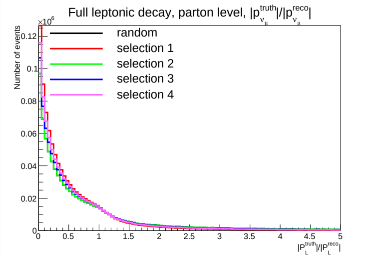


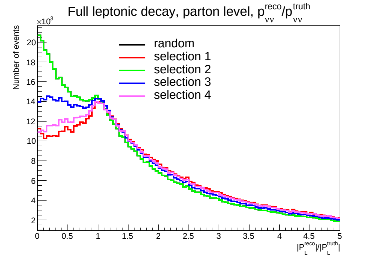
- Exact solution performs better as we have peak at 1.
- M_{T2} has a sharp edge at m_W

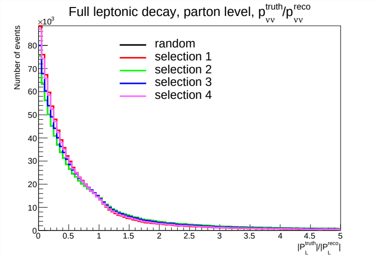
Performance of different selections

Some selections improve p^{reco}/p^{truth} ratio but degrade p^{truth}/p^{reco}





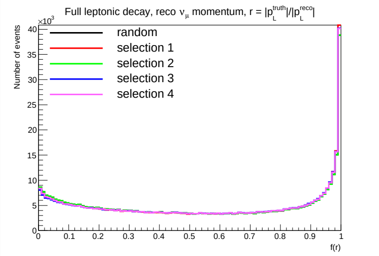


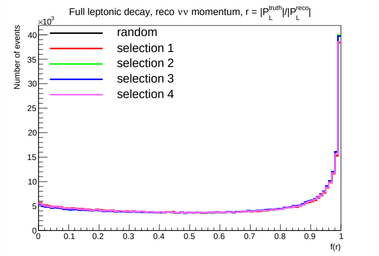


Ljubljana variable

Efficiencies:

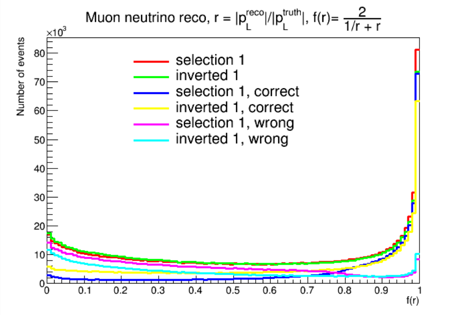
- Random: 50.0 %
- Selection 1: 42.2 %
- Selection 2: 50.0 %
- Selection 3: 48.2 %
- Selection 4: 45.1 %





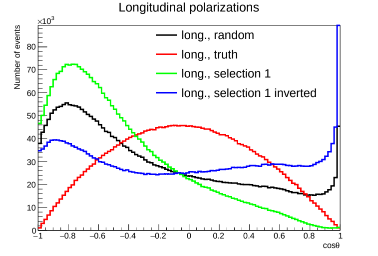
Selection performance

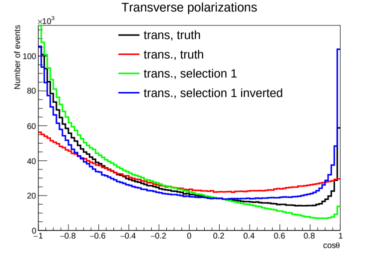
- inverted selection 1 has better efficiency
- inverted selection 1 has worst relative error
- inverted selection 1 has selects better in case of events with bad reconstruction



Angular distributions with selection criteria

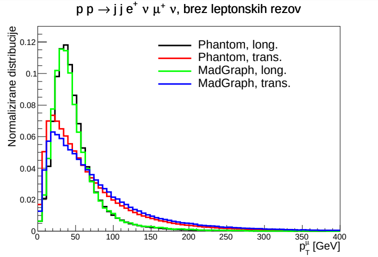
- We are not able to reconstruct truth angular distribution accurate enough.
- inverted selection 1 increase discriminating power for separation of polarization





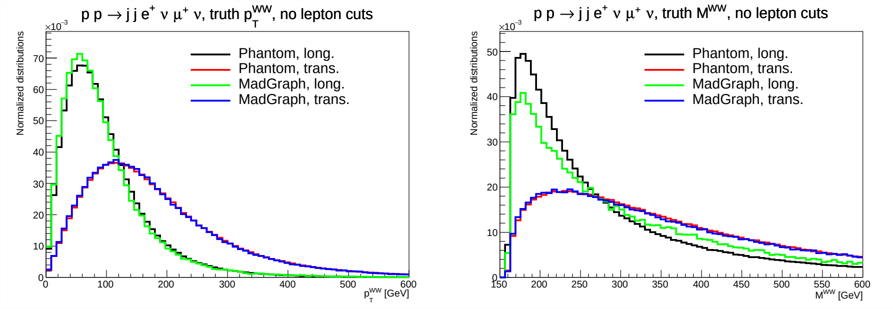
Phantom to MadGraph comparison

Phantom to MadGraph comparison



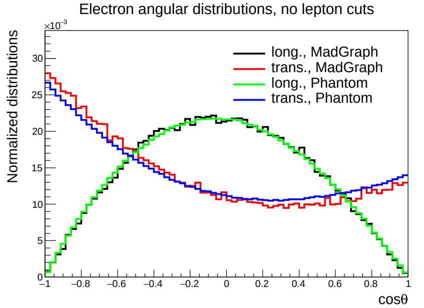
- in MG5 the generation is carried out in 2 steps
  1. polarized $pp \to W^+W^+jj$
  2. decay of the W with MadSpin
- we modified matrix.f file in MadGraph distribution so we produce W_LW_L and W_TW_T pairs
- we used MadSpin in the "bridge" mode using helicity-classified decays
- we needed to fix some python technicalities

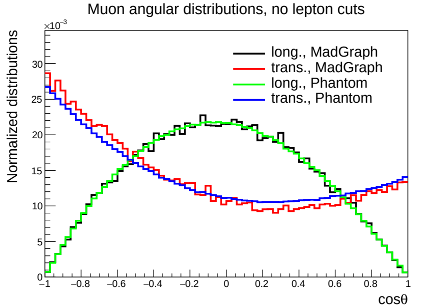
Phantom to MadGraph comparison



Phantom to MadGraph comparison

- Good agreement can be observed between the two MC generator





Conclusions

CONCLUSIONS: WW reconstruction

- we proposed different approach for evaluate selection criterion
- it is not clear that the use of selection criteria improves performance of reconstruction algorithm
- the use of alternative reconstruction algorithm can be tested in fully-leptonic channel
- we carried out the first comparison of polarized VBS samples between MadGraph and Phantom

Future plans:

- selection of the solutions with smaller invariant mass
- combine different selection criteria as an input to a neural network to take into account the correlations and optimize W reconstruction choosing the correct p_L

References

[1] arXiv:hep-ph/0603011

[2] arXiv:0801.3359 [hep-ph]

[3] arXiv:1710.09339

[4] arXiv:1205.2484

[5] arXiv:hep-ph/9406381

[6] arXiv:hep-ph/9406381

[7] arXiv:0908.0079

[8] B. Hoonhout, K. Oussoren, S. Bentvelse: *Higgs spin analysis in Collins-Soper frame using opening angles of different-flavour final state leptons* (link)

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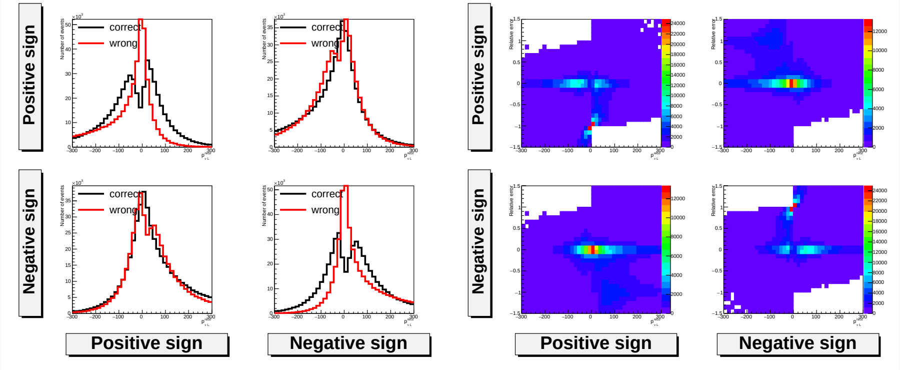
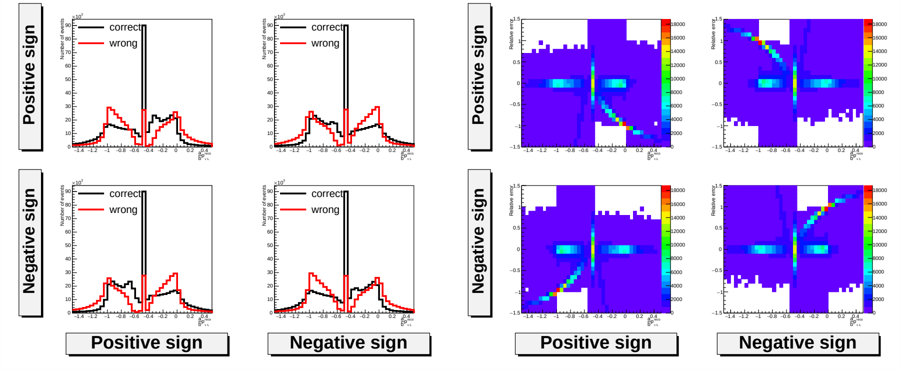
Criteria Selection 1 at Parton Level



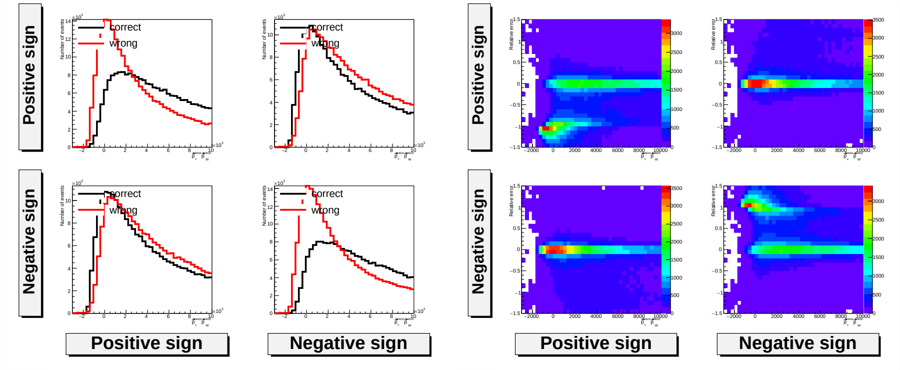
Selection 1 - algorithm discards the solutions which have absolute value smaller than 50 GeV, if both solutions lie under or above 50 GeV, random solution is chosen.

Criteria Selection 2 at Parton Level



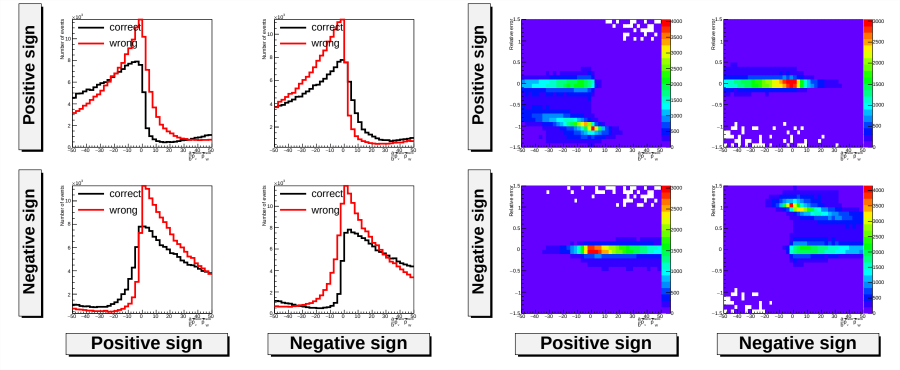
selection 2 - algorithm discards all the solutions for which $-p_L * a/b < 0.5$. If both solutions pass or don't pass this criterium, random solution is chosen.

Criteria Selection 3 at Parton Level



Selection 3 - if the scalar product of the reconstructed neutrino three-momentum (solution is taken for the longitudinal component) with the reconstructed W three-momentum is smaller than $2500 GeV^2$, the solution is discarded. If both solutions lie under or above $2500 GeV^2$, random solution is chosen.

Criteria Selection 4 at Parton Level



selection 4- if the value of the scalar product of the reconstructed neutrino three-momentum (solution is taken for the longitudinal component) with the reconstructed Ws three-momentum (each threaded separately), multiplied by a/b, is smaller than 30 GeV or larger than 25 GeV, the solution is discarded. a and b are the parameters of the