Primordial Black Holes and Gravitational Waves in a stiff pre-BBN epoch

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1912.xxxx with

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Introduction

- **What are PBHs?**
  Formed in the early universe when the density fluctuations of high amplitude ($\delta > \delta_c$) re-enter the Hubble horizon at post-inflationary epochs and collapse gravitationally.

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- **Why PBHs?**
  • Nonrelativistic and collisionless: Can be a significant component of DM.
  • GW experiments (LIGO, VIRGO, LISA etc.) will look at more binary black hole events: $M > M_\odot$ stellar black holes are rare $\Longrightarrow$ Massive PBH?
  • A tool to probe the epoch from the time the smaller scales ($k > k_{\rm CMB}^{\rm max}$) exit inflationary horizon - - - BBN.

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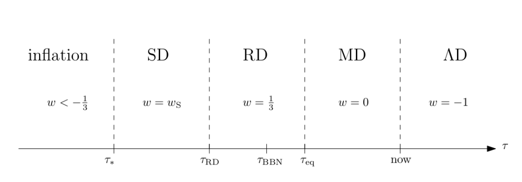
- **Aim**
  • The effect of a modified evolution during stiff-domination $1/3 < w < 1$ on PBH formation.

PBH in a stiff-domination



(*1905.11960*)

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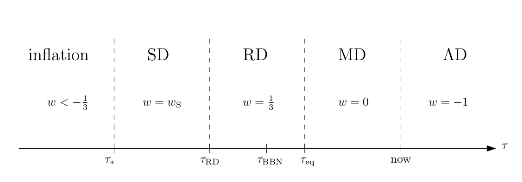

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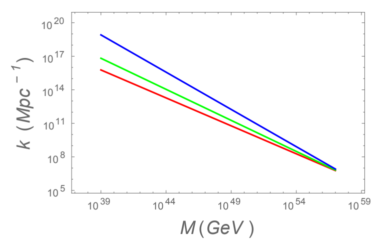
$$M(k) = \left(\frac{\gamma}{2G}\right)\left(2\times\frac{\pi^2 g_*^{eq}}{30}\right)^{\frac{1}{3w+1}}\left(\frac{8\pi G}{3}\right)^{\frac{1}{3w+1}}\left(\frac{g_s(T_{eq})}{g_s(T_1)}\right)^{\frac{3w-1}{3(3w+1)}}$$
$$\times(a_{eq}T_{eq})^{\frac{3(1+w)}{3w+1}}\,T_1^{-\frac{3w-1}{3w+1}}\,k^{-\frac{3(1+w)}{3w+1}}$$



PBH Abundance: Relevant Quantities

- Critical density contrast: $\delta_c = \frac{3(1+w)}{(5+3w)} \sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right)$
- Fraction of the Horizon mass going into PBH: $\gamma = 0.2$
- Mass fraction:

  $\beta(M) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm PBH(M)}}{d\ln M} = 2\int_{\zeta_c}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(M)} e^{-\frac{\zeta^2}{2\sigma(M)^2}} d\zeta = {\rm erfc}\left(\frac{\zeta_c}{\sqrt{2}\sigma(M)}\right)$

  $\zeta_c = \frac{(5+3w)}{2(1+w)}\delta_c$: critical value of curvature perturbation.

  $\sigma(M)$: variance of density contrast.
- Abundance: Fraction of PBH of a particular mass M as DM: $f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{cdm}}$
- Total abundance: Fraction of total PBH as DM:

  $f_{\rm PBH}^{\rm tot} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{cdm}} = \int f_{\rm PBH}(M) d\ln M$.

Dynamics of PBH formation

- Energy density during a single additional pre-BBN epoch:

$$\rho(T) = \left(\frac{3}{8\pi G}\right)\left(\frac{\gamma}{2G}\right)^2 M^{-2} = \frac{\pi^2}{30} g_*(T_1)\left(\frac{g_s(T)}{g_s(T_1)}\right)^{1+w}\left(\frac{T}{T_1}\right)^{3(1+w)} T_1^4$$

- PBH of mass M is formed at temperature T. At formation, $\frac{\rho_{\mathrm{PBH}}(M)}{\rho_T} = \gamma\beta(M)$.

$$f_{\mathrm{PBH}}(M) = \gamma\beta(M)\left(\frac{g_s(T)}{g_s(T_1)}\right)^{w}\left(\frac{g_s(T_1)}{g_s(T_{\mathrm{eq}})}\right)\left(\frac{T}{T_1}\right)^{3w}\left(\frac{T_1}{T_{\mathrm{eq}}}\right)\left(\frac{\Omega_m h^2}{\Omega_c h^2}\right)$$

- $f_{\mathrm{PBH}}^{\mathrm{tot}} = \int f_{\mathrm{PBH}}(M) d\ln M$.
- Gain over PBH formation at radiation domination:

$$g_f \equiv \frac{f_{\mathrm{PBH}}(M)}{f_{\mathrm{PBH}}^{\mathrm{rad}}(M)} \simeq \frac{\beta(M)}{\beta^{\mathrm{rad}}(M)}\left(\frac{T}{T_1}\right)^{3w-1} > 1.$$

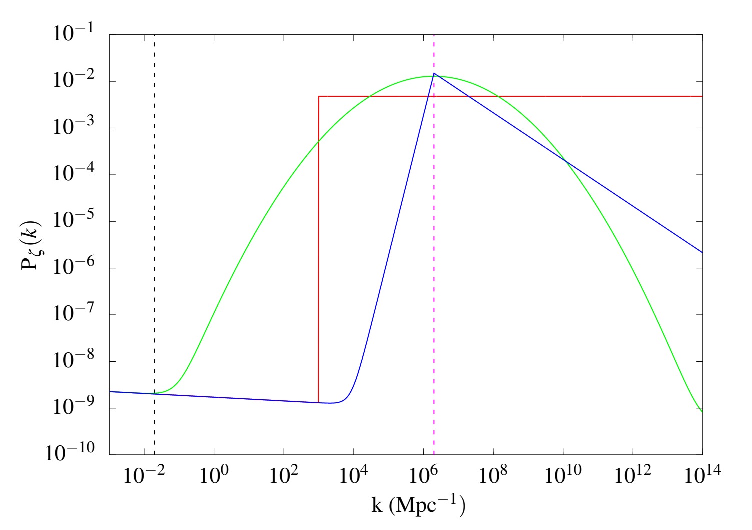
Results: Analysis with different power spectra

- 1. Scale-independent power spectrum: $P_\zeta(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1} + P_p\Theta(k-k_p)$: better understanding of the gain due to $w > 1/3$
- 2. Broken Power Law:

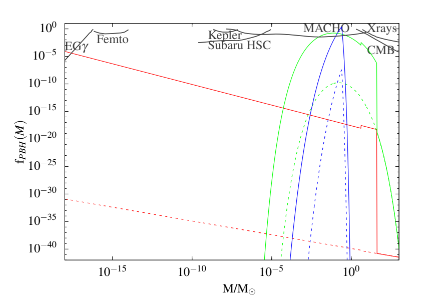
$$\begin{aligned} P_\zeta(k) &= A_s\left(\frac{k}{k_*}\right)^{n_s-1} + P_p\left(\frac{k}{k_p}\right)^{m} \quad k < k_p, \\ &= A_s\left(\frac{k}{k_*}\right)^{n_s-1} + P_p\left(\frac{k}{k_p}\right)^{-n} \quad k \geq k_p \end{aligned}$$

- 3. Gaussian Power Spectrum: $P_\zeta(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1} + P_p \exp\left[-\frac{(N_k-N_p)^2}{2\sigma_p^2}\right]$.
- 2 and 3 are theoretically motivated, e.g. Hybrid inflation leads to power 3.
- Analysis done for $k_p \sim 10^6 Mpc^{-1}$ (near solar mass PBH) and $k_p \sim 10^{12} Mpc^{-1}$ (frequency corresponds to LISA; $M \simeq 10^{-10} M_\odot$ where $f_{\rm PBH}^{\rm tot} = 1$ still allowed).

Power spectra



PBH Mass Spectra



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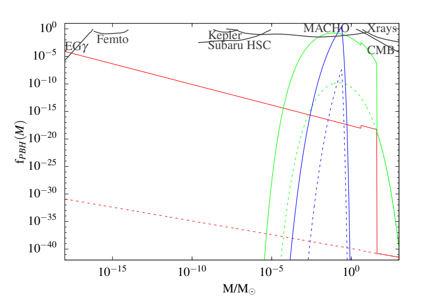
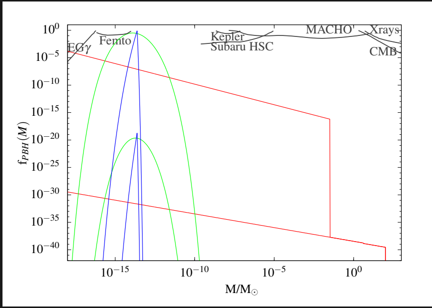
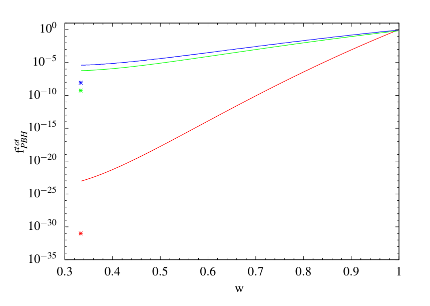


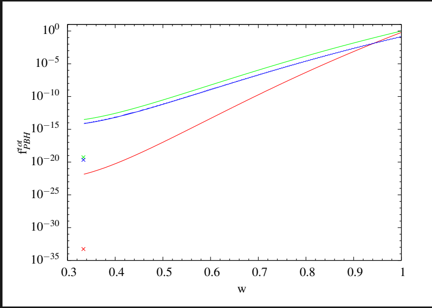


k_p		Scale-inv P_p	Broken Power Law P_p	Gaussian P_p
$10^6 Mpc^{-1}$	RD	0.021	0.0275	0.025
$10^6 Mpc^{-1}$	$w = 1$	0.0048	0.015	0.0129
$10^{12} Mpc^{-1}$	RD			0.0163
$10^{12} Mpc^{-1}$	$w = 1$	0.0048	0.0069	0.0067

for comparison, check *1812.11011*

PBH Total Abundance



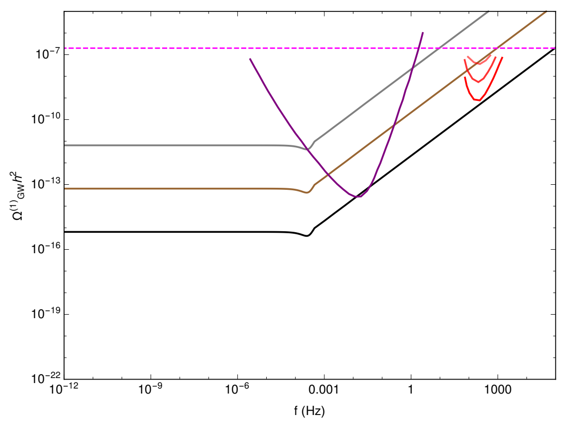


Gravitational Waves

- Modified evolution of the background affects the evolution of the modes that enter during w-domination → modifies the source-free GW (1st order in perturbation).

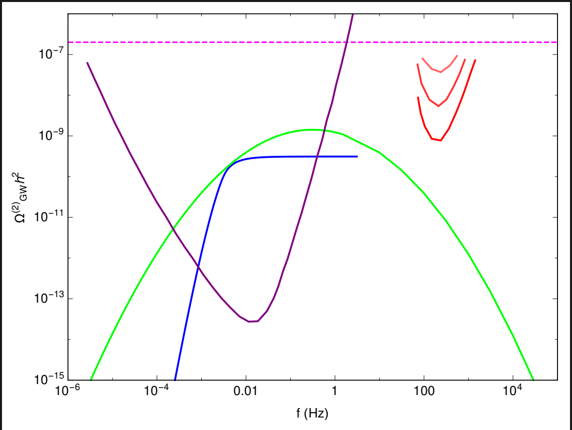
$$\Omega^0_{\rm GW}(k) = \frac{\Omega^{(0)}_{\rm rad}}{12\pi^2}\left(\frac{g_{*,k}}{g_{s,k}}\right)\left(\frac{g_{s,0}}{g_{s,k}}\right)^{4/3}\left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^2\frac{\Gamma^2(\alpha+1/2)}{2^{2(1-\alpha)}\alpha^{2\alpha}\Gamma^2(3/2)}\mathcal{W}(\kappa)\kappa^{2(1-\alpha)}$$

where $\alpha = \frac{2}{1+3w}$ and $\kappa = \frac{k}{k(T_1)} = \frac{f}{f(T_1)}$.



Second order GW

- 1st order scalar perturbations are source for 2nd order tensor perturbations.
- Difference between sourced GW in radiation epoch and kination $w = 1$ epoch: first order scalar transfer functions $\Phi(p,\eta) = \frac{1}{2+(k/p)^{3/2}}$; second order tensor transfer functions $t(k,\eta) = \left(\frac{k(T_1)}{k}\right)^{1/2}\left(\frac{k_{\rm eq}}{k(T_1)}\right)a_{\rm eq}$; evolution of source $\mathcal{S} \sim a^{-8}$.



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- If PBH production takes place during the additional w-dominated epoch AND during radiation domination, then the mass spectra and $f_{\rm PBH}^{\rm tot}$ will be different. Interesting to find the full mass spectrum and corresponding second order GW for different w.
- Spins of PBH formed in this epoch.

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