Quarkonia in QGP

Insights from Lattice QCD

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Introduction

Heavy quarks: important probes of quark-gluon plasma

- quarkonia
- Heavy-light mesons: flow

Can flow of D, B, ... be understood as a diffusive process?
What do we know about quarkonia in equilibrium plasma?
Phenomenology in heavy ion collisions much harder

- Production: color octet passing through plasma?
- Coherent energy loss
- Time evolution of plasma
- Regeneration

In-medium behavior

How to calculate interaction of quarkonia with medium?
Take a meson decay through a certain current, e.g., $J/\psi \to \mu^+\mu^-$

$$\mathcal{M} = e^2 \, \bar{u}(p)\gamma_\mu v(-p) \, \frac{1}{q^2} \, \langle H|J_\mu|0\rangle$$

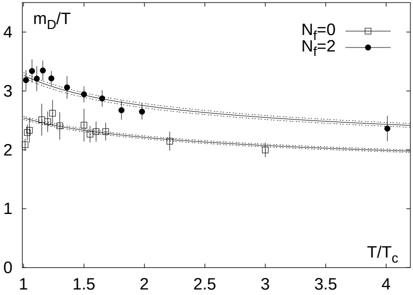
$$\Sigma(\omega) = \frac{\alpha^2}{3\pi^3 q^2} \, n_B(q^0)\sigma^\mu_{V,\mu}(\omega)$$

$$\sigma_H(\omega, \vec{p}) = \int d^4x \, e^{iq.x} \, \langle [J_H(\vec{x}, t), J_H(\vec{0}, 0)]\rangle_T$$

Theoretically most concrete: look at correlator of $\bar{Q}(\vec{x})\gamma_i Q(\vec{x})$.
Thermal width.

Perturbation theory?

HTL perturbation theory:
separation of scales $T \gg m_D$, integrate out T.
Doesn't always work in the temperaturs of interest.



Kaczmarek & Zantow, PRD 71 (2005) 114510.

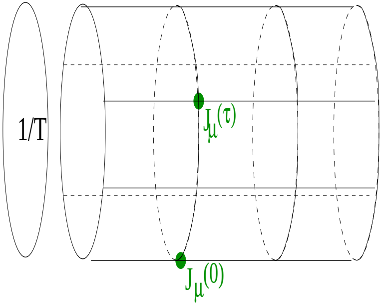
Note that impressive results have been obtained for some thermodynamic quantities.

Kajantie et al.; Vuorinen et al.; Haque et al.; ....

$\rho(\omega; T)$ from lattice

QCD in non-perturbative regime: Lattice QCD

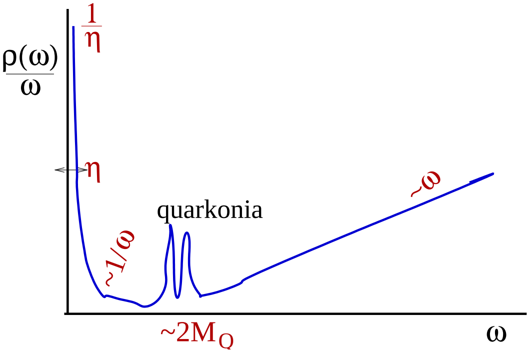
Calculate thermal (Matsubara) correlation function



$$G(\tau, T) = \int_0^\infty \frac{d\omega}{\pi}\ \sigma(\omega, T)\ \frac{\cosh\ \omega(\tau - \frac{1}{2T})}{\sinh\ \frac{\omega}{2T}}$$

Structure of vector current spectral function

$$\frac{\rho_{jj}(\omega)}{\omega}\Big|_{\omega\to 0} \sim \chi \frac{T}{\pi M}\frac{\eta}{\eta^2+\omega^2}$$



Diffusion part: very Difficult to extract from lattice correlator.

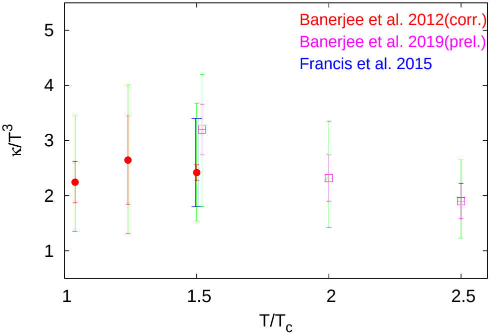
Umeda, 2007; Teaney & Petreczky, 2007

Has been extracted from $E - E$ correlator.

Laine et al. 2009; Banerjee et al. 2012; Kaczmarek et al. 2015

Results for κ

Momentum diffusion coefficient $\kappa = \frac{2T^2}{D}$



Value in right ballpark for explaining experimental data.

Quarkonia from lattice

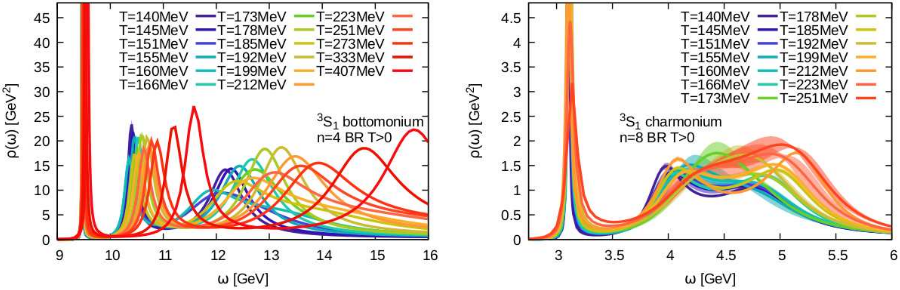
- First studies: charmonia with relativistic charm, Bayesian analysis for ρ.
- 1P states dissolve early, 1S states can survive in QGP beyond 1.5 T_c.

  Datta, Karsch, Petreczky, Wetzorke, 2004. Asakawa & Hatsuda, 2004.
- 1S correlators consistent with dissolution of state, with threshold enhancement.

  Mocsy & Petreczky, 2007-2009.
- At T=350 MeV, 1S $\bar{c}\gamma_5 c$ correlator can be described perturbatively with thermally modified m_Q.

  Burnier et al., 1709.07612.
- NRQCD on lattice promising, in particular for bottomonia.

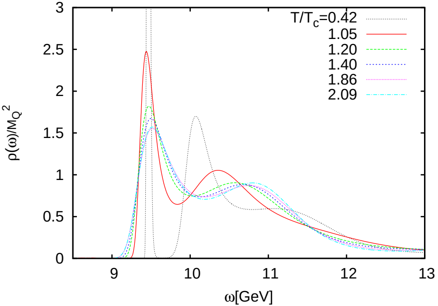
Study with realistic quark masses

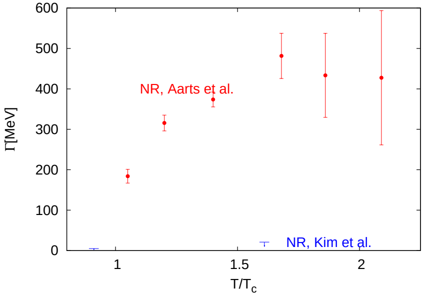


Kim, Petreczky, Rothkopf, 1808.08781

$48^3 \times 12$ lattices.
$T_c \sim 150$ MeV. J/ψ studied till $\sim 1.7 T_c$, $\Upsilon(1S)$ till $\sim 2.7 T_c$.

Bottomonia from NRQCD





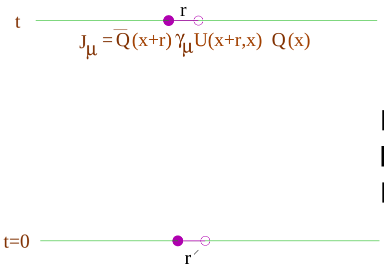
Aarts et al., JHEP 1111 (2011) 103; JHEP 1312 (2013) 064.

$\Upsilon(1S)$ and χ_b survive till $> 2T_c$. But large width for $\Upsilon(1S)$. There was also a recent study with NRQCD, which used extended operators mimicking the various bottomonia wave functions.

S. Mukherjee, S. Meinel, P. Petreczky, 1910.07374

“Potential” at finite T

Can we write down a “thermal potential” that can be used to study the properties of Bottomonia, e.g., the dilepton peak?



$$C(t) = \int d^3x \langle J_\mu(\vec{x}, t) J_\mu(\vec{0}, 0)\rangle_T$$

Leads to a time-like Wilson loop at large M.
Define $V(r, T)$ through

$$\lim_{t\to\infty} i\partial_t C(r, t) = V(r) C(r, t)$$

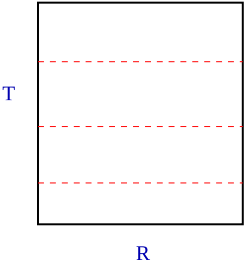
A calculation with the scale separation $T \gg \alpha M \gg gT$ leads to the potential

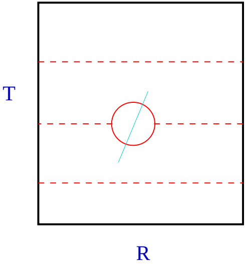
$$V(r) = -\frac{4}{3}\alpha_s \frac{e^{-m_D r}}{r} + i\frac{8}{3}\alpha_s T \int_0^\infty dz \frac{z}{(z^2+1)} \frac{\sin zr}{zr}$$

M. Laine, O. Philipsen, P. Romatschke and M. tassler, JHEP 0703 (2007) 054.

What is it good for?

- The potential captures the thermal effect at leading order in $1/M$, but resums all orders in α.





- The imaginary part is associated with decay width., and leads to the widening of the spectral function.
- The physics captured is that of Landau damping.
- The perturbative calculation assumes the scale separation

$$T \gg \frac{1}{r} \gg m_D \gg \Lambda_{QCD}$$

Issues with potential

- The potential can be calculated non-perturbatively by analytical continuation of the Wilson loop calculated on lattice.

$$W(R,t) = \mathcal{Z} \int d\omega \; e^{-\omega\tau} \; \rho(R,\omega)$$

Rothkopf et al., 1108.1579; Burnier et al., 1509.07366; Petreczky & Weber, 2018.

- In the literature, usually the potential is calculated from Coulomb gauge fixed Wilson lines.
- To connect to the point current correlator one needs potential from Wilson loops.
- Also, the extraction of the potential usually involves assuming a form of the spectral function/ giving it as input in a bayesian analysis, which has large systematics.

Potential from Lattice

- Earlier studies: either fit to (unjustified) Breit-Wigner or Gaussian form, or use Bayesian analysis with associated systematics.
- For a potential to exist via $W^M_T(\tau, \vec{r}) \sim e^{-i\, V_T(\vec{r})\, t}$ ladder of time-ordered gluon propagators needs to be resummed. Then $W^M_T(\tau, \vec{r})$ will have the structure

$$W_M(t, \vec{r_1}, \vec{r_2}) \sim e^{-i \int_0^t dt_1 \int_0^t dt_2\, D^{00}_T(t_1 - t_2, \vec{r_1} - \vec{r_2})}.$$

- Then the Euclidean loop has the structure

$$W_T(\tau, \vec{r}) \sim e^{-\int_0^\tau d\tau_1 \int_0^\tau d\tau_2 \Delta(\tau_1 - \tau_2, \vec{r})},$$

$$\Delta(\tau, \vec{r}) = \int \frac{d\omega}{2\pi}\, e^{-\omega\tau}\, \rho_D(\omega, \vec{r})\, [\theta(\tau) + n_B(\omega)]\,.$$

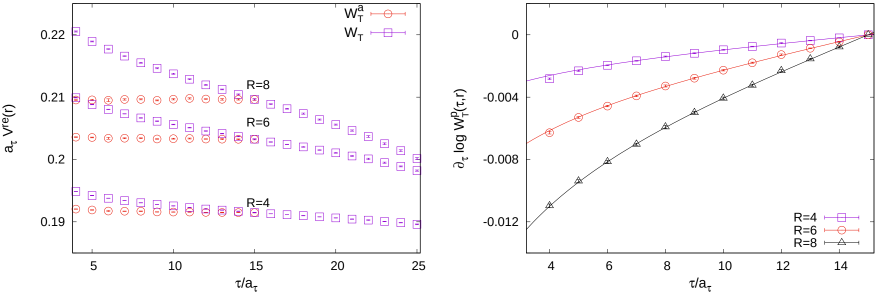
A. Beraudo, J-P. Blaizot & C. Ratti, Nucl. Phys. A 806 (2008) 312.

Potential from lattice

Analysis of this structure leads to a beautiful simplification. Splitting the Euclidean Wilson loop as

$$W_T(\tau, \vec{r}) = W_T^a(\tau, \vec{r}) \times W_T^p(\tau, \vec{r})$$

where $\log W^{a,p}(\tau) = \mp \log W^{a,p}(\beta - \tau)$, $V_T^{\mathrm{re}}(\vec{r})$ and $V_T^{\mathrm{im}}(\vec{r})$ can be extracted from (smeared) $W_T^a(\tau, \vec{r})$ and $W_T^p(\tau, \vec{r})$.



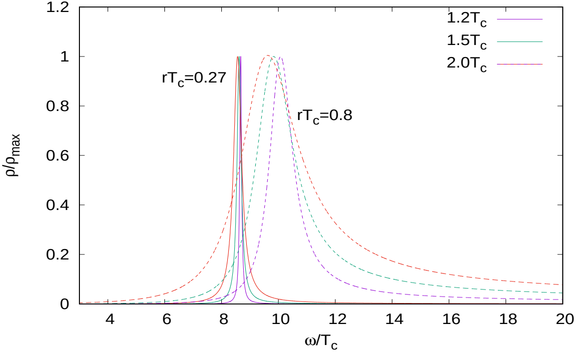
D. Bala & S. Datta, 1909.10548

Potential extraction

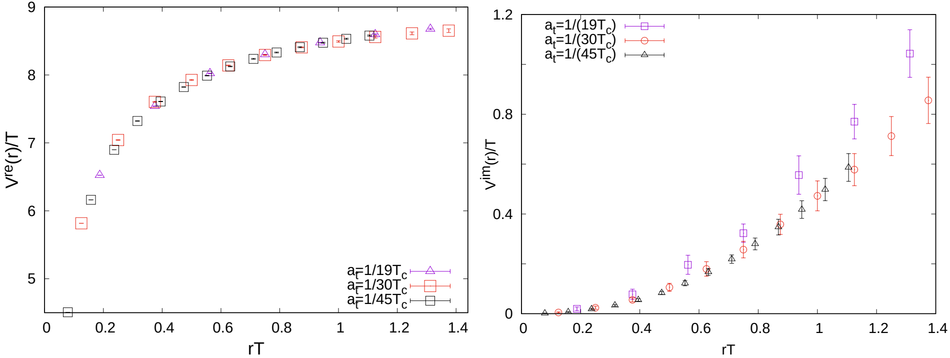
This leads to a structure of the Euclidean loop like

$$W_T(\tau,\vec{r}) \;=\; e^{-V_T^{\rm re}(\vec{r})\left(\tau-\frac{\beta}{2}\right)\,-\,\frac{\beta}{\pi}\,V_T^{\rm im}(\vec{r})\,\log\sin\left(\frac{\pi\,\tau}{\beta}\right)-\sum_l c_l\int_{\frac{\beta}{2}}^{\tau}\;\tilde{G}_l(\tau)}\;W_T(\beta/2,\vec{r})$$

The low-ω structure is different from what has been used in the literature before.

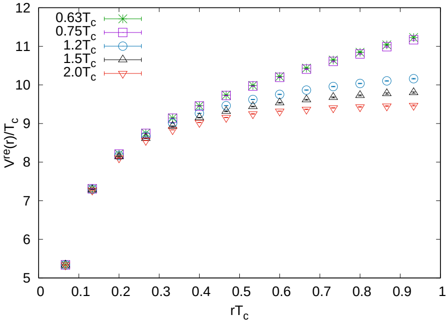


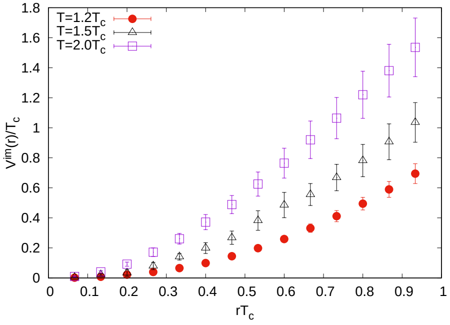
Potential:continuum limit



Potential at 1.2 T_c. The linear part has been screened.

Potential: temperature dependence





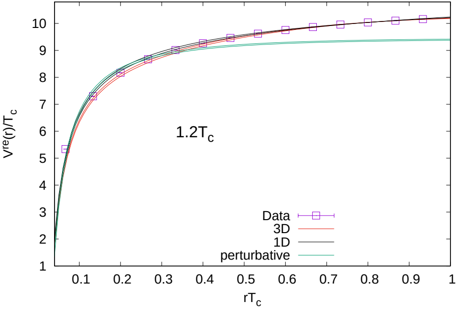
D. Bala & S. Datta, 1909.10548

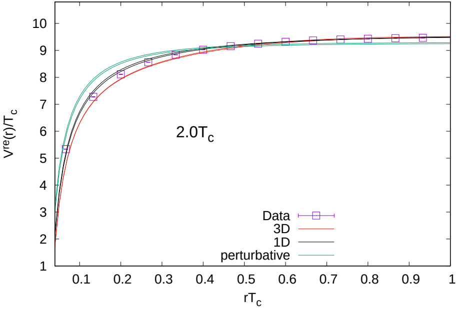
Parametrization of potential

While $V_T^{\rm re}(\vec{r})$ shows screening, it does not agree quantitatively with the perturbative Debye screened potential.
Agrees quite well with a screened string potential

$$V_{1D}^{\rm re}(\vec{r}, T) = -\frac{\alpha}{r} e^{-m_D r} + \frac{\sigma}{m_D}\left(1 - e^{-m_D r}\right) + C$$

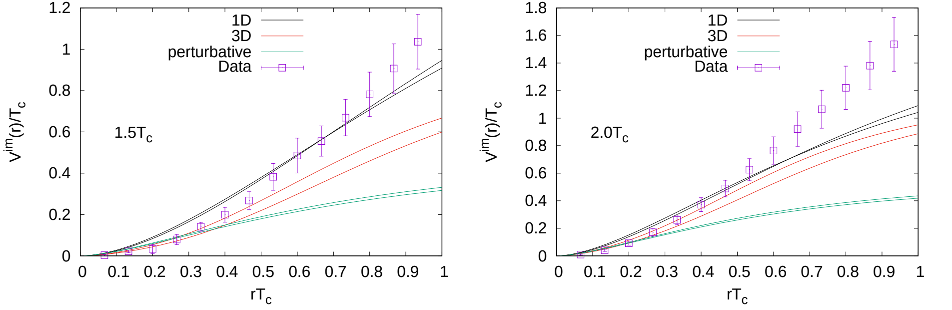
or a 3D Debye screened form of the Cornell potential.





Parametrization of V^{im}

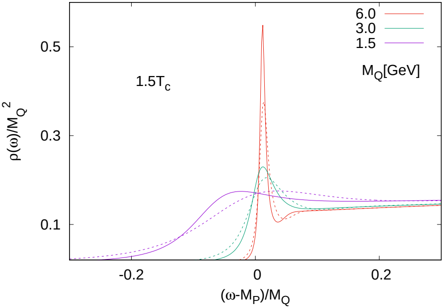
The imaginary part of the potential does not fit with the simple versions of the Debye screened potential.

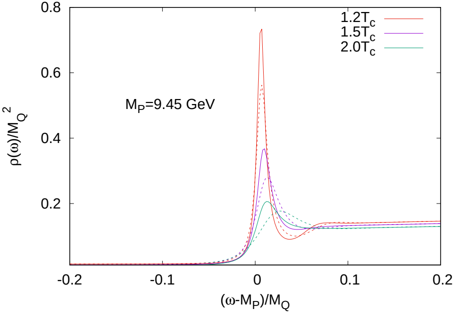


Parametrized by linear combination of 1D screening and perturbative terms (1909.10548).

Spectral function of vector current

We calculate the spectral function for the vector current $\bar{Q}\gamma^i Q$ by calculating the time-evolution of the correlator.
The potential below T_c gives a number of sharp peaks, corresponding to various vector quarkonia.
Above T_c the peak structure modified drastically.





Is thermal potential interesting?

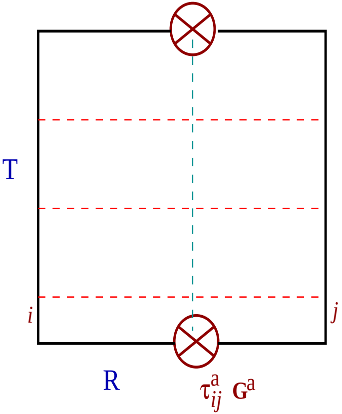
- Spectral function calculated from potential only captures part of the thermal effect.
- Also this spectral function captures modification of $\bar{Q}Q$ system for a given thermal medium.
- A better description of the physics in language of open quantum systems.

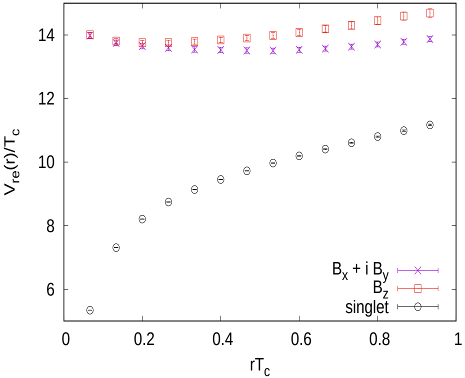
Akamatsu, PRD, 2013, 2015, 2019; Blaizot et al., 2015

- But an "abelianized" description (in the sense that thermal effect captured through thermal gluon 2-point function) has $V_T^{\rm re}(\vec{r})$ and $V_T^{\rm im}(\vec{r})$ as essential ingredients.
- We also need potentials for the Q and $\bar{Q}$ in an octet representation.

Potential for Q and $\bar{Q}$ in color octet configuration

For phenomenological study of quarkonia in QGP, we also need to understand evolution of $\bar{Q}Q$ in color octet configuration.
A gauge invariant state will have the color octet $\bar{Q}Q$ with some gluonic operator.





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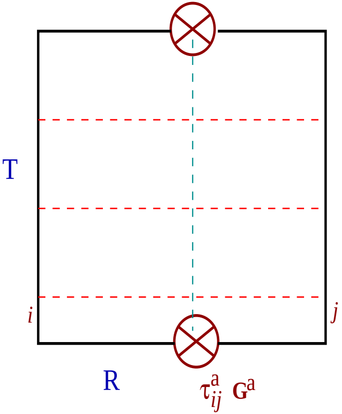
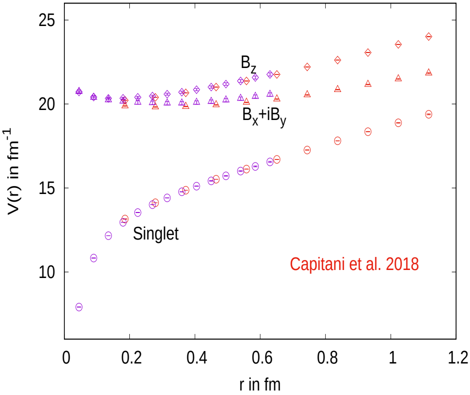
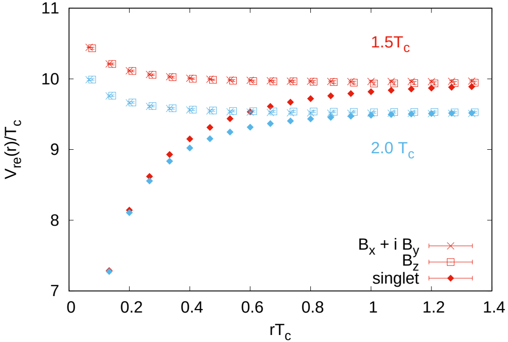




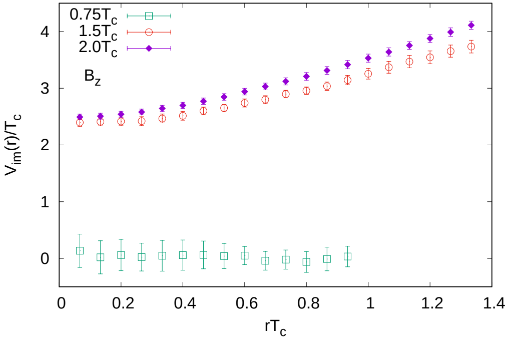
Finite temperature



The $r \to 0$ behavior similar to that seen for octet free energy. There it was interpreted as an entropy contribution. (Petreczky, hep-lat/0502008.)

D. Bala & S. Datta, in prep.

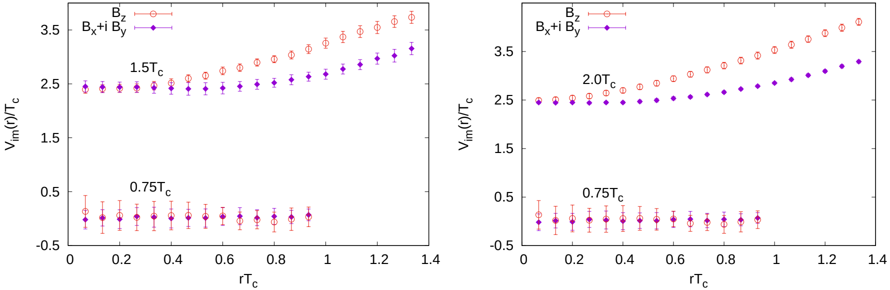
Finite temperature



In perturbation theory, $V_o^{im}(\vec{r};\,T) \;=\; T\left(\frac{4}{3} + \frac{m_D^2}{3}\int dk \;\frac{\sin kr}{r(k^2+m_D^2)}\right)$

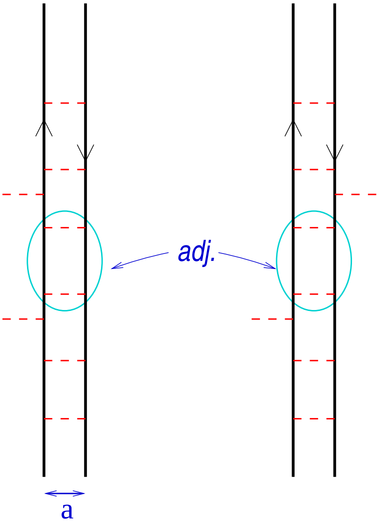
D. Bala & S. Datta, in prep.

$V_o^{\rm im}$



B_z
B_x+i B_y
1.5T_c
0.75T_c
$V_{im}(r)/T_c$
rT_c
3.5
2.5
1.5
0.5
-0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
B_z
B_x+i B_y
2.0T_c
0.75T_c
$V_{im}(r)/T_c$
rT_c

Thermal gluon scattering



- Scattering with external gluons will cluster into insertion of a color electric operator.

  Peskin, 1979

- E.g., the two top A^0 insertions lead to $a\partial_i A^0$
- Sum of these diagrams lead to operator insertion

$$\frac{1}{2N_c} \int dt \int_0^\infty d\tau \Big\langle \vec{a}.g\vec{E}_a(t) \; e^{-i\Delta\tau} \; \vec{a}.g\vec{E}_a(t-\tau) \Big\rangle$$

This can be systematized by writing an effective theory for $\bar{Q}Q$.

Effective theory for $\bar{Q}Q$: pNRQCD

Binding energy of $\bar{Q}Q$ state is $\sim Mv^2 \ll p \sim Mv$, so can integrate gluon energy scale Mv. Leads to a theory non-local in r.
Decomposing the $\bar{Q}Q$ in single and octet, one gets

$$\mathcal{L}_{pNRQCD} = \mathrm{Tr} \left[S^\dagger \left(i\partial_0 - V_s(r) \right) S + O^\dagger \left(iD_0 - V_o(r) \right) O \right] + V_1 \mathrm{Tr} \left[S^\dagger \vec{r}.g\vec{E} O + h.c. \right] + V_2 \, \mathrm{Tr}\, \mathrm{O}^\dagger \tilde{\mathrm{r}}.\mathrm{g}\tilde{\mathrm{E}}\mathrm{O} + \mathcal{O}(\frac{1}{\mathrm{M}})$$

Pineda & Soto (1998); Brambilla et al., RMP 77 (2005) 1423

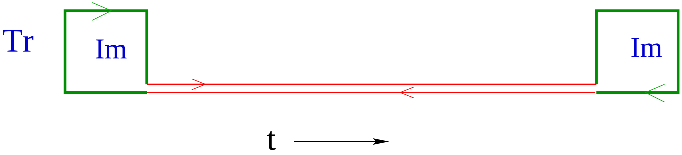
The singlet decay width to leading order in pNRQCD is the gluodissociation width.
The thermal contribution to the SS self energy comes from $r.gE \; r.gE$ correlator.

pNRQCD at finite T

For $1/r \gg T$ one gets

$$\Sigma_s \;=\; -i \frac{1}{2N_c} \frac{\langle r^2 \rangle}{2} \int_{\infty}^{\infty} dt \, e^{-i(V_o - V_s)t} \; \langle E_i^a(t) U_{ab}(t,0) E_i^b(0) \rangle_T$$

Brambilla et al, PRD 78 (2008) 014017



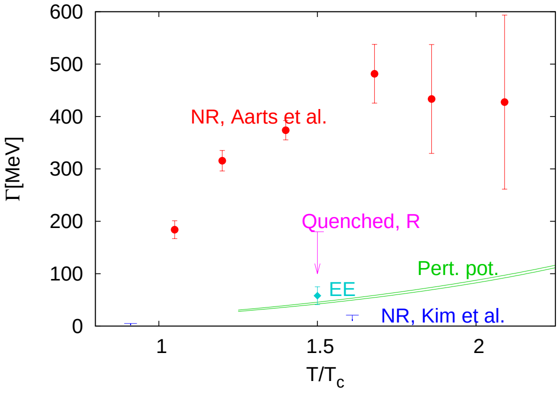
In pert. th. $V_o - V_s \sim \mathcal{O}(\alpha)$. For a leading order estimate let us ignore this.
Note that at this order the above correlator can be related to κ.

Brambilla et al., 1612.07248

From Im $i\Sigma_s$ we can make an estimate of upsilon decay width.

Datta, 1803.11557

Various estimates of $\Upsilon(1S)$ width



Summary

- Lattice QCD has given important insights into study of quarkonia in QGP.
- Quantitative predictions of, e.g., change of the spectral function with temperature remain a challenge, however.
- A combination of effective theory insights and nonperturbative lattice calculations allows us to make further progress.
- Progress in nonperturbative understanding of thermal potential: physics of Debye screening and Landau damping.
- First results on thermal octet potential.
- Essential nonperturbative input for open quantum system study of in-medium quarkonia.
- A rough estimate of Υ gluodissociation width can be obtained by connecting it to the EE correlator.
- A open quantum system treatment of quarkonia in QGP has been initiated using pNRQCD.

Brambilla et al., 2017, 2018. A. Tiwari & R. Sharma, in prep.

Use of lattice QCD will allow going beyond pert. theory.