

Detecting gravitational waves with pulsars as Weber Detectors

Ajit M. Srivastava
Institute of Physics
Bhubaneswar, India

Outline:

1. Pulsar observations: extremely accurate measurements of pulse timings: **Highly sensitive to structure of neutron star**
2. Our earlier work: Density fluctuations during phase transitions in the pulsar core: imprints on pulsar rotation dynamics: changes in moment of inertia (MI) and quadrupole moment Q
Observational implications: **Pulsar timing, Gravitational waves**
Partha Bagchi, Arpan Das, Biswanath Layek, AMS, PLB 747, 120 (2015).
3. Next: Use extreme sensitivity of pulsar to external GWs
Pulsars as remotely stationed Weber GW detectors
Arpan Das, Shreyansh S. Dave, Oindrila Ganguly, AMS, PLB 791, 167 (2019).

A New type of detector: Sensitive to different binary mass ranges:

Relevant for Primordial Black hole mergers ?

4. **Re-visiting gravitational wave events via pulsars**
Minati Biswal, Shreyansh S. Dave, AMS, arXiv:1909.0447
5. **Outlook: importance of extra-galactic pulsar observations**²

Neutron stars:

Neutron stars typically form in supernova explosions.

Mass *of the order of* 1 -2 solar mass; Radius *about* 10 km

Central density *as high as* 5 to 10 times the nuclear equilibrium density *of* $0.16/\text{fm}^3$. About $10^{14}\text{grams}/\text{cm}^3$

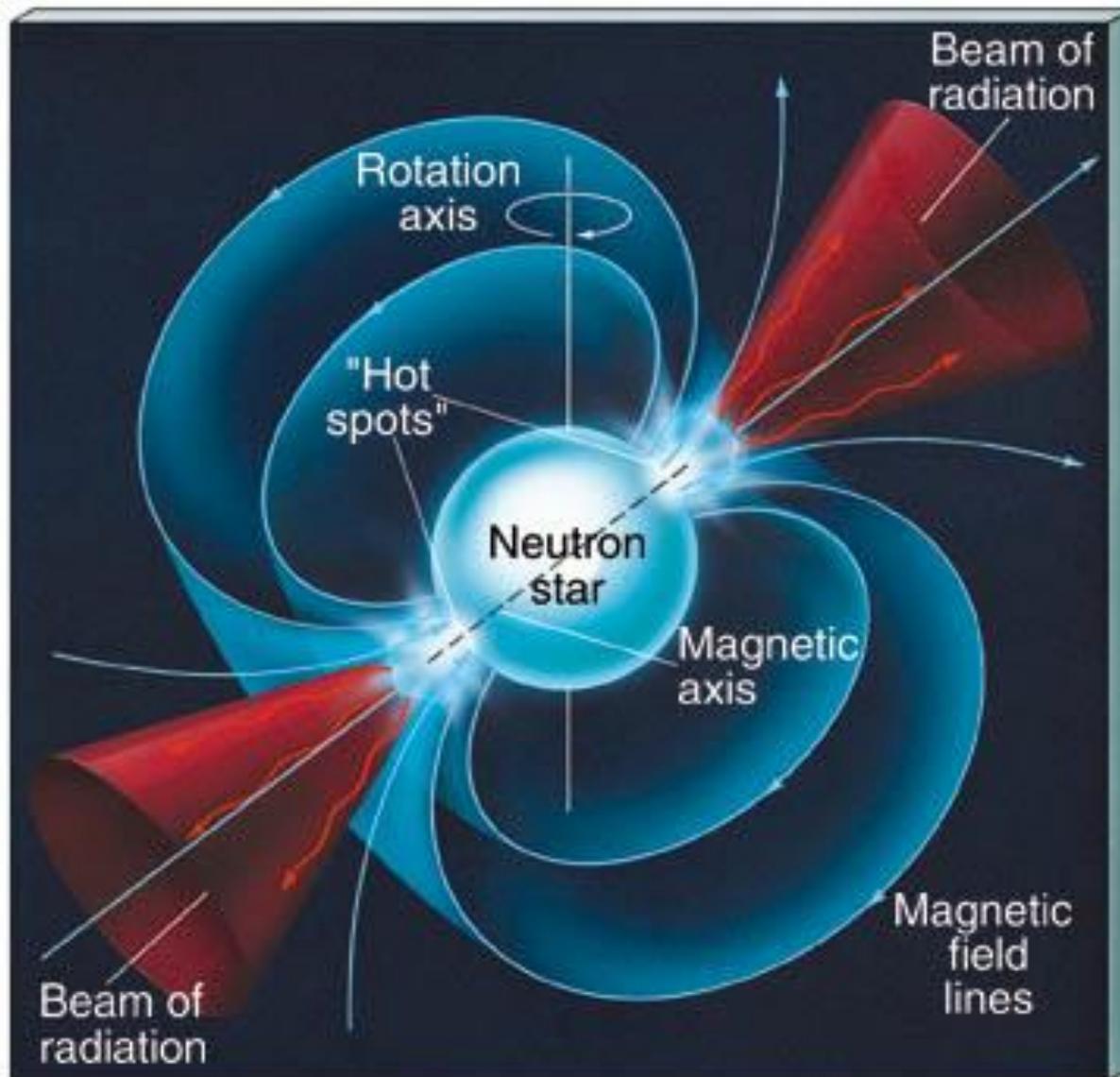
Initial temperature \sim 10 -30 MeV (10^{11} K)

Rapidly cools to T below 0.1 MeV (10^9 K) within minutes by neutrino emission. Subsequent slow cooling.

A neutron star is the densest forms of matter in the observable universe.

Pulsars are rotating neutron stars

Observations: Extremely precise measurements of Pulsar timings (recall: first detection of gravitational waves)



If this radiation cone intersects Earth, Periodic pulses are observed

Pulsars are incredibly accurate clocks

Example: period of the first discovered "millisecond pulsar" is:

$$P = 0.00155780644887275 \text{ sec}$$

It is slowing down at a rate of 1.051054×10^{-19} sec/sec

Pulsar: J0437-4715

$$P = 0.005757451936712637 \text{ sec}$$

Error of 1.7×10^{-17} sec.

Our work: use this incredible precision for detecting changes occurring in the configuration of a neutron star.

Signals of phase transitions from Neutron stars

(Partha Bagchi, Arpan Das, Biswanath Layek, AMS, PLB, 747, 120 (2015))

Use extreme accuracy of pulsar timings to probe various phase transitions occurring inside the pulsar core, for example:
Transitions to exotic phase of QCD, or nucleonic superfluidity

We used association of Phase transitions with density fluctuations:

Density fluctuations inevitably arise during phase transitions:

First order transition:

random bubble nucleations lead to density fluctuations, with typical distance scales of inter-bubble separation

Second order transition, or smooth cross-over:

Density fluctuations with size scale of correlation domains, scale invariant density fluctuations in critical regime

In any transition/cross-over: large scale density fluctuations inevitably arise if topological defects are produced

Any density fluctuations in the neutron star will have
Observational effects:

If density fluctuations arise in the core of a pulsar, it will affect
its moment of inertia (MI) and quadrupole moment Q :

Can be detected by precision measurements of pulse shape/timing

Changes in MI and Q are completely generic

They happen for all types of density fluctuations produced
in phase transitions.

Further: these changes in MI and Q can have both signs, + and -

Implications of phase transition in the pulsar core:

Rapid changes in Pulsar timings and Gravitational waves

Implications of these density fluctuation induced changes:

Change in diagonal components of MI:

This will result in rapid changes in rotation of pulsar.

As density fluctuations dissipate away, leading to a uniform new phase in the core, some part of change in MI will be restored, But not fully.

This is exactly the pattern of glitches and anti-glitches where often only few percent of the change in rotation is recovered.

Also, as we find changes of both + and - sign, glitches and anti-glitches are both naturally accommodated in this picture.

(no such uniform explanation present in the literature for both).

Transient changes in the off-diagonal components of MI and Q

These are distinctive predictions of our model.

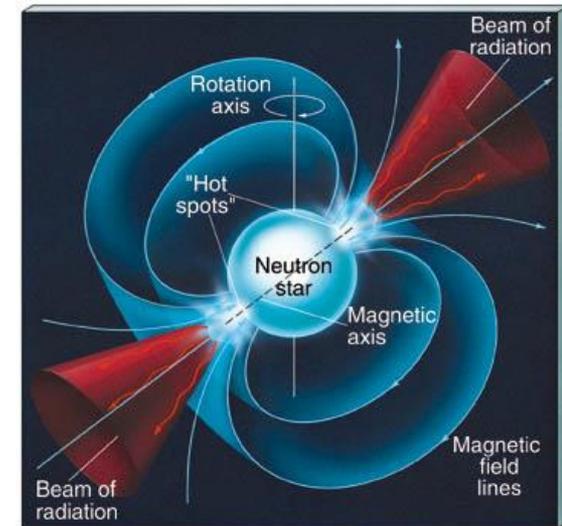
Changes in Off-diagonal components will lead to wobbling of star (on top of any present initially).

This will lead to modulation of pulse intensity as the direction of radiation emission wobbles.

Thus, our model predicts that in association with rapid changes in pulsar timings, there should be modulation in pulse intensities.

This is distinctive prediction of our model.

Vortex de-pinning model cannot lead to off-diagonal components of MI (all vortices point along the rotation axis).



Fractional change of various components of MI and Q caused by Density fluctuations due to string and /or domain wall network.

The results are obtained by varying the core size R_c , while keeping The correlation length $\xi = 10$ fm fixed

	QCD Strings			QCD Walls			Superfluid Strings		
$\frac{R_c}{\xi}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{\delta Q_{xx}}{I}$
5	5E-10	-3E-10	-1E-10	2E-8	-1E-8	-8E-10	2E-6	-1E-6	-4E-7
50	5E-12	-2E-12	2E-12	1E-10	-8E-11	-1E-11	2E-8	-7E-9	7E-9
200	1E-13	2E-14	-7E-14	5E-12	-4E-12	-6E-12	5E-10	6E-11	-2E-10
400	-3E-15	-5E-14	-9E-14	3E-12	-2E-12	3E-14	-1E-11	-2E-10	-3E-10

As R_c is increased, fractional changes in various moment components seem to stabilize, with values ranging from 10^{-14} to 10^{-10} .

Interestingly: Superfluid leads to much larger changes even with small energy density changes of order $0.1 \text{ MeV}/\text{fm}^3$. This is due To larger region (\sim few Km) undergoing superfluid transition

Rapid changes in quadrupole moment Q will lead to gravitational waves.

Note: small value of Q/I arising from density fluctuations (about 10^{-10}) is more than compensated by the very short time scale of microseconds.

Gravitational wave power $\sim (\ddot{Q})^2$

Fastest time scale for conventional mechanism of gravitational wave emission is milliseconds (from rotation), with Q/I of order 1/1000.

Here Q/I is about 10^{-10} , but time scale is at most microseconds. In fact, for topological defect induced density fluctuations, the time scale can be much shorter as defect network coarsens very fast.

Thus density fluctuations arising during phase transitions in compact astrophysical objects, like neutron stars, may provide a new source of gravitational radiation.

Power in the gravitational wave:

$$\frac{dE}{dt} = -\frac{32G}{5c^5} \Delta Q^2 \omega^6 \approx -(10^{33} \text{ J/s}) \left(\frac{\Delta Q / I_0}{10^{-6}} \right)^2 \left(\frac{10^{-3} \text{ sec.}}{\Delta t} \right)^6$$

I_0 is the MI of the pulsar, $\Delta Q / I_0$ is the change in quadrupole Moment occurring in time interval Δt

$\Delta Q / I_0 \approx 10^{-14} - 10^{-10}$ in our model, about 4-8 orders of magnitude smaller than the usual value for deformed neutron star.

However, Δt here is at least 3 orders of magnitude shorter than the conventional case of fastest pulsar (millisecond pulsar). This more than compensates for the smallness of quadrupole moment.

Strain amplitude from a pulsar r distance away:

$$h = -\frac{4\pi^2 G \Delta Q f^2}{c^4 r} \approx 10^{-24} \left(\frac{\Delta Q / I_0}{10^{-6}} \right) \left(\frac{10^{-3} \text{ sec.}}{\Delta t} \right)^2 \left(\frac{1 \text{ kpc}}{r} \right)$$

These values of strain and power are well within the reach of LIGO.

However, LIGO is tuned for optimum frequency in kilo Hz regime.

It has possibilities of upgrades for Mega Hz frequency
(if motivated by physics considerations, no other known astrophysical sources require Mega Hz frequency).

Important to have these upgrades.

This can open up new possibilities for very short time dynamics happening inside neutron star cores.

It is the accuracy of pulsar observations which allows for precision measurement of very tiny density fluctuations.

Pulsars as Weber gravitational wave detectors:

Arpan das, Shreyansh Dave, Oindrila Ganguly, AMS, PLB, 791, 167 (2019)

So far we discussed internal changes in the pulsar affecting its pulse timing etc. Now we discuss changes due to external influence. A gravitational wave passing through a pulsar will cause (very) tiny deformations in the pulsar shape, affecting its rotation.

This will affect the extremely accurately measured spin rate of the pulsar.

It will also affect the pulse profile due to development of the Off-diagonal components of moment of inertia tensor (Here Due to external GW wave, depending on GW source direction w.r.t pulsar spin).

Effect **most pronounced at resonance.**

May be detectable by accurate observations of the pulsar signal.

Resonance likely with Pulsar EOS and Tidal Deformability constrained by recent BNS merger event

The pulsar, thus, acts as a

Remotely stationed Weber detector

of gravitational waves whose signal can be monitored on earth.

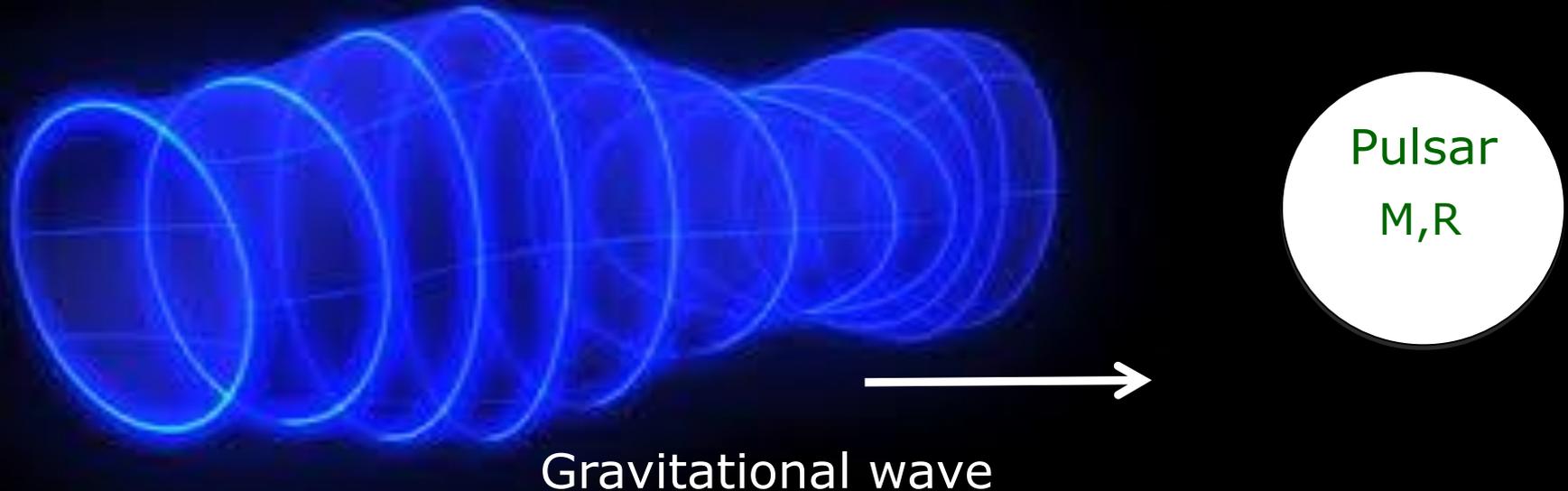


Pulsars as Weber gravitational wave detectors:

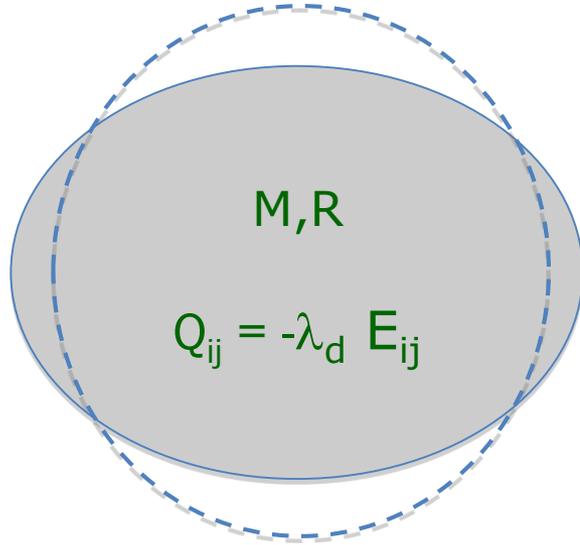
In our earlier work, we discussed internal dynamics of pulsar (e.g. phase transition) leading to its deformations.

Now consider a pulsar under influence of external gravitational waves (GW), coming, say, from a merger event far away.

For simplicity, take a spherical pulsar to begin with



Under the influence of external gravitational wave, the pulsar will undergo quadrupolar deformations



Deformation of neutron star in the Tidal field E_{ij} of gravitational wave is given by

$$Q_{ij} = -\lambda_d E_{ij}$$

λ_d is the tidal deformability :

$$\lambda_d = \frac{2}{3} k_2 \frac{R^5}{G}$$

k_2 is the second Love number with its value for a neutron star lying in the range $k_2 \sim 0.05 - 0.15$ (constrained by recent BNS merger observations)

$E_{ij} = R_{i0j0}$ ($R_{\mu\nu\lambda\rho}$ being the Riemann curvature tensor) can be written in terms of GW strain amplitude for a specific polarization in transverse traceless (TT) gauge

$$R_{\mu 0 \nu 0} = \frac{1}{2} \partial_0 \partial_0 h_{\mu\nu}^{TT}$$

Where

$$h_{\mu\nu}^{TT} = C_{\mu\nu} e^{ik_\sigma x^\sigma}$$

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The suffixes '+' and 'x' denote two different Polarizations of GW

Denoting h_+ by h , the amplitude of resulting E_{ij} is given by

$$E_{xx} = -E_{yy} = \frac{2\pi^2 hc^2}{\lambda^2} \quad \lambda \text{ is the GW wavelength}$$

Resulting change in the quadrupole moment tensor of the neutron star is given by

$$Q_{ij} = -\lambda_d E_{ij}$$

For simplicity, we take the initial NS configuration to be spherically symmetric, and the deformation to be ellipsoidal, with the dimension in the direction of GW propagation remaining unchanged.

Using the above value of Q_{ij} we can calculate changes in the moment of inertia tensor I_{ij} of the NS

Resulting change in the moment of inertia of neutron star is

$$\frac{\Delta I_{xx}}{I} = -\frac{\Delta I_{yy}}{I} \approx \frac{k_2}{3} \frac{R^3 c^2}{GM\lambda^2} 20h$$

We take sample values with $M = M_\odot$ and $R = 10$ km

Highest sensitivity will be reached for smallest values of λ (above equations valid for static case, this requires λ to be much larger than NS radius, range of frequencies we consider are below kHz)

As a typical astrophysical source of GW, we take binary neutron Star (BNS) merger, such as the one detected by LIGO/Virgo

The highest value of GW frequency being about 1 kHz.

Then, we get (with $k_2 = 0.1$ as a sample value)

$$\frac{\Delta I_{xx}}{I} \approx 10^{-2} h$$

LIGO-Virgo detected BNS merger event:

Peak signal strength $h \sim 10^{-19}$

Source distance from earth \sim about 130 million light years.

The Detector neutron star (pulsar) could be very close to it, Suppose it was at 100 light year distance from the BNS merger

(Note: Most neutron stars/pulsars are in globular clusters With very dense cores, so very likely case).

Then the GW strength at the pulsar will be $h \sim 10^{-13}$

Resulting fractional change in moment of inertia (for the Relevant component), hence change in spin rate, will be

$$\frac{\Delta \nu}{\nu} = \frac{\Delta I}{I} \approx 10^{-15}$$

If detector pulsar was 10000 light years away from BNS merger:

This number will be $\sim 10^{-17}$

Such fractional changes in the spin rate of pulsars should be Detectable by precision measurements of the pulses

For millisecond pulsars, accuracy of pulse timings have been measured to an accuracy better than 10^{-15} - 10^{-17} seconds.

Note: Such remarkable accuracy for pulse timings typically requires folding over large number of pulses, for a short GW pulse is it possible?

Important: Neutron star acting as a Weber detector at resonance will exhibit **Ringing** effect.

(This is how Weber detector achieves very high accuracy).

Thus, even for a GW pulse its effects will be present in the pulsar signal for a much larger duration, hence folding should be Possible.

Possibility of Resonant Enhancement:

Above estimates for pulsar spin rate changes did not account for resonance, which can dramatically increase the effects of GW.

For example: Resonant tidal deformations from orbiting binaries can lead to rupture of NS crust. **It has been argued that viscous Effects may not be very dominant for the relevant time scales.**

More precisely, one needs to know the quality factor Q for NS interior

For specific modes, the resonant frequencies of NS can be in the Range of 100 Hz to 1 kHz, which is precisely the range relevant for a typical BNS merger GW source, also for stellar mass black holes.

(Importantly: There is a wide range of resonant frequencies for neutron stars. This will be important for Primordial Black holes which can come in a very wide range of masses, discuss later...)

Thus: the possibility remains that resonance effects may lead to Significant enhancement of the effect of GW on NS spin rate change

Resonant Enhancement:.....

However, resonance enhancement generally requires sustained Signal, which is unlikely for the case of a burst of GW

Again, we recall: for Weber detector, use of material of very high Quality factor (with Q factor of order 10^6) was important.

This is not only for resonant enhancement effect,

Also, even for a short GW pulse, there is **strong Ringing effect for a Weber detector operating at resonance** which helps in enhancing signal to noise ratio.

Due to this ringing effect, the detector continues to vibrate in The resonance mode for significant time even after the passing Of the GW pulse through the detector (due to energy absorbed From the pulse in the resonant mode).

For example: for a GW pulse lasting a few ms, the resonant bar Can continue to ring for time of order 10 minutes with same Frequency, thereby allowing separation between noise and signal

Resonant enhancement for NS:

Thus, if the pulsar continues to ring for significant time after the GW pulse has passed through it, then the radio pulses will continue to retain this “definite frequency signal” hidden within.

Note: We need to know the Q factor for NS interior
This is a new challenge for QCD calculations, apart from the well known problem of determining the equation of state

Folding of many pulses may be able to separate this Ringing signal.

Both these effects: resonant enhancement, and the Ringing Effect, may significantly improve the accuracy of detection of the effect of GW on pulsar signals. **Then these pulsars can become additional detectors of GW located in space.**

If some GW source is close to the pulsar being observed, the GW signal may be very strong on the pulsar, hence strongly affecting its pulses which can be observed on earth.

However, most pulsars, especially millisecond pulsars, are within our galaxy, so very small chances, may be 1 in 3000 years.

Pulsars: A family of remotely stationed Weber detectors:

What we have argued is that the pulsars spread out in space may act as GW detectors.

They detect GW, then communicate this detection to earth based “**Pulsar Observatories**” by their pulses carrying the imprints of GW (arising from tiny deformations in NS configuration from GW).

These imprints have extremely high degree of fidelity (so: signal transmission from pulsar detector to earth has high quality)

What is needed is to have very accurate measurements of pulsar Timings of far away pulsars.

Reaching out for extra galactic pulsars will be very important. Possibility of some GW source close to the pulsar, while being Very far away from us, will show the real strength of this technique.

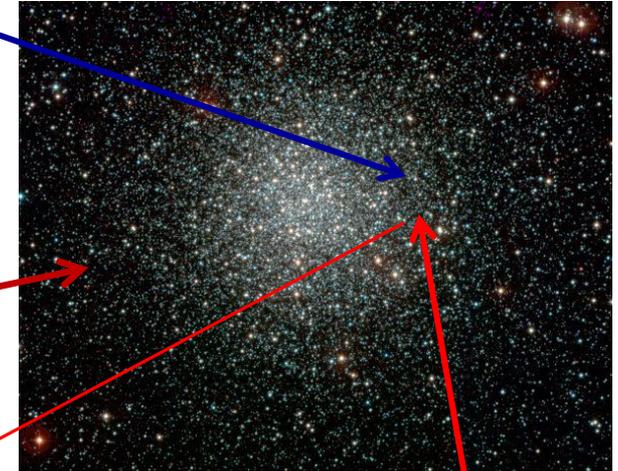
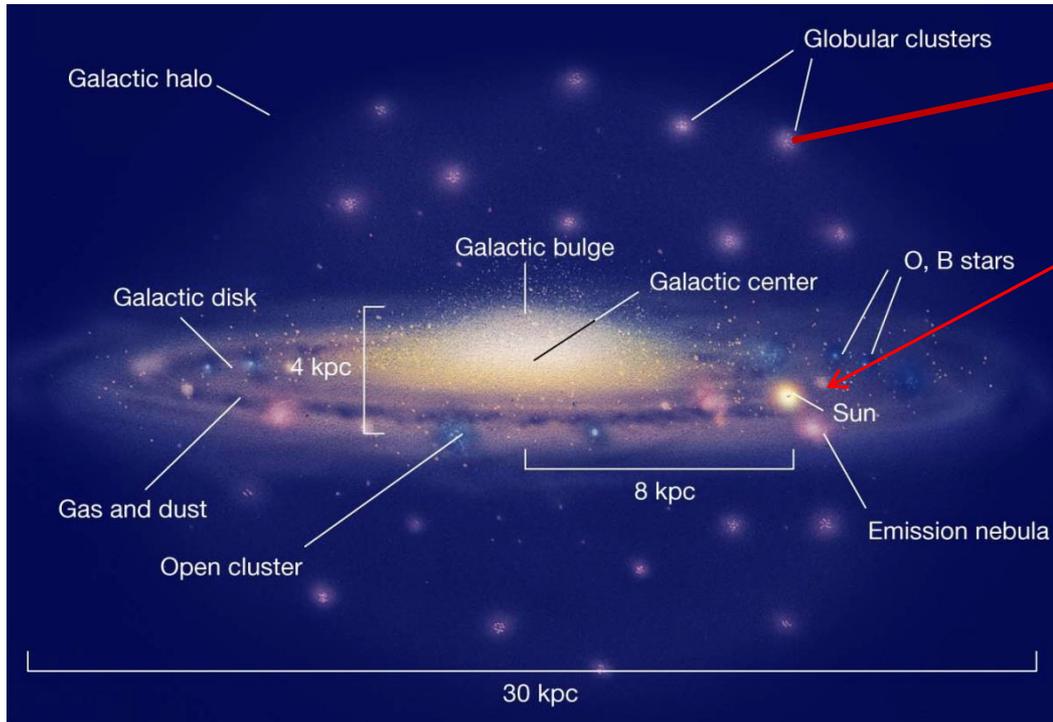
Important: Even if GW event is directly detected at earth, these Pulsars can provide additional detections, helping in **accurate Triangulation of source location. Crucial for Black Hole mergers.**

Pulsars in the neighbourhood of GW sources can provide us a family of remote detectors all of which can be monitored on earth

GW source:
e.g. neutron
star merger

A globular cluster

Milky Way



Pulse with
GW imprint

Pulsar
detector

Caution: This proposal is completely different from the well known proposal of the Pulsar Timing Array for the detection of ultra-low frequency gravitational waves.

Primordial Black hole mergers:

PBH can form abundantly in an early dust dominated era

Should be distinguished from non-relativistic matter dominated case where the last stages of collapse can be halted due to pressure (e.g. Baryonic pressure) with non-zero Jean's length, or anisotropy build up leading to fragmentation of the collapsing region as seen in simulations of gravitational collapse.

The dust case is most favorable to formation of PBH.
This means no other interaction except gravitational.

One of the early papers on this type of scenario:

“Cosmological consequences of gravitationally interacting Planck-mass particles”,

A.M. Srivastava, Phys. Rev. D36, 2368 (1987).

Considered the case of universe initially filled with supermassive, (Planck mass in this case) particles called Geons, postulated by Friedman and Sorkin. These particles have long lifetime, decaying eventually to standard model particles. **Abundant formation of PBH** ²⁸

Primordial Black hole mergers

Primordial black holes can come in very wide range of masses,
From very low masses to super massive ones.

Pulsar detector can have wide range of resonant modes.

Very low frequency modes ~ 1 Hz
to very high frequency modes \sim Tens of kHz

This allows for a much larger range of BH mergers to be detectable with pulsar detectors, e.g. $M_{\text{BH}} \sim 0.1 M_{\text{Sun}}$

Roughly: Peak frequency of GW \sim few kHz ($M_{\text{Sun}}/M_{\text{BH}}$)

This is when we require resonance effects to enhance sensitivity

Light PBH mergers could be more frequent,
hence could occur close to the pulsar detector

In that case even without any resonance effect, pulsar detector might be able to detect it.

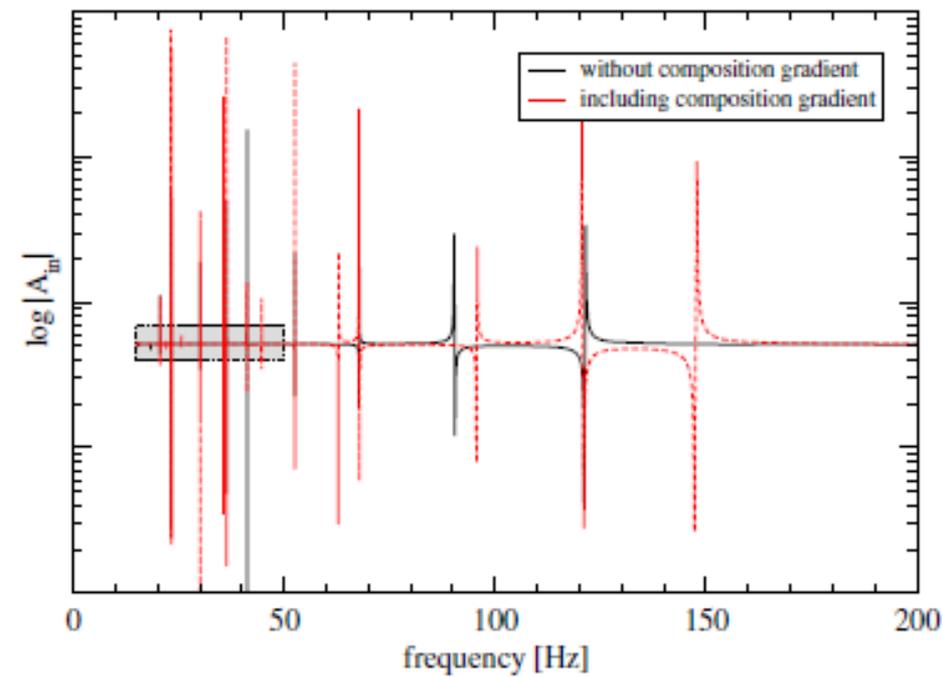
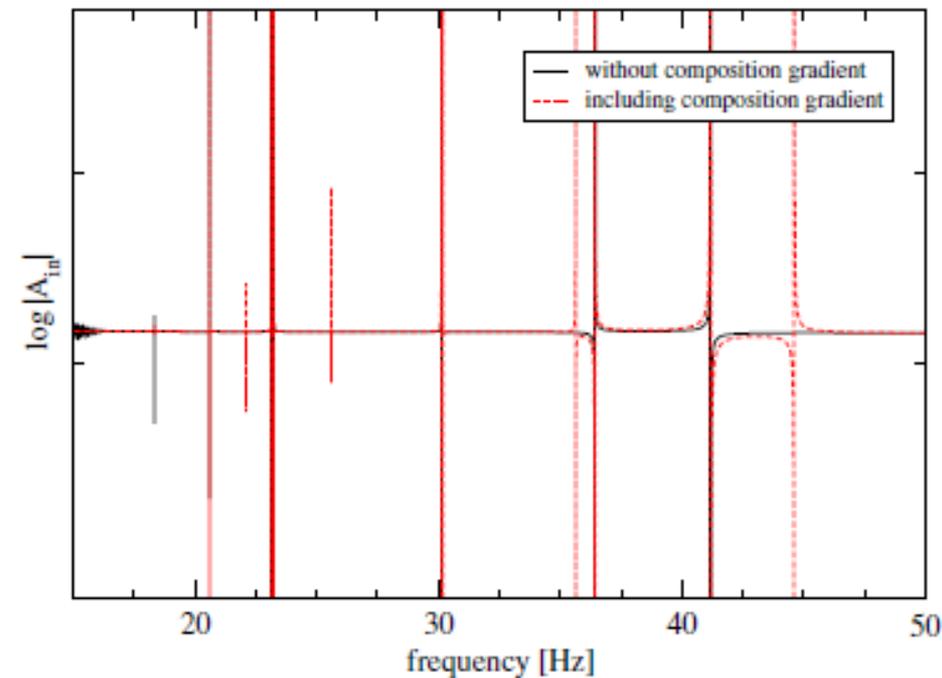


FIG. 3 (color online). The low-frequency spectrum of our neutron star at zero temperature and without a solid crust. The upper panel shows the spectrum up to 200 Hz whereas the lower panel shows a magnification of the grey shaded area. The solid line shows the spectrum of the pure perfect fluid star, while the dashed line includes composition gradients. All spikes in the perfect fluid spectrum are interface modes as there is no composition gradient present; for each of these modes, there is also a mode present in the spectrum of the stratified star (with the exception of the mode at 90.4 Hz; see text). The “new” modes in the stratified spectrum are composition g -modes.

Krüger et. al PRD,92, 063009 (2015)



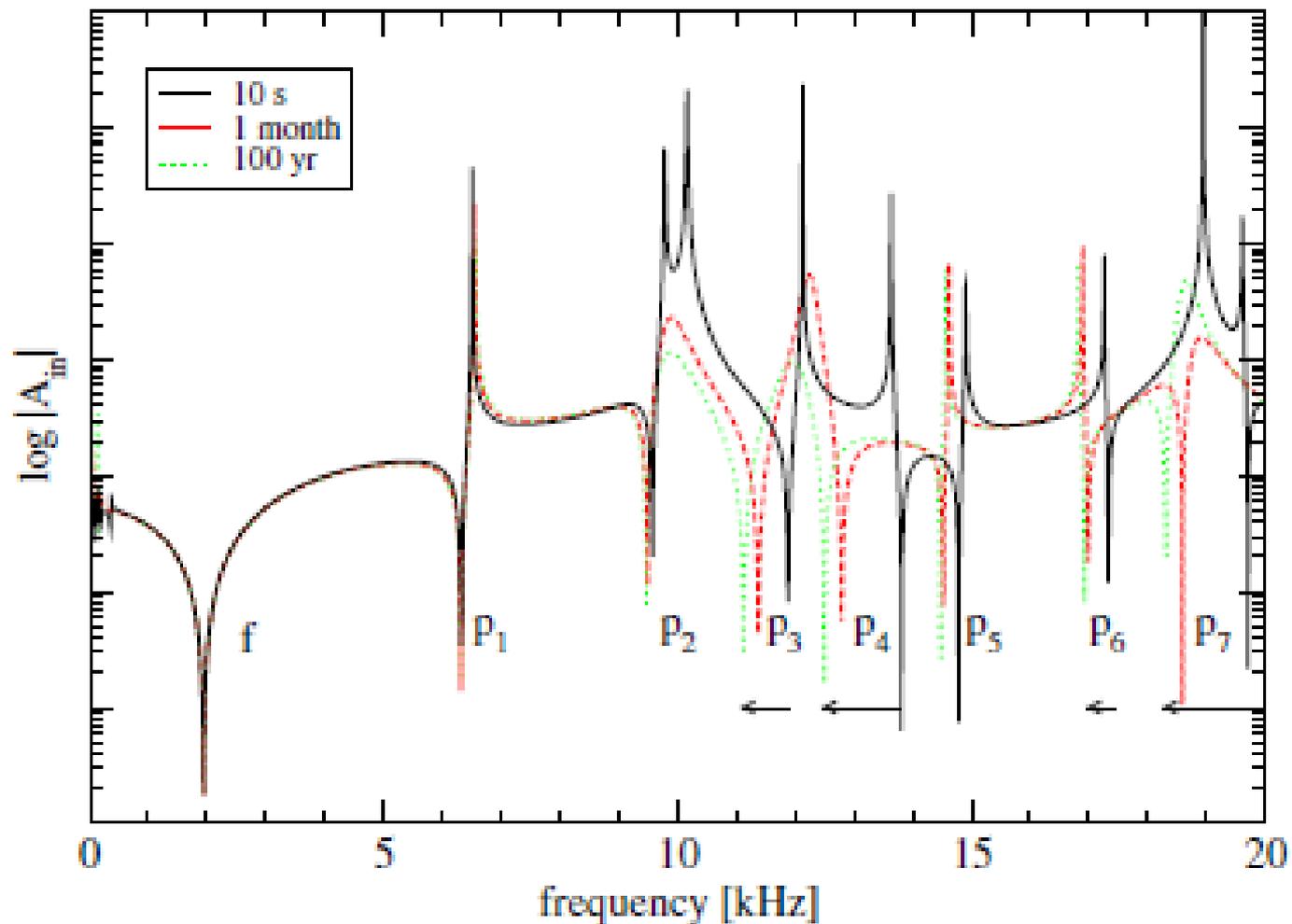
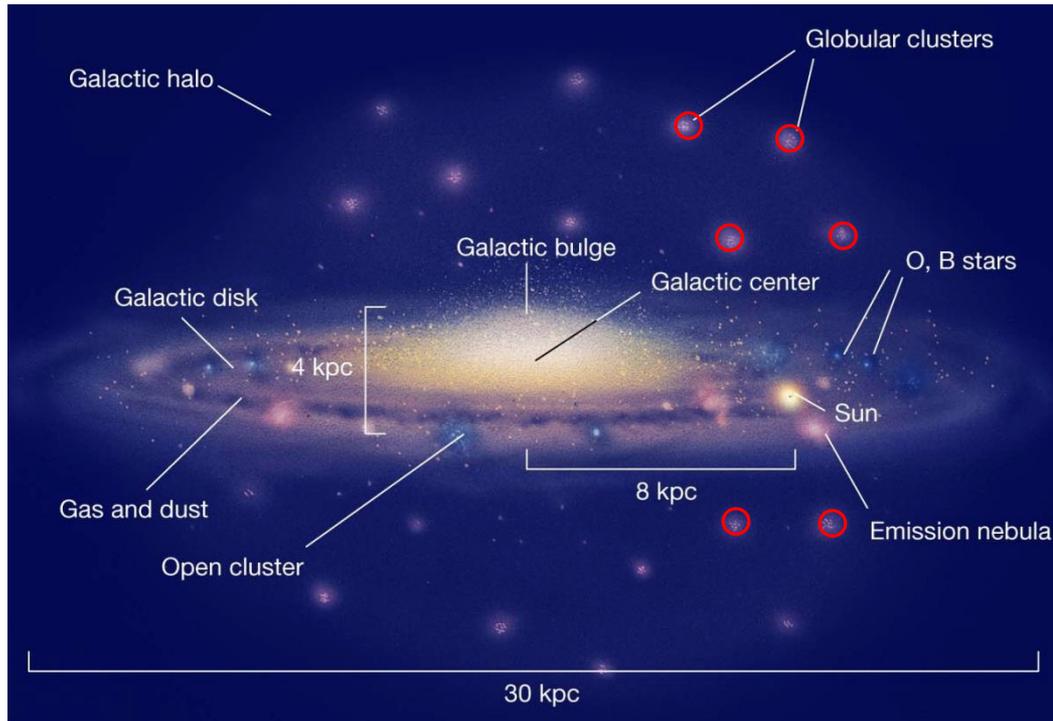


FIG. 7 (color online). The stellar spectrum up to 20 kHz for three different temperature profiles. Clearly visible are the f -mode and first 7 p -modes. Where the frequency of a p -mode varies visibly over time, an arrow indicates this change.

Now consider: cases when gravitational waves have already reached earth in past, either detected, or missed:

**For past GW events, Pulsar mediated signals will reach us in future at precisely determined times:
Opportunity to revisit those events in future, possibly several times.**

Minati Biswal, Shreyansh S. Dave, AMS, arXiv:1909.0447



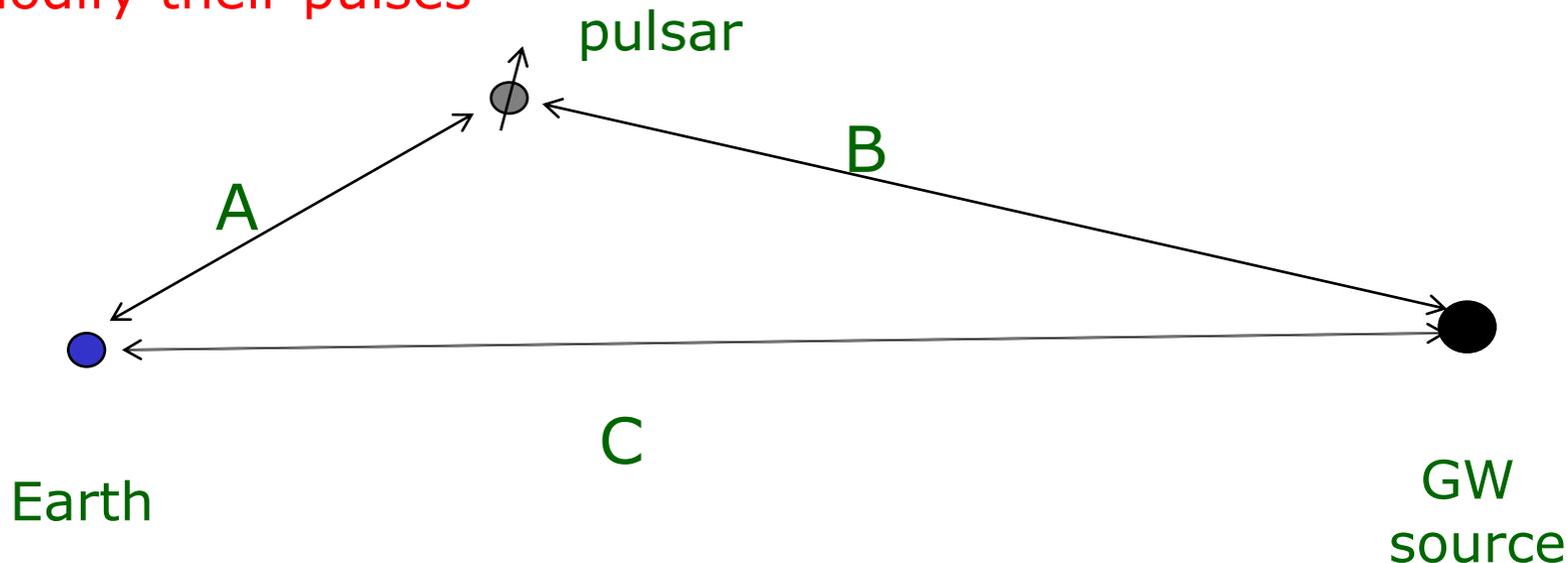
○ Denotes locations of Pulsar detectors which will detect GW at different times.

These modified pulses will reach us in future at predicted times:

Revisit past GW events

Consider the case of those GW events which have already been detected on earth, or the GW signal has passed earth in past, without detection.

Those GW waves will also reach pulsars, say in our galaxy, and Will modify their pulses



The modified pulse of the pulsar will carry the imprints of the original GW signal, which will reach us much later
(total path Being $A + B$)

This will reach us much after the arrival of the original GW signal
Which was along the path C.

Knowing the GW source, and the date of detection of GW on earth, One can determine for each specific pulsars, when its GW-Perturbed signals will reach us on earth.

Remarkable possibility of re-visiting the past GW events

If it works, it will be important: Ability to look at events which occurred in far past

It will allow **repeated** source of information about the GW source with same signal coming again and again from different pulsars

As well as properties of NS and its interiors
(specific to the Pulsar whose signal is being observed)

It will allow multiple detectors, located at astrophysical distance Scales, which can be used for accurate localization of GW source.

This will be of crucial importance for events like BH mergers where No other signals are emitted except GW.

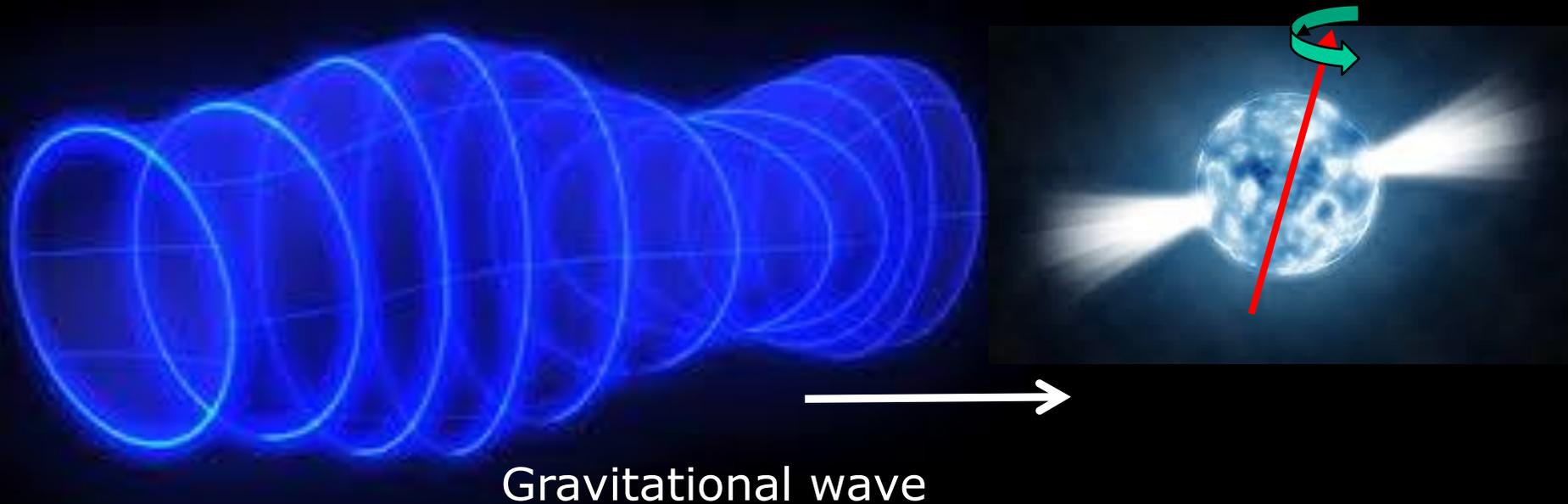
An important feature of pulsar detectors:

Depending on source direction w.r.t pulsar spin, Pulse Profile will be modified. In principle, single pulsar observation will have detailed information about source direction

May also enable detection of any circularly polarized Components in the incoming gravitational wave.

(e.g. from fluid circulation in core collapsed supernova)

Not easy with conventional detectors



We analyzed specific GW events detected by LIGO/Virgo and have identified specific pulsars whose perturbed signals will reach us, say, within next 50 years: Example:

GW source	Pulsar	Signal arrival time mm/dd/year	mean time max time
GW170814 (BH-BH)	J0437-4715	1/14/2035	2038-2043

↓

Closest and brightest pulsar known, $T \sim 5$ ms, distance ~ 500 ly X-ray, in binary with white dwarf, Pictor constellation (South)

Past Supernova events as GW sources:

Estimated GW strain from a type-II supernova (even for a type 1A Supernova) can reach as high as 10^{-20} at a distance of 10 kpc.

We have analyzed recorded supernova events, GWs from these will leave imprints on the pulsars will perturbed signal arrival dates.

Example:

SN1604	J1759-1956	3/1/2020	2039-2060
--------	------------	----------	-----------

(Note: In following tables, errors not available for all cases)

Table I: GW signal arrival dates for next 50 years

Source	Pulsar	Min. date mm/dd/year	Mean year	Max. year	
GW150914	J0736-6304	4/24/2031	2031	2081	
GW170814	J0437-4715	1/14/2035	2038	2043	
GW170817	J1400-1431	BNS 5/21/2048	2048	2052	
GW170818	J2307+2225	12/ 2/2031	2037	2066	
SN185	J0900-3144	9/ 5/2033	2049	2066	
SN1006	B1944+17	4/ 7/2029	2032	2035	
	J0633+1746	4/10/2032	2032	2033	
	J0108-1431	11/28/2033	2034	2036	
	J2124-3358	10/23/2038	2044	2050	
Crab	J1856-3754	3/21/2057	2057	2058	
	J0919-42	6/21/2058	2076	2106	
SN1604	J1734-3058	6/15/2019	2029	2040	
	J1759-1956	3/ 1/2020	2039	2060	
	J1736-3511	4/24/2025	2030	2035	
	B1726-00	2/21/2027	2029	2031	
	J1738+04 (2027-2030), B1734-35 (2031-2040),				
	J1800-0125 (2033-2037), J1832+0029 (2034-2036),				
	J1911-1114 (2035-2038), B1800-21 (2035-2070),				
	J1654-23 (2037-2064), J1851-0053 (2045-2048),				
	J1609-1930 (2054-2060), B1732-02 (2062-2068),				
J1754-3510 (2063-2074), B1804-12 (2064-2078),					
J1758-2630 (2065-2101), J1717+03 (2067-2070).					

Table I: GW signal arrival dates for next 50 years

Source	Pulsar	Min. date mm/dd/year	Mean year	Max. year
SN1885	J0139+33	11/12/2024	2024	2024
	B2310+42	11/19/2030	2030	2030
	J2302+4442	2/22/2031	2031	2031
	J0103+54	4/ 1/2060	2060	2060
	B2334+61	8/ 9/2064	2064	2064
	J2307+2225	7/21/2065	2065	2065
SN1987A	J0709-5923	5/ 9/2023	2023	2023
	B0559-57	7/ 8/2024	2024	2024
	B0923-58	8/ 1/2024	2024	2024
	J0621-55	11/15/2024	2024	2024
	J0437-4715 (2028-2029), B0403-76 (2034), J0656-5449 (2041), J1107-5907 (2050), J1017-7156 (2050), B1055-52 (2052), B1014-53 (2055), J0849-6322 (2060), J1000-5149 (2062), B0901-63 (2064).			

date of earliest signal arrival is given here in days and months, even though the error estimates are in years. It only reflects exactly known date of the GW source, and illustrates the possibility that with more accurate observations (source and pulsar locations), in principle, a very precise date of signal arrival on earth could be predicted.

Table II: GW signal arrival dates for past 50 years

Source	Pulsar	Min. date mm/dd/year	Mean year	Max. year
SN185	J2241-5236	2/ 1/1968	1985	2004
	J1858-2216	6/ 7/2015	2031	2049
SN1006	J0934-4154	3/21/1989	1997	2006
	J0621-55	11/27/1989	1994	1999
	B1118-79	5/ 3/1994	2011	2029
	J1729-2117	3/21/1999	2022	2048
	B1919+21	8/20/2006	2009	2012
Crab	J2322-2650	8/ 5/1996	2005	2017
SN1604	J1755-2534	12/ 8/1968	1983	1998
	J1758-1931	10/13/1970	1986	2004
	J1725-2852	11/23/1970	1983	1997
	J1756-2225	6/30/1971	1991	2013
	B1642-03 (1974-1977), J1744-3130 (1979-1993), J1105-43 (1984), B1919+21 (1984), B1944+17 (1992-1993), J1737-3102 (1994-2011), J1000-5149 (2004-2005), J1813-1246 (2004-2013), B1740-31 (2007-2023), J1759-3107 (2008-2021), B0538-75 (2009), B1612-29 (2010-2015), J1750-28 (2015-2041), J1741-3016 (2016-2038).			

Table II: GW signal arrival dates for past 50 years

Source	Pulsar	Min. date mm/dd/year	Mean year	Max. year
SN1885	J0242+62	4/ 5/1969	1969	1969
	B0052+51	4/26/1987	1987	1987
	J0059+50	11/ 9/1995	1995	1995
	B0045+33	10/15/2000	2000	2000
	J0106+4855	8/13/2001	2001	2001
SN1987A	J0540-7125	6/24/1988	1988	1988
	B0538-75	1/21/1990	1990	1990
	J0711-6830	4/ 1/1991	1991	1991
	J0749-68	12/26/1994	1994	1994
	J0736-6304 (1996), J0834-60 (2007),		J0511-6508 (1996), J0457-6337 (2018).	

How far back in past we can go?

just from the size of the Milky Way, we note that the oldest GW signals one can detect at present using galactic pulsars are those which passed by earth about 200,000 years ago.

Concluding remarks

Pulsar measurements extremely accurate, can detect tiny changes in pulsar structure, either due to internal dynamics, or due to GWs

Pulsars very far away can act as remotely stationed Weber detectors of gravitational waves.

Important to accurately measure signals of pulsars in other galaxies. Reaching out for extra galactic pulsars will be very important.

Possibility of some GW source close to the pulsar, while being very far away from us, will show the real strength of this technique.

Note: for very distant pulsars, timing errors large due to inter-galactic medium changes. This is important for long time stability of signal. But for a GW pulse, it may not be important.

Importantly: Pulsars have a wide frequency range for resonant modes

Could be sensitive to mergers of light primordial black holes

Important to emphasize, various estimates we have made are crude, **it is not very clear if the effects are observable with Present level of accuracy of pulsar measurements.**

However, for past GW events recorded on earth:

The prediction of Dates on which “**perturbed-signals**” from pulsars will reach earth is **beyond question** (apart from error estimates in timing).

It is just trigonometry

It is then worth the effort that we focus on **improving accuracy of predicted dates** and attempt to make best possible measurements of pulses at those times.

After all, we do not know Neutron stars interiors so well

They just might surprise us pleasantly.

Not doing anything on those dates just does not seem right

If you are still not convinced that pulsars make excellent Weber GW detectors, then the following quotation might help

“ 4. Resonant-mass detectors

.....

4.5. Antenna materials :

An ideal resonant bar would consist of a piece of nuclear matter, with high density and a velocity of sound comparable to the velocity of light! **Since this is not available except in neutron stars**, we must find a form of molecular matter which, to maximize coupling to gravitational waves, combines high velocity of sound v_s , and high density ρ . To reduce the thermal noise we require a low acoustic loss Q^{-1} . ”

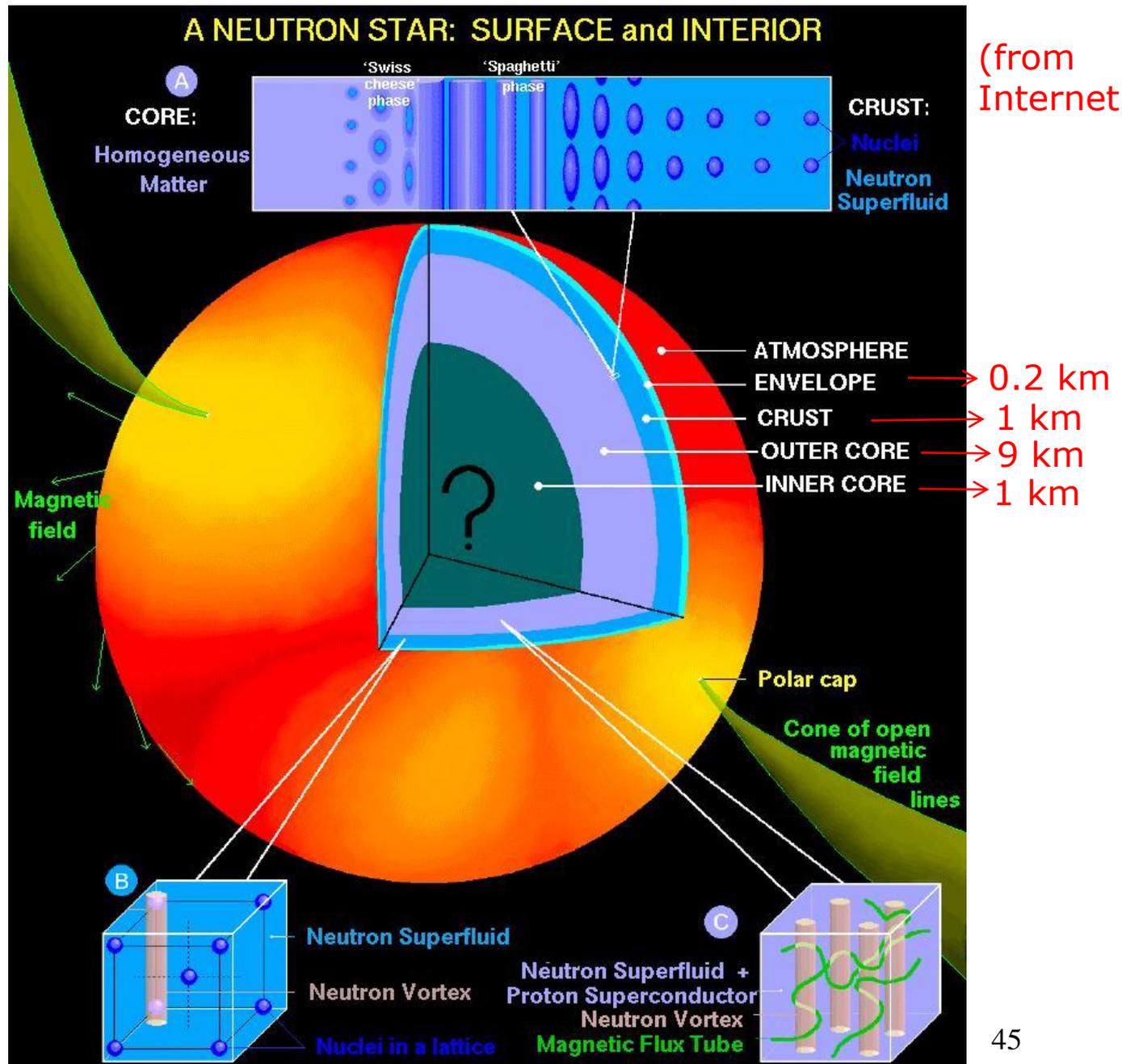
L. Ju, D. G. Blair. and C. Zhao,
“Detection of gravitational waves”,
Rep. Prog. Phys. 63 (2000) 1317–1427

THANK YOU

Inner Core:
 Size ~ 1 Km
 QCD phases:
 QGP, CFL, etc.

Outer Core:
 Size ~ 9 Km
 Superfluid
 Phases of
 Nucleons

Superfluid
 Vortices play
 Important role
 in glitches



(from Internet)

(Conventional) Superfluid interior of neutron star:

Nucleons inside neutron star form a highly degenerate system.
(Temperature of few MeV \ll chemical potential \sim 1 GeV)

Yukawa one pion exchange potential for N-N interaction is attractive:

Cooper problem:

Any attractive interaction destabilizes Fermi surface

Formation of Cooper pairs of neutrons at the Fermi surface.
leading to superfluid state (superconducting state for protons).

Inner Crust region:

Superfluid 1S_0 phase appears in the inner crust region:

Extending from $\rho \sim 4 \times 10^{11}$ g/cm³ to 2×10^{14} g/cm³

Condensation energy/N \sim 0.3 MeV, much smaller than other relevant energy scales.

For the 3P_2 channel, the interaction becomes strongly attractive only at higher energy, this occurs in the core of the neutron star.

Superfluid phase of neutron star:

Observational evidence:

Pulsars are rapidly Rotating neutron stars,

detected by their periodic pulses (electromagnetic waves),
Beamed emission from the magnetic poles of the neutron star.

Superfluid phase in the interior allows for vortex lattice to form.

Vortices are pinned at the interface with crust.

Superfluid vortices play fundamental role in determining rotational properties of neutrons star, hence pulsar timings.

Phenomenon of glitches explained by vortex depinning from crust.

Sudden transfer of angular momentum from superfluid to outer crust
Leads to glitches in pulsar timing.

Pulsars are rotating neutron stars

Observations: Extremely precise measurements of Pulsar timings (recall: first detection of gravitational waves)

Glitches : Sudden increase in rotational frequency,
Slow relaxation, over several months

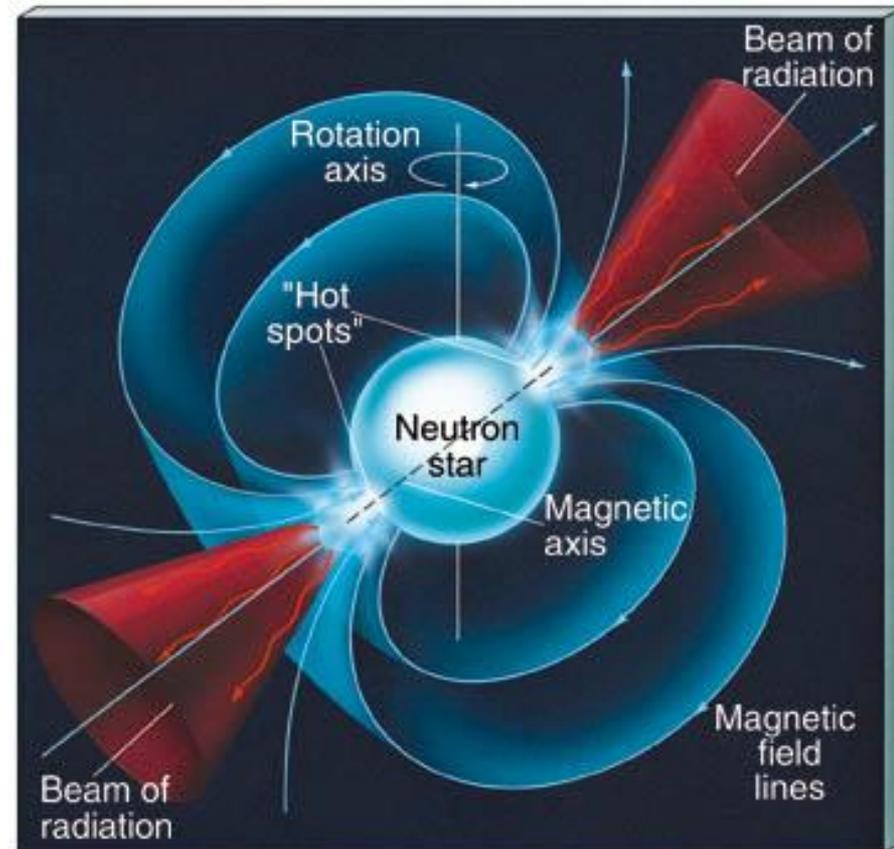
$$\frac{\delta\nu}{\nu} \sim 10^{-10} - 10^{-9}$$

Present understanding in terms of crustquakes, or transfer of angular momentum due to de-pinning of clusters of superfluid vortices

Relatively recent observations:

Anti-glitches : sudden slowing down of rotation

De-pinning of vortices cannot account for these anti-glitches



Neutron star resonant modes: Krüger et. al PRD,92, 063009 (2015)

Different restoring forces result in distinct classes of modes in the NS oscillation spectrum.

f-mode: Due to the presence of neutron star surface

i-modes : Due to interfaces between different layers

Acoustic p-modes: restoring force from pressure

Gravity g-modes: From buoyancy due to internal stratification

Composition g-modes: from composition gradients

which are present also at zero temperature

Thermal g-modes: dominated by thermal effects

In the slow reaction limit (e.g. when relevant weak-interaction processes to restore β -equilibrium is much longer than the typical oscillation period), then buoyancy arises for small fluid element deviations from equilibrium. This leads to Composition g-mode.

In the fast reaction limit, β -equilibrium is maintained at all times and the star is called barotropic. Perturbed fluid elements are not affected by buoyancy and these g-modes are absent.

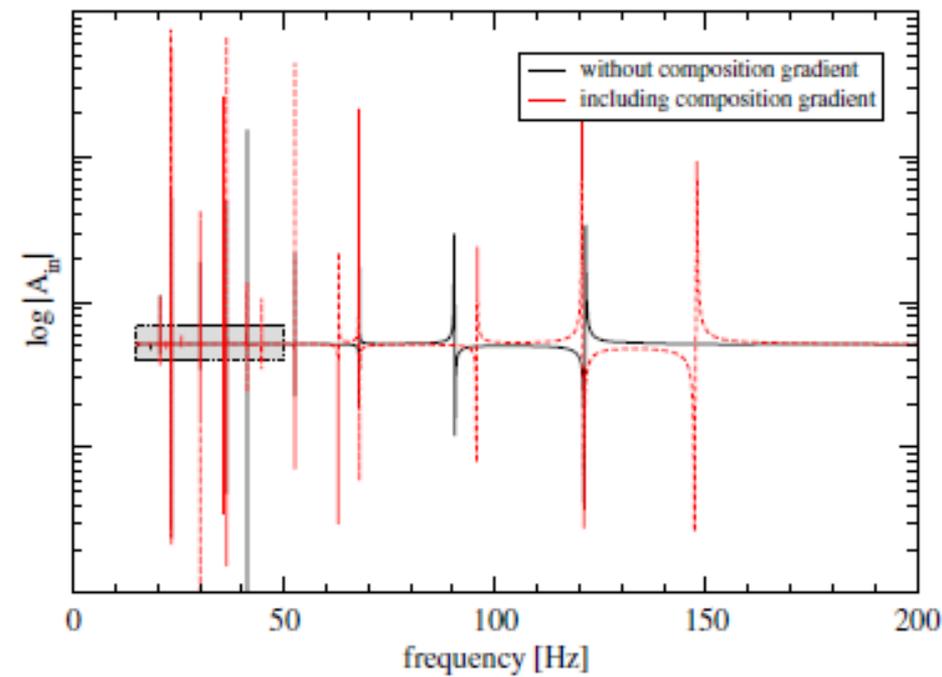
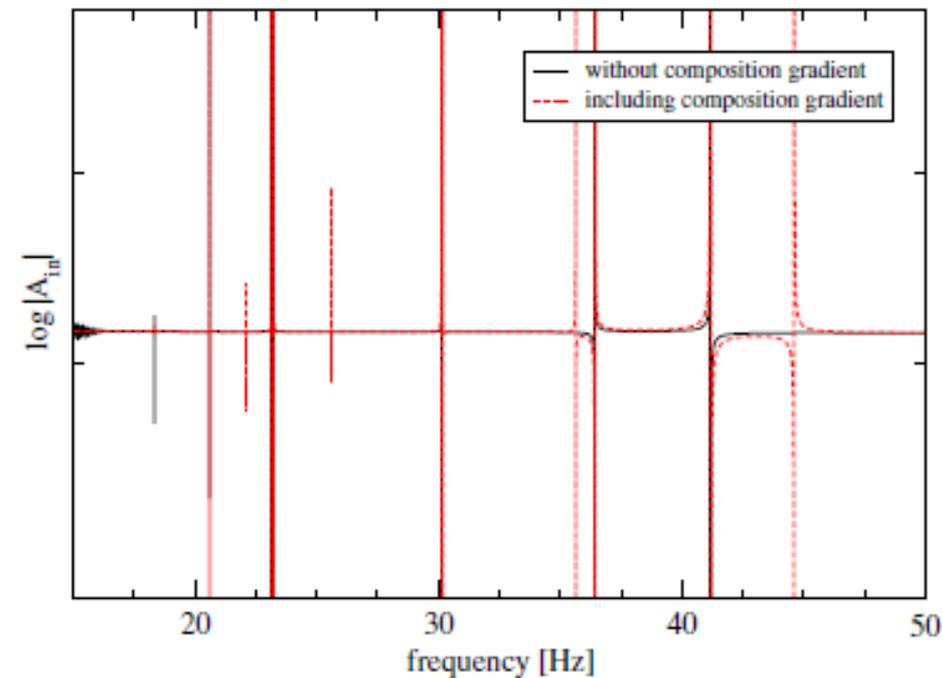


FIG. 3 (color online). The low-frequency spectrum of our neutron star at zero temperature and without a solid crust. The upper panel shows the spectrum up to 200 Hz whereas the lower panel shows a magnification of the grey shaded area. The solid line shows the spectrum of the pure perfect fluid star, while the dashed line includes composition gradients. All spikes in the perfect fluid spectrum are interface modes as there is no composition gradient present; for each of these modes, there is also a mode present in the spectrum of the stratified star (with the exception of the mode at 90.4 Hz; see text). The “new” modes in the stratified spectrum are composition g -modes.

Krüger et. al PRD,92, 063009 (2015)



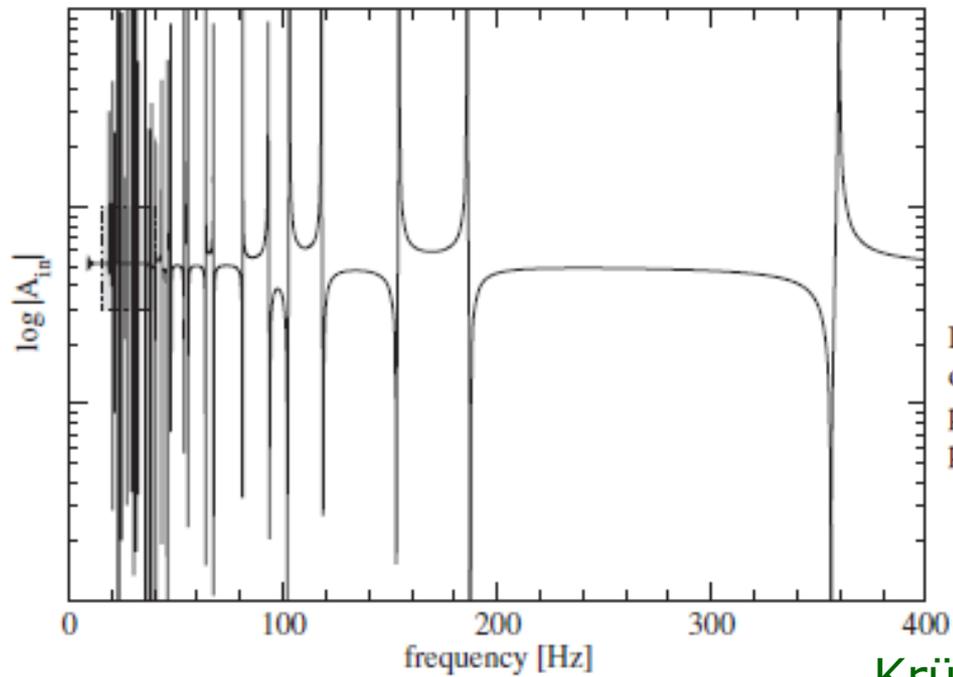
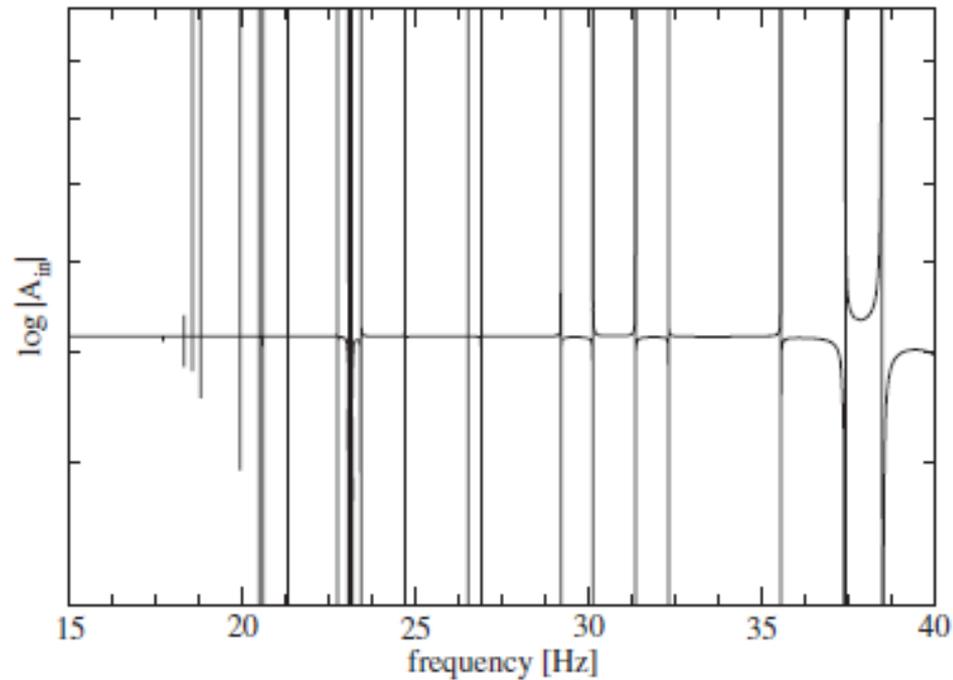


FIG. 5. The low-frequency spectrum when the star is 3 seconds old, including thermal pressure but without solid crust. The lower panel shows a magnification of the grey shaded area in the upper panel.

Krüger et. al PRD,92, 063009 (2015)



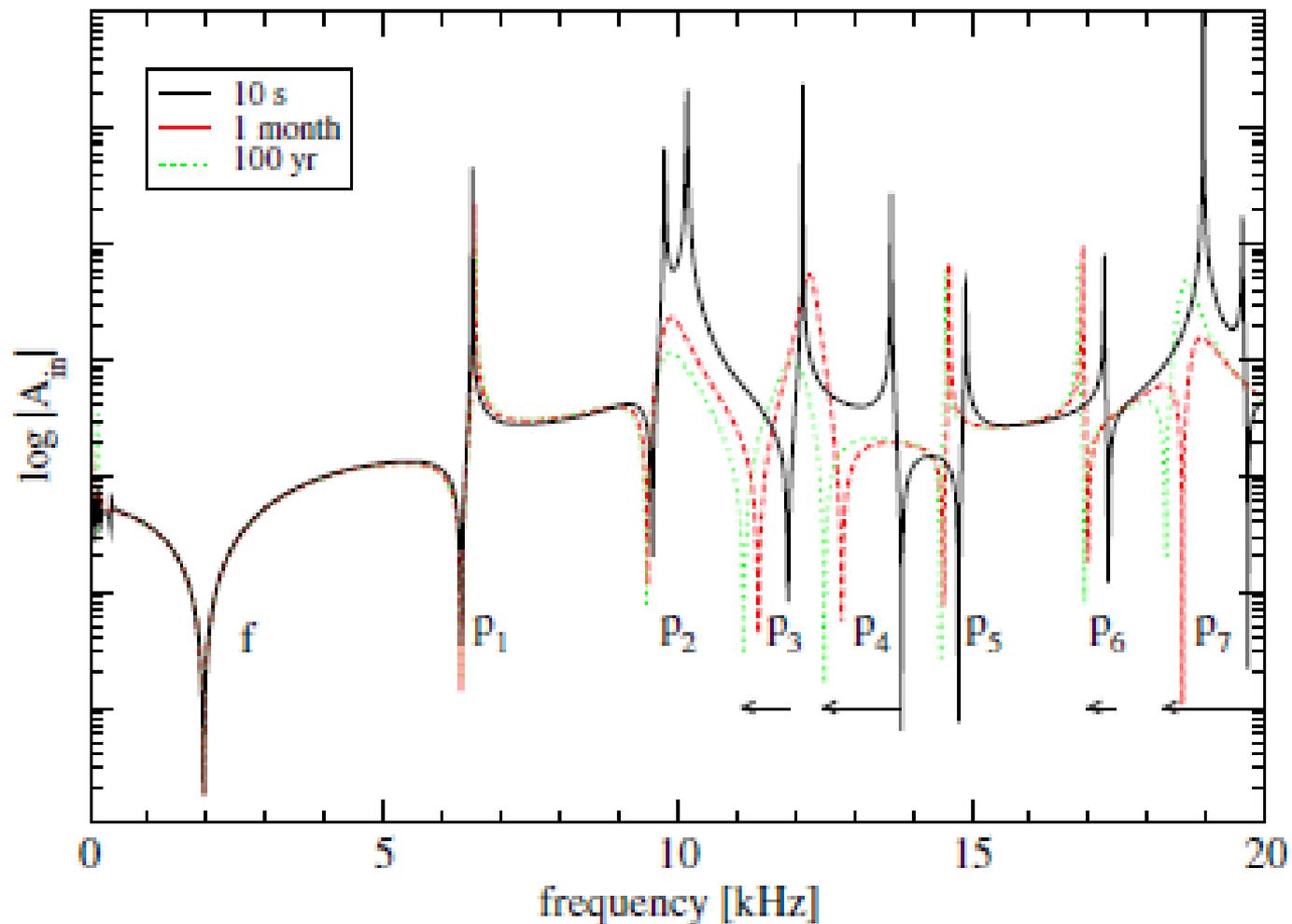


FIG. 7 (color online). The stellar spectrum up to 20 kHz for three different temperature profiles. Clearly visible are the f -mode and first 7 p -modes. Where the frequency of a p -mode varies visibly over time, an arrow indicates this change.

Tidal deformability (T. Hinderer, *Astrophys. J.* 677, 1216 (2008))

Deformation of neutron star in the tidal field E_{ij} of gravitational wave is given by

$$Q_{ij} = -\lambda_d E_{ij}$$

Tidal Deformability λ_d is related to the second Love number k_2

$$\lambda_d = \frac{2}{3} k_2 \frac{R^5}{G}$$

For polytropic pressure-density relation: $P = K\rho^{1+\frac{1}{n}}$

Where K is a constant and n is the polytropic index, numerical results (for $0.5 \leq n \leq 1.0$, and $0.1 \leq (M/R) \leq 0.24$) can be fitted by the formula:

$$k_2 \approx \frac{3}{2} \left(-0.41 + \frac{0.56}{n^{0.33}} \right) \left(\frac{M}{R} \right)^{-0.003} .$$

Tidal field of external gravitational potential:

$$\frac{(1 - g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3} \left(n^i n^j - \frac{1}{3} \delta^{ij} \right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2} \mathcal{E}_{ij} x^i x^j + O(r^3), \quad (1)$$

where $n^i = x^i/r$; this expansion defines \mathcal{E}_{ij} and Q_{ij} . In the Newtonian limit, Q_{ij} is related to the density perturbation $\delta\rho$ by

$$Q_{ij} = \int d^3x \delta\rho(\mathbf{x}) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right), \quad (2)$$

and \mathcal{E}_{ij} is given in terms of the external gravitational potential Φ_{ext} as

$$\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}. \quad (3)$$