New ultraviolet operators in supersymmetric $\mathrm{SO}(10)$ GUT and consistent cosmology

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Outline of the talk

- Summary
- Our method
- Technical details
- Our results

Summary: I

- Supersymmetric Grand Unified Theory based on $\mathrm{SO}(10)$ ($\mathrm{SO}(10)$ MSGUT) has elegant features of both GUT based models and SUSY models e.g. heavy RH neutrinos, smallness of neutrino masses, dark matter candidate,..
- $\mathrm{SO}(10)$ broken down by SUSY preserving vevs to MSSM.
- SUSY preserving vevs isolated but not unique e.g. D-parity applied to SUSY preserving vev breaking $\mathrm{SO}(10)$ to MSSM gives a different SUSY preserving vev
- Leads to an energy barrier between any two distinct SUSY preserving vevs which we call pseudo-defect or pseudo-domain wall (pseudo-DW).
- Can even lead to true topological defects called domain walls for some patterns of breaking $\mathrm{SO}(10)$ MSGUT to MSSM e.g. $\mathrm{SO}(10) \to \mathrm{SO}(6) \times \mathrm{SO}(4) \times D \stackrel{\mathrm{DW}}{\to} \mathrm{SO}(6) \times \mathrm{SO}(4) \to \mathrm{MSSM}.$

Summary: II

- Existence of pseudo-DW conflicts with standard cosmology. So it must be made to vanish by the current epoch.
- Idea: Add Planck suppressed degree 4 non-renormalisable terms to the superpotential so as to create a tiny pressure difference across the pseudo-DW which causes it to go away.
- Turns out $\mathrm{SO}(10)$-invariant higher degree terms will NOT create any pressure difference :-(
- So we add a cleverly chosen degree 4 term that breaks $\mathrm{SO}(10)$ but still preserves SM. That creates a pressure difference which can then remove the pseudo-DW.

Our method: I

- To obtain the additional degree 4 non-renormalisable term, we take inspiration from Gell-Mann and Okubo.
- Gell-Mann and Okubo asked why the 8 pseudoscalar mesons had slightly different masses despite $\mathrm{SU}(3)$ flavour symmetry amongst u, d and s quarks.
- They conjectured that the flavour $\mathrm{SU}(3)$ invariant Hamiltonian $H_0 = \frac{\mu^2}{2}\mathrm{Tr}\,[\Pi^\dagger \Pi]$, Π being the 8-dim adjoint irrep. of $\mathrm{SU}(3)$, was broken by adding a small term H'.
- They took $H' = \frac{\alpha}{2}\mathrm{Tr}\,[\Pi^\dagger \Pi M]$, where 3×3 'coefficient matrix' M was weak isospin $(\mathrm{SU}(2) \times \mathrm{U}(1))$-invariant but not $\mathrm{SU}(3)$-invariant.
- This uniquely fixed M and gave a good fit to the mass relationships amongst the pseudoscalar mesons.

Our method: II

Heavy Higgs part of the renormalisable superpotential of $\mathrm{SO}(10)$ MSGUT

$$W_{\mathrm{ren}} = \frac{m}{4!}\Phi_{ijkl}\Phi_{ijkl} + \frac{\lambda}{4!}\Phi_{ijkl}\Phi_{klmn}\Phi_{mnij} + \frac{M}{5!}\Sigma_{ijklm}\bar{\Sigma}_{ijklm} + \frac{\eta}{4!}\Phi_{ijkl}\Sigma_{ijmno}\bar{\Sigma}_{ijmno},$$

where all indices range independently from 1 to 10.

Φ is 4-index antisym. irrep. $\mathbf{210} = \mathbf{10} \wedge \mathbf{10} \wedge \mathbf{10} \wedge \mathbf{10}$,

Σ is the 5-index self-dual antisym. irrep. $\mathbf{126}$, $\bar{\Sigma}$ is the 5-index anti-self-dual antisym. irrep. $\overline{\mathbf{126}}$,
$\mathbf{126} \oplus \overline{\mathbf{126}} = \mathbf{10} \wedge \mathbf{10} \wedge \mathbf{10} \wedge \mathbf{10} \wedge \mathbf{10}$.

Our method: III

- Extended superpotential

$$W = W_{\rm ren} + \frac{b}{M_{\rm Pl}}\frac{1}{(4!)^4}(\Phi^T M' \Phi)^2,$$

  where $10^4 \times 10^4$ 'coefficient matrix' $M' = M \otimes M \otimes M \otimes M$, M being a carefully chosen element of $\mathrm{SO}(10)$.

- M commutes with the image of SM gauge group inside $\mathrm{SO}(10)$, M does not commute with D-parity. $D \in \mathrm{SO}(10)$, D is a 10×10 diagonal matrix, $D_{ii} = -1$ if $i = 2, 3, 6, 7$, $D_{ii} = 1$ otherwise. Hence extended superpotential is SM-invariant but not $\mathrm{SO}(10)$-invariant.

- In fact, our M commutes with the image of $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_{B-L} \times \mathrm{U}(1)_R$ inside $\mathrm{SO}(10)$.

Our method: IV

- The SUSY preserving vev breaking $\mathrm{SO}(10)$ to MSSM is parametrised by three numbers a, ω, p.
- a occurs in 3 coordinates of the 210-dim. rep. Φ, ω in 6 coordinates, and p in 1 coordinate.
- Applying D-parity operator on above vev gives a new SUSY preserving vev that breaks $\mathrm{SO}(10)$ down to some other gauge group.
- D-parity preserves the 3 occurences of a and 3 occurences of ω. It negates the other 3 occurences of ω and the 1 occurence of p.

Our method: V

- Since the original vev and the D-parity flipped vev are isolated in ‘vev space’ and are both SUSY preserving, any path in ‘vev space’ connecting them must necessarily break SUSY and have a non-zero scalar potential. This creates an energy barrier between the two vevs called pseudo-DW.
- Since the F-terms evaluated at flipped vev are either the same or negated with respect to their evaluations at the original vev, the scalar potential is zero on both sides of the pseudo-DW. So there is no pressure difference across the pseudo-DW.
- Adding degree 4 $\mathrm{SO}(10)$-invariant terms to superpotential does not help because the F-terms evaluated at original vev are either the same or negated wrt being evaluated at flipped vev. So there is still no pressure difference.

Our method: VI

- However, adding $\frac{b}{M_{\rm Pl}}\frac{1}{(4!)^4}(\Phi^T(M^{\otimes 4})\Phi)^2$ to the renormalisable superpotential gives F-terms that evaluate to zero at original vevs (because $M \in \mathrm{SO}(10)$ commutes with SM) and non-zero at flipped vevs (because M does not commute with D).
- This gives rise to a pressure difference across the pseudo-DW which can eventually remove it.
- We get constraints on $|b|$, energy scales of symmetry breaking etc. as a result.

Technical details: I

Renormalisable superpotential:

$$W_{\rm ren} = \frac{m}{4!}\Phi_{ijkl}\Phi_{ijkl} + \frac{\lambda}{4!}\Phi_{ijkl}\Phi_{klmn}\Phi_{mnij} + \frac{M}{5!}\Sigma_{ijklm}\bar{\Sigma}_{ijklm} + \frac{\eta}{4!}\Phi_{ijkl}\Sigma_{ijmno}\bar{\Sigma}_{ijmno},$$

Original vev breaking $\mathrm{SO}(10)$ to MSSM: (Aulakh-Girdhar, Nucl. Phys. B711, 2005), unique charge zero vev conserving SM

$$\Phi_{1234} = \Phi_{1256} = \Phi_{3456} = a,$$
$$\Phi_{1278} = \Phi_{1290} = \Phi_{3478} = \Phi_{3490} = \Phi_{5678} = \Phi_{5690} = \omega,$$
$$\Phi_{7890} = p,$$
$$a = -\frac{m}{\lambda}\frac{1-2x-x^2}{1-x}, \quad \omega = -\frac{m}{\lambda}x, \quad p = -\frac{m}{\lambda}\frac{x(1-5x^2)}{(1-x)^2},$$
$$-8x^3 + 15x^2 - 14x + 3 = (x-1)^2\frac{\lambda M}{\eta m}.$$

Vevs for Σ, $\bar{\Sigma}$ omitted.

Direct breaking of $\mathrm{SO}(10)$ to MSSM for generic x e.g. $x = 0.21$ by Aulakh-Girdhar.

Technical details: II

Original vev breaking $\mathrm{SO}(10)$ to MSSM:

$$\Phi_{1234} = \Phi_{1256} = \Phi_{3456} = a,$$
$$\Phi_{1278} = \Phi_{1290} = \Phi_{3478} = \Phi_{3490} = \Phi_{5678} = \Phi_{5690} = \omega,$$
$$\Phi_{7890} = p,$$

D-parity flipped vev: D diagonal, $D_{ii} = -1$ if $i = 2, 3, 6, 7$, $D_{ii} = 1$ o/w.

$$\Phi_{1234} = \Phi_{1256} = \Phi_{3456} = a,$$
$$\Phi_{1278} = \Phi_{3478} = \Phi_{5678} = \omega, \quad \Phi_{1290} = \Phi_{3490} = \Phi_{5690} = -\omega,$$
$$\Phi_{7890} = -p,$$

F-term	Original vev	D-Flipped vev
$F_{\Phi_{1234}} = F_{\Phi_{1256}} = F_{\Phi_{3456}}$	$2ma + 2\lambda(a^2 + 2\omega^2) + \eta\sigma\bar{\sigma}$	$2ma + 2\lambda(a^2 + 2\omega^2) + \eta\sigma\bar{\sigma}$,
$F_{\Phi_{1278}} = F_{\Phi_{3479}} = F_{\Phi_{5678}}$	$2m\omega + 2\lambda(2a + p)\omega - \eta\sigma\bar{\sigma}$	$2m\omega + 2\lambda(2a + p)\omega - \eta\sigma\bar{\sigma}$,
$F_{\Phi_{1290}} = F_{\Phi_{3490}} = F_{\Phi_{5690}}$	$2m\omega + 2\lambda(2a + p)\omega - \eta\sigma\bar{\sigma}$	$-2m\omega - 2\lambda(2a + p)\omega + \eta\sigma\bar{\sigma}$,
$F_{\Phi_{7890}}$	$2mp + 6\lambda\omega^2 + \eta\sigma\bar{\sigma}$	$-2mp - 6\lambda\omega^2 - \eta\sigma\bar{\sigma}$
Other $F_{\Phi_{ijkl}}$	0	0

Technical details: III

Extended superpotential:

$$W = W_{\mathrm{ren}} + \frac{b}{M_{\mathrm{Pl}}} \frac{1}{(4!)^4} (\Phi^T M^{\otimes 4} \Phi)^2,$$

Want: $M \in \mathrm{SO}(10)$, M commutes with embedding of $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ into $\mathrm{SO}(10)$, M does not commute with D.

SM embedded into $\mathrm{SO}(10)$ via Pati-Salam $\mathrm{SO}(6) \times \mathrm{SO}(4)$:
$\mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L}$ embedded into $\mathrm{SU}(4)$, $\mathrm{su}(4) \cong \mathrm{so}(6)$, thus $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L}$ is embedded into the top left $\mathrm{SO}(6)$.

$\mathrm{so}(4) \cong \mathrm{su}(2)_L \oplus \mathrm{su}(2)_R$, $\mathrm{so}(4)$ consists of 4×4 antisymmetric matrices, can take a basis consisting of 3 anti-self-dual and 3 self-dual antisymmetric matrices.

$\mathrm{su}(2)_L$ isomorphic to anti-self-dual part, $\mathrm{su}(2)_R$ isomorphic to self-dual part. This embeds $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ into the bottom right $\mathrm{SO}(4)$.

Technical details: IV

Self-dual and anti-self-dual parts commute.

So any linear combination $J = j_1 J_1^+ + j_2 J_2^+ + j_3 J_3^+$ of the self-dual generators J_1^+, J_2^+, J_3^+ will commute with $\mathrm{su}(2)_L$. Fix one such J and call it the generator of $\mathrm{u}(1)_R$.

Let $K = \exp(J)$ where j_1, j_2, j_3 chosen suitably so that K is an element of $\mathrm{SO}(4)$. Then

$$M = \left(\begin{array}{c|c} I_{6\times 6} & 0_{6\times 4} \\ \hline 0_{4\times 6} & K_{4\times 4} \end{array} \right)$$

is an element of $\mathrm{SO}(10)$ which commutes with $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ embedded into $\mathrm{SO}(10)$, and hence with SM embedded into $\mathrm{SO}(10)$.

Technical details: V

Finally:

$$M = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cccc} \sqrt{2}\, I_{6\times 6} & & 0_{6\times 4} & & \\ \hline & 0 & 1 & 0 & 1 \\ 0_{4\times 6} & -1 & 0 & 1 & 0 \\ & 0 & -1 & 0 & 1 \\ & -1 & 0 & -1 & 0 \end{array} \right).$$

is an element of $\mathrm{SO}(10)$ which commutes with SM embedded into $\mathrm{SO}(10)$.

Easy to check that M does not commute with D.

Technical details: VI

$$V|_{\text{flipped vevs}} - V|_{\text{original vevs}}$$
$$= \frac{|b|^2}{M_{\rm Pl}^2}(4^2(3a^2+6\omega^2+p^2))((3a^2+6\omega^2+p^2)^2-(3a^2+p^2)^2).$$

We take a generic x close to middle of $(0,1)$ e.g. $x = 0.21$ of Aulakh-Girdhar. Then $\omega \ll \max\{a,p\} \sim m$.

Following Aulakh-Girdhar, we set $m = M_X$, the mass of the lightest superheavy vector particle mediating proton decay and $\omega \sim M_R$, the energy scale of pseudo-DW formation.

Then

$$|V|_{\text{flipped vevs}} - V|_{\text{original vevs}}| \sim \frac{|b|^2}{M_{\rm Pl}^2} M_X^4 M_R^2.$$

Our results: I

For successful pseudo-DW removal, we get

- Removal in radiation dominated era: $b^2 \geq \frac{M_R^4}{M_X^4}$,
- Removal in matter dominated era: $b^2 \geq \frac{M_R^{7/2} M_{\rm Pl}^{1/2}}{M_X^4}$.
- Removal in weak inflation phase of matter dominated era: $|b|^2 \geq \frac{T_D^{12} M_{\rm Pl}^5}{M_R^{13} M_X^4}$, where T_D is the unknown temp. scale where the pseudo-DW begins to experience instability.

Our results: II

Era	M_R	$b \geq$
RD	10^7	10^{-18}
	10^9	10^{-14}
	10^{13}	10^{-6}
MD	10^7	10^{-15}
	10^9	$10^{-11.5}$
	10^{13}	$10^{-4.5}$

Era	M_R	$b \leq 1, \frac{T_D}{M_R} \leq$
WI	10^7	$10^{-3.17}$
	10^9	$10^{-3.3}$
	10^{13}	$10^{-3.67}$

Our results: III

Miniscule values of b suffice to remove pseudo-DW in radiation dominated and matter dominated eras. Our model is hardly constrained in these two scenarios.

Assuming $|b| \leq O(1)$, we get $\frac{T_D}{M_R} < 10^{-3}$ making the weak inflation scenario marginal for our model.

For $T_D > 10\mathrm{MeV}$ (required by Big Bang Nucleosynthesis), the model comfortably accomodates $\gtrsim 10^{10}\mathrm{GeV}$ scale expected of thermal leptogenesis.

Thank you!

Backup: I

Duality in $\mathrm{so}(2n)$: The dual of a k-index antisym. tensor $T_{i_1 \cdots i_k}$ is a $(2n-k)$-index antisym. tensor given by

$$\tilde{T}_{j_1 \cdots j_{2n-k}} = \frac{(-i)^n}{k!} \epsilon_{j_1 \cdots j_{2n-k} i_1 \cdots i_k} T_{i_1 \cdots i_k},$$

where ϵ is the $(2n)$-index antisym. tensor taking values ± 1.

This tells us that, for $k < n$, the $(2n-k)$-index antisym. irrep. is isomorphic to the k-index antisym. irrep.

Furthermore, the n-index antisym. rep. is reducible and is the direct sum of two irreps.

For $n = 5$, the 5-index antisym. rep. is $\binom{10}{5} = 252$ dim., and is the direct sum of two irreps called $\mathbf{126}$ and $\overline{\mathbf{126}}$.

For $n = 2$, the 2-index antisym. rep. is $\binom{4}{2} = 6$ dim. and is the same as the adjoint rep. It is the direct sum of two irreps of dim. 3. This gives the isomorphism $\mathrm{so}(4) \cong \mathrm{su}(2) \oplus \mathrm{su}(2)$.

Backup: II

Self-dual and anti-self-dual in $\mathrm{so}(2n)$: An n-index antisym. tensor T is self-dual if $T = \tilde{T}$ and anti-self-dual if $T = -\tilde{T}$.

Given any such tensor T, we can write it as $T = T^+ + T^-$, where $T^+ = \frac{T+\tilde{T}}{2}$, $T^- = \frac{T-\tilde{T}}{2}$. Now T^+ is self-dual and T^- is anti-self-dual. This gives the decomposition of the n-index antisym. rep. into two irreps of the same dim. viz. self-dual part and anti-self-dual part.

For $n = 2$, start with the standard basis for 4×4 antisym. matrices (same as the 2-index antisym. tensors) viz. M^{ij} is a 4×4 matrix with (i, j)th entry 1, (j, i)th entry -1 and the other entries 0. Then the above process gives a basis for the self-dual part as

$$J_1^+ = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, J_2^+ = -\frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, J_3^+ = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$