XVI WORKSHOP ON

High Energy Physics Phenomenology WHEPP XVI

Indian Institute of Technology Guwahati

Amplitude relations in b-baryon decays based on SU(3)-flavor analysis

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Motivations

- Direct CP violation (A_{CP}) asymmetry measurement in $\Lambda_b^0 \to p^+ K^-$ and $\Lambda_b^0 \to p^+ \pi^-$ first in CDF and subsequently in LHCb.

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- Goal: Formulate a general framework to analyze two body hadronic weak decays of b-baryons.

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- Identify CP violation relations in b-baryon decay modes.

He '15

SU(3) flavor symmetry

- The SU(3) triplet representation ($\mathbf{3}$) of quarks (q_i) and its conjugate ($\overline{\mathbf{3}}$) denoting the anti-quarks ($\overline{q_i}$) consist of the flavor states;

$$q_i = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad \overline{q_i} = \begin{pmatrix} \overline{d} \\ -\overline{u} \\ \overline{s} \end{pmatrix} \tag{1}$$

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- The ground state b-baryon has two light quarks (ud, us, ds) in addition to the b quark. Under SU(3) flavor, these b-baryons transform an SU(3) anti-triplet ($\overline{\mathbf{3}}$).

SU(3)..

- According to the sign convention chosen in Eq. (1), the pseudoscalar meson wavefunctions are given as,

$$K^+ = u\bar{s}, \qquad K^- = -s\bar{u}, \qquad K^0 = d\bar{s}, \qquad \overline{K}^0 = s\bar{d}$$

$$\pi^+ = u\bar{d}, \qquad \pi^- = -d\bar{u}, \qquad \pi^0 = \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$$

$$\eta_8 = -\frac{1}{2\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad \eta_1 = -\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

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- η_1 is an SU(3) flavor singlet.

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- The most general effective Hamiltonian for a two-body hadronic decay of a ground state b-baryon transforms as a,

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- $\mathcal{H}_{\text{gen}} = \mathbf{3} \oplus \overline{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{15}' \oplus \mathbf{24} \oplus \mathbf{42}$

SU(3)-decomposition of decay amplitudes

- The amplitude of the process $i \to f_b f_m$,

$$\mathcal{A}(i \to f_b f_m) = (-1)^{I_3+\frac{Y}{2}+\frac{T}{3}} \sum_{\substack{\{f,R\} \\ Y^b+Y^m=Y^f,\, Y^f-Y^i=Y^H \\ I_3^b+I_3^m=I_3^f,\, I_3^f-I_3^i=I_3^H}} C^{I_3^b\, I_3^m\, I_3^f}_{I^b\, I^m\, I^f} \quad C^{I_3^f\, -I_3^i\, I_3^H}_{I^f\, I^i\, I^H} \langle \mathbf{f} \parallel \mathbf{R_I} \parallel \mathbf{i} \rangle$$
$$\times \begin{pmatrix} \mathbf{f_b} & \mathbf{f_m} & \mathbf{f} \\ (Y^b, I^b, I_3^b) & (Y^m, I^m, I_3^m) & (Y^f, I^f, I_3^f) \end{pmatrix}$$
$$\times \begin{pmatrix} \mathbf{f} & \bar{\mathbf{i}} & \mathbf{R} \\ (Y^f, I^f, I_3^f) & (-Y^i, I^i, -I_3^i) & (Y^H, I^H, I_3^H) \end{pmatrix}, \tag{3}$$

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- By construction, the set of SU(3)-reduced matrix elements form a complete orthonormal basis.
- Apriori, all decay modes are independent, hence no relations among each other.

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- Use

$$\langle \mathbf{f} \parallel \mathbf{R_I} \parallel \mathbf{i} \rangle = \underbrace{\mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}} \sqrt{\frac{\text{dim f}}{\text{dim R}}}}_{\text{dynamical Coeff. of } \mathcal{H}} \langle \mathbf{f} \parallel \mathbf{R} \parallel \mathbf{i} \rangle. \tag{4}$$

where

$$\mathcal{H}_{\text{eff}}^{\text{dim=6}} = \sum_{\substack{\{Y,I,I_3\} \\ \mathbf{R}}} \mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}} \mathbf{R_I}, \tag{5}$$

Unbroken dim-6 effective Hamiltonian

$$\mathcal{H}^{\rm dim=6} = \underbrace{(\overline{q_i}b)(\overline{q_j}q_k)}_{\mathbf{3}\otimes\mathbf{3}\otimes\overline{\mathbf{3}}\equiv\mathbf{3}^{(\overline{\mathbf{3}})}\oplus\mathbf{3}^{(\mathbf{6})}\oplus\overline{\mathbf{6}}\oplus\mathbf{15}} \quad (6)$$

The effective Hamiltonian for charmless b decays,

- Tree operators:

$$\frac{\sqrt{2}\mathcal{H}_{\rm T}}{4G_F} = \Bigg\{\lambda_u^s\left[\frac{(C_1+C_2)}{2}\left(-\mathbf{15}_1-\frac{1}{\sqrt{2}}\mathbf{15}_0-\frac{1}{\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\right)+\frac{(C_1-C_2)}{2}\left(\mathbf{6}_1+\mathbf{3}_0^{(\overline{\mathbf{3}})}\right)\right]$$
$$+\lambda_u^d\left[\frac{(C_1+C_2)}{2}\left(-\frac{2}{\sqrt{3}}\mathbf{15}_{3/2}-\frac{1}{\sqrt{6}}\mathbf{15}_{1/2}-\frac{1}{\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\right)+\frac{(C_1-C_2)}{2}\left(-\mathbf{6}_{1/2}+\mathbf{3}_{1/2}^{(\overline{\mathbf{3}})}\right)\right]\Bigg\},$$

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- Electroweak penguin operators:

$$\frac{\sqrt{2}\mathcal{H}_{\text{EWP}}}{4G_F} = \Bigg\{-\lambda_t^s\left[\frac{(C_9+C_{10})}{2}\left(-\frac{3}{2}\mathbf{15}_1-\frac{3}{2\sqrt{2}}\mathbf{15}_0+\frac{1}{2\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\right)+\frac{(C_9-C_{10})}{2}\left(\frac{3}{2}\mathbf{6}_1+\frac{1}{2}\mathbf{3}_0^{(\overline{\mathbf{3}})}\right)\right]$$
$$-\lambda_t^d\left[\frac{(C_9+C_{10})}{2}\left(-\sqrt{3}\,\mathbf{15}_{3/2}-\frac{1}{2}\sqrt{\frac{3}{2}}\mathbf{15}_{1/2}+\frac{1}{2\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\right)+\frac{(C_9-C_{10})}{2}\left(-\frac{3}{2}\mathbf{6}_{1/2}+\frac{1}{2}\mathbf{3}_{1/2}^{(\overline{\mathbf{3}})}\right)\right]\Bigg\}.$$

- Project out the coefficients corresponding to the **15** part of the Hamiltonian. Relations between reduced matrix elements regardless of the initial and final states

$$\frac{\langle \mathbf{f} \parallel \mathbf{15_0} \parallel \mathbf{i} \rangle}{\langle \mathbf{f} \parallel \mathbf{15_1} \parallel \mathbf{i} \rangle} = \frac{1}{\sqrt{2}}, \quad \frac{\langle \mathbf{f} \parallel \mathbf{15_{\frac{1}{2}}} \parallel \mathbf{i} \rangle}{\langle \mathbf{f} \parallel \mathbf{15_{\frac{3}{2}}} \parallel \mathbf{i} \rangle} = \frac{1}{2\sqrt{2}},$$
$$\frac{\lambda_t^d}{\lambda_t^s} \frac{\langle \mathbf{f} \parallel \mathbf{15_0} \parallel \mathbf{i} \rangle_{\text{EWP}}}{\langle \mathbf{f} \parallel \mathbf{15_{\frac{1}{2}}} \parallel \mathbf{i} \rangle_{\text{EWP}}} = \sqrt{3}, \quad \frac{\lambda_u^d}{\lambda_u^s} \frac{\langle \mathbf{f} \parallel \mathbf{15_0} \parallel \mathbf{i} \rangle_{\text{T}}}{\langle \mathbf{f} \parallel \mathbf{15_{\frac{1}{2}}} \parallel \mathbf{i} \rangle_{\text{T}}} = \sqrt{3} \tag{7}$$

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- If more than one operator structure contributes to the Hamiltonian;

$$\frac{\langle \mathbf{f} \parallel \mathbf{R_I} \parallel \mathbf{i} \rangle}{\langle \mathbf{f} \parallel \mathbf{R_{I'}} \parallel \mathbf{i} \rangle} = \frac{\sum_l \mathcal{C}_l C_l}{\sum_m \mathcal{C}_m C'_m}, \tag{8}$$

where, the $C_i^{(')}$ are the coefficients of the different components of the Hamiltonian and $\mathcal{C}_j$'s are the CG coefficients and the sums extend over all the corresponding contributions to the Hamiltonian.

- Factor out the CKM elements $\lambda_{u,t}^{s,d}$ and write the decay amplitude in terms of tree and penguin reduced amplitudes

$$\begin{aligned} \mathcal{A}^{\mathcal{S}} =& \lambda_u^q \mathcal{A}_{\mathrm{T}}^{\mathcal{S}} + \lambda_t^q \mathcal{A}_{\mathrm{P}}^{\mathcal{S}}, \\ \mathcal{A}^{\mathcal{P}} =& \lambda_u^q \mathcal{A}_{\mathrm{T}}^{\mathcal{P}} + \lambda_t^q \mathcal{A}_{\mathrm{P}}^{\mathcal{P}}, \end{aligned} \tag{9}$$

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  where $q = s, d$ denote the $\Delta S = -1, 0$ process, $\mathcal{S}$ and $\mathcal{P}$ denote the S wave and P wave amplitudes of the decay.
- The $\Delta S = -1$ and $\Delta S = 0$ decay amplitudes and the reduced SU(3) elements are expressed as column matrices $\mathcal{A}$ and $\mathcal{R}$ respectively and related by the matrix equation,

$$\mathcal{A}_T = \mathcal{T}\mathcal{R} \qquad \mathcal{A}_P = \mathcal{P}\mathcal{R} \tag{10}$$

Finding amplitude relations in $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \otimes \mathbf{8}_{\mathcal{M}}$

- Identify the identical rows of the $\mathcal{T}$ and $\mathcal{P}$ matrices which readily gives the simplest amplitude relations for the tree part and the penguin part,

$$\begin{aligned}
\mathcal{T}(\Lambda_b^0 \to \Sigma^- K^+) &= \mathcal{T}(\Xi_b^0 \to \Xi^- \pi^+), \\
\mathcal{T}(\Lambda_b^0 \to p^+ \pi^-) &= \mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-), \\
\mathcal{T}(\Xi_b^- \to n K^-) &= \mathcal{T}(\Xi_b^- \to \Xi^0 \pi^-), \\
\mathcal{T}(\Xi_b^- \to \Xi^- K^0) &= \mathcal{T}(\Xi_b^- \to \Sigma^- \overline{K}^0), \\
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\mathcal{T}(\Xi_b^0 \to n \overline{K}^0) &= \mathcal{T}(\Lambda_b^0 \to \Xi^0 K^0), \\
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\end{aligned}$$

$$\begin{aligned}
\mathcal{P}(\Lambda_b^0 \to \Sigma^- K^+) &= \mathcal{P}(\Xi_b^0 \to \Xi^- \pi^+), \\
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\mathcal{P}(\Xi_b^- \to n K^-) &= \mathcal{P}(\Xi_b^- \to \Xi^0 \pi^-), \\
\mathcal{P}(\Xi_b^- \to \Xi^- K^0) &= \mathcal{P}(\Xi_b^- \to \Sigma^- \overline{K}^0), \\
\mathcal{P}(\Xi_b^0 \to \Xi^- K^+) &= \mathcal{P}(\Lambda_b^0 \to \Sigma^- \pi^+), \\
\mathcal{P}(\Xi_b^0 \to \Sigma^- \pi^+) &= \mathcal{P}(\Lambda_b^0 \to \Xi^- K^+), \\
\mathcal{P}(\Xi_b^0 \to \Sigma^+ \pi^-) &= \mathcal{P}(\Lambda_b^0 \to p^+ K^-), \\
\mathcal{P}(\Xi_b^0 \to n \overline{K}^0) &= \mathcal{P}(\Lambda_b^0 \to \Xi^0 K^0), \\
\mathcal{P}(\Xi_b^0 \to p^+ K^-) &= \mathcal{P}(\Lambda_b^0 \to \Sigma^+ \pi^-), \\
\mathcal{P}(\Xi_b^0 \to \Xi^0 K^0) &= \mathcal{P}(\Lambda_b^0 \to n \overline{K}^0).
\end{aligned}$$

arXiv 1911.01121

More relations

Triangle relations connecting

- $\Delta S = -1$ decays modes;

$$
\begin{aligned}
\mathcal{T}(\Lambda_b^0 \to \Sigma^+\pi^-) + \mathcal{T}(\Lambda_b^0 \to \Sigma^-\pi^+) + 2\mathcal{T}(\Lambda_b^0 \to \Sigma^0\pi^0) &= 0, \\
\mathcal{T}(\Xi_b^- \to \Xi^-\pi^0) - \sqrt{3}\mathcal{T}(\Xi_b^- \to \Xi^-\eta_8) + \sqrt{2}\mathcal{T}(\Xi_b^- \to \Sigma^-\overline{K^0}) &= 0, \\
\mathcal{T}(\Xi_b^- \to \Sigma^0 K^-) - \sqrt{3}\mathcal{T}(\Xi_b^- \to \Lambda^0 K^-) + \sqrt{2}\mathcal{T}(\Xi_b^- \to \Xi^0\pi^-) &= 0, \\
\mathcal{T}(\Xi_b^0 \to \Xi^-\pi^+) - \mathcal{T}(\Lambda_b^0 \to \Xi^- K^+) + \mathcal{T}(\Lambda_b^0 \to \Sigma^-\pi^+) &= 0, \\
\mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-) - \mathcal{T}(\Lambda_b^0 \to p^+ K^-) + \mathcal{T}(\Lambda_b^0 \to \Sigma^+\pi^-) &= 0,
\end{aligned}
$$

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\mathcal{T}(\Xi_b^0 \to \Xi^-\pi^+) - \mathcal{T}(\Lambda_b^0 \to \Xi^- K^+) + \mathcal{T}(\Lambda_b^0 \to \Sigma^-\pi^+) &= 0, \\
\mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-) - \mathcal{T}(\Lambda_b^0 \to p^+ K^-) + \mathcal{T}(\Lambda_b^0 \to \Sigma^+\pi^-) &= 0,
\end{aligned}
$$

- $\Delta S = 0$ decay modes;

$$
\begin{aligned}
\mathcal{T}(\Xi_b^- \to \Sigma^0\pi^-) - \sqrt{3}\mathcal{T}(\Xi_b^- \to \Lambda^0\pi^-) - \sqrt{2}\mathcal{T}(\Xi_b^- \to n K^-) &= 0, \\
\mathcal{T}(\Xi_b^- \to \Sigma^-\pi^0) - \sqrt{2}\mathcal{T}(\Xi_b^- \to \Xi^- K^0) - \sqrt{3}\mathcal{T}(\Xi_b^- \to \Sigma^-\eta_8) &= 0, \\
\mathcal{T}(\Xi_b^0 \to \Sigma^-\pi^+) - \mathcal{T}(\Xi_b^0 \to \Xi^- K^+) - \mathcal{T}(\Lambda_b^0 \to \Sigma^- K^+) &= 0, \\
\mathcal{T}(\Xi_b^0 \to p^+ K^-) - \mathcal{T}(\Xi_b^0 \to \Sigma^+\pi^-) + \mathcal{T}(\Lambda_b^0 \to p^+\pi^-) &= 0.
\end{aligned}
$$

Amp. relations for the case of $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \otimes \mathbf{1}_{\mathcal{M}}$

$$\begin{aligned} \mathcal{T}(\Xi_b^0 \to \Xi^0 \eta_1) &= \mathcal{T}(\Lambda_b^0 \to n\eta_1), \\ \mathcal{T}(\Xi_b^- \to \Xi^- \eta_1) &= \mathcal{T}(\Xi_b^- \to \Sigma^- \eta_1), \end{aligned} \tag{11}$$

- Triangle relations for $\Delta S = -1$ processes,

$$\mathcal{T}(\Lambda_b^0 \to \Lambda\eta_1) - \tfrac{1}{\sqrt{3}}\mathcal{T}(\Lambda_b^0 \to \Sigma^0\eta_1) - \tfrac{\sqrt{2}}{\sqrt{3}}\mathcal{T}(\Xi_b^0 \to \Xi^0\eta_1) = 0$$

- Triangle relations for $\Delta S = 0$ processes,

$$\mathcal{T}(\Lambda_b^0 \to n\eta_1) + \frac{\sqrt{3}}{\sqrt{2}}\mathcal{T}(\Xi_b^0 \to \Lambda^0\eta_1) - \frac{1}{\sqrt{2}}\mathcal{T}(\Xi_b^0 \to \Sigma^0\eta_1) = 0$$

Amp. relations for the case of $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \otimes \mathbf{8}_{\mathcal{M}}$

$$\mathcal{T}(\Xi_b^0 \to \Lambda_s^{0*}\overline{K^0}) = \mathcal{T}(\Lambda_b^0 \to \Lambda_s^{0*}K^0)$$
$$\mathcal{T}(\Xi_b^0 \to \Lambda_s^{0*}\eta_8) = \mathcal{T}(\Xi_b^- \to \Lambda_s^{0*}K^-)$$

- Triangle $\Delta S = -1$ relations:

$$\mathcal{T}(\Lambda_b^0 \to \Lambda_s^{0*}\pi_0) - \frac{1}{\sqrt{3}}\mathcal{T}(\Lambda_b^0 \to \Lambda_s^{0*}\eta_8) + \frac{\sqrt{2}}{\sqrt{3}}\mathcal{T}(\Xi_b^0 \to \Lambda_s^{0*}\overline{K^0}) = 0,$$

- Triangle $\Delta S = 0$ relations:

$$-\frac{1}{\sqrt{3}}\mathcal{T}(\Xi_b^0 \to \Lambda_s^{0*}\eta_8) + \mathcal{T}(\Xi_b^0 \to \Lambda_s^{0*}\pi_0) - \frac{\sqrt{2}}{\sqrt{3}}\mathcal{T}(\Lambda_b^0 \to \Lambda_s^{0*}K^0) = 0$$

The same set of relations hold for the penguin part of the all the above amplitude relations.

SU(3)-breaking effects & general SU(3) relations

To the first order in strange quark mass,

$$\mathcal{H}_{\epsilon}^{\text{dim}=6} \subset (\mathbf{3} \oplus \overline{\mathbf{6}} \oplus \mathbf{15}) \otimes (\mathbf{1} + \epsilon\, \mathbf{8}) = (\mathbf{3} \oplus \overline{\mathbf{6}} \oplus \mathbf{15}) + \epsilon(\mathbf{3}_i \oplus \overline{\mathbf{6}}_i \oplus \mathbf{15}_1 \oplus \mathbf{15}_2 \oplus \mathbf{15}_3^1 \oplus \mathbf{15}_3^2 \oplus \mathbf{15}' \oplus \mathbf{24} \oplus \mathbf{42}),$$

where the subscript $i = 1, 2, 3$ indicates the origin of that representation from $\mathbf{3}$,$\overline{\mathbf{6}}$, $\mathbf{15}$ respectively.

- Result: More SU(3) reduced amplitudes

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  - Less relations.

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- Result: More SU(3) reduced amplitudes
  - Less relations.
- Sole isospin relation that survives,

$$\mathcal{T}(\Lambda_b^0 \to \Sigma^+\pi^-) + \mathcal{T}(\Lambda_b^0 \to \Sigma^-\pi^+) + 2\mathcal{T}(\Lambda_b^0 \to \Sigma^0\pi^0) = 0$$

- Recall that, Isospin symmetry of the unbroken Hamiltonian forbids a $\Delta I = 2$ and $\Delta I = 5/2$ transition.

$$\begin{aligned}
&\frac{\mathcal{T}(\Xi_b^0 \to \Sigma^0 \bar{K}^0)}{3} + \frac{\mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^0 \to \Xi^0 \pi^0)}{3} + \frac{\mathcal{T}(\Xi_b^0 \to \Xi^- \pi^+)}{3\sqrt{2}} \\
&\quad + \frac{\mathcal{T}(\Xi_b^- \to \Sigma^0 K^-)}{3} + \frac{\mathcal{T}(\Xi_b^- \to \Sigma^- \bar{K}^0)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^- \to \Xi^0 \pi^-)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^- \to \Xi^- \pi^0)}{3} \\
&\quad + \frac{\sqrt{2}\mathcal{T}(\Lambda_b^0 \to \Sigma^0 \pi^0)}{3} + \frac{\mathcal{T}(\Lambda_b^0 \to \Sigma^- \pi^+)}{3\sqrt{2}} + \frac{\mathcal{T}(\Lambda_b^0 \to \Sigma^+ \pi^-)}{3\sqrt{2}} = 0, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&\frac{\mathcal{T}(\Xi_b^0 \to \Sigma^0 \bar{K}^0)}{\sqrt{6}} + \frac{\mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^0 \to \Xi^0 \pi^0)}{\sqrt{6}} - \frac{\mathcal{T}(\Xi_b^0 \to \Xi^- \pi^+)}{2\sqrt{3}} \\
&\quad + \frac{\mathcal{T}(\Xi_b^- \to \Sigma^0 K^-)}{\sqrt{6}} + \frac{\mathcal{T}(\Xi_b^- \to \Sigma^- \bar{K}^0)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^- \to \Xi^0 \pi^-)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^- \to \Xi^- \pi^0)}{\sqrt{6}} = 0. \qquad (13)
\end{aligned}$$

CP relations

- Decay rate

$$\Gamma(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M}) = \frac{|\mathbf{p}_{\mathcal{B}}|}{8\pi m_{\mathcal{B}_b}^2}\left[|S|^2 + |P|^2\right]$$

Brown '83, Donoghue '85, Dunietz '92

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- Factor out the kinematic factors,

$$\begin{aligned} S &= \sqrt{2m_{\mathcal{B}_b}(E_{\mathcal{B}} + m_{\mathcal{B}})}\mathcal{A}^{\mathcal{S}} \\ P &= \sqrt{2m_{\mathcal{B}_b}(E_{\mathcal{B}} - m_{\mathcal{B}})}\mathcal{A}^{\mathcal{P}} \end{aligned}$$

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- Define A_{CP} as

$$\begin{aligned} A_{CP} =& \frac{\Gamma(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M}) - \Gamma(\overline{\mathcal{B}}_b \to \overline{\mathcal{B}}\,\overline{\mathcal{M}})}{\Gamma(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M}) + \Gamma(\overline{\mathcal{B}}_b \to \overline{\mathcal{B}}\,\overline{\mathcal{M}})} \\ =& \frac{\Delta_{CP}(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M})}{2\tilde{\Gamma}(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M})}, \end{aligned}$$

Δ_{CP} and δ_{CP}

- δ_{CP} written in terms of decay amplitudes,

$$\delta^a_{CP}(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M}) = -4\mathbf{J} \times \mathrm{Im}\Big[\mathcal{A}^{a*}_{\mathrm{T}}(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M})\mathcal{A}^{a}_{\mathrm{P}}(\mathcal{B}_b \to \mathcal{B}\,\mathcal{M})\Big].$$

- $\mathrm{Im}(V_{ub}V^*_{ud}V^*_{tb}V_{td}) = -\mathrm{Im}(V_{ub}V^*_{us}V^*_{tb}V_{ts}) = \mathbf{J}$
- Ten δ^a_{CP} relations are obtained,

$$\begin{aligned}
\delta^a_{CP}(\Lambda^0_b \to \Sigma^- K^+) &= -\,\delta^a_{CP}(\Xi^0_b \to \Xi^- \pi^+),\\
\delta^a_{CP}(\Lambda^0_b \to p^+ \pi^-) &= -\,\delta^a_{CP}(\Xi^0_b \to \Sigma^+ K^-),\\
\delta^a_{CP}(\Xi^-_b \to n K^-) &= -\,\delta^a_{CP}(\Xi^-_b \to \Xi^0 \pi^-),\\
\delta^a_{CP}(\Xi^-_b \to \Xi^- K^0) &= -\,\delta^a_{CP}(\Xi^-_b \to \Sigma^- \overline{K}^0),\\
\delta^a_{CP}(\Xi^0_b \to \Xi^- K^+) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Sigma^- \pi^+),\\
\delta^a_{CP}(\Xi^0_b \to \Sigma^- \pi^+) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Xi^- K^+),\\
\delta^a_{CP}(\Xi^0_b \to \Sigma^+ \pi^-) &= -\,\delta^a_{CP}(\Lambda^0_b \to p^+ K^-),\\
\delta^a_{CP}(\Xi^0_b \to n \overline{K}^0) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Xi^0 K^0),\\
\delta^a_{CP}(\Xi^0_b \to p^+ K^-) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Sigma^+ \pi^-),\\
\delta^a_{CP}(\Xi^0_b \to \Xi^0 K^0) &= -\,\delta^a_{CP}(\Lambda^0_b \to n \overline{K}^0),
\end{aligned}$$

for both $a = S$ and $a = P$.

- The relation between A_{CP} and δ_{CP} is,

$$\Delta_{CP} = \frac{|\mathbf{p}_{\mathcal{B}}|}{4\pi}\frac{(E_{\mathcal{B}} + m_{\mathcal{B}})}{m_{\mathcal{B}_b}}\left[\delta^S_{CP} + \left(\frac{|\mathbf{p}_{\mathcal{B}}|}{E_{\mathcal{B}} + m_{\mathcal{B}}}\right)^2 \delta^P_{CP}\right]$$

- Δ_{CP} depends on the masses of the initial and final baryons as well as the final state meson
  - Ignore $\mathbf{p}_{\mathcal{B}}$ and $m_{\mathcal{B}}$ differences between the various modes,
- Alternatively, measure the longitudinal polarization (α) of the daughter baryon from an angular distribution study of the final states.

$$\alpha = \frac{2\mathrm{Re}(\mathcal{A}^{\mathcal{S}*}\mathcal{A}^{\mathcal{P}})|\mathbf{p}_{\mathcal{B}}|/E_{\mathcal{B}} + m_{\mathcal{B}}}{|\mathcal{A}^{\mathcal{S}}|^2 + |\mathcal{A}^{\mathcal{P}}|^2(|\mathbf{p}_{\mathcal{B}}|/E_{\mathcal{B}} + m_{\mathcal{B}})^2} \quad (14)$$

- To be verified in experiments

$$\frac{A_{CP}(\mathcal{B}_{bi} \to \mathcal{B}_j\mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bl} \to \mathcal{B}_m\mathcal{M}_n)} \simeq -\frac{\tau_{\mathcal{B}_{bi}}}{\tau_{\mathcal{B}_{bl}}}\frac{\mathcal{BR}(\mathcal{B}_{bl} \to \mathcal{B}_m\mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{B}_j\mathcal{M}_k)}, \quad (15)$$

Conclusion

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- Proposed a general framework to analyze hadronic beauty baryon decays using SU(3) flavor symmetry.
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Thank You

Effective Hamiltonian for charmless b decays

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}}\Big[\lambda_u^{(s)}\Big(C_1(Q_1^{(u)} - Q_1^{(c)}) + C_2(Q_2^{(u)} - Q_2^{(c)})\Big) - \lambda_t^{(s)}\sum_{i=1,2} C_i Q_i^{(c)} - \lambda_t^{(s)}\sum_{i=3}^{10} C_i Q_i$$

$$+\lambda_u^{(d)}\Big(C_1(Q_1^{(u)} - Q_1^{(c)}) + C_2(Q_2^{(u)} - Q_2^{(c)})\Big) - \lambda_t^{(d)}\sum_{i=1,2} C_i Q_i^{(c)} - \lambda_t^{(d)}\sum_{i=3}^{10} C_i Q_i\Big],$$

where $V_{ub}V_{us}^* = \lambda_u^s$, $V_{ub}V_{ud}^* = \lambda_u^d$, $V_{tb}V_{ts}^* = \lambda_t^s$, $V_{tb}V_{td}^* = \lambda_t^d$ are the CKM elements and C_i s are the Wilson coefficients. matrix elements: