# One-Loop Angularity Distributions with Recoil using SCET

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in collaboration with:
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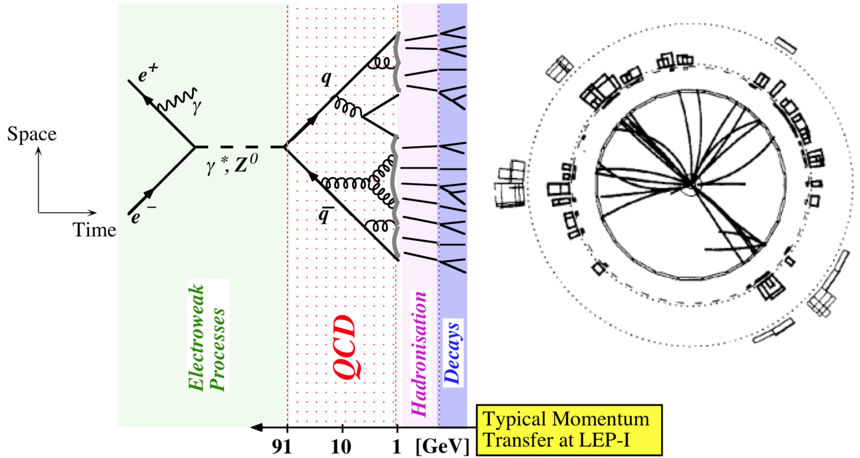
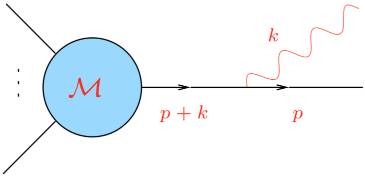
# Jets in $e^+ e^-$ annihilations




- Electroweak Processes: hard scattering process $e^+ e^- \to \gamma^*, Z^0 \to q\,\bar{q}$
- QCD: shower development calculated in perturbation theory (fixed order; (N)LL)
- Hadronization: phenomenological models of string-, cluster- or dipole fragmentation/or appropriate shape functions




$$iM = -ig\bar{u}(p)\not{\epsilon}(k)\, t^a \frac{i(\not{p}+\not{k})}{(p+k)^2}\mathcal{M} \tag{1}$$

- Singularities: $(p+k)^2 \to 0 \Rightarrow 2p^0k^0(1-cos\theta_{pk}) \to 0$
- Three potential regions of enhancement:
  1. $k^0 \to 0 \Rightarrow k^\mu \to 0$ — *Soft/Infrared*
  2. $cos\theta_{pk} \to 1$ — *Collinear*
  3. $p^0 \to 0$ — *Never a problem*

*Jet formation governed by infrared dynamics of QCD. Need inclusive observables*

$$T = \max_{\hat{n}} \frac{1}{Q} \sum_{i \epsilon X} |\vec{p}_i \cdot \hat{n}| \tag{2}$$

- $\hat{n}$ is the *thrust axis*, giving the direction of net momentum flow.
- For two back-to-back particles of equal mass m,

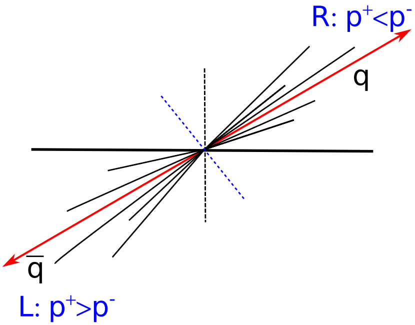
$$\begin{aligned} T &= \max_{\hat{n}} \frac{1}{Q} \sum_{i \epsilon X} |\vec{p}_i \cdot \hat{n}| \\ &= \frac{2p}{2E} \approx 1 - \frac{m^2}{2E^2} \end{aligned}$$

  $\Rightarrow$ the more pencil-like an event is $T \to 1$, while for a spread out multi-jet event, $T$ is farther away from 1 (with $T \sim 0.5$ for spherical distribution of particles).
- In literature, commonly used $\tau = 1 - T \Rightarrow \tau \to 0$ specifies collimated back-to-back jets.

$$\mathrm{B} = \frac{1}{Q} \sum_{i \epsilon X} |\vec{p}_{\perp}^{\,i}| = \frac{1}{Q} \sum_{i \epsilon X} |\vec{p}_i \times \hat{n}| \tag{3}$$

- $p_T$ defined relative to the thrust axis, $\hat{n}$.
- Jet broadening probes the transverse spread in a jet.
- For collimated back-to-back jets, B is close to 0.

$$p^{\pm} = E \mp p_z$$




$$\textbf{Thrust: } \tau = \frac{1}{Q}\left[\sum_{i\epsilon L}|p_i^-| + \sum_{i\epsilon R}|p_i^+|\right]$$

$$\textbf{Broadening: } \mathrm{B} = \frac{1}{Q}\left[\sum_{i\epsilon L}\sqrt{p_i^+ p_i^-} + \sum_{i\epsilon R}\sqrt{p_i^- p_i^+}\right]$$

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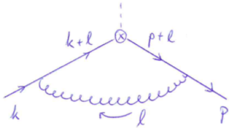
- **Berger, Kucs, Sterman, 03**

$$\tau_b = \frac{1}{Q}\left[\sum_{i\epsilon L}(p_i^+)^{\frac{1-b}{2}}(p_i^-)^{\frac{1+b}{2}} + \sum_{i\epsilon R}(p_i^+)^{\frac{1+b}{2}}(p_i^-)^{\frac{1-b}{2}}\right] \quad (4)$$

- For Infrared safety : $-1 < b < \infty$.
- Generalization to *'thrust'* ($b = 1$) and jet *'broadening'* ($b = 0$).
- Varying *'b'* changes the sensitivity of the observable to the substructure of the jet.




k and p are massless particles, introduce light-like vectors

$n^\mu = (1,0,0,1)$, $\bar{n}^\mu = (1,0,0,-1)$ s.t.

$n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$

$\Rightarrow k^\mu = E\bar{n}^\mu$ and $p^\mu = En^\mu$

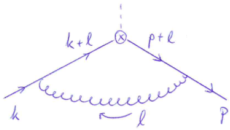
**Light-cone coordinates**

Any 4-vector $p^\mu$ can be decomposed as

$$p^\mu = \underbrace{n \cdot p}_{p_+} \frac{\bar{n}^\mu}{2} + \underbrace{\bar{n} \cdot p}_{p_-} \frac{n^\mu}{2} + p_\perp^\mu = (p_+, p_-, p_\perp) \quad , \quad p^2 = p_+ p_- + p_\perp^2 .$$

- If $p^\mu$ is mainly in the $n^\mu$ direction, $p^- \gg p^+$ and $p^- \gg p_\perp$
- Defining a dimensionless parameter $\lambda$,

$$(p_+, p_-, p_\perp) \sim Q(\lambda^2, 1, \lambda).$$

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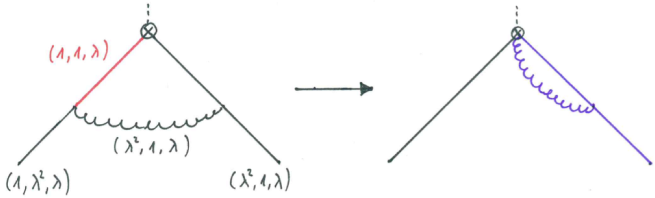
$$\Rightarrow p_n^\mu \sim Q(\lambda^2, 1, \lambda),\ k_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$$

# EFT contd.

- The possible allowed scalings for momentum $l^\mu$

| | |
|---|---|
| $l \sim Q(\lambda^2, 1, \lambda)$ | $n$-collinear |
| $l \sim Q(1, \lambda^2, \lambda)$ | $\bar{n}$-collinear |
| $l \sim Q(\lambda^2, \lambda^2, \lambda^2)$ | ultra-soft |
| $l \sim Q(\lambda, \lambda, \lambda)$ | soft |



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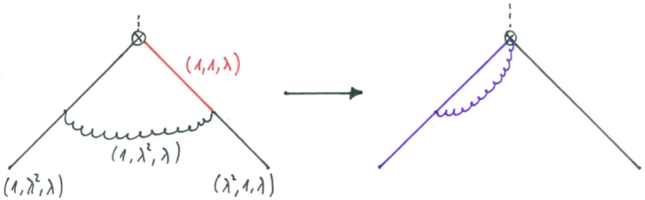

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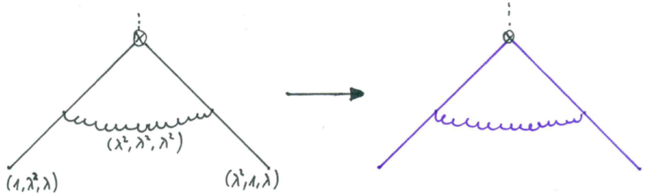


$$p^{\mu} \equiv (p^{+}, p^{-}, p_{\perp})$$

**Thrust:**

- $\tau = \frac{1}{Q}\left[\sum_{i\epsilon L}|p_i^{-}| + \sum_{i\epsilon R}|p_i^{+}|\right]$
- On-shell $\Rightarrow p^{+}\,p^{-} = p_{\perp}^{2}$
- $p_i^{+} \sim \tau \qquad \forall\ i\,\epsilon\, R$

  & $p_i^{-} \sim \tau \qquad \forall\ i\,\epsilon\, L$
- $\lambda \sim \sqrt{\tau}$

$$\Rightarrow p_{\bar{n}}(p^{-}>p_{\perp}>p^{+}) \sim Q(\lambda^2, 1, \lambda),$$
$$p_{n}(p^{-}<p_{\perp}<p^{+}) \sim Q(1, \lambda^2, \lambda),$$
$$p_{s}(p^{-}\sim p_{\perp}\sim p^{+}) \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

$\mathrm{SCET_{I}}$

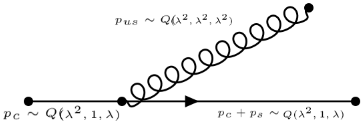
**Broadening:**

- $\mathrm{B} = \frac{1}{Q}\left[\sum_{i\epsilon L}\sqrt{p_i^{+}p_i^{-}} + \sum_{i\epsilon R}\sqrt{p_i^{-}p_i^{+}}\right]$
- On-shell $\Rightarrow p^{+}\,p^{-} = p_{\perp}^{2}$
- $p_i^{\perp} \sim \mathrm{B} \qquad \forall\ i\,\epsilon\, L, R$
- $\lambda \sim \mathrm{B}$

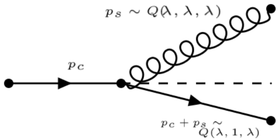
$$\Rightarrow p_{\bar{n}}(p^{-}>p_{\perp}>p^{+}) \sim Q(\lambda^2, 1, \lambda),$$
$$p_{n}(p^{-}<p_{\perp}<p^{+}) \sim Q(1, \lambda^2, \lambda),$$
$$p_{s}(p^{-}\sim p_{\perp}\sim p^{+}) \sim Q(\lambda, \lambda, \lambda)$$

$\mathrm{SCET_{II}}$

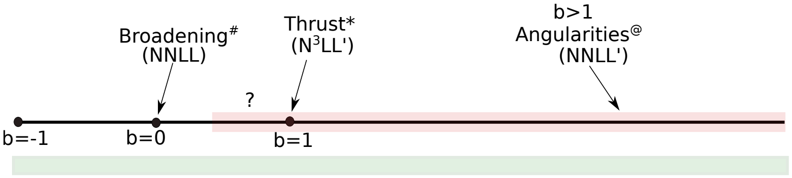
- **Thrust:**




- **Broadening:**



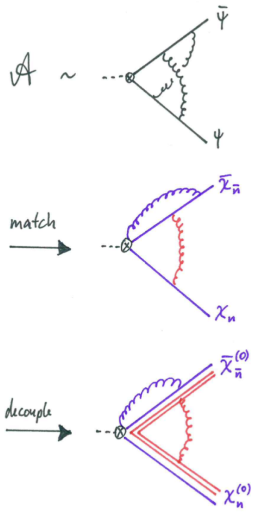

**Jet Angularities are novel observables that allow us to transform between recoil-insensitive to recoil-sensitive observables in a continuous manner making a unified framework particularly challenging.**




*Catani, Trentadue, Turnock, Webber, 93; Florian, Grazzini, 04; Schwartz, 07; Becher, Schwartz, 08; Abbate, Fickinger, Hoang, Mateu, Stewart, 10

#Dokshitzer, Lucenti, Marchesini, Salam, 98; Becher, Bell, Neubert, 11; Chiu, Jain, Neill, Rothstein, 11; Becher and Bell, 12

@Hornig, Lee, Ovanesyan, 09; Bell, Hornig, Lee, Talbert, 18




The amplitude for the $e^+e^- \to 2$ jets process involves the QCD current $\bar{\psi}\Gamma\psi$.

First, we match onto SCET, with matching coefficient $C(Q^2, \mu^2)$.

$$O_2 = \bar{\xi}_{\bar{n}} W_{\bar{n}} \Gamma W_n^\dagger \xi_n = \bar{\mathcal{X}}_{\bar{n}} \Gamma \mathcal{X}_n$$

The decoupling transformation renders the collinear fields inert to soft interactions.

$$\mathcal{X}_n \to S_n \mathcal{X}_n^{(0)} \quad , \quad A_n^\mu \to S_n A_n^{(0)\mu} S_n^\dagger$$
$$\mathcal{X}_{\bar{n}} \to S_{\bar{n}} \mathcal{X}_{\bar{n}}^{(0)} \quad , \quad A_{\bar{n}}^\mu \to S_{\bar{n}} A_{\bar{n}}^{(0)\mu} S_{\bar{n}}^\dagger$$

So now we need the matrix element of

$$O_2 = \bar{\mathcal{X}}_{\bar{n}}^{(0)} S_{\bar{n}}^\dagger \Gamma S_n \mathcal{X}_n^{(0)} .$$

[Image Courtesy: B.O. Lange]

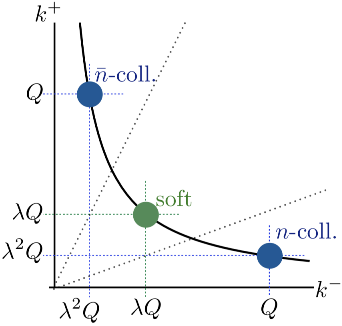
- **Thrust factorization**:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_L\mathrm{d}\tau_R} = H(Q;\mu)\int \mathrm{d}\tau_n\,\mathrm{d}\tau_{\bar{n}}\,\mathrm{d}\tau_n^s\,\mathrm{d}\tau_{\bar{n}}^s\,\delta(\tau_R-\tau_n-\tau_n^s)\,\delta(\tau_L-\tau_{\bar{n}}-\tau_{\bar{n}}^s)$$
$$\mathcal{J}(\tau_n;\mu)\,\mathcal{J}(\tau_{\bar{n}};\mu)\,\mathcal{S}(\tau_n^s,\tau_{\bar{n}}^s;\mu)$$

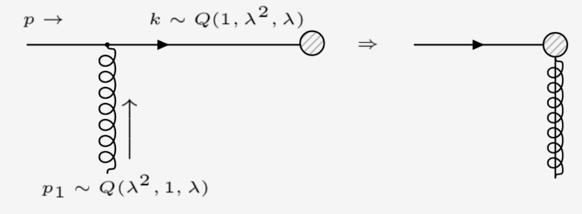
- **Broadening factorization**:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_L\mathrm{d}\tau_R} = H(Q;\mu)\int \mathrm{d}\tau_n\,\mathrm{d}\tau_{\bar{n}}\,\mathrm{d}\tau_n^s\,\mathrm{d}\tau_{\bar{n}}^s\,\delta(\tau_R-\tau_n-\tau_n^s)\,\delta(\tau_L-\tau_{\bar{n}}-\tau_{\bar{n}}^s)$$
$$\int \mathrm{d}\vec{p}_t^{\,2}\,\mathrm{d}\vec{k}_t^{\,2}\,\mathcal{J}(\tau_n,\vec{p}_t^{\,2};\mu,\nu)\,\mathcal{J}(\tau_{\bar{n}},\vec{k}_t^{\,2};\mu,\nu)\,\mathcal{S}(\tau_n^s,\tau_{\bar{n}}^s,\vec{p}_t^{\,2},\vec{k}_t^{\,2};\mu,\nu)$$

- Appears when $k^+ k^-$ is fixed but the ratio $k^+/k^-$ diverges.
- Dimensional regularization does not suffice.
- Need a regulator that essentially breaks boost-invariance.
- No rapidity divergence in the full theory.



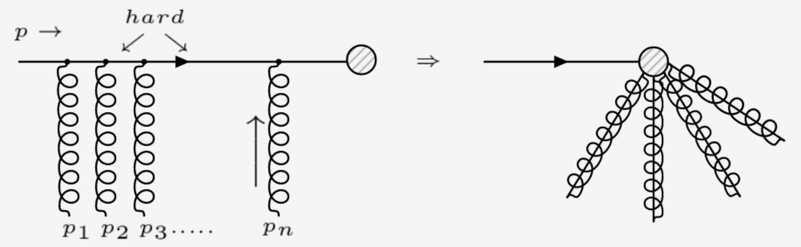
Consider the diagram:




$$p = p_1 + k \sim Q(1, 1, \lambda) \Rightarrow hard$$

$$\begin{aligned} &\left(\frac{n^\mu}{2}\bar{n}\cdot A^a\right)\frac{i(\not{p}_1 + \not{k})}{(p_1+k)^2}(i\,g\,\gamma_\mu\,T^a) \\ =& -g(\bar{n}\cdot A^a)\frac{(\not{p}_1 + \not{k})}{(p_1+k)^+(p_1+k)^- - (p_{1\perp}+k_\perp)^2}\frac{\not{n}}{2}T^a \\ =& -g(\bar{n}\cdot A^a)\frac{(\frac{\not{n}}{2}\bar{n}\cdot p_1 + \frac{\not{\bar{n}}}{2}n\cdot k)}{k^+\,p_1^-}\frac{\not{n}}{2}T^a = -g\frac{\bar{n}\cdot A}{\bar{n}\cdot p_1} \end{aligned} \tag{5}$$

$$\mathrm{I} = \int_{\mu_S}^{\mu_L}\frac{\mathrm{d}p_1^-}{p_1^-} = \int_{\mu_S}^{\Lambda}\frac{\mathrm{d}p_1^-}{p_1^-} + \int_{\Lambda}^{\mu_L}\frac{\mathrm{d}p_1^-}{p_1^-} = \int_{\mu_S}^{\infty}\frac{\mathrm{d}p_1^-}{p_1^-} + \int_{0}^{\mu_L}\frac{\mathrm{d}p_1^-}{p_1^-}$$




$$W_n = \left[ \sum_{perm} \exp\left( -g \frac{\bar{n} \cdot A_n}{\bar{n} \cdot p} \right) \right]$$

$\Rightarrow$ For regulating the divergence,

$$W_n = \left[ \sum_{perm} \exp\left( -\frac{g}{\bar{n} \cdot p} w \frac{|\bar{n} \cdot p|^{-\eta}}{\nu^{-\eta}} \bar{n} \cdot A_n \right) \right] \tag{5}$$

# Broadening-like factorization for Jet Angularities

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_L \mathrm{d}\tau_R} = H(Q;\mu)\int \mathrm{d}\tau_n\,\mathrm{d}\tau_{\bar{n}}\,\mathrm{d}\tau_n^s\,\mathrm{d}\tau_{\bar{n}}^s\,\delta(\tau_R-\tau_n-\tau_n^s)\,\delta(\tau_L-\tau_{\bar{n}}-\tau_{\bar{n}}^s)$$
$$\int \mathrm{d}\vec{p}_t^{\,2}\,\mathrm{d}\vec{k}_t^{\,2}\,\mathcal{J}(\tau_n,\vec{p}_t^{\,2};\mu,\nu)\,\mathcal{J}(\tau_{\bar{n}},\vec{k}_t^{\,2};\mu,\nu)\,\mathcal{S}(\tau_n^s,\tau_{\bar{n}}^s,\vec{p}_t^{\,2},\vec{k}_t^{\,2};\mu,\nu)$$

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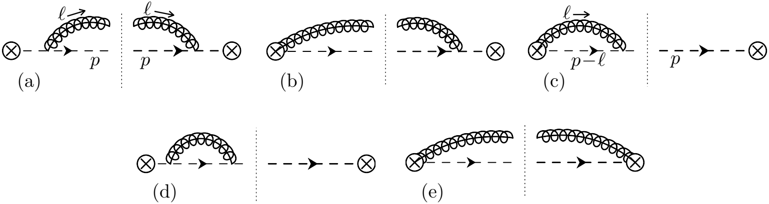
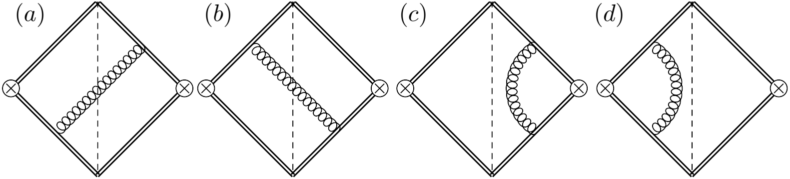



$$\mathcal{J}(\tau_n, p_t) = \frac{1}{\eta} f_J(\tau_n, p_t) + \frac{1}{\epsilon^2} g_J(\tau_n, p_t) + \frac{1}{\epsilon} h_J(\tau_n, p_t) + \text{ finite-terms}$$




$$\mathcal{S}(\tau_n^s, \tau_{\bar{n}}^s, p_t, k_t) = \frac{1}{\eta} f_S(\tau_n, \tau_{\bar{n}}^s, p_t, k_t) + \frac{1}{\epsilon^2} g_S(\tau_n, \tau_{\bar{n}}^s, p_t, k_t) + \frac{1}{\epsilon} h_S(\tau_n, \tau_{\bar{n}}^s, p_t, k_t) + \text{finite-terms}$$

- $b > 0$

$$\left[\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{d\tau_b}\right]^{\mathrm{NLO}} = \frac{\alpha_s\, C_F}{\pi}\left\{\underbrace{-\frac{3}{(1+b)}\frac{1}{\tau_b} - \frac{4}{1+b}\frac{\ln\tau_b}{\tau_b}}_{\text{thrust}} \underbrace{-\frac{4}{(1+b)}\frac{\ln(1-r)}{\tau_b}}_{\text{recoil}}\right\}$$

thrust ⇓ leading-singular

recoil ⇓ **?**

where, $r$ is given by the solution of

$$\frac{r}{(1-r)^{1+b}} = (\tau_b)^b \tag{6}$$

$$\frac{r}{(1-r)^{1+b}} = \tau^b$$

- **Small-$\tau$ limit:**

$$r = a_1\tau^b + a_2\tau^{2b} + a_3\tau^{3b} + a_4\tau^{4b} + \dots \tag{7}$$

$$\Rightarrow \frac{\ln(1-r)}{\tau} = \sum_{n=1}^{\infty} \frac{c_n}{\tau^{1-n\,b}}$$
$$= \sum_{m=1}^{\lceil 1/b \rceil - 1} \frac{c_m}{\tau^{1-m\,b}} + \text{power} - \text{corrections} \tag{8}$$

$$\frac{r}{(1-r)^{1+b}} = \tau^b$$

- **Small-$b$ limit:**

$$r = r_0 + b\, r_1 + b^2\, r_2 + \ldots \qquad \text{where,} \frac{r_0}{1-r_0} = \tau^b$$

$$\Rightarrow \frac{ln(1-r)}{\tau} = -\ln 2 \left[\frac{1}{\tau}\right]_+ - \frac{b}{2}\left[\frac{\ln \tau}{\tau}\right]_+ + \frac{b\, \ln 2}{2}\left[\frac{1}{\tau}\right]_+ + \mathcal{O}(b^2) \qquad (7)$$

⇒Recoil term provides a leading singular contribution for jet broadening (b=0) and reduces to only power corrections for thrust (b=1) while other angularities $0 < b < 1$ contain new additional singularities.

| $b$ | % correction for $\tau_b = 0.05$ | % correction for $\tau_b = 0.1$ |
|---|---|---|
| 1 | 2 | 6 |
| 0.5 | 8 | 16 |
| 0.25 | 16 | 26 |
| 0 | 31 | 45 |
| -0.2 | 15 | 24 |
| -0.5 | 2 | 5 |

Table: Relative size of the extra singular contribution compared to the leading singular contribution in the peak region for the $\tau_b$ distribution, for various values of $b$. A $2-6\%$ correction for $b=1$ or $-0.5$ shows the typical size of the power corrections due to the additional term.

# Comparison against Event2

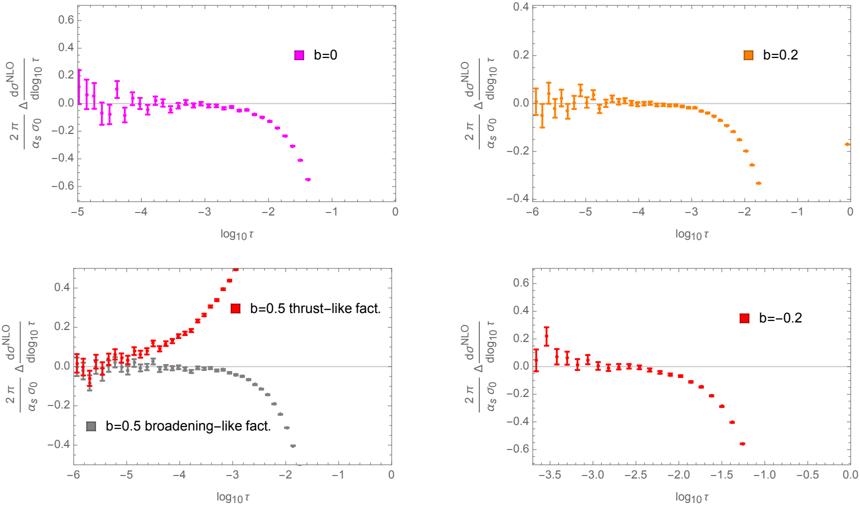



Figure: Differences between Event2 and our results from broadening-like factorization at NLO for $\mathrm{d}\sigma/\mathrm{d}\log_{10}\tau$ for different $b$ values.

# Summary

- Jet angularities provide a novel way of looking into the substructure which remains unexposed while looking at a single event shape observable.
- A broadening-like factorization for angularities provides the correct distribution for all $b > -1$ angularities while a thrust-like factorization works only in a certain range.
- Our analysis allows to smoothly interpolate between the thrust and jet broadening limits which have been defined within different effective theories so far.
- The fixed order angularity distributions with a broadening-like factorization suggest that the recoil effects are always important for $b < 1$ angularities.
- The recoil contributions, in the form of sub-leading singular terms, for $0 < b < 1$ provide a non-negligible contribution in the peak region. This is expected to effect the resummation of these observables.

- The fixed order results provided by SCET framework contain large logarithms of the angularity exponent which become dominant in the $\tau \to 0$ region and an all-order resummation is essential for obtaining predictive theoretical results, which is my next immediate goal.
- Resummation of these recoil-sensitive angularities will shed further light on the yet unexplored recoil effects.
- This analysis will open up the $b < 0.5$ range of angularities exponents which can then be utilized for high precision $\alpha_s$ extractions from angularities, using the present and future LEP data.

- J. Chiu, A. Jain, D. Neill, and I. Z. Rothstein, *The Rapidity Renormalization Group*, *Phys. Rev. Lett.* **108**.151601, arXiv:1104.0881 [hep-ph].
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- A. Hornig, C. Lee, and G. Ovanesyan, *Effective Predictions of Event Shapes: Factorized, Resummed, and Gapped Angularity Distributions*, *JHEP* **05** (2009) 122, arXiv0901.3780 [hep-ph].


Thank you