Dark matter capture in celestial bodies: Improved treatment of multiple scattering and constraints from white dwarfs

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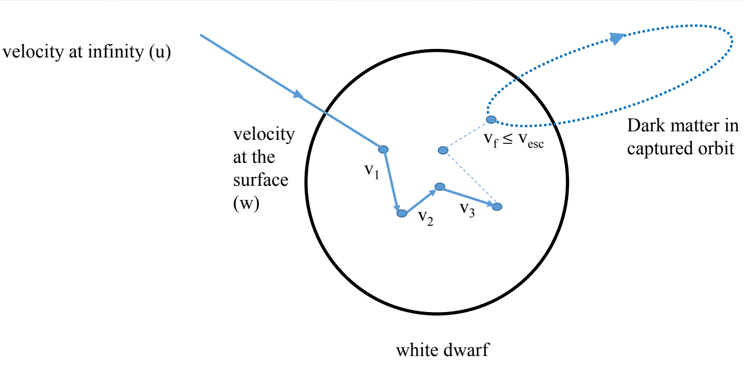
Motivations

- Till date WIMPs have remained the most popular dark matter candidate.
- Null results from different direct detection experiments have constrained the WIMP parameter space severely especially in the tens of GeV mass range.
- Hence we find growing interests in probing sub-GeV dark matters.
- However direct detection experiments are not sensitive to dark matters $\lesssim 100$ MeV.
- Hence, we devise a complimentary strategy to constrain light dark matters using astrophysical probes e.g., studying effects of DM capture in white dwarfs.

DM capture

- Dark matter capture, schematically:



- Number of scatterings inside a star is typically given by: $\sim R/\lambda_{\rm fs} = n\,\sigma\,R \simeq \sigma/R^2$

Recap of capture by single scattering

- The capture rate is given by :

$$C_1 = \sigma N_T n_{DM} \int_u \frac{f(u)du}{u} w^2 g_1(u)$$

where, N_T is the total number of target particles and n_{DM} is the number density of the incoming DM particles.

- Here, w is the initial dark matter velocity and can be written as:

$$w^2 = u^2 + v_{\rm esc}^2.$$

- $g_1(u) \rightarrow$ probability of getting captured after single scatter ($v_{\rm final} \leq v_{\rm esc}$).

- The fractional loss in kinetic energy lies in the interval:

$$0 \leq \frac{\Delta E}{E} \leq \beta$$

where, $\beta = 4 m_{DM} m_T / (m_{DM} + m_T)^2$.

- On the other hand, for capture we require:

$$\frac{\Delta E}{E} \geq \frac{m_{DM} w^2/2 - m_{DM} v_{\rm esc}^2/2}{m_{DM} w^2/2} = \frac{u^2}{w^2}$$

- Hence, the probability of capture is given by:

$$g_1(u) = \frac{1}{\beta} \left\{ \left(\beta - \frac{u^2}{u^2 + v_{\rm esc}^2} \right) \right\} \Theta \left(\beta - \frac{u^2}{u^2 + v_{\rm esc}^2} \right)$$

- An important assumption that goes into this is that the distribution of energy loss is uniform $\rightarrow$ Is this always true ?
- It is true **only** when $\frac{d\sigma}{d\Omega} = \text{const}$.
- The distribution function of θ_{CM} in the CM frame is: $p(\Omega_{CM}) = \frac{1}{\Omega_{CM}} \frac{d\sigma}{d\Omega_{CM}}$.
- Using $\theta_{recoil} = \pi/2 - \theta_{CM}/2$ we can hence find the distribution of θ_{recoil} and consequently the distribution of $\Delta E/E$ where $\Delta E/E = \beta \cos^2 \theta_{\text{recoil}}$.
- For constant $\frac{d\sigma}{d\Omega}$, $p(\Omega)$ is constant too

  $\Rightarrow$ The distribution of $\frac{\Delta E}{E}$ is uniform (and vice-versa).

- But if $\frac{d\sigma}{d\Omega_{CM}} \sim \frac{1}{\theta_{CM}^4}$, then the distribution of $\cos^2\theta_{recoil}$ goes as $\frac{\delta}{z^2}$ (δ is some small angle), instead of just being constant.
- A concrete example : DM self-scattering (mediated by a vector or scalar) $\Rightarrow$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha_D^2 M_{\rm DM}^2}{\left(M_{\rm DM}^2 v_{\rm rel}^2 \sin^2(\theta_{CM}/2) + M_{\rm med}^2\right)^2}$$

- Limiting cases of the above yields :

  Massive mediator $\Rightarrow \frac{d\sigma}{d\Omega_{CM}} =$ const.

  Light Mediator $\Rightarrow \frac{d\sigma}{d\Omega_{CM}} \sim \frac{1}{\theta_{CM}^4}$

Generalising to multiple scatterings

- The capture rate generalises to :

$$C_N = \underbrace{\pi R^2}_{\text{area of the object}} \times \underbrace{p_N(\tau)}_{\text{probability for } N \text{ collisions}} \times \underbrace{n_{\rm DM} \int \frac{f(u)du}{u} (u^2 + v_{\rm esc}^2)}_{\text{DM flux}} \times \underbrace{g_N(u)}_{\text{probability that } v_f \leq v_{\rm esc} \text{ after } N \text{ collisions}} .$$

- The probability for a dark matter with optical depth $\tau = \frac{3\sigma N_T}{2\pi R^2}$ to participate in N actual scatters is given by Poisson(τ,N).

- Taking all incidence angle into account :

$$p_N(\tau) = 2\int_0^1 dy \, \frac{y e^{-y\tau}(y\tau)^N}{N!}$$

- Using $p_1(\tau) \sim \frac{2}{3}\tau$, we recover C_1 as expected.
- We know, $\Delta E = z\beta E$, where, $z = \cos^2\theta_{recoil}$.
- The capture probability g_N reduces to :

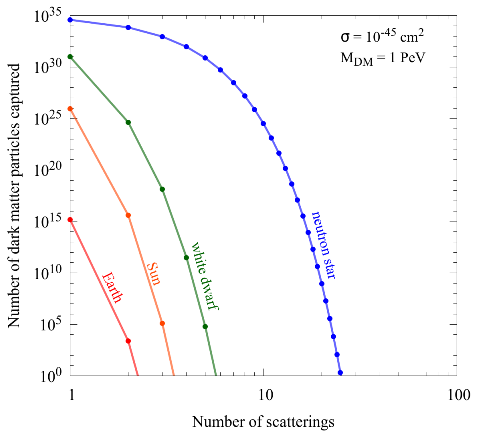
$$g_N(u) = \int_0^1 dz_1 \int_0^1 dz_2 \ldots \int_0^1 dz_N \, \theta\left(v_{\rm esc} - \underbrace{w \prod_{i=1}^{N} (1 - z_i\beta)^{1/2}}_{\mathrm{v}_f} \right)$$

$$= \frac{1}{\beta} \frac{v_{\rm esc}^2}{u^2 + v_{esc}^2} \left[\frac{1}{\beta} \log \frac{1}{1-\beta} \right]^{N-1} - \left(\frac{1}{\beta} - 1 \right)$$

- $g_N(u) \leq 1$ and $g_N(u) \geq 0$ gives lower and upper limit on u.

Number of scatterings actually allowed

- Total capture rate: $C_{tot} = \sum C_N$
- $\tau_{lifetime} \times C_N$ is an integer
  $\Rightarrow$ we truncate at that N for which $\tau_{lifetime} \times C_N < 1$.

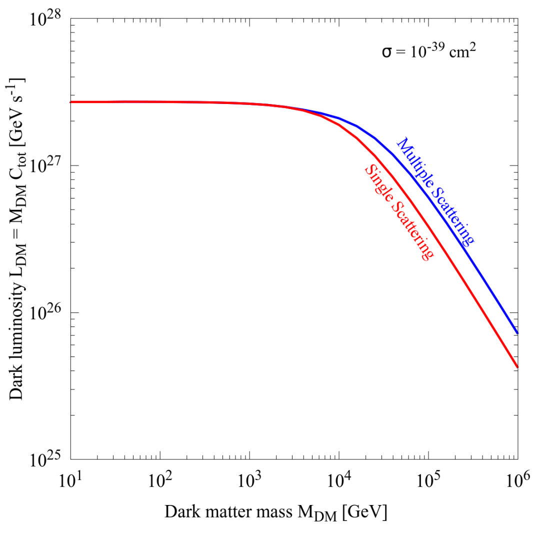


Limits on DM-proton cross section

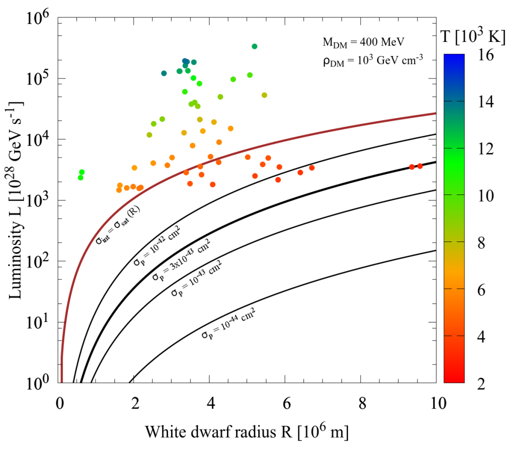
- We can constrain σ_p from luminosity and temperature measurements of white dwarfs in the M4 cluster.
- The captured dark matters annihilates to SM particles and add to the luminosity of the stellar object.
- When the capture and annihilation rates have equilibrated, the additional dark luminosity is given by $L_{DM} = M_{DM} \times C_{tot}$.
- But $L_{DM} < L_{observed} \Rightarrow$ this helps us to put an upper limit on the cross section.
- DM primarily loses energy by colliding with carbon nuclei.
- To convert it to the proton cross section we use:

$$\sigma_{\rm Carbon} = F_{Helm}^2 \frac{\mu_N^2}{\mu_p^2} A^2 \sigma_{\rm proton} \,.$$

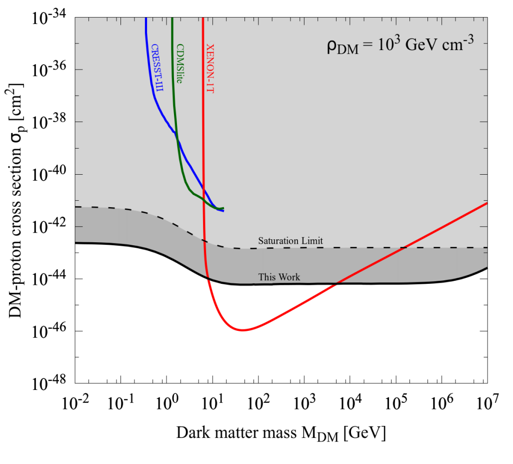
Single vs. multiple scattering



- $L_{DM} = f\left(\sigma_p, M_{DM}, R_{WD}, \rho_{DM}^{halo}\right)$.
- Independently knowing luminosity and temperature of the WD allows us calculate R_{WD} via $L = 4\pi\,\sigma_{SB} R_{WD}^2 T^4$.



- Finally, taking one of the low luminosity white dwarfs for providing the most stringent limits we get:



- Remember: $\sigma \leq \sigma_{\rm sat}$, where $\sigma_{\rm sat} = \pi R_{WD}^2 / N_T$.

Conclusions

- We have improved upon the previous treatment of multiple scattering by deriving an analytical expression for the capture probability.
- We provided a better understanding about the actual number of scatterings that can actually take place inside a celestial object.
- Using actual white dwarf data from the M4 globular cluster we were able to constrain DM-proton spin independent cross section.
- In both high and low mass regime this is the most stringent constraint available till date.

THANK YOU