Grand Unification: Proton Lifetime and Topological Defects

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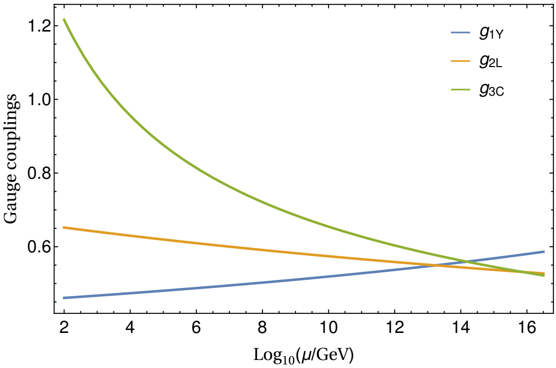
Outline

The Standard Model

- Known elementary particles are described by the Standard Model gauge symmetry : $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$.
- The gauge couplings evolve with the renormalization scale:



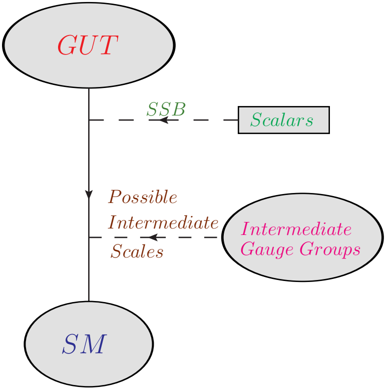
Unanswered questions in SM

- There are too many free parameters in SM.
- Electric charge quantization can not be understood.
- Can not produce enough baryon asymmetry of the universe.
- Neutrino is massless in SM (no left-handed anti-neutrino $(\nu^c)_L$).
- Gravity has not been included in the Standard Model.

Grand Unification beyond the SM

- The basic idea in a Grand Unified Theory (GUT) is that the SM, $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$, is embedded in a larger simple group, $\mathcal{G}$.

Schematic view



Motivation towards the Grand Unification

- Renormalization Group Evolution of Standard Model gauge couplings gives the hint of the possibility of the Grand Unified Theories.
- Imposition of higher symmetry may constraint some free parameters.
- All fermions including $(\nu^c)_L$ can be put in one representation in GUT.
- One way to understand the charge quantization.

Prediction of GUTs and Constraints

- Presence of lepto-quark bosons $\Rightarrow$ Prediction of proton decay. But it is yet to be observed!

- Super-Kamiokande[1] experiment puts a stringent constraint on the partial lifetime for the Golden Channel $(p \rightarrow \pi^0 e^+)$:

$$\boxed{\tau(p \rightarrow \pi^0 e^+) > 1.6 \times 10^{34} \text{ years.}}$$

- Formation of topological defects like domain wall, monopole.

[1] **Super-K Collaboration, K. Abe et al., PRL 113 (Sep, 2014), PRD 95 (2017).**

Proton decay operators

- Integrate out the heavy lepto-quark gauge bosons $\Rightarrow$ $(B-L)$-conserving dimension-six proton decay operators:

$$\mathcal{O}_{I}^{d=6} = \Omega_1^2 \epsilon^{ijk} \epsilon^{ab} \overline{u_{i\alpha}^C} \gamma^\mu Q_{ja\alpha} \overline{e_\beta^C} \gamma_\mu Q_{kb\beta}$$

$$\mathcal{O}_{II}^{d=6} = \Omega_1^2 \epsilon^{ijk} \epsilon^{ab} \overline{u_{i\alpha}^C} \gamma^\mu Q_{ja\alpha} \overline{d_{k\beta}^C} \gamma_\mu L_{b\beta}$$

$$\mathcal{O}_{III}^{d=6} = \Omega_2^2 \epsilon^{ijk} \epsilon^{ab} \overline{d_{i\alpha}^C} \gamma^\mu Q_{jb\alpha} \overline{u_{k\beta}^C} \gamma_\mu L_{a\beta}$$

$$\mathcal{O}_{IV}^{d=6} = \Omega_2^2 \epsilon^{ijk} \epsilon^{ab} \overline{d_{i\alpha}^C} \gamma^\mu Q_{jb\alpha} \overline{\nu_\beta^C} \gamma_\mu Q_{ka\beta} \ .$$

Ω_1 $[\Omega_2]$ are Wilson coefficients after integrating out the heavy gauge bosons $(X,Y) = (2, 5/6, 3)$ $[(X', Y') = (2, -1/6, 3)]$.

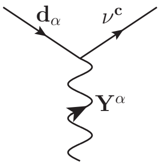
Prediction of Proton Decay

- Mediation of lepto-quark gauge bosons$\Rightarrow$ proton decay into meson plus antilepton :

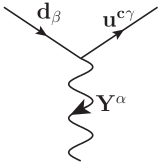
$$\boxed{p \to M + \bar{l}}$$

$M \in \{\pi^+, \pi^0, K^+, K^0, \eta\}$ and $l \in \{e, \mu, \nu_{e,\mu,\tau}\}$.

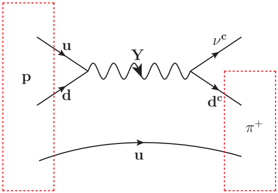
- Example:



Lepto-quark vertex



Di-quark vertex



$\mathbf{p} \to \pi^+ \nu$

- Selection rules : $\boxed{\Delta B = \Delta L = -1}$ and $\boxed{\Delta S = 0, 1}$.

Lifetime bounds for nucleon decay into meson plus antilepton

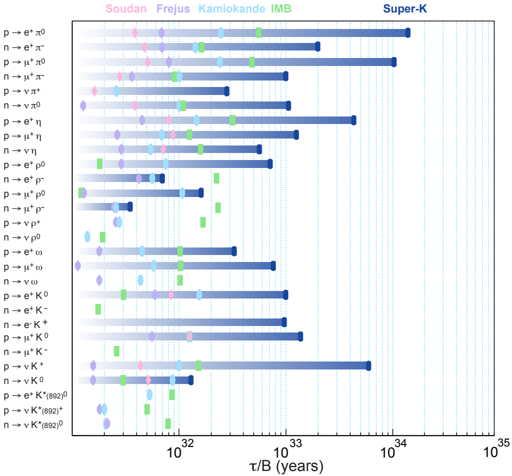


Image source : *Working Group Report: Baryon Number Violation*, K. S. Babu et al.

Proton Lifetime for $p \to \pi^0 e^+$

- $p \to \pi^0 e^+$ and $p \to \pi^+ \bar{\nu}$ have the most dominating branching ratios in the context of GUTs, whereas $p \to \pi^0 e^+$ has the highest experimental exclusion $\Rightarrow$ We need to look into $\tau(p \to \pi^0 e^+)$ only.

$$\tau(p \to \pi^0 e^+) = \left[\frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2} \right)^2 A_L^2 \frac{g_U^4}{4M_X^4} (1 + |V_{ud}|^2)^2 \right. \\ \left. \times \left(A_{SR}^2 |\langle \pi^0 | (ud)_R u_L | p \rangle|^2 + A_{SL}^2 |\langle \pi^0 | (ud)_L u_L | p \rangle|^2 \right) \right]^{-1},$$

where, $A_{SL(R)}$ are short-range renormalization factors, and A_L is the long-range renormalization factor of the proton decay operators. The matrix elements from lattice QCD[2] :

$$\langle \pi^0 | (ud)_R u_L | p \rangle = -0.131, \quad \langle \pi^0 | (ud)_L u_L | p \rangle = 0.134.$$

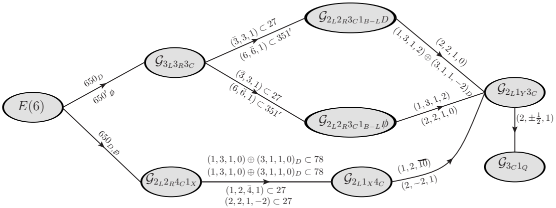
[2] **Y. Aoki, T. Izubuchi, E. Shintani and A. Soni, PRD 96, 1, 014506 (2017).**

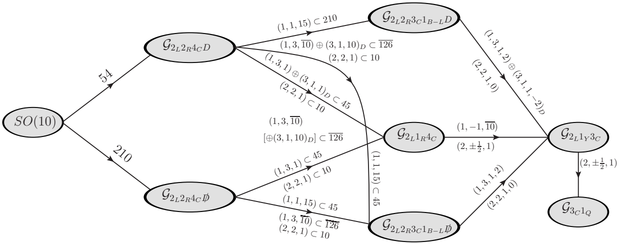
Pattern of symmetry breaking

- Minimal direct breaking of GUT, based on $SU(5)$, suffers from non-unification of gauge couplings, excluded by proton lifetime bound and so on.
- Either, non-minimal GUTs with exotic SM multiplets.
- Or, we have to look into one or more intermediate step breaking from GUT to SM based on higher rank gauge groups like $SO(10)$ and $E(6)$.
- Unification through left-right symmetric intermediate step $\Rightarrow$

Left-right symmetric theory P conserved	$\xrightarrow{\langle\phi\rangle}$	Left Chiral theory P and CP violated

Breaking paths of E(6) and SO(10) to SM





Symmetry breaking and Threshold Correction

- The matching condition[3] for the symmetry breaking, $\mathcal{G}_p \to \mathcal{G}_d$, at the scale μ

$$\boxed{\frac{1}{\alpha_d(\mu)} - \frac{C_2(\mathcal{G}_d)}{12\pi} = \left(\frac{1}{\alpha_p(\mu)} - \frac{C_2(\mathcal{G}_p)}{12\pi}\right) - \frac{\Lambda_d(\mu)}{12\pi},}$$

with threshold correction:

$$\boxed{\Lambda_d(\mu) = -21\ \mathrm{Tr}(t_{dV}^2 \ln \frac{M_V}{\mu}) + 2\eta\ \mathrm{Tr}(t_{dS}^2 \ln \frac{M_S}{\mu}) + 8\kappa\ \mathrm{Tr}(t_{dF}^2 \ln \frac{M_F}{\mu}).}$$

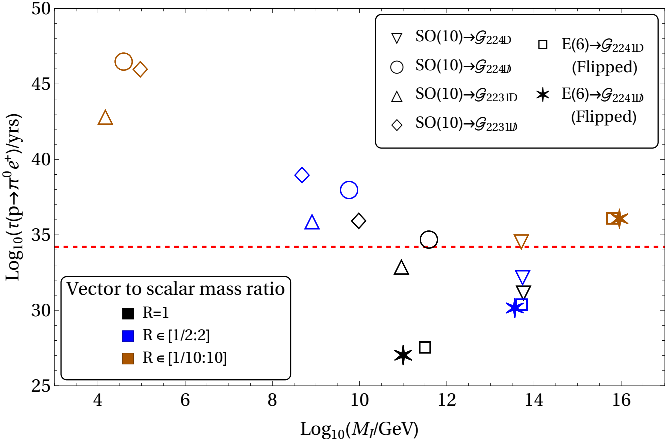
M_V, M_S and M_F are masses of heavy vector, scalar and fermion fields.

- Parameterize threshold effect by: $\boxed{R = \frac{M_{S/F}}{M_V}}$,

  with M_V is degenerate with μ.

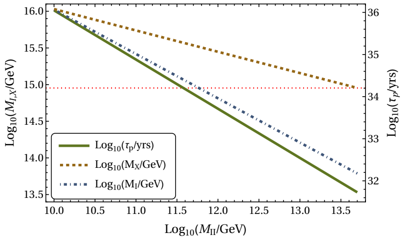
[3] **S. Weinberg, Phys. Lett. 91B, 51 (1980).**

One Intermediate Scale and Proton Lifetime

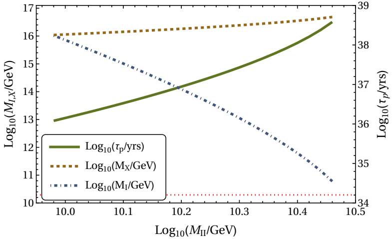


Chakrabortty, King, RM 1901.05867

Two step breaking and τ_p compatible with $R = 1$

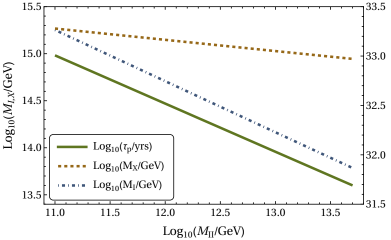


$\mathrm{SO}(10) \rightarrow \mathcal{G}_{2_L 2_R 4_C D} \rightarrow \mathcal{G}_{2_L 2_R 3_C 1_{B-L}} \not{D} \rightarrow \mathrm{SM}$

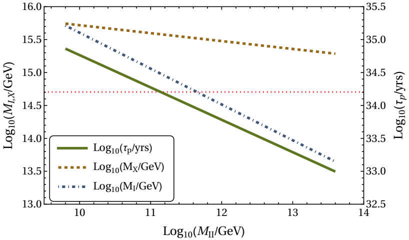


$\mathrm{SO}(10) \rightarrow \mathcal{G}_{2_L 2_R 4_C} \not{D} \rightarrow \mathcal{G}_{2_L 2_R 3_C 1_{B-L}} \not{D} \rightarrow \mathrm{SM}$

Two step breaking and τ_p compatible with $R \in [1/2 : 2]$



$R = 1$

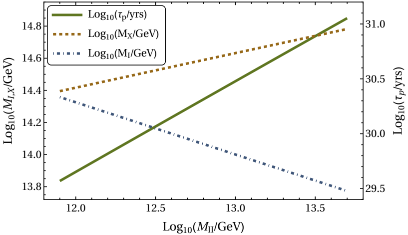


$R \in [1/2 : 2]$

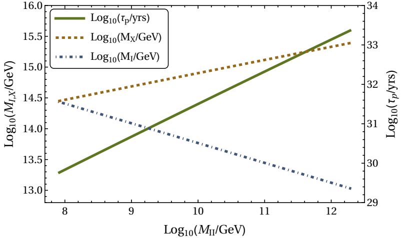
$$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} D} \to \text{SM.}$$

Chakrabortty, King, RM 1901.05867

Two step breaking and τ_p incompatible even with $R \in [1/10 : 10]$



$R = 1$



$R \in [1/10 : 10]$

$$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 1_R 4_C} \to \text{SM}.$$

Effect of dimension-5 operator on the unification scenarios

- At high scale (M_X) there can be correction due to non - renormalizable dimensional-5 opeartor:

$$-\frac{\eta}{M_{\text{Pl}}}\left[\frac{1}{4}\text{Tr}(\text{F}^{\mu\nu}\phi_{\text{D}}\text{F}_{\mu\nu})\right]$$

ϕ_D comes from the symmetric product of two adjoints.

- Gauge couplings will get modified as:

$$g_U^2 = g_i^2(M_X)(1+\epsilon\delta_i)$$

$\epsilon = \eta v_D/(2M_{\text{Pl}}) \sim \mathcal{O}(M_X/M_{\text{Pl}})$ and δ depends on intermediate gauge group (group theoretic factor).

Example: Group theoretic factors: δ_i

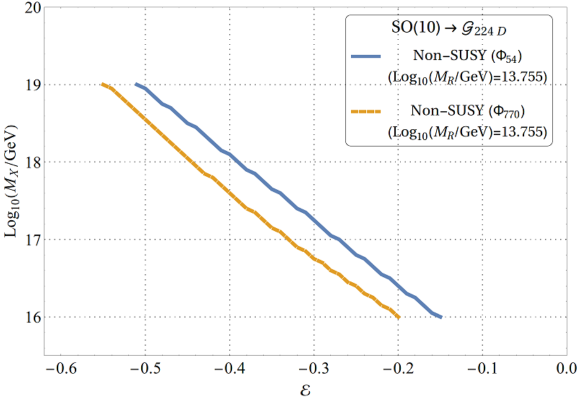
Group	Scalar Representation	δ_{3L}	δ_{3R}	δ_{3C}
$E(6)$	650	$\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
$E(6)$	$650_{\not D}$	$\frac{3}{2\sqrt{6}}$	$\frac{-3}{2\sqrt{6}}$	0

Table: $E(6) \to \mathcal{G}_{3_L 3_R 3_C}$.

Group	Scalar Representation	δ_{2L}	δ_{2R}	δ_{4C}
$SO(10)$	54	$\frac{3}{2\sqrt{15}}$	$\frac{3}{2\sqrt{15}}$	$\frac{-1}{\sqrt{15}}$
$SO(10)$	210	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	0
$SO(10)$	770	$\frac{5}{3\sqrt{5}}$	$\frac{5}{3\sqrt{5}}$	$\frac{2}{3\sqrt{5}}$

Table: $SO(10) \to \mathcal{G}_{2_L 2_R 4_C}$.

Example: Effect of Dim-5 operator



Chakrabortty, Raychaudhuri 0812.2783
Chakrabortty, RM, Mohanty, Patra, Srivastava 1711.11391

Topological defects associated with spontaneous symmetry breaking

- Topological defects may appear during the SSB of a group $\mathcal{G}$ down to its subgroup $\mathcal{H}$.
- The vacuum manifold: $\mathcal{M} = \mathcal{G}/\mathcal{H}$.
- Non-trivial homotopy group $\Pi_k(\mathcal{M})$ of the vacuum manifold implies formation of topological defects.
- Various types of topological defects which can be formed are : domain walls (k = 0), cosmic strings (k = 1), monopoles (k = 2) etc.

GUT $\to \mathcal{G}_{\rm I} \to \mathcal{G}_{\rm II} \to$ SM	Topological defects		
	GUT $\to \mathcal{G}_{\rm I}$	$\mathcal{G}_I \to \mathcal{G}_{II}$	$\mathcal{G}_{II} \to$ SM
$E(6) \to \mathcal{G}_{3_L 3_R 3_C D} \to$ $\mathcal{G}_{2_L 2_R 3_C 1_{LR} D} \to$ SM	Unstable $\mathbb{Z}_2$-strings $+\mathbb{Z}_3$-monopoles	Stable monopoles	Domain walls + embedded strings
$E(6) \to \mathcal{G}_{3_L 3_R 3_C} \to$ $\mathcal{G}_{2_L 2_R 3_C 1_{LR} \not{D}} \to$ SM	$\mathbb{Z}_3$-monopoles	Stable monopoles	Embedded strings
$E(6) \to \mathcal{G}_{2_L 2_R 4_C 1_X D}$ $\to \mathcal{G}_{2_L 1_X 4_C} \to$ SM	Unstable $\mathbb{Z}_2$-strings + stable monopoles + unstable $\mathbb{Z}_2$-monopoles	Domain walls	Embedded strings
$E(6) \to \mathcal{G}_{2_L 2_R 4_C 1_X \not{D}}$ $\to \mathcal{G}_{2_L 1_X 4_C} \to$ SM	Stable monopoles + unstable $\mathbb{Z}_2$-monopoles	No defects	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to$ $\mathcal{G}_{2_L 2_R 3_C 1_{B-L} D} \to$ SM	$\mathbb{Z}_2$-strings (stable upto M_{II}) $+\mathbb{Z}_2$-monopoles	Stable monopoles	Domain walls + embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to$ $\mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to$ SM	Unstable $\mathbb{Z}_2$-strings + $\mathbb{Z}_2$-monopoles	Domain walls + stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C \not{D}} \to$ $\mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to$ SM	$\mathbb{Z}_2$-monopoles	Stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to$ $\mathcal{G}_{2_L 1_R 4_C} \to$ SM	Unstable $\mathbb{Z}_2$-strings + $\mathbb{Z}_2$-monopoles	Domain walls + stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C \not{D}} \to$ $\mathcal{G}_{2_L 1_R 4_C} \to$ SM	$\mathbb{Z}_2$-monopoles	Stable monopoles	Embedded strings

Chakrabortty, King, RM 1901.05867; Lazarides, Shafi 1904.06880

Cosmological constraints

- Topological defects like stable domain walls and monopoles should be formed before the end of inflation.
- Upper bound on monopole number per comoving volume $Y_M = n_M/s < 10^{-27}$ as obtained from MACRO.
- The monopole number density after inflation: $\sim H^3 \exp[-3N]$, N be the number of e-fold after the formation of monopoles.
- The MACRO bound gives: $\boxed{N > \log(H/T_R) + 20}$, $T_R =$ reheat temperature.

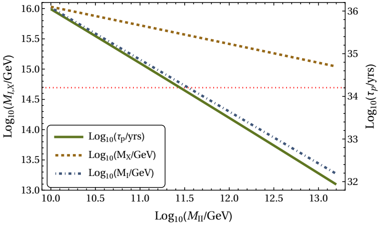
Nefer, Shafi 1510.04442

Summary

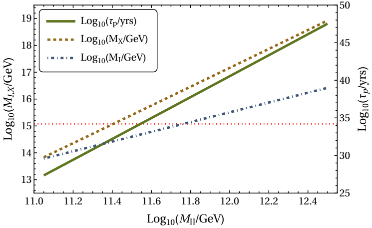
- Many Non-SUSY GUTs are incompatible with the Super-K proton lifetime bound unless we take the effect of threshold correction into account.
- Choosing two different sets, $R \in [1/2 : 2]$ and $R \in [1/10 : 10]$, we have noticed that most of the scenarios can be made safe apart from $SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 1_R 4_C} \to$ SM.
- Effect of dimension-5 operators can improve the proton lifetime for specific breaking paths.
- Topological defects e.g., domain walls, monopoles conflict cosmological observations unless they are inflated away.

Thank You

Two step breaking and τ_p compatible with $R = 1$



$E(6) \to \mathcal{G}_{3_L 3_R 3_C} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to SM$



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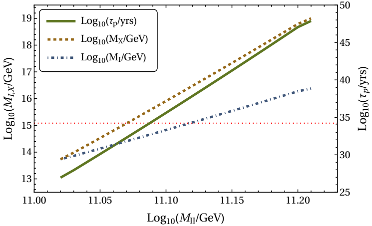

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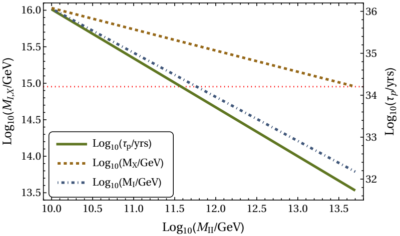

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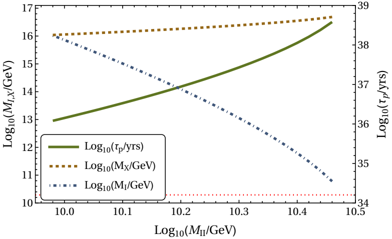

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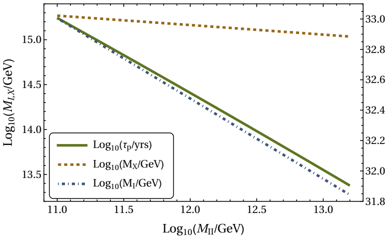
Two step breaking and τ_p compatible with $R \in [1/2 : 2]$



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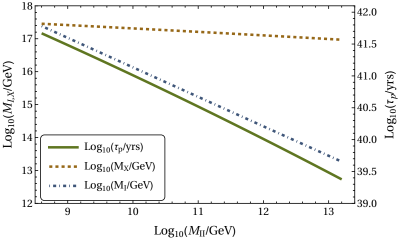

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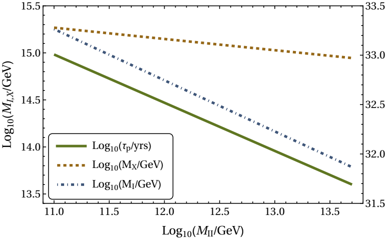

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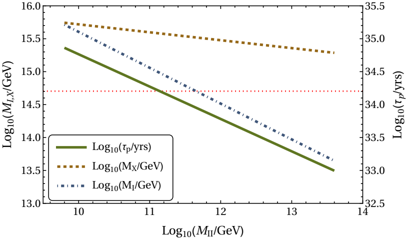

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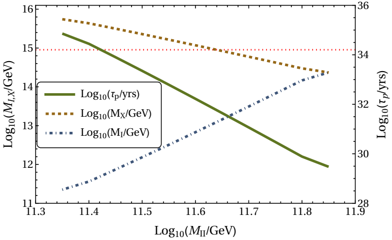

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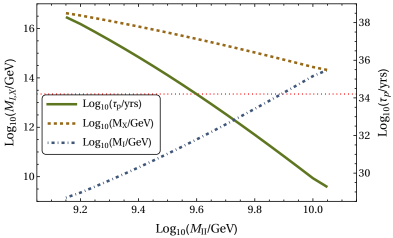

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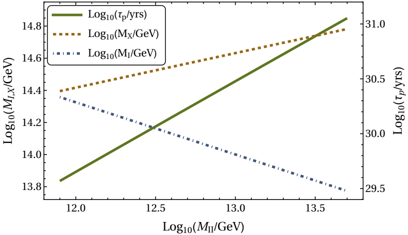

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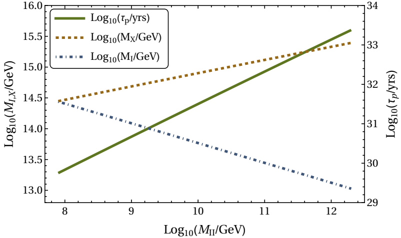

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$$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 1_R 4_C} \to \text{SM}.$$

For Further Reading I

- J. Chakrabortty, S. F. King and R. Maji, arXiv:1901.05867 [hep-ph].
- J. Chakrabortty, R. Maji, S. K. Patra, T. Srivastava and S. Mohanty, Phys. Rev. D **97**, no. 9, 095010 (2018) [arXiv:1711.11391 [hep-ph]].