Chiral Susceptibility in Quark Matter

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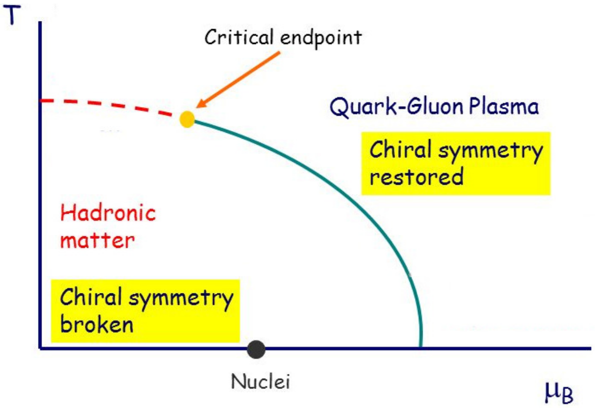
Plan of the Talk:

- Introduction
- NJL Model
- Wigner Function
- Chiral Condensate
- Chiral Susceptibility
- Results
- Summary

INTRODUCTION



INTRODUCTION

- Exploration of the QCD phase diagram is the one of the very exciting topic in the field of strong interaction.
- QCD exhibits a nonperturbative phenomenon: Chiral Symmetry breaking
- Breaking of the chiral symmetry is associated with the chiral condensate.

Introduction

- The chiral susceptibility measures the response of the chiral condensate with respect to the current quark mass.
- Chiral susceptibility can be defined as

$$\boxed{\chi_c = \frac{\partial}{\partial m}\langle \bar{\Psi}\Psi \rangle}$$

$$\Psi = (u \;\; d)^{\mathrm{T}}$$

$$\boxed{\chi_c = \frac{\partial}{\partial m}\langle \bar{u}u \rangle + \frac{\partial}{\partial m}\langle \bar{d}d \rangle}$$

- It is an order parameter to represent the phase transition.

NJL MODEL

- The 2 flavour NJL Lagrangian is

$$\mathscr{L} = \bar{\Psi}(i\not{D} - m + \mu_5\gamma^0\gamma_5)\Psi + G_1\left[(\bar{\Psi}\tau^a\Psi)^2 + (\bar{\Psi}i\gamma_5\tau^a\Psi)^2\right]$$
$$+ G_2\left[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\boldsymbol{\tau}\Psi)^2 - (\bar{\Psi}i\gamma_5\Psi)^2 + (\bar{\Psi}i\gamma_5\boldsymbol{\tau}\Psi)^2\right]$$

  Where $G_1 = (1-\alpha)g$, $G_2 = \alpha g$, $\Psi = (u\ d)^{\mathrm{T}}$, $a = 0, 1, 2, 3$ and $m = \mathrm{diag}(m_u\ m_d)$, $m_u = m_d = m_0$.

- Taking the mean field approximation, the lagrangian transforms as

$$\mathscr{L} = \bar{u}(i\not{D} - M_u + \mu_5\gamma^0\gamma^5)u + \bar{d}(i\not{D} - M_d + \mu\gamma^0 + \mu_5\gamma^0\gamma^5)d$$
$$- 2G_1\left[\langle\bar{u}u\rangle^2 + \langle\bar{d}d\rangle^2\right] - 4G_2\left[\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right]$$

  where M_u and M_d are the constituent masses of u and d quarks in terms of chiral condensates.

$$\boxed{M_u = m_0 - 4G_1\langle\bar{u}u\rangle - 4G_2\langle\bar{d}d\rangle} \text{ and } \boxed{M_d = m_0 - 4G_1\langle\bar{d}d\rangle - 4G_2\langle\bar{u}u\rangle}$$

WIGNER FUNCTION

- The Wigner function is the quantum analogue of the classical distribution function *Phys. J. A 54, 21 (2018)*
- The gauge invariant Wigner function in presence of the background gauge field, A_μ is defind using the gauge link between the fermion operators

$$W_{\alpha\beta}(x,p) = \frac{1}{h^4}\int d^4x' \langle \bar{\Psi}_\beta(x_+)U(x_+,x_-,A)\Psi_\alpha(x_-)\rangle e^{-ipx'/\hbar}$$

where

$$U(x_+,x_-,A) = \exp[-i\frac{q}{\hbar}\int_{x_-}^{x_+} A_\mu(z)dz^\mu] \text{ and } x_\pm = x \pm \frac{x'}{2}$$

- Using the solutions of the Dirac equation in the presence of the background gauge field $A_\mu = (0,-By,0,0)$

$$W(p) = \sum_{n,s}\Big\{ f_{FD}(E^{(n)}_{p_z,s})\,\delta(p_0 - E^{(n)}_{p_z,s})W^{(n)}_{+,s}(p) + \big[1-\bar{f}_{FD}(E^{(n)}_{p_z,s})\big]\delta(p_0+E^{(n)}_{p_z,s})W^{(n)}_{-,s}(p)\Big\}$$

where

$$E^{(0)}_{p_z} = \sqrt{M^2+(p_z-\mu_5)^2}\;,\; E^{(n)}_{p_z,s} = \sqrt{M^2+\left(\sqrt{p_z^2+2neB}-s\mu_5\right)^2}$$

Chiral Condensate

- The Wigner fucntion can be decomposed into 16 independent generators of the Clifford algebra as

$$W = \frac{1}{4}\left(F + i\gamma^5 P + \gamma^\mu V_\mu + \gamma^5\gamma^\mu A_\mu + \frac{1}{2}\sigma^{\mu\nu} S_{\mu\nu}\right)$$

- The chiral condensate in presence of external magnetic field and chiral chemical potential

$$\langle\bar{\Psi}\Psi\rangle = \int d^4p\, F = \int d^4p\, \mathrm{Tr}\,(W)$$

Ann. Phys. (N.Y.) 245, 445 (1996).

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$$\langle\bar{\Psi}\Psi\rangle = \frac{N_c\, eB}{2\pi^2}\Bigg[\int dp_z \frac{M}{E^{(0)}_{p_z}}\left(f_{FD}(E^{(0)}_{p_z}) + \bar{f}_{FD}(E^{(0)}_{p_z}) - 1\right) + \sum_{n=1}^{\infty}\sum_s \int dp_z \frac{M}{E^{(n)}_{p_z,s}}\left(f_{FD}(E^{(n)}_{p_z,s}) + \bar{f}_{FD}(E^{(n)}_{p_z,s}) - 1\right)\Bigg]$$

where

$$E^{(0)}_{p_z} = \sqrt{M^2 + (p_z - \mu_5)^2}\ ,\ E^{(n)}_{p_z,s} = \sqrt{M^2 + \left(\sqrt{p_z^2 + 2neB} - s\mu_5\right)^2}$$

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The UV divergent terms are

$$-\frac{N_c\, eB}{2\pi^2}\left[\int dp_z \frac{M}{E^{(0)}_{p_z}} + \sum_{n=1}^{\infty}\sum_s \int dp_z \frac{M}{E^{(n)}_{p_z,s}}\right]$$

Chiral Susceptibility

Susceptibility measures the effect on the medium by the external field. The chiral susceptibility is formulated as

$$\chi_c = \frac{\partial}{\partial m}\langle\bar{\Psi}\Psi\rangle = \left[\frac{\partial\langle\bar{u}u\rangle}{\partial m} + \frac{\partial\langle\bar{d}d\rangle}{\partial m}\right] = \chi_{cu} + \chi_{cd}$$

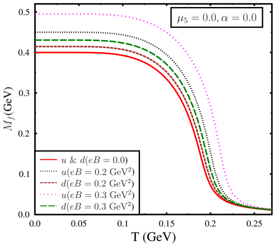
where m is the bare mass of quark and $\langle\bar{\Psi}\Psi\rangle$ is the chiral condensate.

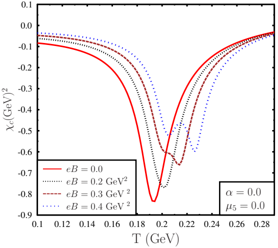
$$\chi_{cu} = \frac{\partial\langle\bar{u}u\rangle}{\partial M_u}\left[\frac{1+4(G_1-G_2)\,\frac{\partial\langle\bar{d}d\rangle}{\partial M_d}}{\left(1+4G_1\frac{\partial\langle\bar{u}u\rangle}{\partial M_u}\right)\left(1+4G_1\frac{\partial\langle\bar{d}d\rangle}{\partial M_d}\right) - 16G_2^2\,\frac{\partial\langle\bar{u}u\rangle}{\partial M_u}\,\frac{\partial\langle\bar{d}d\rangle}{\partial M_d}}\right]$$

$$\chi_{cd} = \frac{\partial\langle\bar{d}d\rangle}{\partial M_d}\left[\frac{1+4(G_1-G_2)\,\frac{\partial\langle\bar{u}u\rangle}{\partial M_u}}{\left(1+4G_1\frac{\partial\langle\bar{u}u\rangle}{\partial M_u}\right)\left(1+4G_1\frac{\partial\langle\bar{d}d\rangle}{\partial M_d}\right) - 16G_2^2\,\frac{\partial\langle\bar{u}u\rangle}{\partial M_u}\,\frac{\partial\langle\bar{d}d\rangle}{\partial M_d}}\right]$$

RESULTS

- The parameters are chosen as $m_u = m_d = 6$ MeV, $\Lambda = 590$ MeV and $g = 2.435/\Lambda^2$
- Constituent quark mass varies with temperature. As temperature rises, the mass of quark decreases.
- Magnetic field enhances the chiral condensate. This is called magnatic catalysis.
- **Chiral symmetry is approximatly restored in high temp limit.**
- **The chiral phase transition temperature increases with magenatic field.**

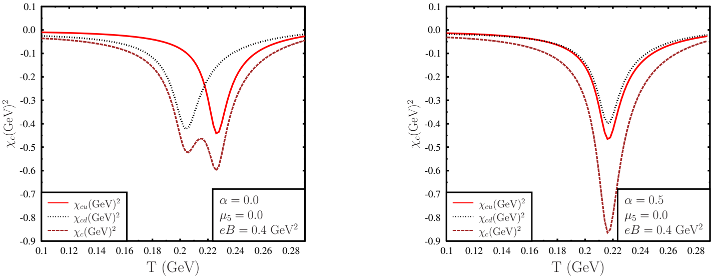




Phys. Rep. 407, 205 (2005)

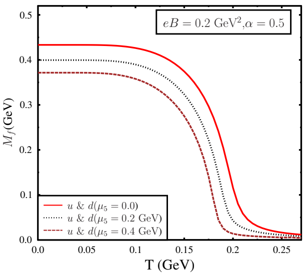
RESULTS

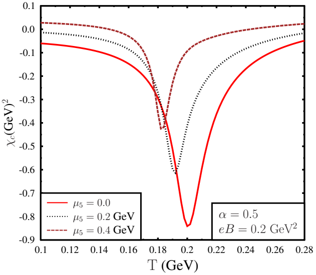
- Magnetic field interact with the quraks and breaks the isospin symmetry.
- But the mixing again restores the isospin symmetry.
- **In the presence of the magnetic field, the chiral phase transition temperature is different for the u and d quarks.**



RESULTS

- $\alpha = 0.5$ defines the maximum mixing of flavours.
- $G_1 = G_2 = 0.5g \Rightarrow M_u = M_d = m_0 - 2g\big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\big)$
- **The constituent quark mass decreses with temperature.**
- **The chiral phase transition temperature decreases with chiral chemical potential.**





SUMMARY

- we have seen that the constituent quark masses of the u and d quarks **decreases** as temperature gets increase.
- The constituent quark mass **decreases** with increase of chiral chemical potential.
- The chiral transition temperature **increases** with magnetic field.
- The chiral transition temperature **decreases** with chiral chemical potential.

SUMMARY

- we have seen that the constituent quark masses of the u and d quarks **decreases** as temperature gets increase.
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THANKYOU