Charged scalars confronting neutrino mass and muon g-2 anomaly

Nabarun Chakrabarty
Centre for High Energy Physics, IISc, Bangalore

WHEPP 2019, IIT Guwahati
Dec 09, 2019

Based on JHEP 1812 (2018) 104
NC, Cheng-Wei Chiang (National Taiwan U.), Takahiro Ohata (Kyoto U.), Koji Tsumura (Kyoto U.)

New physics in muon g-2

- No compelling evidence of new physics from the energy frontier (LEP, Tevatron, LHC) so far.
- Other experimental observations necessiate new physics.

1) Neutrino mass: Impossible to generate non-zero neutrino masses within the SM alone $\longrightarrow$ Seesaw mechanism!

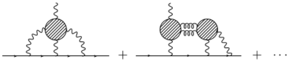
2) Muon anomaly: A 3.6 σ discrepancy between SM prediction and experimental data (from BNL) gives

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(63)(48) \times 10^{-11} \ .$$

- Our motive: To search for a common framework to accomodate the two aforementioned issues.

New physics in muon g-2

- Hadronic light-by-light scattering amplitudes are difficult to evaluate.



- Lattice QCD techniques have enabled more accurate evaluation of such amplitudes (arXiv 1911.08123)

The hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD

Thomas Blum,[1,2] Norman Christ,[3] Masashi Hayakawa,[4,5] Taku Izubuchi,[6,2]
Luchang Jin,[1,2,*] Chulwoo Jung,[6] and Christoph Lehner[7,6]

[1] *Physics Department, University of Connecticut, 2152 Hillside Road, Storrs, CT, 06269-3046, USA*

[2] *RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

[3] *Physics Department, Columbia University, New York, New York 10027, USA*

[4] *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

[5] *Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan*

[6] *Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

[7] *Universität Regensburg, Fakultät für Physik, 93040, Regensburg, Germany*

We report the first result for the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment with all errors systematically controlled. Several ensembles using 2+1 flavors of physical mass Möbius domain-wall fermions, generated by the RBC/UKQCD collaborations, are employed to take the continuum and infinite volume limits of finite volume lattice QED+QCD. We find $a_\mu^{\mathrm{HLbL}} = 7.20(3.98)_{\mathrm{stat}}(1.65)_{\mathrm{sys}} \times 10^{-10}$. Our value is consistent with previous model results and leaves little room for this notoriously difficult hadronic contribution to explain the difference between the Standard Model and the BNL experiment.

- A generic New Physics (NP) contribution to muon g-2 $\sim \mathcal{O}((m_\mu/\mathrm{M})^2)$
- Sensitivity to 10 MeV $<$ M $<$ 1 TeV

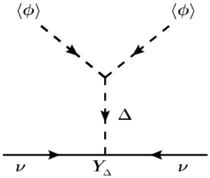
A Type-II seesaw solution?

- Features a complex scalar triplet with mass scale M_Δ (See the talks by E.J.Chun and M.Mitra)

$$\Delta = \begin{pmatrix} \frac{H^+}{\sqrt{2}} & \delta^{++} \\ \frac{1}{\sqrt{2}}(v_\Delta + \delta_0 + i\delta_1) & -\frac{H^+}{\sqrt{2}} \end{pmatrix},$$

$$\mathcal{L} \supset -\mu\phi^T(i\sigma_2)\Delta^\dagger\phi - y_\Delta^{ij}\,\overline{L_i^c}\,(i\sigma_2)\Delta L_j$$

- UV-complete realisation of the Weinberg operator



- Majorana neutrino mass $m_\nu^{ij} = \sqrt{2}Y_\Delta^{ij}v_\Delta \simeq \mu Y_\Delta^{ij}\frac{v^2}{M_\Delta^2}$

A Type-II seesaw solution?

- Leptophillic $H^+,\ \delta^{++} \Rightarrow$ Additional contributions to Δa_μ
- The contribution turns out to be negative. (Tsumura et al, 2003)
- $\Delta a_\mu = \Delta a_\mu^{\text{singly charged}} + \Delta a_\mu^{\text{doubly charged}}$
  $= -\frac{(m_\nu^2)^{\mu\mu}}{96\pi^2}\frac{m_\mu^2}{v_\Delta^2 m_{H^+}^2} - \frac{(m_\nu^2)^{\mu\mu}}{12\pi^2}\frac{m_\mu^2}{v_\Delta^2 m_{H^{++}}^2}$
- Opt for non-minimal framework

Extending the Type-II seesaw

- A possible remedy(?): Two doubly charged scalars H_1^{++}, H_2^{++}
  $\Downarrow$
  Interference between the 1-loop diagrams can generate a positive contribution
- A second doubly charged Higgs can emerge from various multiplets
- We choose the simplest: An $SU(2)_L$ singlet k^{++}
- A δ^{++}-k^{++} mixing can be arranged by the operator $\left(\tilde{\phi}^\dagger \Delta \phi k^{--} + \text{H.c.}\right)$

$$\Downarrow$$

$$H_1^{++}, H_2^{++}$$

Type II + more charged scalars

- We augment the Higgs Triplet model with the charged scalars $k^{++}, k_e^{+}, k_\mu^{+}, k_\tau^{+}$.
- A $\mathbb{Z}_3$ symmetry is introduced.

Field	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$\mathbb{Z}_3$
ϕ	$(\mathbf{1}, \mathbf{2}, 1/2)$	1
L_e, L_μ, L_τ	$(\mathbf{1}, \mathbf{2}, -1/2)$	$1, \omega, \omega^2$
e_R, μ_R, τ_R	$(\mathbf{1}, \mathbf{1}, -1)$	$1, \omega, \omega^2$
Δ	$(\mathbf{1}, \mathbf{3}, 1)$	1
k^{++}	$(\mathbf{1}, \mathbf{1}, 2)$	1
$k_e^{+}, k_\mu^{+}, k_\tau^{+}$	$(\mathbf{1}, \mathbf{1}, 1)$	$1, \omega, \omega^2$

Scalar sector

- The scalar potential $V = V_2 + V_3 + V_4$ $(\alpha = e, \mu, \tau)$:

$$
\begin{aligned}
V_2 &= \mu_\phi^2(\phi^\dagger\phi) + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger\Delta) + m_k^2|k^{++}|^2 + M_{\alpha\alpha}^2 k_\alpha^+ k_\alpha^-, \\
V_3 &= \mu_1\, \phi^T(i\sigma_2)\Delta^\dagger\phi + \mu_2\, \mathrm{Tr}\big(\Delta^\dagger\Delta^\dagger\big)k^{++} + \mu_{\alpha\beta}\, k_\alpha^+ k_\beta^+ k^{--} + \mathrm{H.c.} \\
V_4 &= \lambda(\phi^\dagger\phi)^2 + \lambda_1\phi^\dagger\phi\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_2[\mathrm{Tr}(\Delta^\dagger\Delta)]^2 + \lambda_3\mathrm{Tr}[(\Delta^\dagger\Delta)^2] + \lambda_4\phi^\dagger\Delta\Delta^\dagger\phi \\
&\quad + \lambda_5\phi^\dagger\phi|k^{++}|^2 + \lambda_6\,\mathrm{Tr}(\Delta^\dagger\Delta)|k^{++}|^2 + \lambda_7\big(\tilde{\phi}^\dagger\Delta\phi k^{--} + \mathrm{H.c.}\big) + \lambda_8|k^{++}|^4 \\
&\quad + \lambda_9\phi^\dagger\phi k_\alpha^+ k_\alpha^- + \lambda_{10}\,\mathrm{Tr}(\Delta^\dagger\Delta)k_\alpha^+ k_\alpha^- + \lambda_{11} k_\alpha^+ k_\alpha^- k^{++} k^{--} \\
&\quad + \lambda_{12}\phi^\dagger\Delta^\dagger\phi k_e^+ + \lambda_{13} k_\alpha^+ k_\alpha^- k_\beta^+ k_\beta^- .
\end{aligned}
$$

- Assumption 1: *Soft* $\mathbb{Z}_3$ violation through off-diagonal entries of $\mu_{\alpha\beta}$

- Assumption 2: $\lambda_{12} \to 0$

$\delta^{++} - k^{++}$ mixing

- $\lambda_7\big(\tilde{\phi}^\dagger \Delta \phi k^{--} + \text{H.c.}\big) \xrightarrow{\text{EWSB}}$ Mixing between δ^{++} and k^{++}

$$\begin{aligned} \delta^{++} &= c_\theta H_1^{++} + s_\theta H_2^{++} \\ k^{++} &= -s_\theta H_1^{++} + c_\theta H_2^{++} \end{aligned}$$

- $M_{1,2}^{++}$ are the masses of the $H_{1,2}$
- We choose a real λ_7
- Assumption 1 and 2:
  No mixing amongst $(H^+, k_e^+, k_\mu^+, k_\tau^+)$

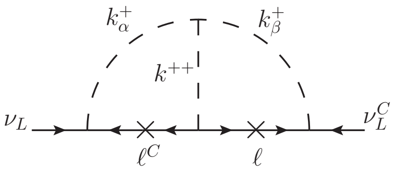
Yukawa sector

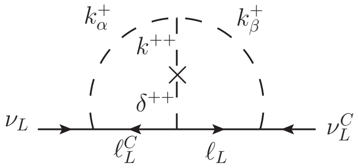
- Seesaw-like Yukawa interactions get flavor-restricted + New terms:

$$\mathcal{L}_{\rm Y} = -y_\Delta^{ee}\,\overline{L_e^c}\,(i\sigma_2)\Delta L_e - y_S^{ee}\,\overline{e_R^c}\,e_R k^{++} - 2\,y_\Delta^{\mu\tau}\,\overline{L_\mu^c}\,i\sigma_2\Delta L_\tau - 2y_S^{\mu\tau}\,\overline{\mu_R^c}\,\tau_R k^{++} - \sum_{\alpha=e,\mu,\tau} y_A^\alpha\,\epsilon^{\alpha\beta\gamma}\,\overline{L_\beta^c}\,i\sigma_2 L_\gamma k_\alpha^+ + \mathrm{H.c}$$

Radiative contributions to ν-mass

- $\mathbb{Z}_3$ symmetry $\Rightarrow$ Δ contributes to the *ee* and $\mu\tau$ elements of m_ν
- Two-loop contributions arise:





- $\mathbb{Z}_3$ breaking $\mu_{\alpha\beta}$ enter these amplitudes.
- Diagram on the left is similar to what is seen in the Zee-Babu model.
- Diagram on the right an artefact of $\delta^{++} - k^{++}$ mixing.

Chirality flip and Δa_μ

- Δ and k^{++} can couple to leptons of specific chiralities only

$$\mathcal{L}_\mathrm{Y} \supset -2\, y_\Delta^{\mu\tau}\, \overline{L_\mu^c}\, i\sigma_2 \Delta L_\tau - 2\, y_S^{\mu\tau}\, \overline{\mu_R^c}\, \tau_R k^{++} + \mathrm{H.c}$$

- The mass eigenstates couple to mixed chiralities

$$\mathcal{L}_Y \supset \sum_i \overline{\ell_\alpha^c} (y_{iL}^{\alpha\beta} P_L + y_{iR}^{\alpha\beta} P_R) \ell_\beta\, H_i^{++} + \text{H.c.}$$

where,

$$\begin{aligned} y_{1L}^{\alpha\beta} &= y_\Delta^{\alpha\beta} c_\theta , \\ y_{1R}^{\alpha\beta} &= y_S^{\alpha\beta} s_\theta \;, \\ y_{2L}^{\alpha\beta} &= y_\Delta^{\alpha\beta} s_\theta \;, \\ y_{2R}^{\alpha\beta} &= -y_S^{\alpha\beta} c_\theta \;. \end{aligned}$$

Chirality flip and Δa_μ

- $\Delta a_\mu^{\text{singly charged}} \simeq -\frac{m_\mu^2 (y_\Delta^{\mu\tau})^2}{48\pi^2 M_{H^+}^2} - \frac{m_\mu^2 (y_A^e)^2}{48\pi^2 (M_e^+)^2} - \frac{m_\mu^2 (y_A^\tau)^2}{48\pi^2 (M_\tau^+)^2}$

- $\theta \neq 0 \Rightarrow$ Chirality flipping effect in the 1-loop diagrams for muon g-2

$$\mathcal{L}_Y \supset -2\, y_\Delta^{\mu\tau}\, \overline{L_\mu^c}\, i\sigma_2 \Delta L_\tau - 2\, y_S^{\mu\tau}\, \overline{\mu_R^c}\, \tau_R k^{++} + \mathrm{H.c}$$

- For $M_2^{++} = M_1^{++} + \Delta M$ the chirality flipping contribution is

$$\Delta a_\mu^{\text{doubly charged}} \simeq \frac{y_\Delta^{\mu\tau} y_S^{\mu\tau}}{16\pi^2} \frac{m_\mu m_\tau}{(M_1^{++})^3} \Delta M s_\theta c_\theta \log \frac{m_\tau^2}{(M_1^{++})^2} \,.$$

- Size of the chirality flip is $\sim O(m_\tau / m_\mu) \Rightarrow \Delta a_\mu > 0$

LFV

- $\mathbb{Z}_3$ symmetry $\longrightarrow$ $H_{1,2}^{++}$ couple only to ee and $\mu\tau$ bilinears
  $\longrightarrow$ k_e^+ couples to $\mu\nu_\tau$; k_μ^+ to $\tau\nu_e$; k_τ^+ to $e\nu_\mu$
  $\Downarrow$

  (1) $l_i \to l_j\gamma$ are absent

  (2) Only $\tau \to \bar{\mu}ee$ has non-zero rates among the $l_i \to \bar{l}_j l_k l_l$ LFV processes

$$\frac{\mathrm{BR}_{\tau\to\bar{\mu}ee}}{\mathrm{BR}_{\tau\to\mu\nu\nu}} = \frac{1}{4G_F^2}\Big\{\big(|y_S^{\tau\mu}|^2|y_\Delta^{ee}|^2 + |y_\Delta^{\tau\mu}|^2|y_S^{ee}|^2\big)s_\theta^2 c_\theta^2\Big(\frac{1}{(M_1^{++})^2} - \frac{1}{(M_2^{++})^2}\Big)^2$$
$$+|y_S^{\tau\mu}|^2|y_S^{ee}|^2\Big(\frac{s_\theta^2}{(M_1^{++})^2} + \frac{c_\theta^2}{(M_2^{++})^2}\Big)^2$$
$$+|y_\Delta^{\tau\mu}|^2|y_\Delta^{ee}|^2\Big(\frac{c_\theta^2}{(M_1^{++})^2} + \frac{s_\theta^2}{(M_2^{++})^2}\Big)^2\Big\},$$

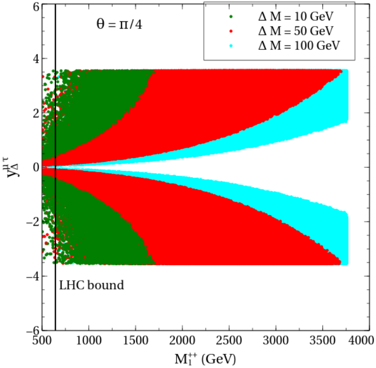
- We take all Yukawa couplings real. The EDM amplitudes vanish at one-loop.

Parameter scans

- We choose the following parameters: $(v_\Delta, M_{H^+}, M_i^{++}, M_\alpha^+, y_A^\alpha, y_\Delta^{ee}, y_S^{ee}, y_\Delta^{\mu\tau}, y_S^{\mu\tau}, \theta)$ as independent.
- $1.2 \times 10^{-9} \leq \Delta a_\mu \leq 4.4 \times 10^{-9}$ (2σ range)
- *Constraints*
  (a) $BR_{\tau \to \bar{\mu} ee} < 8.4 \times 10^{-9}$ (most recent limit)
  (b) $\lambda_7 = 2\, s_\theta\, c_\theta \left[(M_2^{++})^2 - (M_1^{++})^2\right]/v^2$ remains perturbative, *i.e.*, $|\lambda_7| \leq 4\pi$

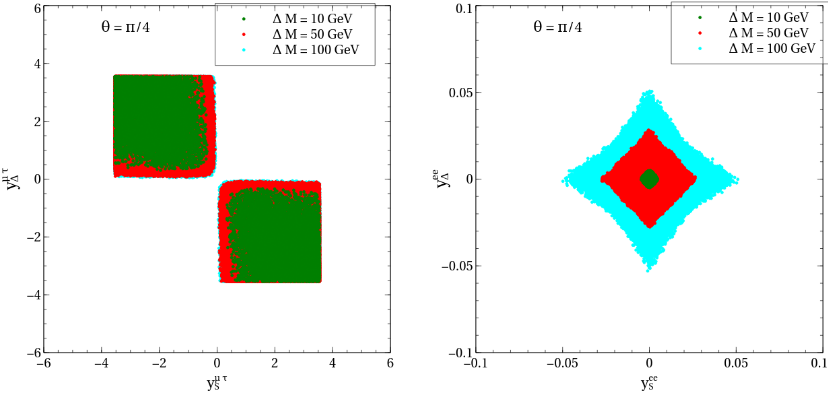
Parameter scans

- $M_{H^+} = M_1^{++},\ M_\alpha^+ \simeq 800$ GeV, $v_\Delta = 10^{-15}$ GeV
- $y_\Delta^{\mu\tau}$ is free from the $m_\nu^{\mu\tau}$ constraint.



Parameter scans

- Parameter space most relaxed for maximal mixing ($\theta = \frac{\pi}{4}$)



- $BR_{\tau \to \bar{\mu} ee}$ can be suppressed by choosing y_{Δ}^{ee} and y_{S}^{ee} to be small.

Chirality flip and ν-mass

- m_ν = $\text{Tree}_{\text{Type-II}} + \text{Loop}_{\text{ZB}} + \text{Loop}_{\delta^{++}-k^{++}}$

$$m_\nu^{\alpha\beta} = \sqrt{2} y_\Delta^{\alpha\beta} v_\Delta$$
$$-16 \sum_{\alpha'\beta'\alpha''\beta''} \mu_{\alpha''\beta''} y_A^{\alpha''} \epsilon^{\alpha\alpha'\alpha''} y_A^{\beta''} \epsilon^{\beta\beta'\beta''} \Big\{ y_S^{\alpha'\beta'} \Big[s_\theta^2 I_{k1}^{\alpha''\beta''\alpha'\beta'} + c_\theta^2 I_{k2}^{\alpha''\beta''\alpha'\beta'} \Big]$$
$$+ y_\Delta^{\alpha'\beta'} s_\theta c_\theta \Big[- I_{\Delta 1}^{\alpha''\beta''\alpha'\beta'} + I_{\Delta 2}^{\alpha''\beta''\alpha'\beta'} \Big] \Big\},$$

$$I_k(m_1, m_2, m, m_c, m_d)$$
$$= \int \frac{d^d p_E}{(2\pi)^d} \frac{d^d q_E}{(2\pi)^d} \frac{m_c m_d}{(p_E^2 + m_1^2)(p_E^2 + m_c^2)(q_E^2 + m_2^2)(q_E^2 + m_d^2)((p_E + q_E)^2 + m^2)}$$

$$I_\Delta(m_1, m_2, m, m_c, m_d)$$
$$= - \int \frac{d^d p_E}{(2\pi)^d} \frac{d^d q_E}{(2\pi)^d} \frac{p_E . q_E}{(p_E^2 + m_1^2)(p_E^2 + m_c^2)(q_E^2 + m_2^2)(q_E^2 + m_d^2)((p_E + q_E)^2 + m^2)}$$

$m_{1,2} \to M_\alpha^+$, $m \to M_{1,2}^{++}$, $m_{c,d} \to$ lepton mass

Fitting with neutrino data

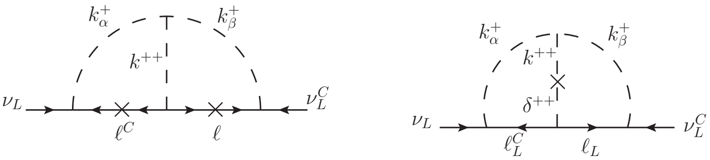
$$
\begin{aligned}
m_\nu &= U^*_{\rm PMNS}\, m_\nu^{\rm diag}\, U^T_{\rm PMNS}\,, \\
U_{\rm PMNS} &= V_{\rm PMNS} \times {\rm diag}(1, e^{i\alpha_{21}/2},, e^{i\alpha_{31}/2}) \text{ and} \\
V_{\rm PMNS} &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}
\end{aligned}
$$

We take the following for the neutrino sector parameters

$$
\begin{aligned}
&\sin^2\theta_{12} = 0.307\ , \quad \sin^2\theta_{23} = 0.510\ , \quad \sin^2\theta_{13} = 0.021\ , \\
&\Delta m^2_{21} = 7.45 \times 10^{-5}\ \text{GeV}^2\ , \ \Delta m^2_{32} = 2.53 \times 10^{-3}\ \text{GeV}^2\ , \\
&\delta_{CP} = 1.41\pi\ , \quad \alpha_{21} = \alpha_{31} = 0\ .
\end{aligned}
$$

- The six complex $m_\nu^{\alpha\beta}$ can be expressed as linear combinations of the six $\mu_{\alpha\beta}$
  Example: $m_\nu^{e\mu} \propto \mu_{e\tau}$, $m_\nu^{e\tau} \propto \mu_{e\mu}$
- $\mu_{\alpha\beta}$ have to be complex
- $\mu_{\alpha\beta}$ do not enter into the one-loop muon g-2 and LFV amplitudes.

Fitting with neutrino data



- I_Δ integrand has a different momentum structure.
- $I_\Delta \sim (m^2_{\text{scalar}}/m^2_{\text{lepton}})\ I_k$
- $\theta \neq 0$ causes the new 2-loop amplitude to be dominant.
  $\Downarrow$
- $\mu_{\alpha\beta} << $ Trilinear parameter in the Zee-Babu model

Conclusions + Future directions

- FNAL is aiming to measure the muon g-2 more precisely. The muon anomaly (as long it persists) signals new physics.
- $\delta^{++} - k^{++}$ mixing $\Rightarrow$ Chirality flip leading to a possible explanation of the muon anomaly

$\Rightarrow$ A two-loop amplitude different from the Zee-Babu case.

$\Rightarrow$ Zero EDM at one-loop.

$\Rightarrow$ LFV within bounds

Possible extensions

- Collider search of doubly charged scalar that has sizeable couplings to $\mu - \tau$
- Extending and using such a framework to explain the flavor anomalies (Leptoquarks?)

Thank you
for your attention

$$
\begin{aligned}
\Delta a_\mu^{\Delta^+} &= -\frac{m_\mu^2}{8\pi^2(1+2v_\Delta^2/v_\phi^2)}(y_\Delta^{\mu\tau})^2 \int_0^1 dx \frac{x(1-x)}{M_{H^+}^2 - m_\mu^2(1-x)} \ , \\
\Delta a_\mu^{k^+} &= -\frac{m_\mu^2}{16\pi^2} \sum_{\alpha=e,\tau} (y_A^\alpha)^2 \int_0^1 dx \frac{x(1-x)}{(M_\alpha^+)^2 - m_\mu^2(1-x)}, \\
\Delta a_\mu^{H_i^{++}} &= -\frac{m_\mu^2}{4\pi^2} \int_0^1 dx \ x^2 \frac{[(y_{iL}^{\mu\tau})^2 + (y_{iR}^{\mu\tau})^2](1-x) + 2\, y_{iL}^{\mu\tau} y_{iR}^{\mu\tau}(m_\tau/m_\mu)}{m_\mu^2 x^2 + (m_\tau^2 - m_\mu^2)x + (M_i^{++})^2(1-x)} \\
&\quad - \frac{m_\mu^2}{2\pi^2} \int_0^1 dx \ x(1-x) \frac{[(y_{iL}^{\mu\tau})^2 + (y_{iR}^{\mu\tau})^2]x + 2\, y_{iL}^{\mu\tau} y_{iR}^{\mu\tau}(m_\tau/m_\mu)}{m_\mu^2 x^2 + ((M_i^{++})^2 - m_\mu^2)x + m_\tau^2(1-x)},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{BR}_{\tau\to\bar{\mu}ee}}{\mathrm{BR}_{\tau\to\mu\nu\nu}} = \frac{1}{4G_F^2}\Big\{ & \big(|y_S^{\tau\mu}|^2|y_\Delta^{ee}|^2 + |y_\Delta^{\tau\mu}|^2|y_S^{ee}|^2\big)s_\theta^2 c_\theta^2 \Big(\frac{1}{(M_1^{++})^2} - \frac{1}{(M_2^{++})^2}\Big)^2 \\
& + |y_S^{\tau\mu}|^2|y_S^{ee}|^2 \Big(\frac{s_\theta^2}{(M_1^{++})^2} + \frac{c_\theta^2}{(M_2^{++})^2}\Big)^2 \\
& + |y_\Delta^{\tau\mu}|^2|y_\Delta^{ee}|^2 \Big(\frac{c_\theta^2}{(M_1^{++})^2} + \frac{s_\theta^2}{(M_2^{++})^2}\Big)^2 \Big\} \ ,
\end{aligned}
$$

$$(m_1|m_2|m) = \int d^d p_E d^d q_E \; \frac{1}{(p_E^2+m_1^2)(q_E^2+m_2^2)((p_E+q_E)^2+m^2)} \; ,$$

$$(2m_1|m_2|m) = \int d^d p_E d^d q_E \; \frac{1}{(p_E^2+m_1^2)^2(q_E^2+m_2^2)((p_E+q_E)^2+m^2)}$$

$$I_\Delta(m_1, m_2, m, m_c, m_d)$$
$$= -\int \frac{d^d p_E}{(2\pi)^d}\frac{d^d q_E}{(2\pi)^d}\frac{p_E.q_E}{(p_E^2+m_1^2)(p_E^2+m_c^2)(q_E^2+m_2^2)(q_E^2+m_d^2)((p_E+q_E)^2+m^2)}$$

We define

$$D_1 = p_E^2 + m_1^2$$
$$D_2 = q_E^2 + m_2^2$$
$$D_c = p_E^2 + m_c^2$$
$$D_d = q_E^2 + m_d^2$$
$$D = (p_E + q_E)^2 + m^2$$

and

$$I_\Delta(m_1, m_2, m, m_c, m_d)$$
$$= -\frac{1}{2}\int \frac{d^d p_E}{(2\pi)^d}\frac{d^d q_E}{(2\pi)^d}\left[\frac{(D - m^2 - D_1 + m_1^2 - D_2 + m_2^2)}{D_1 D_c D_2 D_d D}\right]$$

$$
\begin{aligned}
I_\Delta(m_1,m_2,m,m_c,m_d) = & -\frac{1}{2}\int \frac{d^dp_E}{(2\pi)^d}\frac{d^dq_E}{(2\pi)^d}\left[\frac{1}{D_1D_cD_2D_d}\right] \\
& -\frac{1}{2}\frac{1}{(2\pi)^8}\frac{1}{(m_2^2-m_d^2)}\Big[(m_c|m_2|m)-(m_c|m_d|m)\Big] \\
& -\frac{1}{2}\frac{1}{(2\pi)^8}\frac{1}{(m_1^2-m_c^2)}\Big[(m_1|m_d|m)-(m_c|m_d|m)\Big] \\
& -\frac{1}{2}\frac{1}{(2\pi)^8}\frac{(m_1^2+m_2^2-m^2)}{(m_1^2-m_c^2)(m_2^2-m_d^2)}\Big[(m_1|m_2|m)-(m_1|m_d|m) \\
& -(m_c|m_2|m)+(m_c|m_d|m)\Big] \\
= & -\frac{1}{2}\int \frac{d^dp_E}{(2\pi)^d}\frac{d^dq_E}{(2\pi)^d}\left[\frac{1}{D_1D_cD_2D_d}\right] \\
& -\frac{1}{2}\frac{1}{(2\pi)^8}\frac{1}{(m_1^2-m_c^2)(m_2^2-m_d^2)}\Bigg[(m_1^2+m_2^2-m^2)(m_1|m_2|m) \\
& +(m^2-m_2^2-m_c^2)(m_c|m_2|m)+(m^2-m_1^2-m_d^2)(m_1|m_d|m) \\
& +(m_c^2+m_d^2-m^2)(m_c|m_d|m)\Bigg]
\end{aligned}
$$

$$= -\frac{1}{2}\int \frac{d^d p_E}{(2\pi)^d}\frac{d^d q_E}{(2\pi)^d}\left[\frac{1}{D_1 D_c D_2 D_d}\right] - \frac{1}{2}\frac{1}{(3-d)}\frac{1}{(2\pi)^8}\frac{1}{(m_1^2-m_c^2)(m_2^2-m_d^2)}$$

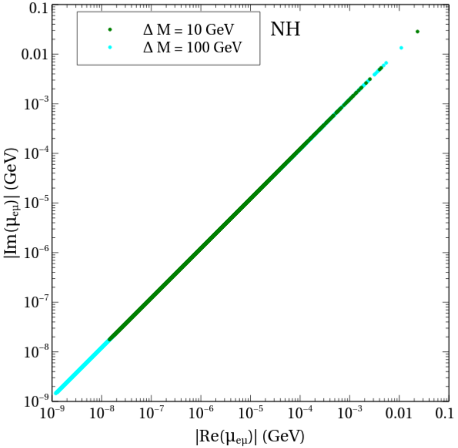
$$\Big[(m_1^2+m_2^2-m^2)\Big(m_1^2(2m_1|m_2|m)+m_2^2(2m_2|m_1|m)+m^2(2m|m_1|m_2)\Big)$$

$$+(m^2-m_2^2-m_c^2)\Big(m_c^2(2m_c|m_2|m)+m_2^2(2m_2|m_c|m)+m^2(2m|m_c|m_2)\Big)$$

$$+(m^2-m_1^2-m_d^2)\Big(m_1^2(2m_1|m_d|m)+m_d^2(2m_d|m_1|m)+m^2(2m|m_1|m_d)\Big)$$

$$+(m_c^2+m_d^2-m^2)$$

$$\Big(m_c^2(2m_c|m_d|m)+m_d^2(2m_d|m_c|m)+m^2(2m|m_c|m_d)\Big)\Big] \qquad \text{(A.18c)}$$



Δ M = 10 GeV
Δ M = 100 GeV
NH
|Im(μ_eμ)| (GeV)
|Re(μ_eμ)| (GeV)
0.1
0.01
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9

Model B: Introduction

- A new model comprising $k^{++}, \Delta_e, \Delta_\mu, \Delta_\tau$.
- Again a $\mathbb{Z}_3$ symmetry

Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$\mathbb{Z}_3$
k^{++}	$(1,1,2)$	1
Δ_e	$(1,3,1)$	1
Δ_μ	$(1,3,1)$	ω
Δ_τ	$(1,3,1)$	ω^2

Table: Quantum numbers of the additional scalar fields in Model B under the SM gauge group and $\mathbb{Z}_3$.

- *A complimentarity to Model A noted.*

Model B: Scalar potential

- The scalar potential $V = V_2 + V_3 + V_4$:

$$\begin{aligned}
V_2 =& \mu_\phi^2(\phi^\dagger\phi) + M^2_{\Delta\alpha\beta}\mathrm{Tr}(\Delta^\dagger_\alpha\Delta_\beta) + M_k^2|k^{++}|^2, \\
V_3 =& \mu_e\,\phi^{\mathrm{T}}(i\sigma_2)\Delta^\dagger_e\phi + \mu_\mu\,\phi^{\mathrm{T}}(i\sigma_2)\Delta^\dagger_\mu\phi + \mu_\tau\,\phi^{\mathrm{T}}(i\sigma_2)\Delta^\dagger_\tau\phi \\
&+ \mu_2\,k^{++}\mathrm{Tr}(\Delta^\dagger_e\Delta^\dagger_e) + \mathrm{H.c.},
\end{aligned}$$

$$\begin{aligned}
V_4 =& \lambda(\phi^\dagger\phi)^2 + \lambda_{1\alpha}(\phi^\dagger\phi)\mathrm{Tr}(\Delta^\dagger_\alpha\Delta_\alpha) \\
&+ (\lambda_{2\alpha\beta\gamma\delta}\mathrm{Tr}(\Delta^\dagger_\alpha\Delta_\beta)\mathrm{Tr}(\Delta^\dagger_\gamma\Delta_\delta) + \mathrm{H.c.}) \\
&+ (\lambda_{3\alpha\beta\gamma\delta}\mathrm{Tr}(\Delta^\dagger_\alpha\Delta_\beta\Delta^\dagger_\gamma\Delta_\delta) + \mathrm{H.c.}) \\
&+ \lambda_{4\alpha}\phi^\dagger\Delta_\alpha\Delta^\dagger_\alpha\phi + \lambda_5\phi^\dagger\phi|k^{++}|^2 + \lambda_{6\alpha}\mathrm{Tr}(\Delta^\dagger_\alpha\Delta_\alpha)|k^{++}|^2 \\
&+ \lambda_7(\tilde{\phi}^\dagger\Delta_e\phi k^{--} + \mathrm{H.c.}) + \lambda_8|k^{++}|^4.
\end{aligned}$$

- The trilinear $\mathbb{Z}_3$-breaking terms with $\mu_\mu, \mu_\tau \neq 0$ ensure all the triplets acquire VEVs.

- $< \Delta_\alpha >= v_\alpha$ $(\alpha = e, \mu, \tau)$, $v_\Delta^2 = v_e^2 + v_\mu^2 + v_\tau^2$

Model B: Scalar mixings

$$\Delta_\alpha = \begin{pmatrix} \frac{\delta_\alpha^+}{\sqrt{2}} & \delta_\alpha^{++} \\ \frac{1}{\sqrt{2}}(v_\alpha + \delta_{0\alpha} + i\delta_{1\alpha}) & -\frac{\delta_\alpha^+}{\sqrt{2}} \end{pmatrix} .$$

- For $v_\Delta \ll v_\phi$

$$\begin{pmatrix} G^+ \\ H_e^+ \\ H_\mu^+ \\ H_\tau^+ \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \delta_e^+ \\ \delta_\mu^+ \\ \delta_\tau^+ \end{pmatrix} , \quad \begin{pmatrix} k^{++} \\ \delta_e^{++} \\ \delta_\mu^{++} \\ \delta_\tau^{++} \end{pmatrix} = \begin{pmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} H_1^{++} \\ H_2^{++} \\ H_\mu^{++} \\ H_\tau^{++} \end{pmatrix}$$

- $H_{1(2)}^{++}$ has mass $M_{1(2)}^{++}$.

Model B: Yukawa sector

- Seesaw interactions get restricted by $\mathbb{Z}_3$

$$\mathcal{L}_{\rm Y} = -y_\Delta^{ee}\,\overline{L_e^c}\,(i\sigma_2)\Delta_e L_e - y_S^{ee}\,\overline{e_R^c}\,e_R k^{++} - 2\,y_\Delta^{\mu\tau}\,\overline{L_\mu^c}\,i\sigma_2\Delta_e L_\tau - 2\,y_S^{\mu\tau}\,\overline{\mu_R^c}\,\tau_R k^{++}$$
$$-y_\Delta^{\mu\mu}\,\overline{L_\mu^c}\,(i\sigma_2)\Delta_\mu L_\mu - 2\,y_\Delta^{e\tau}\,\overline{L_e^c}\,(i\sigma_2)\Delta_\mu L_\tau - y_\Delta^{\tau\tau}\,\overline{L_\tau^c}\,(i\sigma_2)\Delta_\tau L_\tau - 2\,y_\Delta^{e\mu}\,\overline{L_e^c}\,(i\sigma_2)\Delta_\tau L_\mu$$
$$+{\rm H.c.} \tag{2.16}$$

- Neutrino mass generated at the tree level

$$m_\nu = \sqrt{2}\begin{pmatrix} y_\Delta^{ee}\,v_e & y_\Delta^{e\mu}\,v_\tau & y_\Delta^{e\tau}\,v_\mu \\ y_\Delta^{e\mu}\,v_\tau & y_\Delta^{\mu\mu}\,v_\mu & y_\Delta^{\mu\tau}\,v_e \\ y_\Delta^{e\tau}\,v_\mu & y_\Delta^{\mu\tau}\,v_e & y_\Delta^{\tau\tau}\,v_\tau \end{pmatrix}\,.$$

- Complex $y_\Delta^{\alpha\beta}$ and $v_\alpha \neq 0$ needed to satisfy neutrino data

Model B: Similarities with Model A

- $k^{++} - \delta^{++}$ mixing in model A $\Longleftrightarrow$ $k^{++} - \delta_e^{++}$ mixing in model B
  $\Downarrow$
- The chirality flipped contribution to Δa_μ remains the same
- The branching fraction in the $\tau \to \bar{\mu} ee$ rate is also the same

Model B: Differences with Model A

- Radiative neutrino mass in Model A vs. Tree level neutrino mass in Model B
- Real $y_\Delta^{\alpha\beta}$ in Model A vs. complex $y_\Delta^{\alpha\beta}$ in Model B
  $$\Downarrow$$
  $$\text{EDM} \neq 0$$
- $\tau \to \bar{e}\mu\mu$ is turned on in Model B (apart from $\tau \to \bar{\mu}ee$)

Model B: EDM

Model prediction for electron EDM (for instance):

$$d_e = -\frac{2em_e \sin\theta \cos\theta y_S^{ee}\ \mathrm{Im}(y_\Delta^{ee})}{(4\pi)^2} \times \left\{ \frac{-2\log\left((M_1^{++})^2/m_e^2\right) + 1}{(M_1^{++})^2} + \frac{2\log\left((M_2^{++})^2/m_e^2\right) - 1}{(M_2^{++})^2} \right\}$$

Latest constraints:

$$|d_e| < 1.1 \times 10^{-29}\ e\text{-cm},$$
$$|d_\mu| < 1.9 \times 10^{-19}\ e\text{-cm},$$
$$|d_\tau| < (-0.22 - 0.45) \times 10^{-16}\ e\text{-cm}.$$

Model B: Parameter scans

- The independent parameters here are $(M_\alpha^+, M_i^{++}, v_\alpha, y_S^{\mu\tau}, y_S^{ee}, \theta)$
- $y_\Delta^{ee} = \frac{m_\nu^{ee}}{\sqrt{2}v_e}$ and $y_\Delta^{\mu\tau} = \frac{m_\nu^{\mu\tau}}{\sqrt{2}v_e}$

$$\Downarrow$$

- v_e must be in the optimum ball-park to appropriately enhance (suppress) $y_\Delta^{\mu\tau}$ (y_Δ^{ee})

$$\mathrm{BR}_{\tau\to\bar{e}\mu\mu} = \frac{|y_\Delta^{e\tau}|^2 |y_\Delta^{\mu\mu}|^2}{4G_F^2 (M_\tau^{++})^4} .$$

$\mathrm{BR}_{\tau\to\bar{e}\mu\mu} < 10^{-8}$ gives

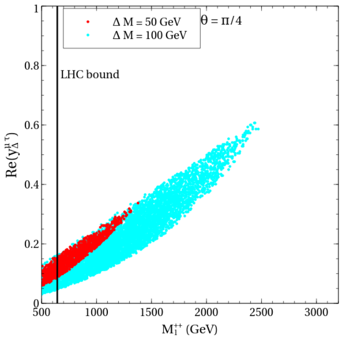
$$v_\mu \gtrsim \frac{v}{M_\tau^{++}} \sqrt{\frac{|m_\nu^{e\tau}|}{1 \text{ eV}} \frac{|m_\nu^{\mu\mu}|}{1 \text{ eV}}} \times 10^{-7} \text{ GeV} .$$

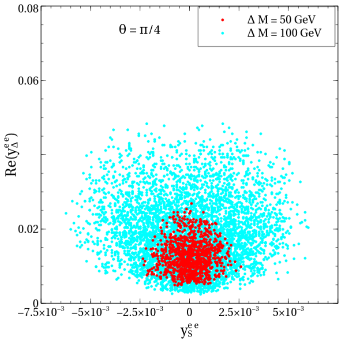
$$\Downarrow$$

- $v_\mu = v_\tau = 10^{-8}$ GeV evades the constraint

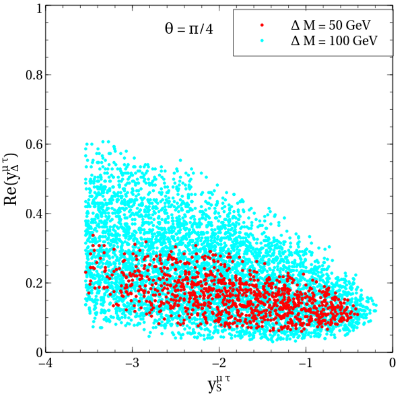
Model B: Parameter scans

Assuming normal hierarchy:

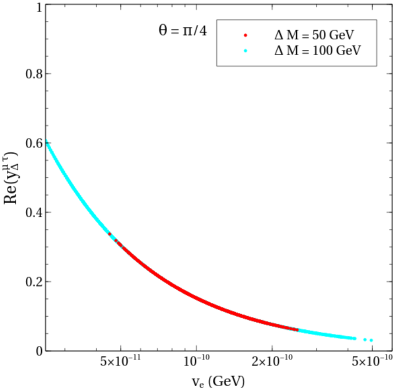




Model B: Parameter scans

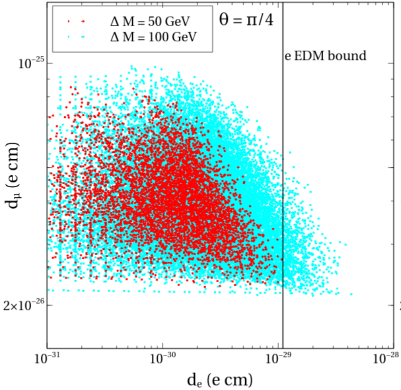


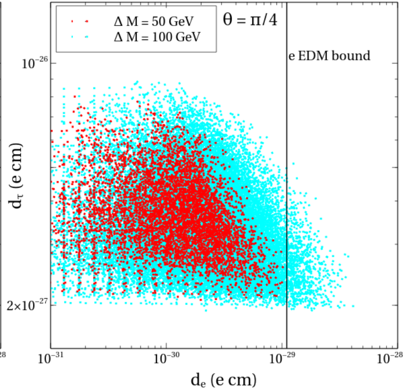
Model B: Parameter scans



- The VEV of Δ_e gets bounded from both ends.

Model B: Parameter scans





- Parameter region mostly compatible with EDM constraints.

Model B: Conclusions

- Δ_e–k^{++} mixing paves the way for a chirality flipping contribution in the muon $g-2$ loops.
- In the case of a normal neutrino mass hierarchy, the parameter space favoring an enhanced muon $g-2$ also complies with the bounds on the branching fractions of $\tau \to \bar{\mu}ee$ and $\tau \to \bar{e}\mu\mu$.
- The present scenario disfavors an inverted neutrino mass hierarchy.
- Unlike in Model A, the triplet VEV gets bounded from both ends in the process of reconciling the muon $g-2$ anomaly with LFV constraints.