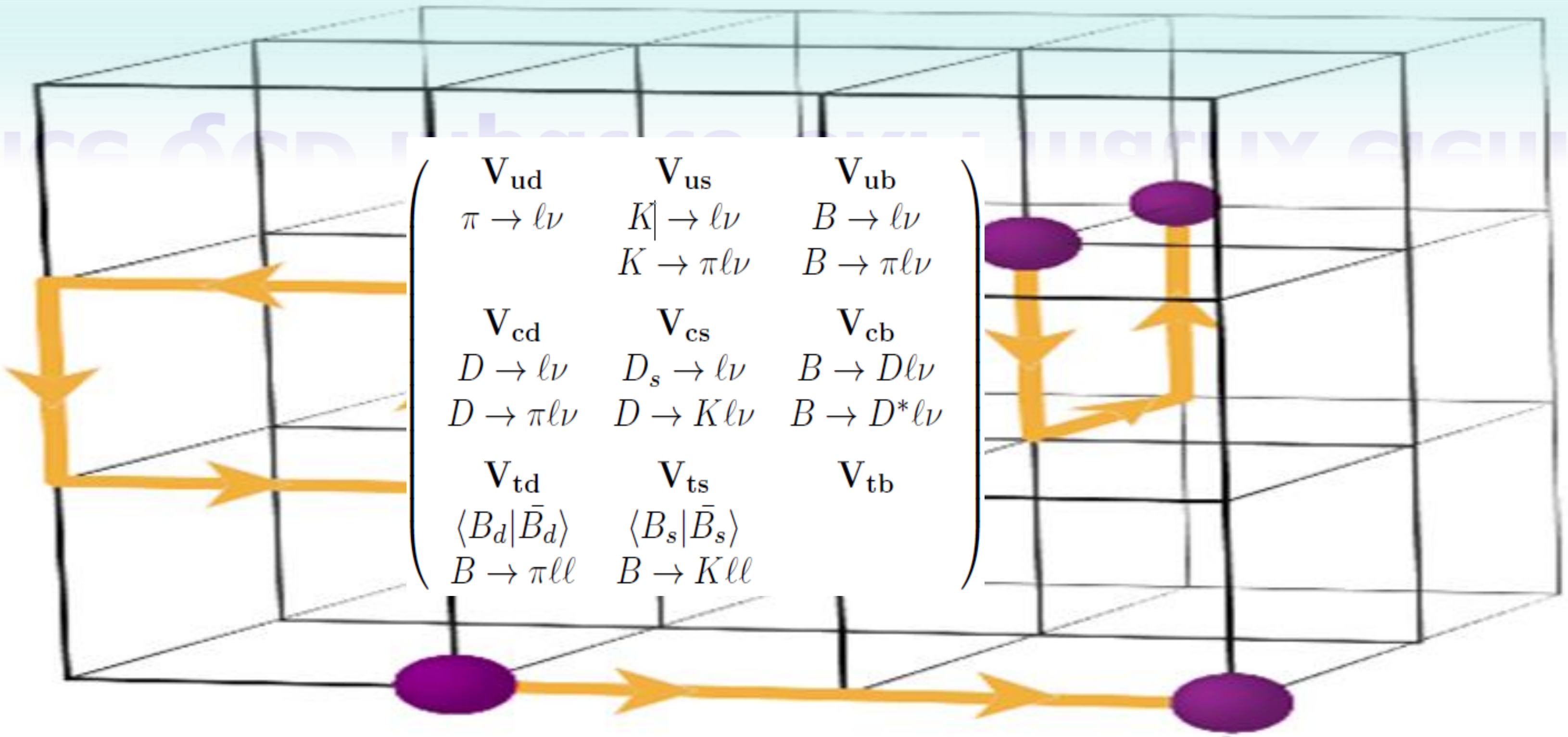
Lattice QCD input to CKM matrix elements



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Most of the information are taken from:

1. FLAG Review 2019 (Aoki et al, 1902.08191)
2. Lattice 2019 talk by S. Gottlieb

Weak interaction and Lattice QCD

- Lattice QCD is needed
  - to interpret flavour physics data
  - to extract the values of CKM matrix elements

- Most extensions of the Standard Model contain new CP- violating phases, new quark flavour-changing interactions
  - ➔ New Physics effects expected in the quark flavour sector

- To describe weak interaction involving quarks, one must include effects of confining quarks into hadrons.

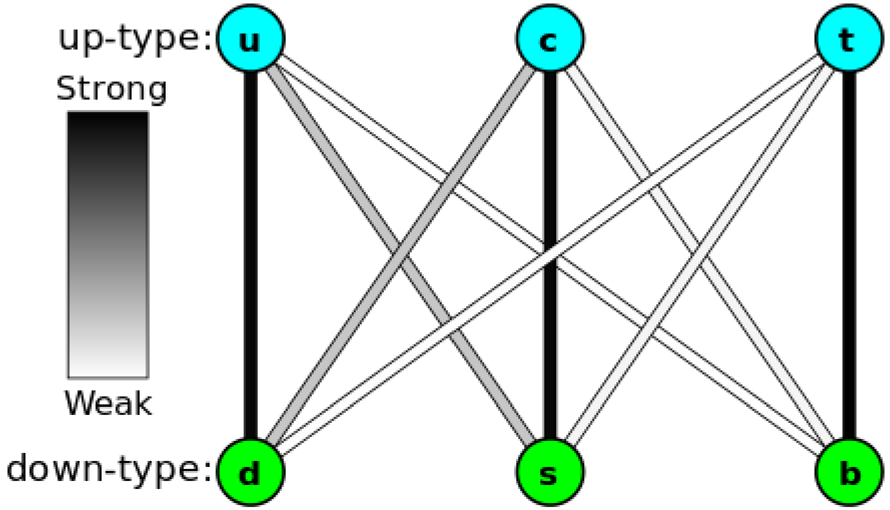
- Typically most non-perturbative QCD effects get absorbed into hadronic matrix elements such as decay constants, form factors and bag parameters

- So far, Lattice QCD is the best tool to calculate non-perturbative QCD effects with **controlled systematics.**

Using LQCD we can calculate two, three and four point functions with control systematics

CKM Matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Wikipedia

- CKM matrix is unitary
  - Each row and column is a complex unit vector
  - Each row (column) is orthogonal to other rows (columns)

- Violation of unitarity is evidence of new-physics

- If two different processes produce two different values of the matrix elements, that could also be evidence for new physics

Some relevant processes corresponding to CKM matrix elements:

$$\begin{pmatrix} \begin{array}{c}\mathbf{V_{ud}}\\ \pi\to\ell\nu\\ \\ \end{array} & \begin{array}{c}\mathbf{V_{us}}\\ K\to\ell\nu\\ K\to\pi\ell\nu\end{array} & \begin{array}{c}\mathbf{V_{ub}}\\ B\to\ell\nu\\ B\to\pi\ell\nu\end{array}\\ \begin{array}{c}\mathbf{V_{cd}}\\ D\to\ell\nu\\ D\to\pi\ell\nu\end{array} & \begin{array}{c}\mathbf{V_{cs}}\\ D_s\to\ell\nu\\ D\to K\ell\nu\end{array} & \begin{array}{c}\mathbf{V_{cb}}\\ B\to D\ell\nu\\ B\to D^*\ell\nu\end{array}\\ \begin{array}{c}\mathbf{V_{td}}\\ \langle B_d|\bar{B}_d\rangle\\ B\to\pi\ell\ell\end{array} & \begin{array}{c}\mathbf{V_{ts}}\\ \langle B_s|\bar{B}_s\rangle\\ B\to K\ell\ell\end{array} & \begin{array}{c}\mathbf{V_{tb}}\\ \\ \\ \end{array} \end{pmatrix}$$

CKM matrix elements and lattice calculations

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

"Gold plated" processes on the lattice → **CKM matrix elements**

- **One hadron in the initial state and zero or one hadron in the final state**
- **Stable hadrons (that is narrow or far from threshold → easier to study on lattice)**
- **Chiral extrapolation is controllable**

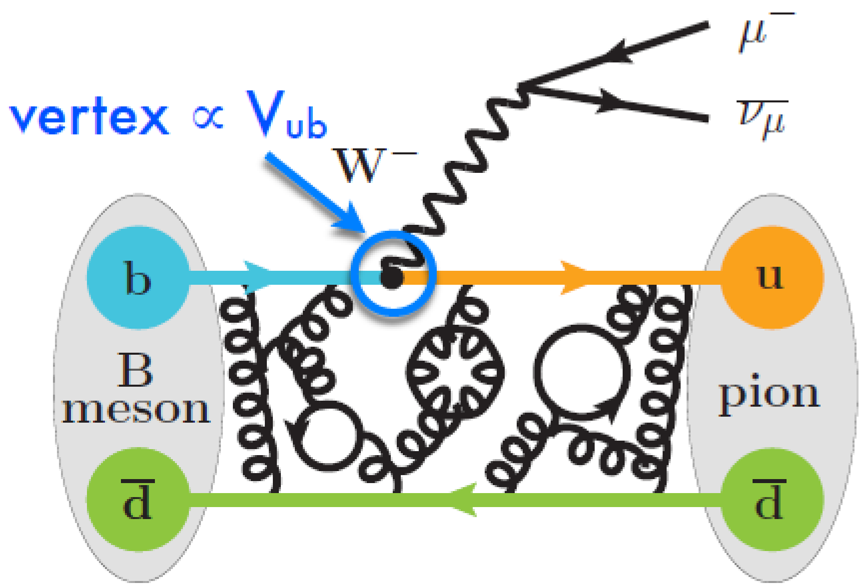
First row: Light quarks

Leptonic

$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\boxed{\pi \to \ell\nu} & \boxed{K \to \ell\nu} & B \to \ell\nu \\
 & \boxed{K \to \pi\ell\nu} & B \to \pi\ell\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \\
B \to \pi\ell\ell & B \to K\ell\ell &
\end{pmatrix}
$$

Semileptonic

Weak matrix elements



$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2}, \frac{d\Gamma(B\to D^{(*)}\ell\nu)}{dw}, \ldots$$

Compute nonperturbative QCD parameters (decay constants, form factors, B-parameters,...) numerically with LATTICE QCD

(Experiment) = (known) x (CKM factors) × (Hadronic Matrix Element)

@Van de Water

Decay constants from Lattice QCD

In SM :

$$\Gamma(H \to \ell\nu) = \frac{M_H}{8\pi} f_H^2 \left|G_F V_{Qq}^* m_\ell\right|^2 \left(1 - \frac{m_\ell^2}{M_H^2}\right)^2 ,$$

Pseudoscalar to vacuum matrix element
of the axial current ⟹ pseudoscalar decay constant

$$\langle 0|\mathcal{A}^\mu|H(p)\rangle = ip^\mu f_H ,$$

$$\langle 0|\mathcal{A}^\mu|H(p)\rangle \, (M_H)^{-1/2} = i(p^\mu/M_H)\phi_H$$

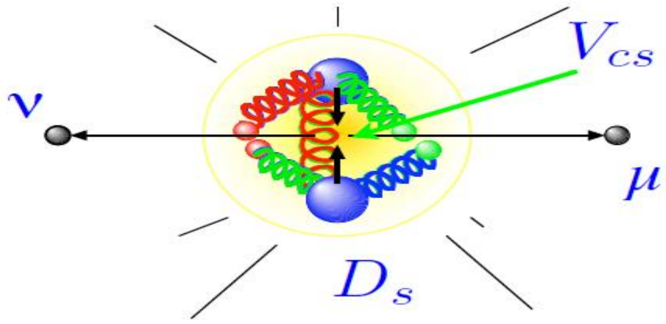
$$f_H = \phi_H/\sqrt{M_H}$$

H	$\mathcal{A}^\mu$	V
D	$\bar{d}\gamma^\mu\gamma^5 c$	V_{cd}^*
D_s	$\bar{s}\gamma^\mu\gamma^5 c$	V_{cs}^*
B	$\bar{b}\gamma^\mu\gamma^5 u$	V_{ub}
B_s	$\bar{b}\gamma^\mu\gamma^5 s$	—

Renormalization constant (to match with continuum physics) :

$$Z_{A^\mu} A^\mu \doteq \mathcal{A}^\mu + \mathrm{O}\left(\alpha_s a\Lambda f_i(m_Q a)\right) + \mathrm{O}\left(a^2\Lambda^2 f_j(m_Q a)\right)$$

Leptonic decay constants



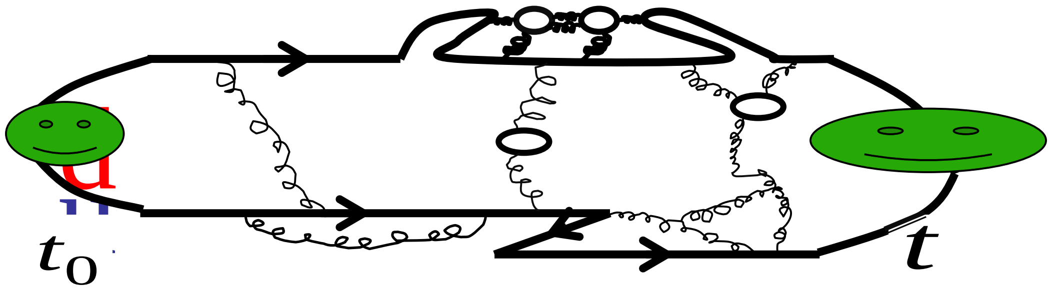
Need to calculate two point correlation functions :

$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

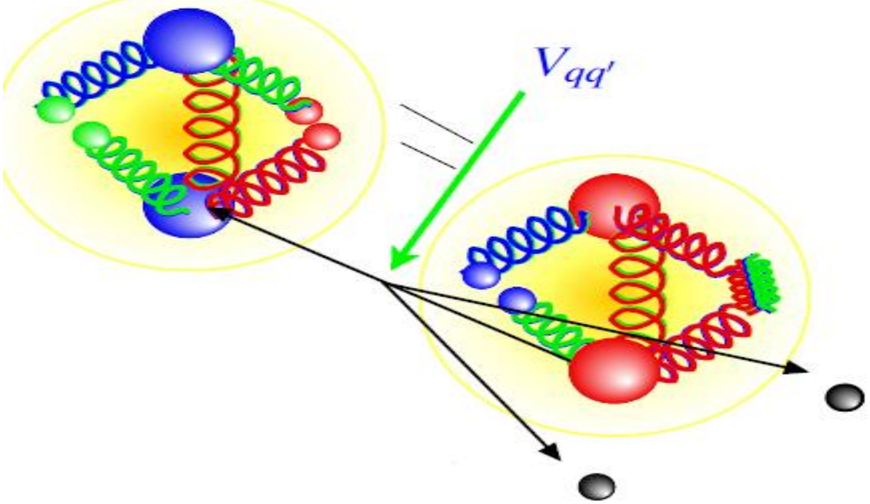
$$= \sum_n e^{-E_p^n(t-t_0)} \left|\langle 0|\varphi(x_0)|n,\vec{p}\rangle\right|^2$$

$$= \sum_n W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$



Two point function

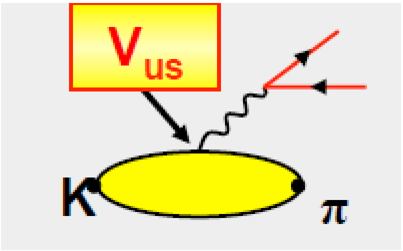
Semileptonic form factors



$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$ $K \to \pi\ell\nu$	$B \to \pi\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$ $D \to \pi\ell\nu$	$D_s \to \ell\nu$ $D \to K\ell\nu$	$B \to D\ell\nu$ $B \to D^*\ell\nu$
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$B_d \leftrightarrow \overline{B}_d$	$B_s \leftrightarrow \overline{B}_s$	

SM :

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{cx}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_D^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_D^2(E_P^2-m_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_D^2-m_P^2)^2|f_0(q^2)|^2\right]$$



$$\langle P|V_\mu|D\rangle = f_+(q^2)\left(p_{D\mu}+p_{P\mu}-\frac{m_D^2-m_P^2}{q^2}q_\mu\right)+f_0(q^2)\frac{m_D^2-m_P^2}{q^2}q_\mu\,,$$

$$V_\mu = \bar{x}\gamma_\mu c$$

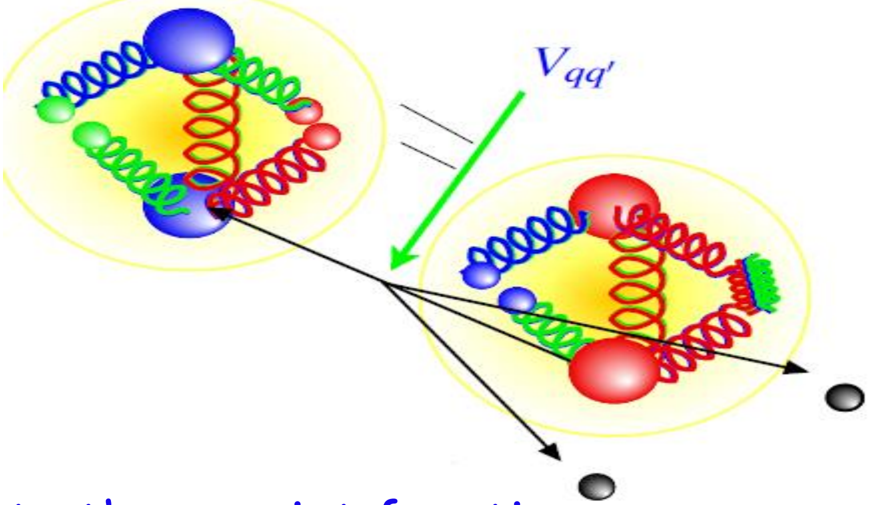
$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_P|^3|V_{cx}|^2|f_+(q^2)|^2$$

$|V_{cd}|$ and $|V_{cs}|$

$$\langle P|T_{\mu\nu}|D\rangle = \frac{2}{m_D+m_P}[p_{P\mu}p_{D\nu}-p_{P\nu}p_{D\mu}]\,f_T(q^2)$$

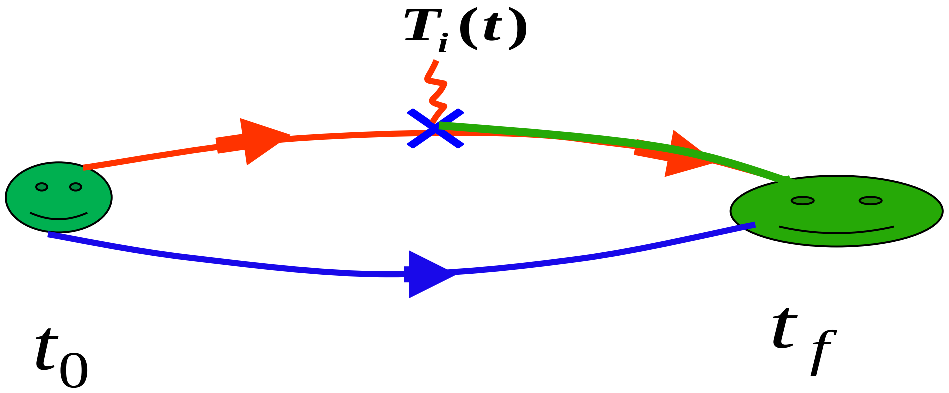
Parity even current: BSM physics

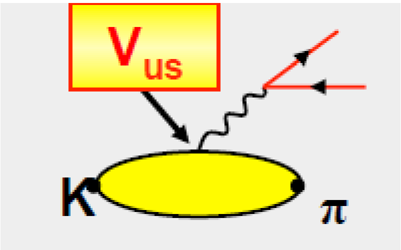
Semileptonic form factors



$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

Need to calculate three point function :

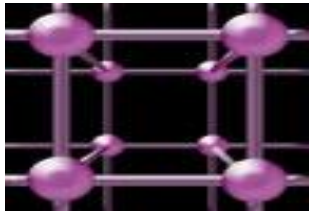




$$G^{\alpha\beta}_{PT_\mu P}(t_2, t_1, \vec{p}, \vec{p'}) = \sum_{\vec{x_2}, \vec{x_1}} e^{-i\vec{p}\cdot\vec{x_2}} e^{-i\vec{q}\cdot\vec{x_1}}$$

$$< 0|\, \mathrm{T}\left(\chi^\alpha(x_2)\, T_\mu\, (x_1)\, \bar{\chi}^\beta(0)\right) |0 >$$

Observables in LQCD



$$< \hat{O} > = \mathrm{Lim}_{\beta\to\infty} \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} \hat{O}(U,\bar{\psi},\psi)]$$

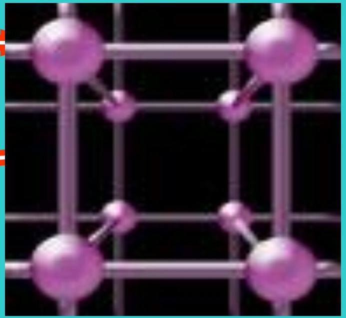
$$= \mathrm{Lim}_{\beta\to\infty} \frac{\int DU\, D\bar{\psi}\, D\psi\; O[U,\bar{\psi},\psi]\; e^{-S_g[U]-S_F[U,\bar{\psi},\psi]}}{\int DU\, D\bar{\psi}\, D\psi\; e^{-S_g[U]-S_F[U,\bar{\psi},\psi]}}$$

Integrating out the Grassmann variables is possible since $S_F = \bar{\psi} D \psi$

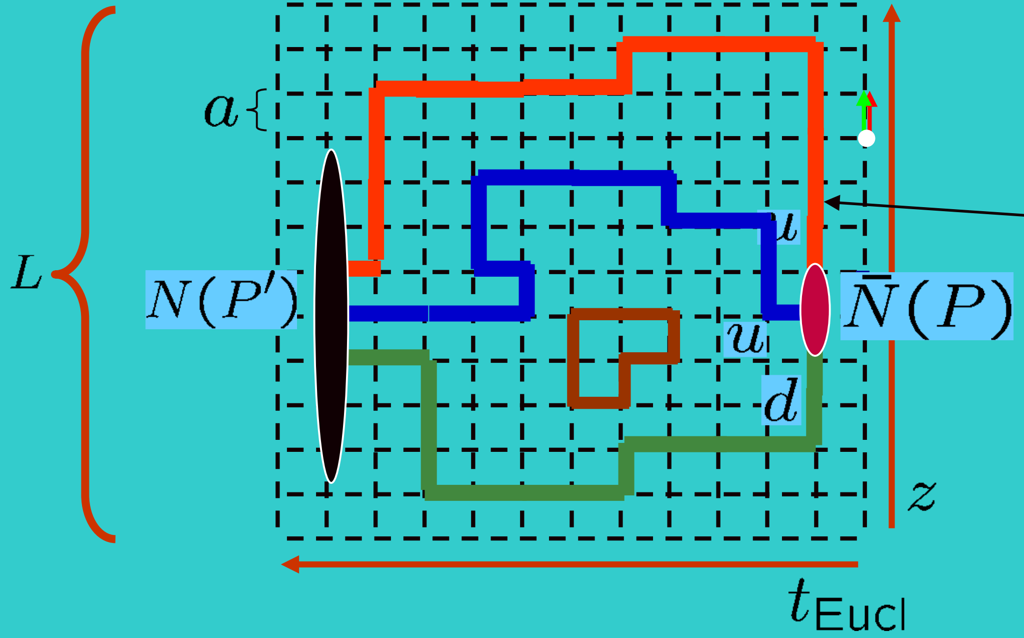
$$< \hat{O} > = \frac{\int DU \{\det D\}^{n_f}\; O[U, D^{-1}]\, e^{-S_g[U]}}{\int DU\; \{\det D\}^{n_f}\; e^{-S_g[U]}} = \prod_n \int \underbrace{dU_n \frac{1}{Z} \{\det D(U)\}^{n_f}\; e^{-S_g[U]}}\; O[U, D^{-1}]$$

Quark
(on Lattice sites)

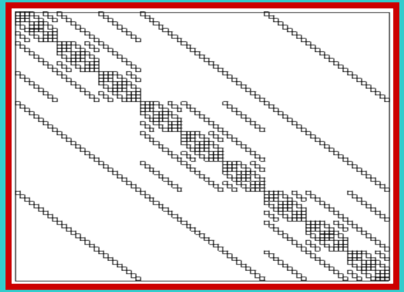
Gluon
(on Links)

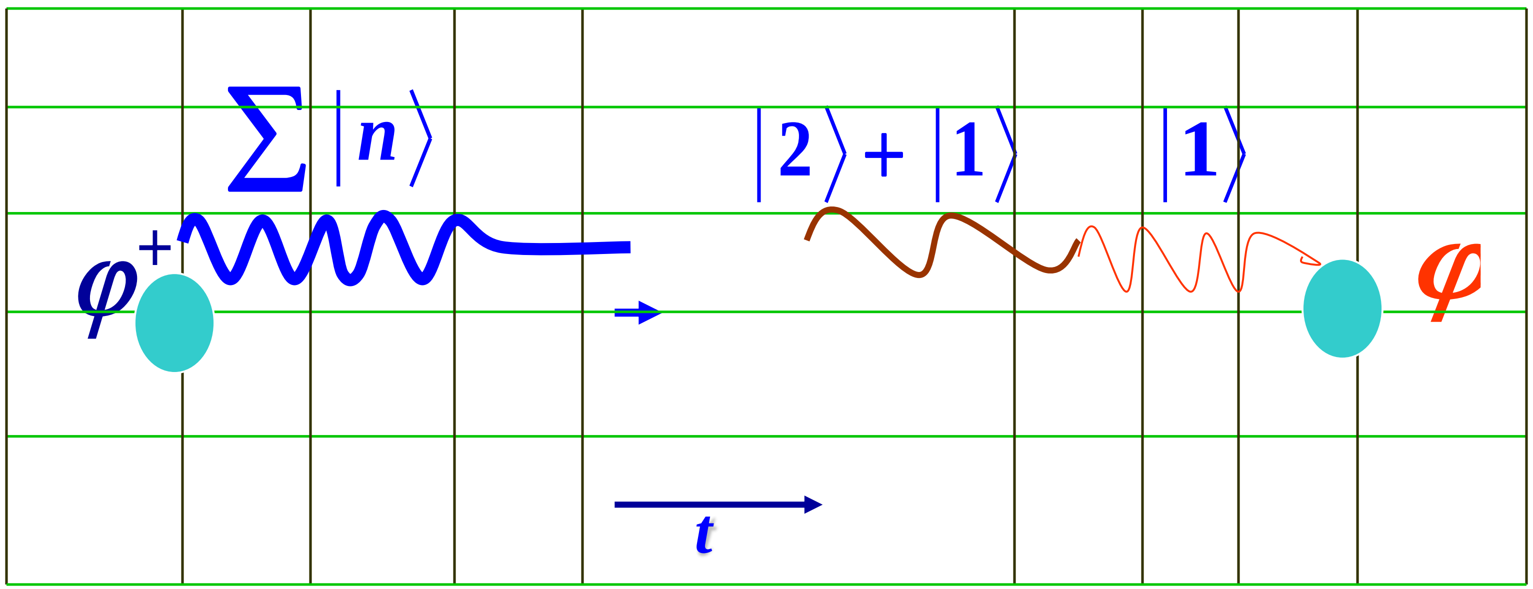


Quark Jungle Gym



quark propagators :
Inverse of very large matrix of space-time, spin and color





$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_n W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$

Analysis (Extraction of Mass)

$$G(\tau) = \sum_{i=1}^{N} W_i \, e^{-m_i \tau} \underset{\tau\to\infty}{\approx} W_1 \, e^{-m_1 \tau}$$

Effective mass :

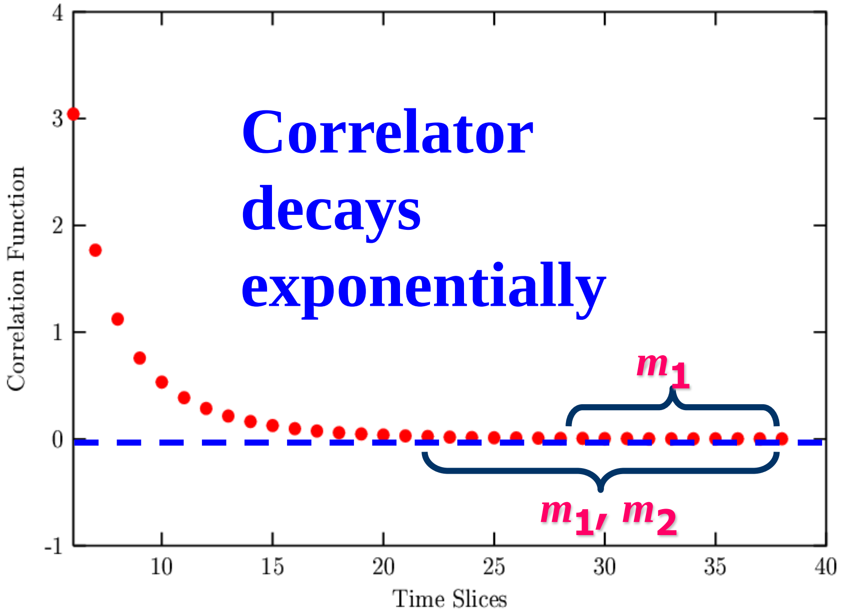
$$\frac{G(\tau)}{G(\tau+1)} = e^{-m_1\tau + m_1(\tau+1)}$$

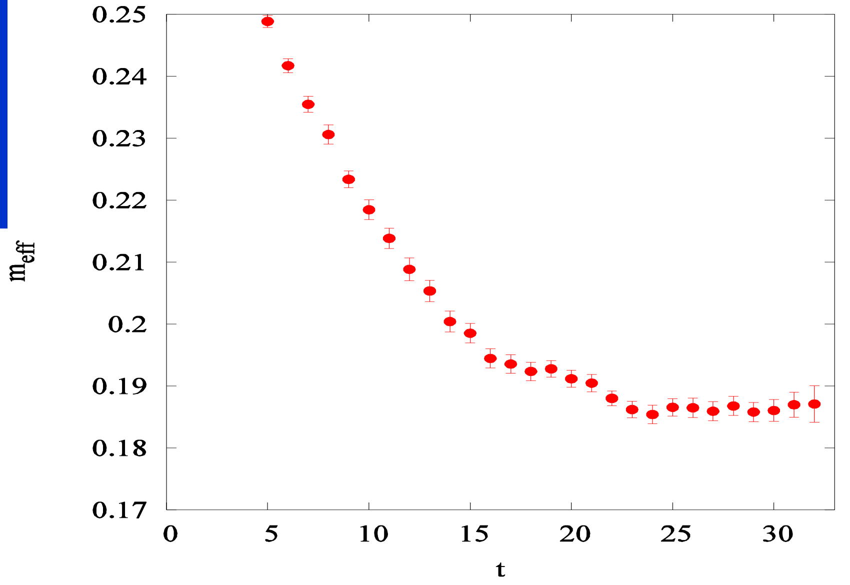
$$m(\tau) = \ln\left[\frac{G(\tau)}{G(\tau+1)}\right]$$

$$\chi^2 = \sum_{i=1}^{N}\left[\frac{f(t_i) - \langle G(t_i)\rangle}{\varepsilon(t_i)}\right]$$

$$e^{-mt} = e^{-(ma)(t/a)}$$

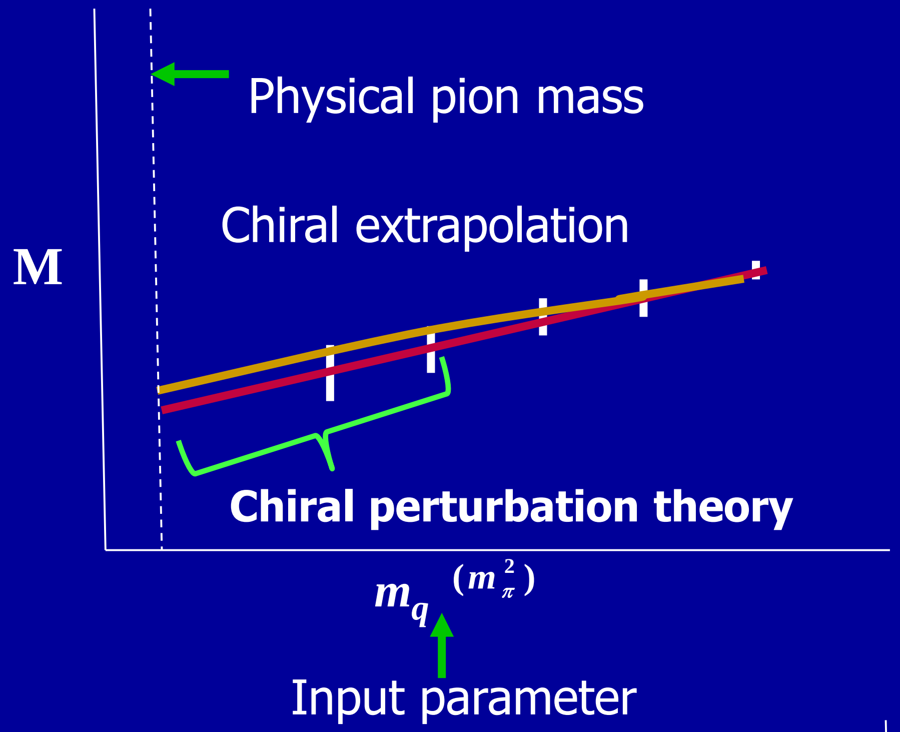
dimensionless mass — integer timeslices

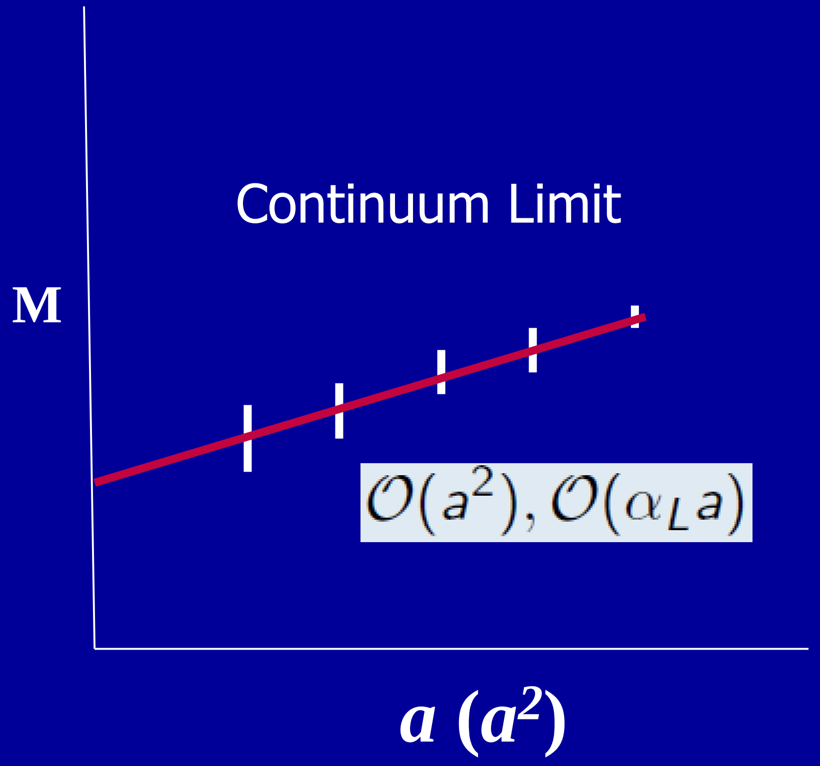


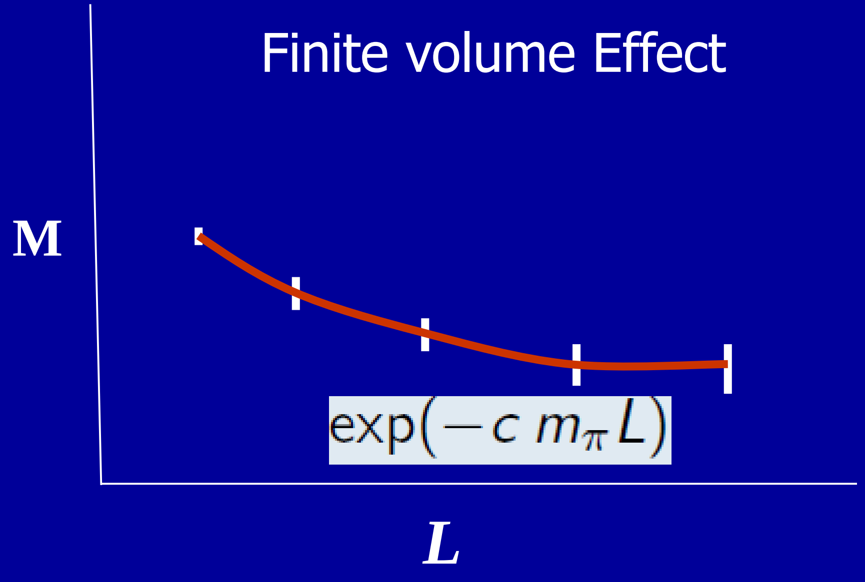


Determine *a* by measuring some physical quantity and compare that to expt, like parameter tuning in any renormalized field theory

Control of Sytemetics







Two and three point functions

$$C_2^{\eta_c}(t) = \sum_i \left(A^i_{\eta_c}\right)^2 e^{-E^i_{\eta_c} t}$$

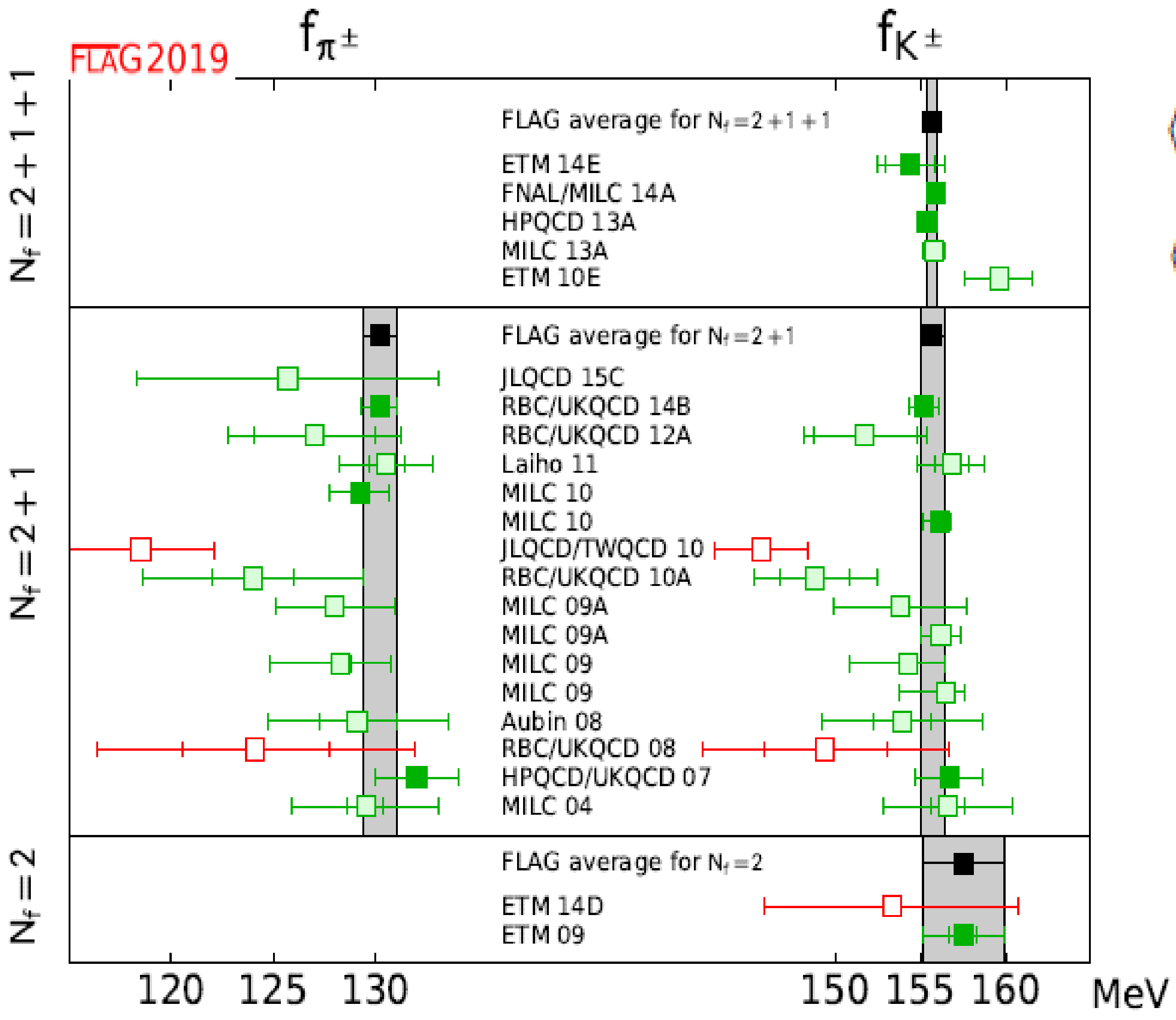
$$C_2^{B_c}(\tau) = \sum_i \left(A^i_{B_c}\right)^2 e^{-E^i_{B_c} \tau}$$

$$C_{3,m}^{B_c \to \eta_c}(t,\tau) = \sum_{i,j} A^i_{\eta_c} \varphi^m A^j_{B_c} e^{-E^i_{\eta_c} t} e^{-E^i_{B_c} \tau}$$

φ^m: Can be obtained by

- fitting these two and three point function simultaneously
- constructing appropriate ratios

$f_{\pi^\pm}$ and $f_{K^\pm}$

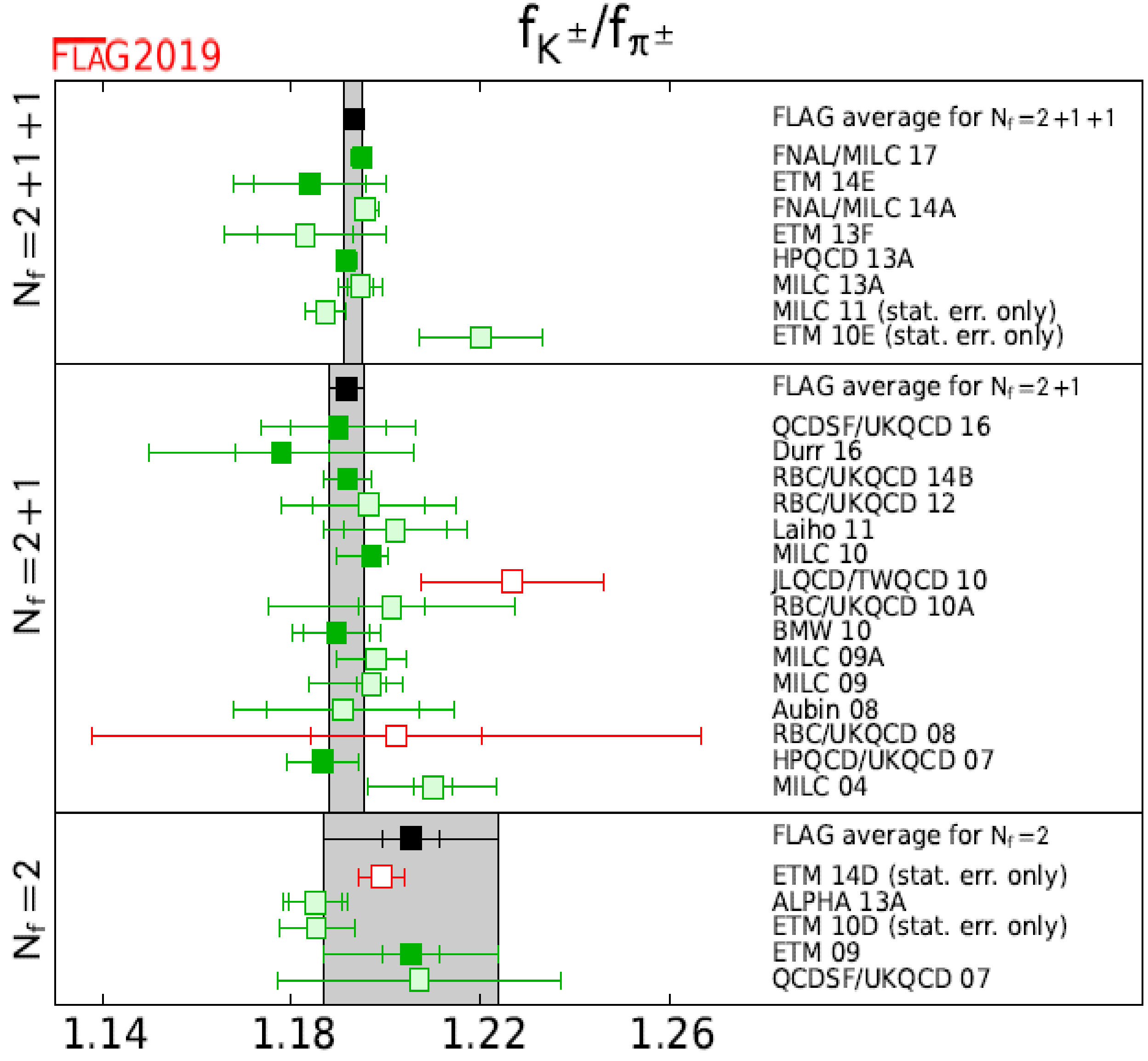


$$\langle 0|\,\bar{d}\gamma_\mu\gamma_5\,u|\pi^+(p)\rangle = i\,p_\mu f_{\pi^+}$$

$$\langle 0|\,\bar{s}\gamma_\mu\gamma_5\,u|K^+(p)\rangle = i\,p_\mu f_{K^+}$$

FLAG'19

$f_{K^\pm}/f_{\pi^\pm}$



Ratio of meson decay constants are easy to calculate and a good way to test unitarity

PDG:

$$\left|\frac{V_{us}}{V_{ud}}\right|\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4)$$

Error: 0.15%

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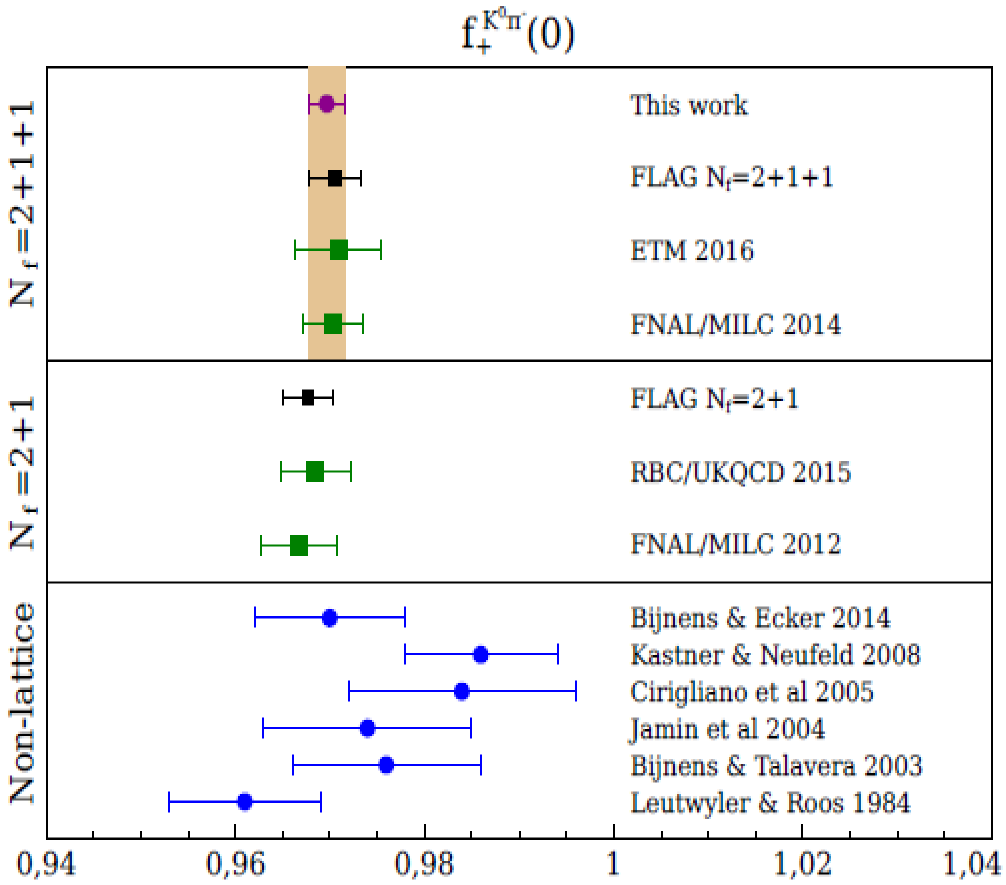
K semileptonic decay

$$\langle \pi^+ | V^\mu | K^0 \rangle = f_+^{K^0\pi^-}(q^2)[p_K^\mu + p_\pi^\mu] + f_-^{K^0\pi^-}(q^2)[p_K^\mu - p_\pi^\mu]$$
$$= f_+^{K^0\pi^-}(q^2)\left[p_K^\mu + p_\pi^\mu - \frac{m_K^2 - m_\pi^2}{q^2} q^\mu\right]$$
$$+ f_0^{K^0\pi^-}(q^2)\frac{m_K^2 - m_\pi^2}{q^2} q^\mu,$$



Experiments tell us:

$|V_{us}|\, f_+(0) = 0.21654(41)$: PDG

We just need the form factor $f_+(q^2 = 0)$ to extract $|V_{us}|$

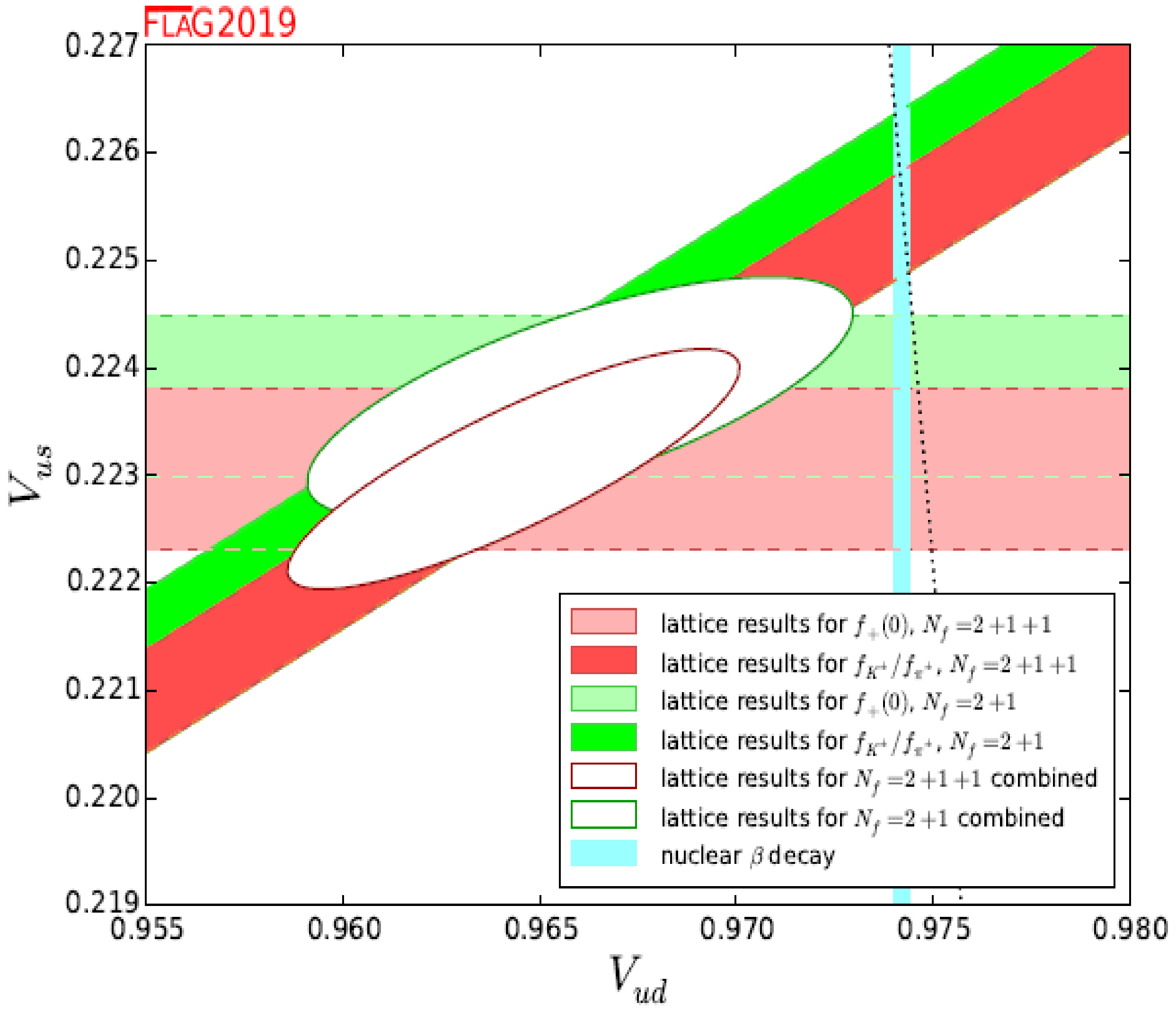
$$f_+(0) \equiv f_+^{K^0\pi^-}(0) = f_0^{K^0\pi^-}(0) = q^\mu \langle \pi^-(p') | \bar{s}\gamma_\mu u | K^0(p) \rangle / (M_K^2 - M_\pi^2) \Big|_{q^2 \to 0}$$

$f_+(0)$ from Lattice QCD:

0.9696(19) MILC/FNAL

0.9707(27) FLAG'19

S.Gottlieb'Lat19


FLAG 2019
0.227
0.226
0.225
0.224
0.223
0.222
0.221
0.220
0.219
V_{us}
0.955
0.960
0.965
0.970
0.975
0.980
V_{ud}
lattice results for $f_+(0)$, $N_f = 2+1+1$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1+1$
lattice results for $f_+(0)$, $N_f = 2+1$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1$
lattice results for $N_f = 2+1+1$ combined
lattice results for $N_f = 2+1$ combined
nuclear β decay

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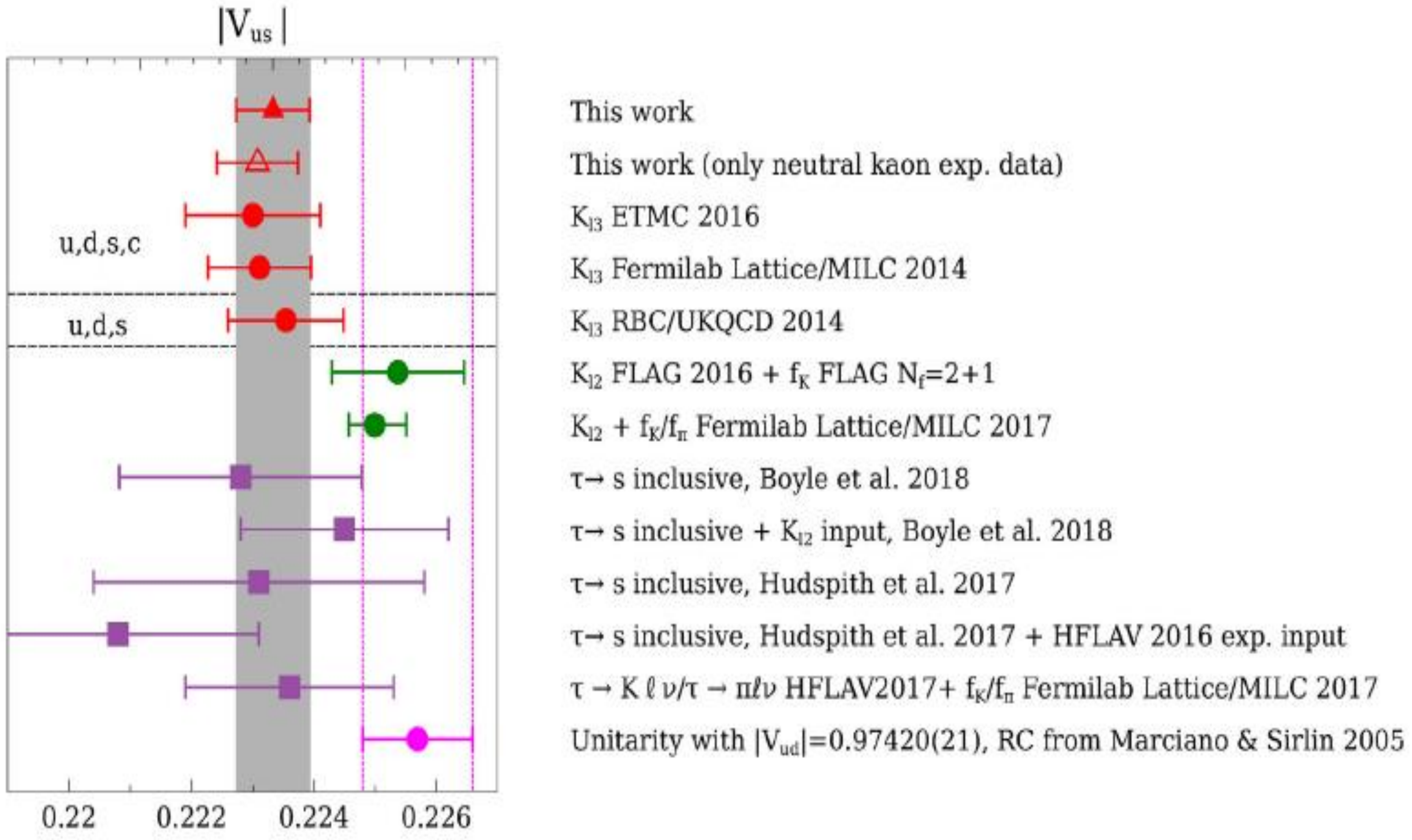


$|V_{us}|f_+^{K^0\pi^-} = 0.21654(41)$ Moulson (CKM2017)

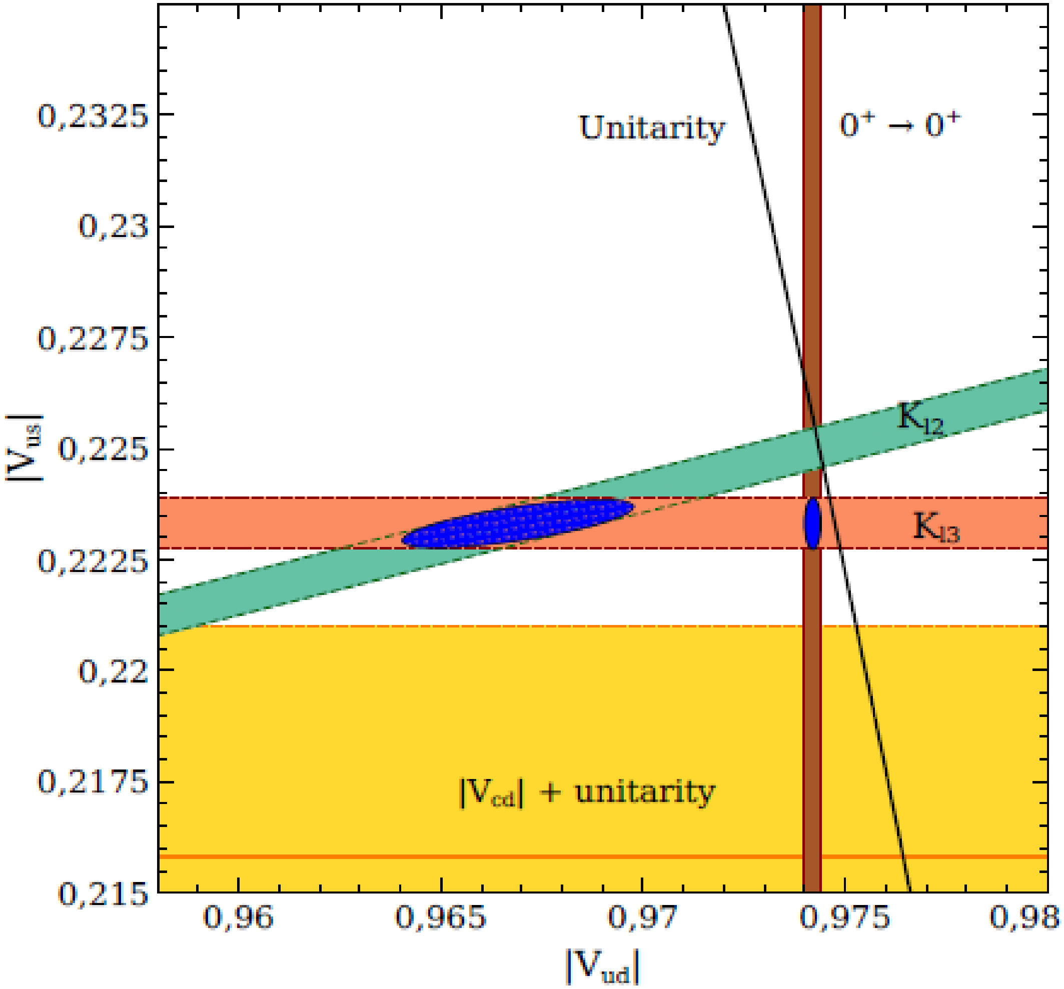
$|V_{us}| = 0.22333(61)$ (MILC/FERMILAB: PRD99,114509(2019)

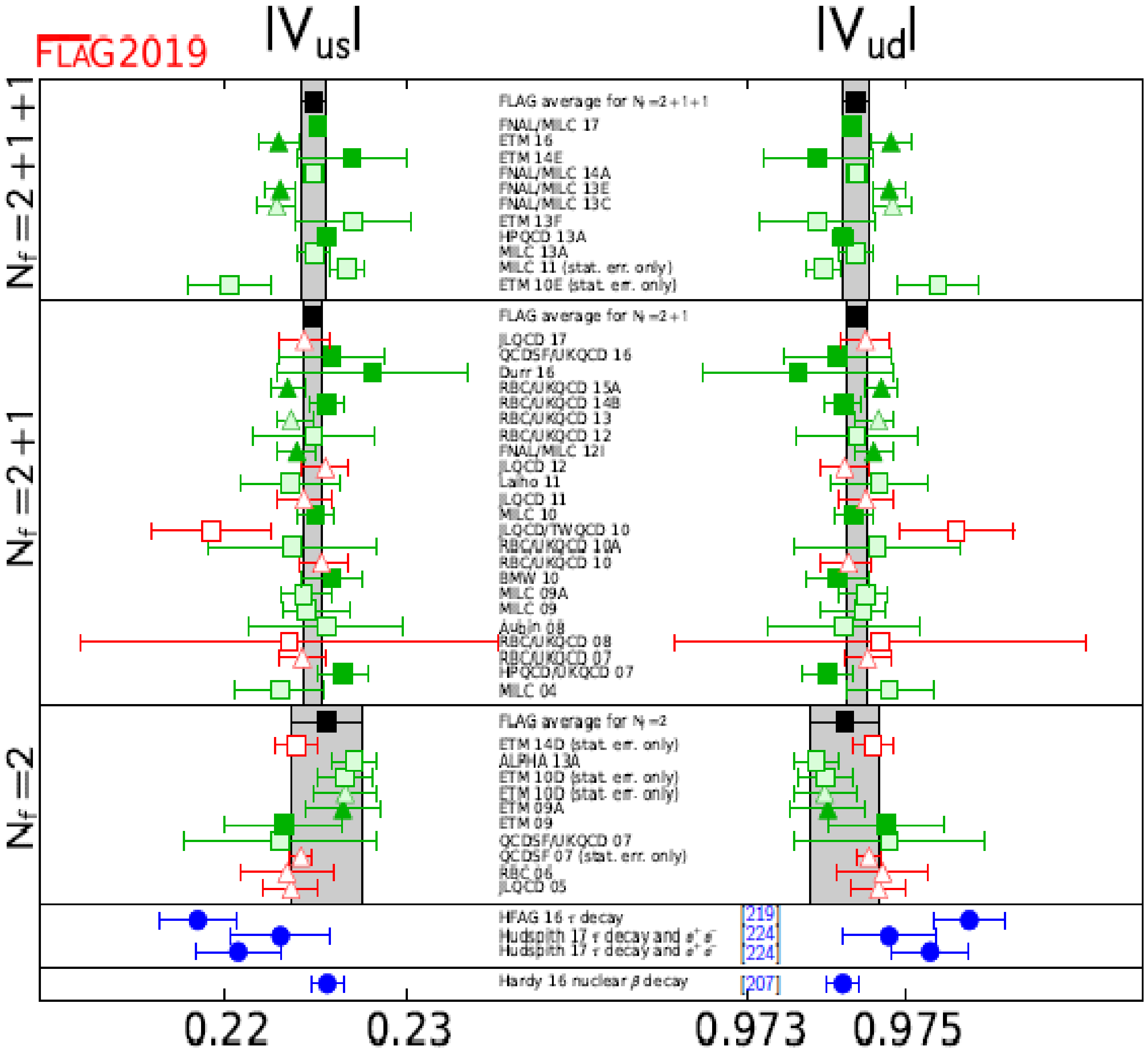
S.Gottlieb'Lat19

Is there a tension?

Results from K_{l2}, K_{l3} and the value of $|V_{ud}|$= 0.0.97420()21) from superallowed nuclear β decays (1807.01146) implies a tension

MILC/Fermilab: Phys.Rev D99, 114509 (2019)





Taking PDG value of

$|V_{ub}| = 3.94(36)10^{-3}$

$|\boldsymbol{V_u}|^2 = |\boldsymbol{V_{ud}}|^2 + |\boldsymbol{V_{us}}|^2 + |\boldsymbol{V_{ub}}|^2$

$2+1+1$

$= 0.99884(53);\ 2.2\ \sigma$ $\quad f_+(0)$

$= 0.99986(46)$ $\quad f_{K^\pm}/f_{\pi^\pm}$

$2+1$

$= 0.99914(53);\ 1.6\ \sigma$ $\quad f_+(0)$

$= 0.99999(54)$ $\quad f_{K^\pm}/f_{\pi^\pm}$

FLAG’19

Second row: charm quark

Leptonic → $D \to \ell\nu$, $D_s \to \ell\nu$

Semileptonic → $B \to D\ell\nu$, $D \to \pi\ell\nu$, $D \to K\ell\nu$, $B \to D^*\ell\nu$

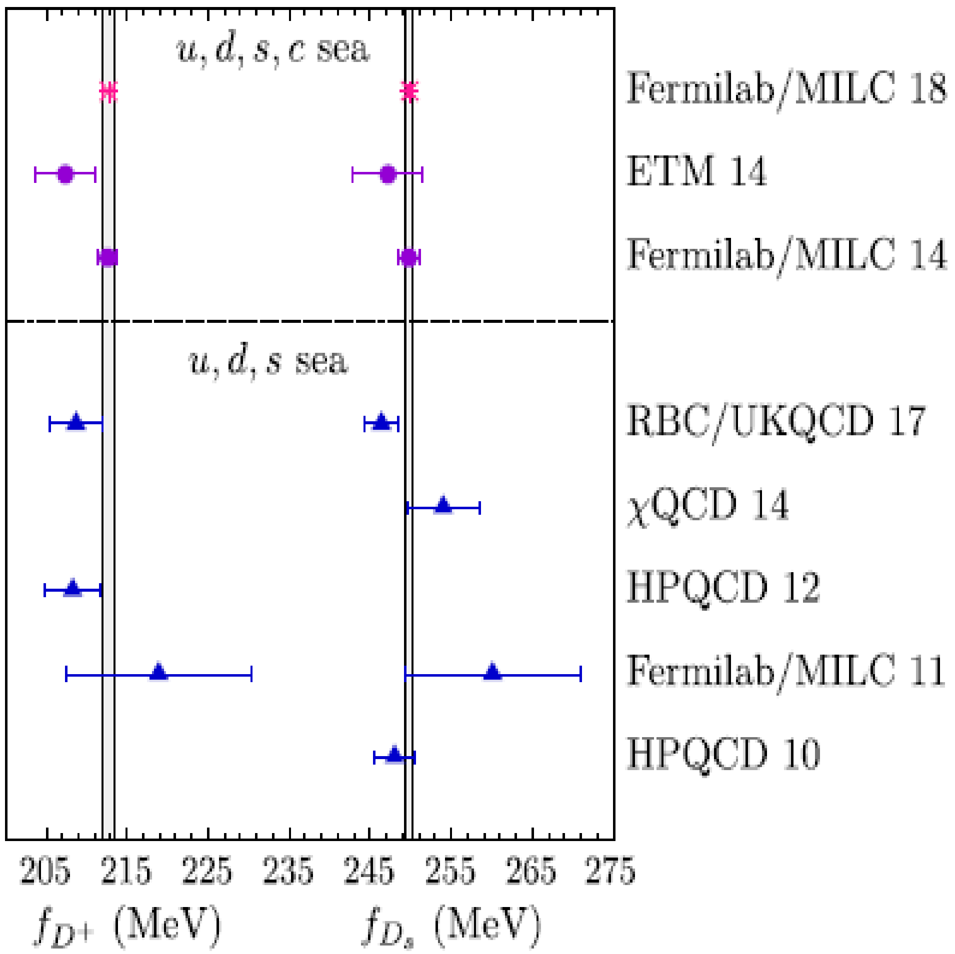
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

Charmed meson Decay Constants

$$f_{D^0} = 211.6(0.3)_{\rm stat}(0.5)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}[0.2]_{\rm EM\ scheme}\ \text{MeV},$$

$$f_{D^+} = 212.7(0.3)_{\rm stat}(0.4)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}[0.2]_{\rm EM\ scheme}\ \text{MeV},$$

$$f_{D_s} = 249.9(0.3)_{\rm stat}(0.2)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}[0.2]_{\rm EM\ scheme}\ \text{MeV},$$

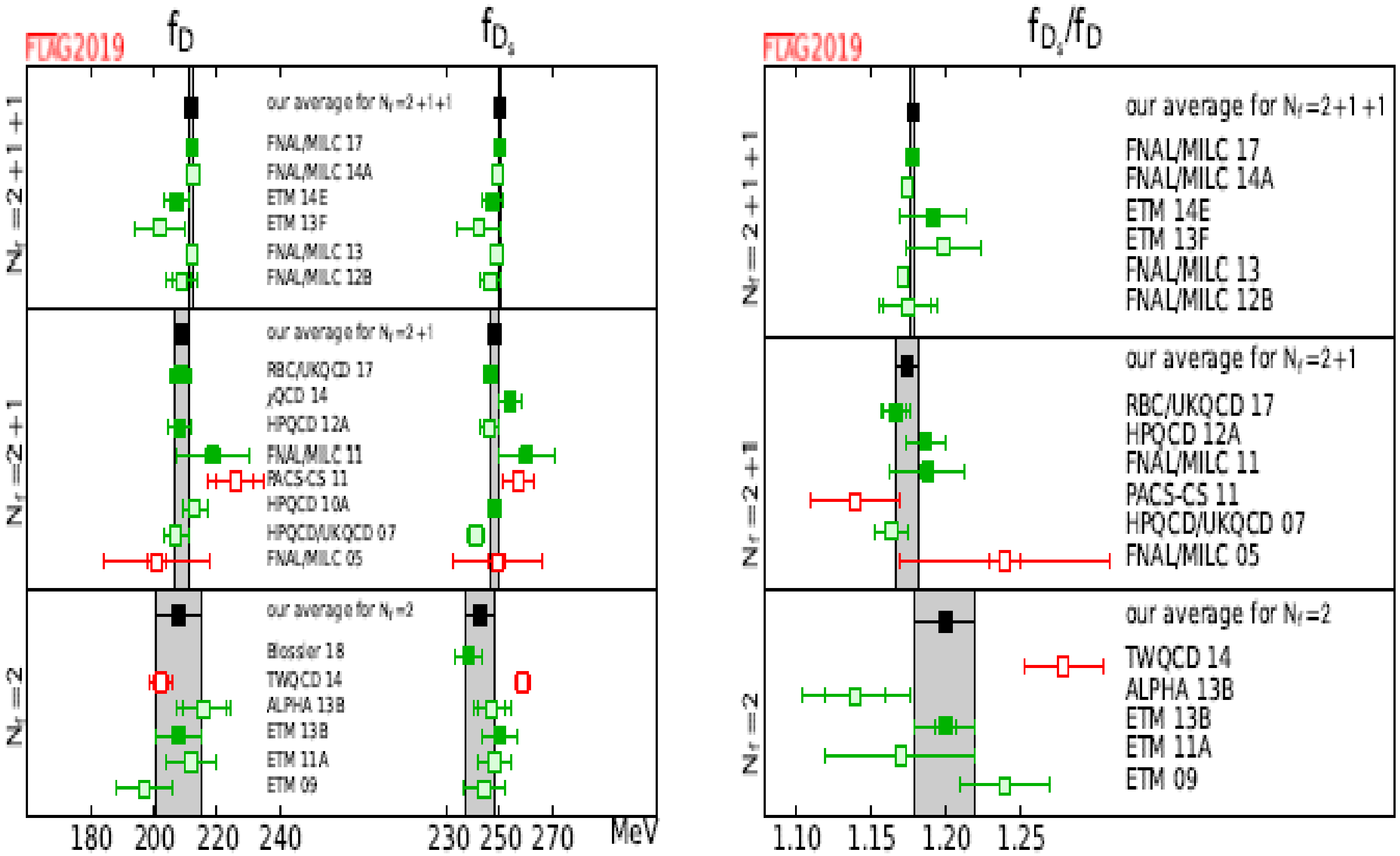


MILC : PRD98 (2018) no.7, 074512, arXiv:1810.00250

$$f_D = 212.0(0.7)\ \mathrm{MeV}$$

$$f_{D_s} = 249.9(0.5)\ \mathrm{MeV}$$

$$\frac{f_{D_s}}{f_D} = 1.1783(0.0016)$$



FLAG'19

V_{cd} and V_{cs}

PDG: $f_D|V_{cd}| = 45.91(1.05)\ \text{MeV}$

$f_{D_s}|V_{cs}| = 250.9(4.0)\ \text{MeV}$

$|V_{cd}| = 0.2152\ (5)_{f_D}(49)_{expt}(6)_{EM}$ MILC/FERMILAB

$= 0.2166(7)(50)$ FLAG'19

$|V_{cs}| = 1.001\ (2)_{f_{D_s}}(16)_{expt}(3)_{EM}$ MILC/FERMILAB

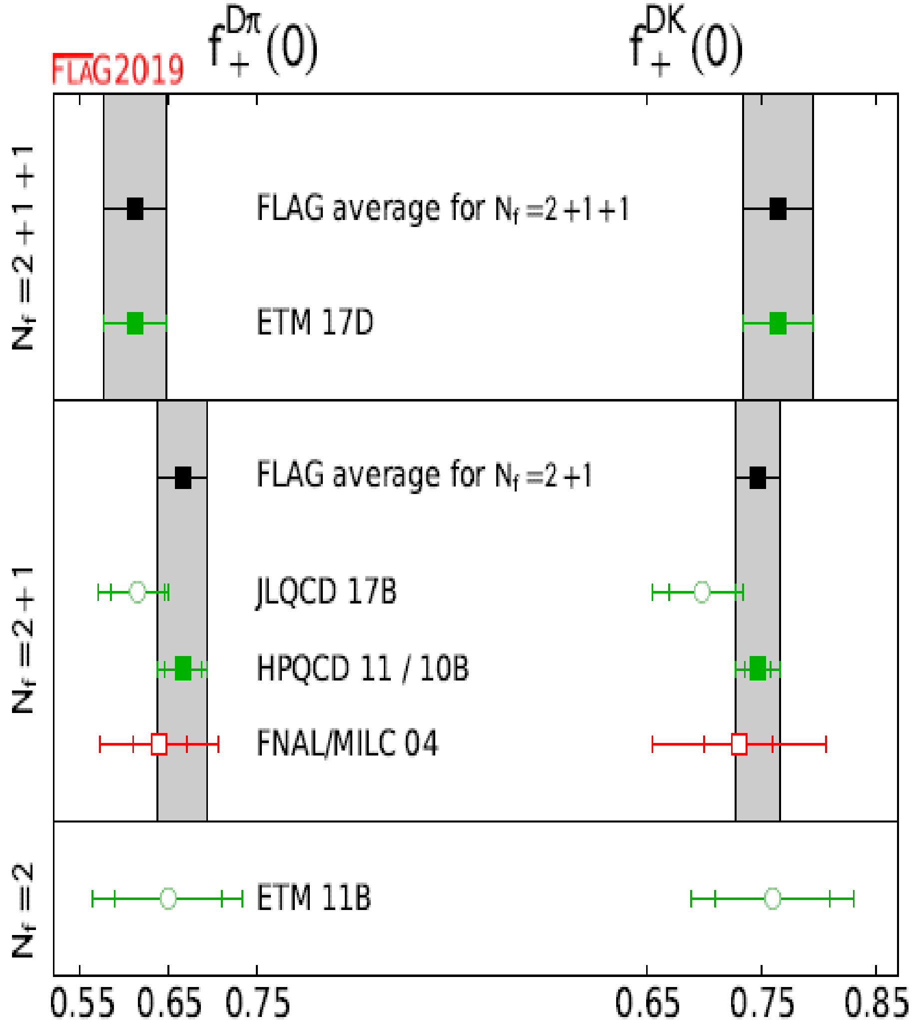
$= 1.004(2)(16)$ FLAG'19

BES-III: $D_s^+ \to \mu^+\nu_\mu$ **Phys. Rev. Lett. 122, 071802 (2019)**

$$f_{D_s}|V_{cs}| = 246.2 \pm 3.6_{\rm stat} \pm 3.5_{\rm sys} = 246.2(5.0)\text{MeV}$$

$$|V_{cs}|_{\rm SM,\, f_{D_s}} = 0.9822(2)_{f_{D_s}}(20)_{\rm expt}(3)_{\rm EM}$$

Charm semileptonic decay



$N_f = 2+1:$

$$f_+^{D\pi}(0) = 0.666(29)$$
$$f_+^{DK}(0) = 0.747(19)$$

HPQCD: Phys.Rev. D82 (2010) 114506,
Phys.Rev. D84 (2011) 114505

$N_f = 2+1+1:$

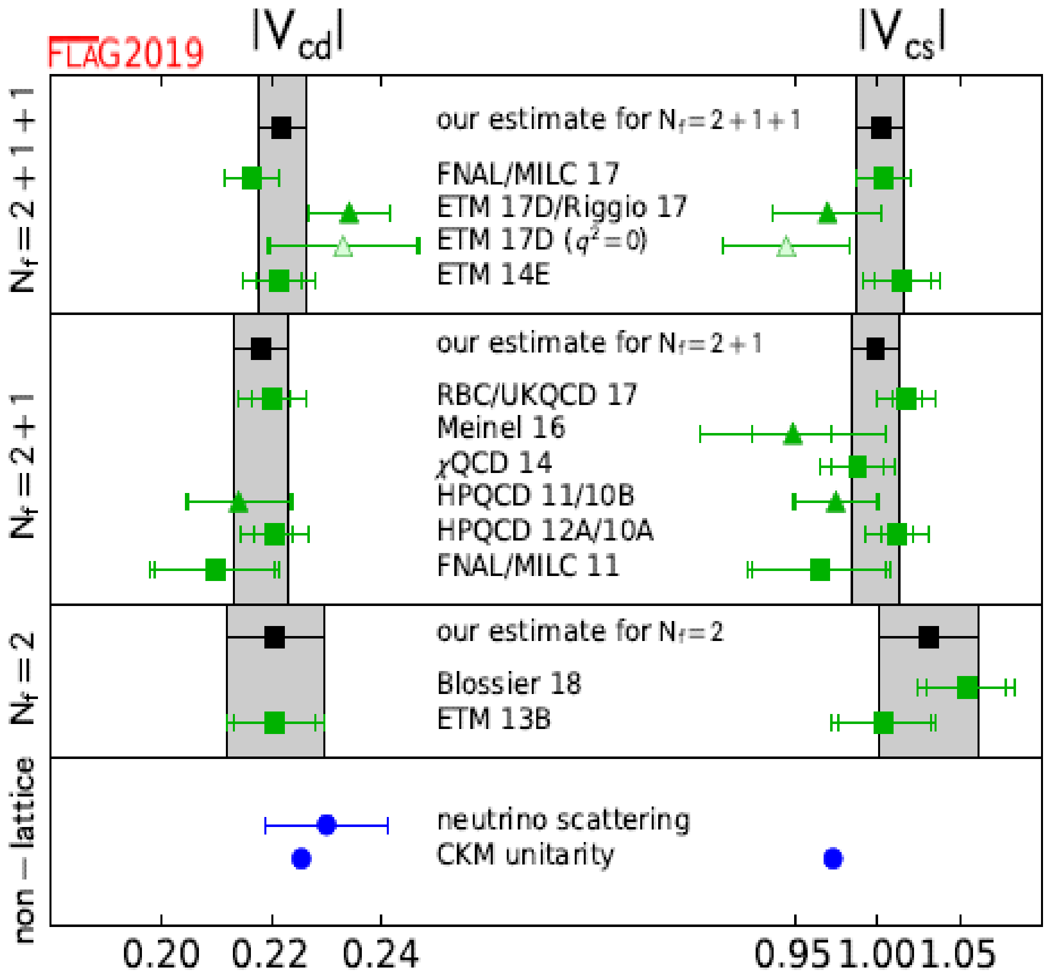
$$f_+^{D\pi}(0) = 0.612(35)$$
$$f_+^{DK}(0) = 0.765(31)$$

ETM: Phys. Rev. D96 (2017) 054514

Squares: leptonic
Triangles: semileptonic

Dominant errors:

Experimental (leptonic)
Theory (semileptonic)



FLAG'19

Summary of $|V_{cd}|$ and $|V_{cs}|$

leptonic decays, $N_f = 2+1+1$:	$\|V_{cd}\| = 0.2166(7)(50)$,	$\|V_{cs}\| = 1.004(2)(16)$,
leptonic decays, $N_f = 2+1$:	$\|V_{cd}\| = 0.2197(25)(50)$,	$\|V_{cs}\| = 1.012(7)(16)$,
semileptonic decays, $N_f = 2+1+1$:	$\|V_{cd}\| = 0.2341(74)$,	$\|V_{cs}\| = 0.970(33)$,
semileptonic decays, $N_f = 2+1$:	$\|V_{cd}\| = 0.2141(93)(29)$,	$\|V_{cs}\| = 0.967(25)(5)$,
semileptonic Λ_c decay, $N_f = 2+1$:		$\|V_{cs}\| = 0.949(24)(51)$,
FLAG2019, $N_f = 2+1+1$:	$\|V_{cd}\| = 0.2219(43)$,	$\|V_{cs}\| = 1.002(14)$,
FLAG2019, $N_f = 2+1$:	$\|V_{cd}\| = 0.2182(50)$,	$\|V_{cs}\| = 0.999(14)$,

Errors : 1.9-2.4% and 1.5%

Lattice QCD needs to improve semilleptonic results

S.Gottlieb'Lat19

Second row unitarity

$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2$: $1.050(2)_{|V_{cd}|}(32)_{|V_{cs}|}(0)_{|V_{cb}|}$ **1.6 sigma effect**

: 0.996(64) (semileptonic decays…ETMC 1706.03657)
semileptonic decays yield smaller value of $|V_{cs}|$,
hence better agreement with unitarity

Third column: Bottom quark

$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\
 & K \to \pi\ell\nu & B \to \pi\ell\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \\
B \to \pi\ell\ell & B \to K\ell\ell &
\end{pmatrix}
$$

Leptonic

Semileptonic

B hadron decays

- Leptonic and semileptonic decays are being studied in LQCD
  - Mesons are extensively studied
  - Baryon results are also coming

- Rare decays involving flavour changing neutral currents are also studied
  - FCNC vanish at tree level in SM, so a good place to look for new physics
  - Some tension between SM predictions from Lattice and LHCb measurement
  - Alternative to B-meson mixing for determining $|V_{td}|$ and $|V_{ts}|$

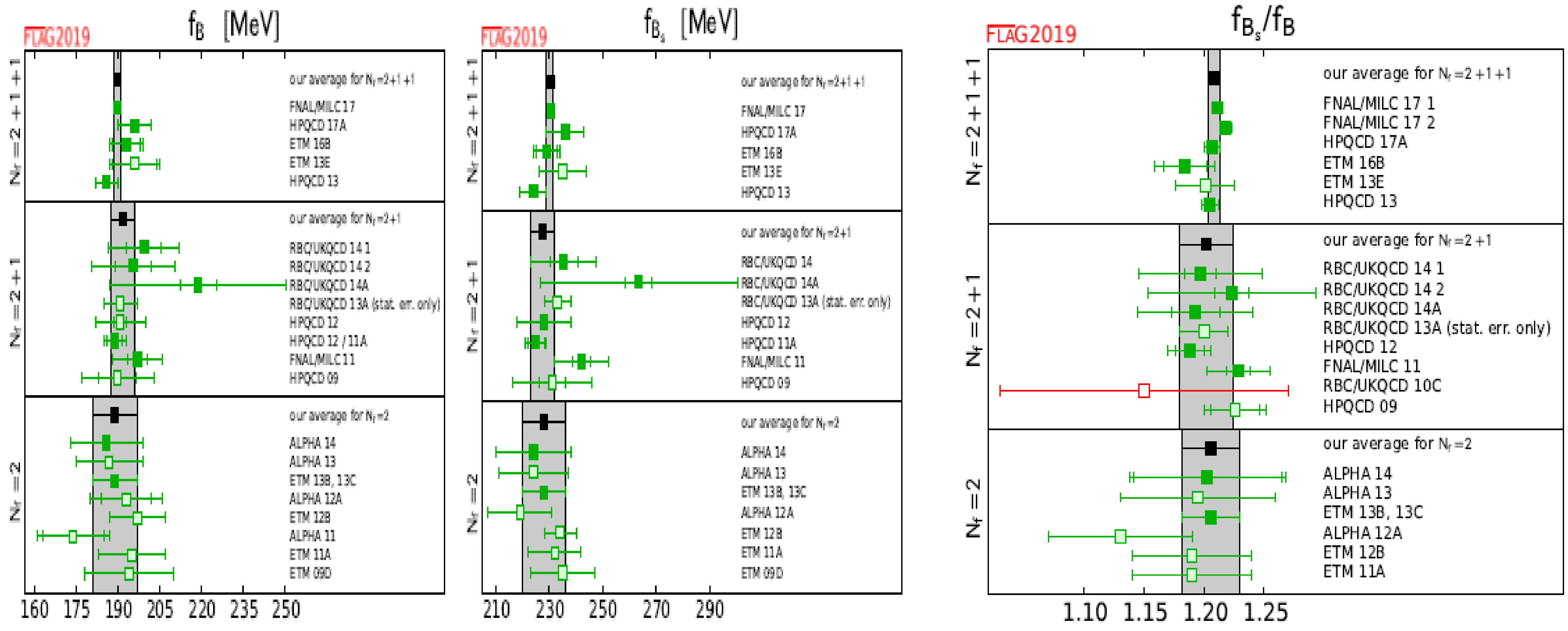
Leptonic decay constants of B(q,s,c) mesons

$$\Gamma(B \to l\nu) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2$$

$$\begin{aligned} B(B^- \to \tau\bar{\nu}) &= 0.91 \pm 0.22 \text{ from Belle,} \\ &= 1.79 \pm 0.48 \text{ from BaBar,} \\ &= 1.06 \pm 0.33 \text{ average,} \end{aligned}$$

$$\begin{aligned} |V_{ub}| f_B &= 0.72 \pm 0.09 \text{ MeV from Belle,} \\ &= 1.01 \pm 0.14 \text{ MeV from BaBar,} \\ &= 0.77 \pm 0.12 \text{ MeV average,} \end{aligned}$$

FLAG’19


FLAG2019
f_B [MeV]
our average for N_f=2+1+1
FNAL/MILC 17
HPQCD 17A
ETM 16B
ETM 13E
HPQCD 13
our average for N_f=2+1
RBC/UKQCD 14 1
RBC/UKQCD 14 2
RBC/UKQCD 14A
RBC/UKQCD 13A (stat. err. only)
HPQCD 12
HPQCD 12 / 11A
FNAL/MILC 11
HPQCD 09
our average for N_f=2
ALPHA 14
ALPHA 13
ETM 13B, 13C
ALPHA 12A
ETM 12B
ALPHA 11
ETM 11A
ETM 09D
160 175 190 205 220 235 250
FLAG2019
f_{B_s} [MeV]
our average for N_f=2+1+1
FNAL/MILC 17
HPQCD 17A
ETM 16B
ETM 13E
HPQCD 13
our average for N_f=2+1
RBC/UKQCD 14
RBC/UKQCD 14A
RBC/UKQCD 13A (stat. err. only)
HPQCD 12
HPQCD 11A
FNAL/MILC 11
HPQCD 09
our average for N_f=2
ALPHA 14
ALPHA 13
ETM 13B, 13C
ALPHA 12A
ETM 12B
ETM 11A
ETM 09D
210 230 250 270 290
FLAG2019
f_{B_s}/f_B
our average for N_f=2+1+1
FNAL/MILC 17 1
FNAL/MILC 17 2
HPQCD 17A
ETM 16B
ETM 13E
HPQCD 13
our average for N_f=2+1
RBC/UKQCD 14 1
RBC/UKQCD 14 2
RBC/UKQCD 14A
RBC/UKQCD 13A (stat. err. only)
HPQCD 12
FNAL/MILC 11
RBC/UKQCD 10C
HPQCD 09
our average for N_f=2
ALPHA 14
ALPHA 13
ETM 13B, 13C
ALPHA 12A
ETM 12B
ETM 11A
1.10 1.15 1.20 1.25
N_f = 2 + 1 + 1
N_f = 2 + 1
N_f = 2

FLAG'19

$$f_{B^+} = 189.4(0.8)_{\text{stat}}(1.1)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}}[0.1]_{\text{EM scheme}} \text{ MeV},$$

$$f_{B^0} = 190.5(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.3)_{f_{\pi,\text{PDG}}}[0.1]_{\text{EM scheme}} \text{ MeV},$$

$$f_{B_s} = 230.7(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.2)_{f_{\pi,\text{PDG}}}[0.2]_{\text{EM scheme}} \text{ MeV}.$$

$$f_{B_s}/f_{B^+} = 1.2180(33)_{\text{stat}}(33)_{\text{syst}}(05)_{f_{\pi,\text{PDG}}}[03]_{\text{EM scheme}},$$

$$f_{B_s}/f_{B^0} = 1.2109(29)_{\text{stat}}(25)_{\text{syst}}(04)_{f_{\pi,\text{PDG}}}[03]_{\text{EM scheme}},$$

$$f_{B_s}/f_{D_s} = 0.9233(25)_{\text{stat}}(42)_{\text{syst}}(02)_{f_{\pi,\text{PDG}}}[03]_{\text{EM scheme}}.$$

$$N_f = 2+1+1: \qquad f_B = 190.0(1.3) \text{ MeV}$$

$$N_f = 2+1+1: \qquad f_{B_s} = 230.3(1.3) \text{ MeV}$$

$$N_f = 2+1+1: \qquad \frac{f_{B_s}}{f_B} = 1.209(0.005)$$

FLAG'19

$$N_f = 2 \qquad \text{Belle } B \to \tau\nu_\tau : \qquad |V_{ub}| = 3.83(14)(48) \times 10^{-3} ,$$
$$N_f = 2+1 \qquad \text{Belle } B \to \tau\nu_\tau : \qquad |V_{ub}| = 3.75(8)(47) \times 10^{-3} ,$$
$$N_f = 2+1+1 \qquad \text{Belle } B \to \tau\nu_\tau : \qquad |V_{ub}| = 3.79(3)(47) \times 10^{-3} ;$$

$$N_f = 2 \qquad \text{Babar } B \to \tau\nu_\tau : \qquad |V_{ub}| = 5.37(20)(74) \times 10^{-3} ,$$
$$N_f = 2+1 \qquad \text{Babar } B \to \tau\nu_\tau : \qquad |V_{ub}| = 5.26(12)(73) \times 10^{-3} ,$$
$$N_f = 2+1+1 \qquad \text{Babar } B \to \tau\nu_\tau : \qquad |V_{ub}| = 5.32(4)(74) \times 10^{-3} ,$$

$$N_f = 2 \qquad \text{average } B \to \tau\nu_\tau : \qquad |V_{ub}| = 4.10(15)(64) \times 10^{-3} ,$$
$$N_f = 2+1 \qquad \text{average } B \to \tau\nu_\tau : \qquad |V_{ub}| = 4.01(9)(63) \times 10^{-3} ,$$
$$N_f = 2+1+1 \qquad \text{average } B \to \tau\nu_\tau : \qquad |V_{ub}| = 4.05(3)(64) \times 10^{-3} ,$$

FLAG’19

B meson semileptonic and rare decays

Semileptonic decays :

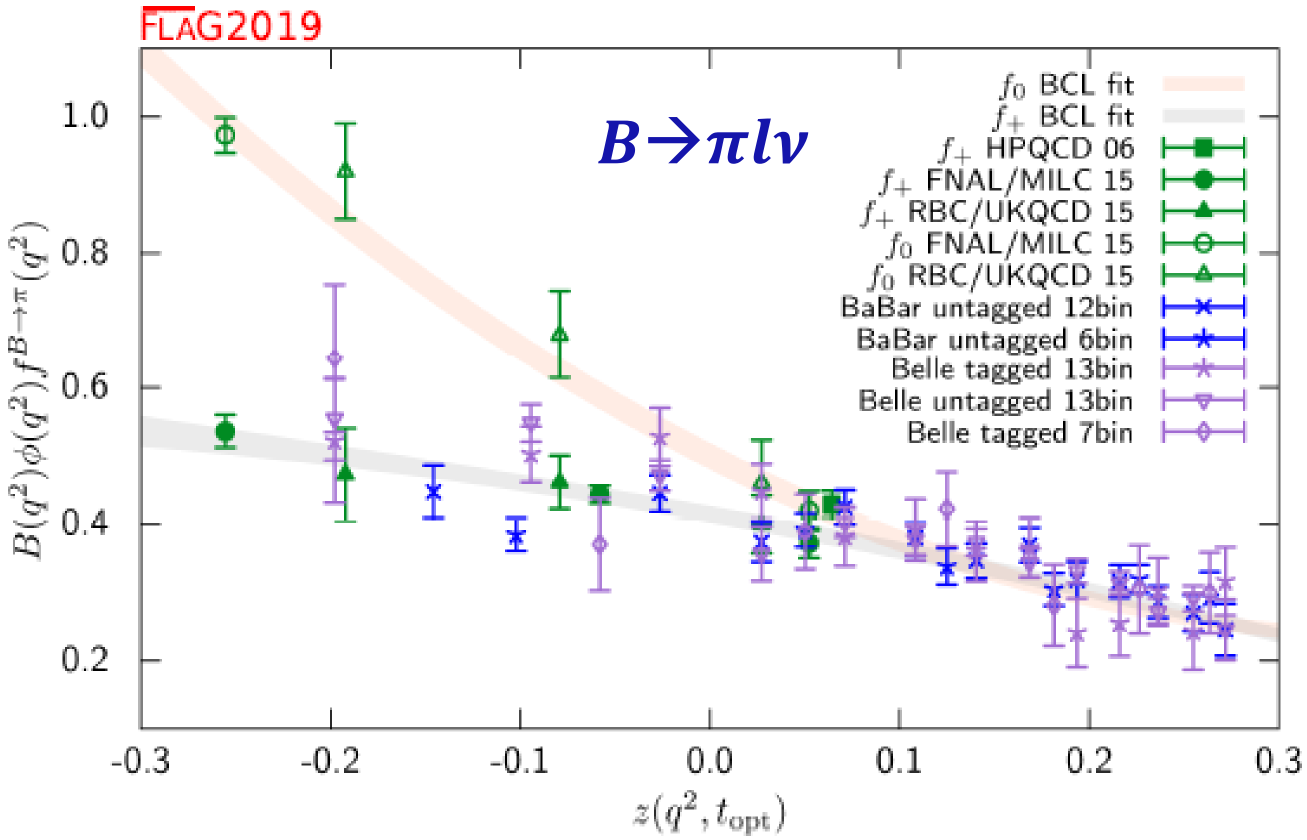
$$B \to \pi \ell \nu,\ B_s \to K \ell \nu,\ B_s \to K^* \ell \nu,\ \Lambda_b \to p \ell \nu$$

$$B \to D \ell \nu,\ B \to D^* \ell \nu,\ B_s \to D_s^{(*)} \ell \nu,\ \Lambda_b \to \Lambda_c \ell \nu$$

$$\frac{d\Gamma(B_{(s)} \to P\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 \right.$$
$$\left. + \frac{3m_\ell^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

Rare decays (FCNC) :

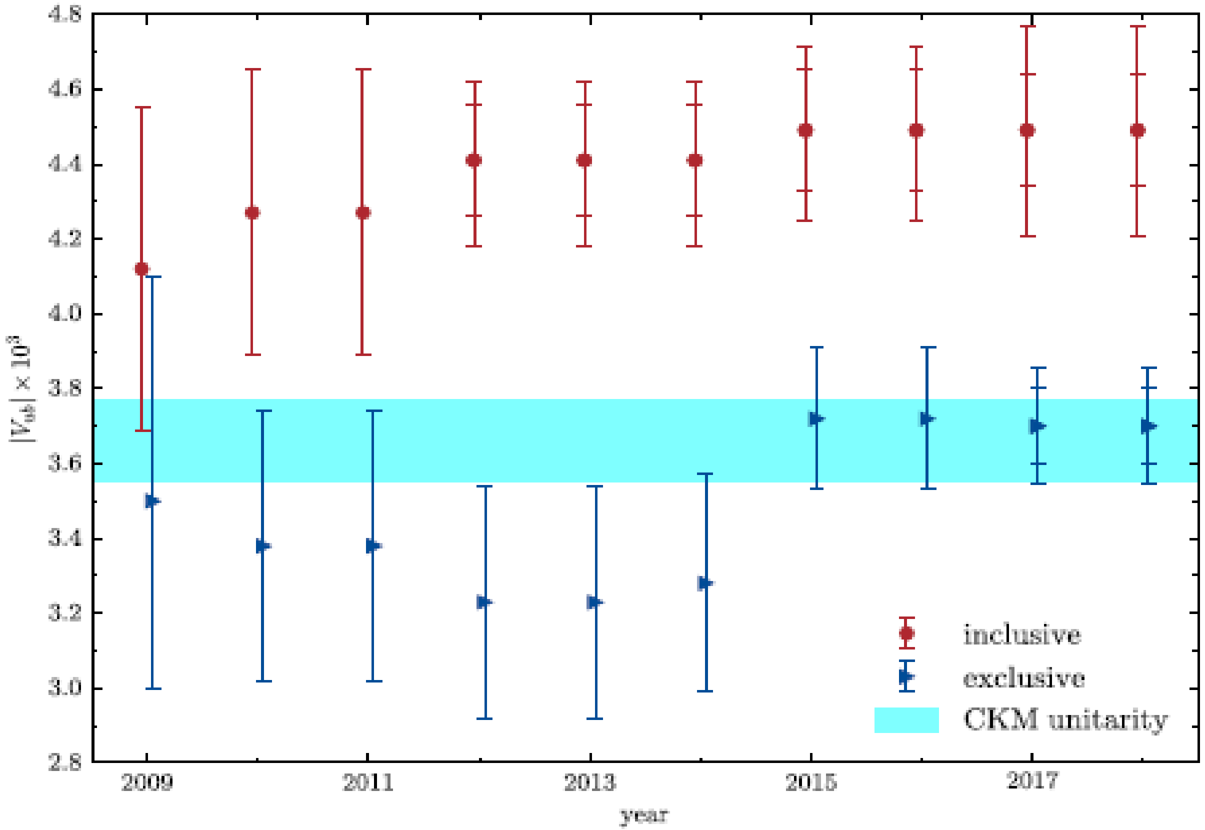
$$B^0 \to \mu^+ \mu^-,\ B_s \to \mu^+ \mu^-,\ B \to K \ell^+ \ell^-$$


FLAG2019
B→πlν
f_0 BCL fit
f_+ BCL fit
f_+ HPQCD 06
f_+ FNAL/MILC 15
f_+ RBC/UKQCD 15
f_0 FNAL/MILC 15
f_0 RBC/UKQCD 15
BaBar untagged 12bin
BaBar untagged 6bin
Belle tagged 13bin
Belle untagged 13bin
Belle tagged 7bin
$B(q^2)\phi(q^2)f^{B\to\pi}(q^2)$
$z(q^2, t_{\rm opt})$
1.0
0.8
0.6
0.4
0.2
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3

FLAG'19

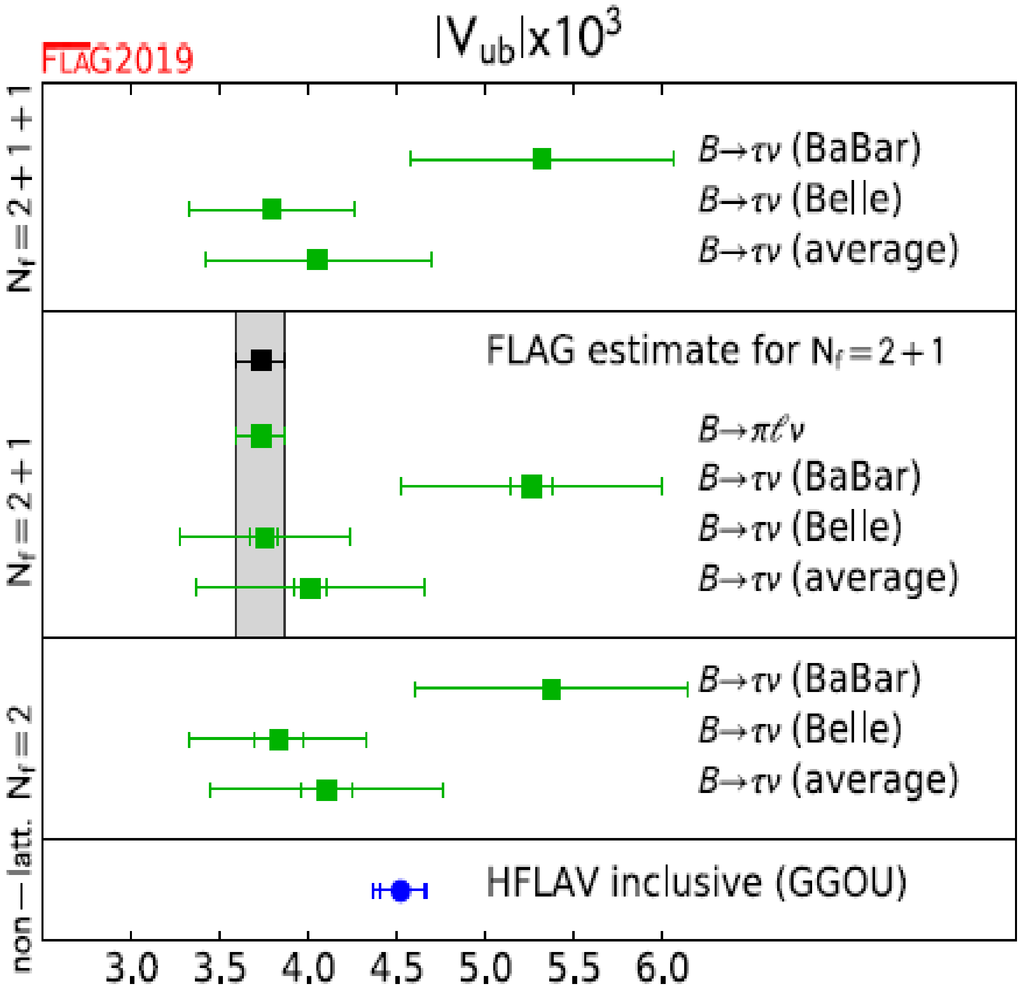
Inclusive Vs exclusive decays

Long standing difference in the determination of $|V_{ub}|$



S.Gottlieb'Lat19

$|V_{ub}|$ from FLAG



- BaBar and Belle leptonic decays results don't agree very well.
- Leptonic error totally dominated by experiment.
- Semileptonic result is more precise.
- Tension between inclusive and exclusive determinations.
- Critical role for Belle II for both leptonic and semileptonic results
- Lattice semileptonic will improve in the next couple of years as N_f=2+1+1 results become available.

$$V_{ub} = 3.73(0.14) \times 10^{-3},\ i.e., 3.8\%$$

S.Gottlieb'Lat19

$B \rightarrow Dl\nu$

Scalar and vector form factors : 2

$$\frac{\langle D(p_D)|i\bar{c}\gamma_\mu b|B(p_B)\rangle}{\sqrt{m_D m_B}} = h_+(w)(v_B + v_D)_\mu + h_-(w)(v_B - v_D)_\mu \, ,$$

$$f_+(q^2) = \frac{1}{2\sqrt{r}} \left[(1+r)h_+(w) - (1-r)h_-(w)\right] \, ,$$

$$f_0(q^2) = \sqrt{r}\left[\frac{1+w}{1+r} h_+(w) + \frac{1-w}{1-r} h_-(w)\right] \, ,$$

$B \rightarrow D^{*} l\nu$

Vector and axial-vector form factors : 4

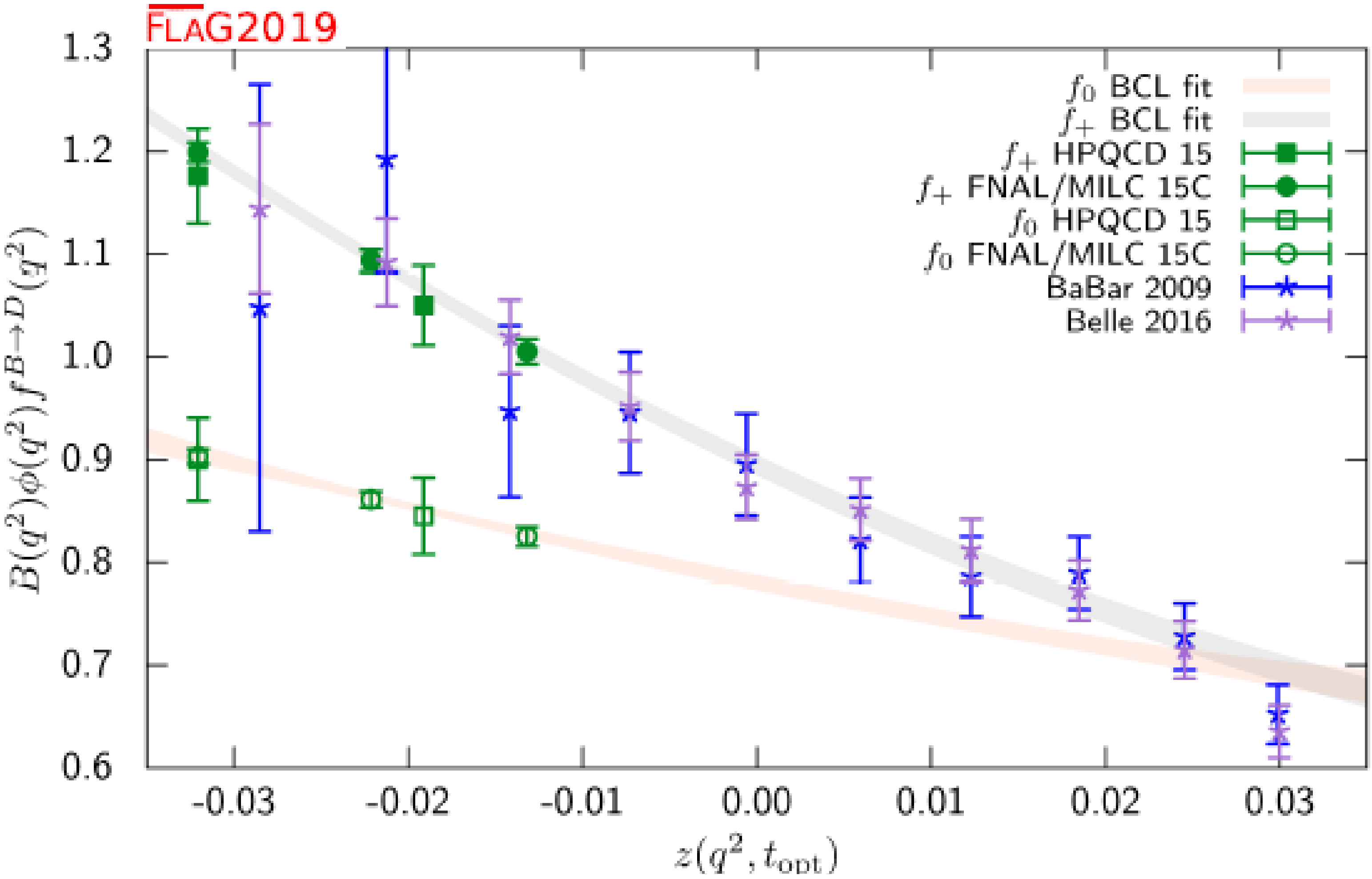
$$\langle D^*|V_\mu|B\rangle = \sqrt{m_B m_{D^*}}\, h_V(w)\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} v^\alpha_{D^*} v^\beta_B\,,$$

$$\langle D^*|A_\mu|B\rangle = i\sqrt{m_B m_{D^*}}\left[h_{A_1}(w)(1+w)\epsilon^{*\mu} - h_{A_2}(w)\epsilon^*\cdot v_B v_{B\mu} - h_{A_3}(w)\epsilon^*\cdot v_B v_{D^*\mu}\right]$$

$$\frac{d\Gamma_{B^-\to D^0\ell^-\bar{\nu}}}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B+m_D)^2(w^2-1)^{3/2}|\eta_{EW}|^2|V_{cb}|^2|\mathcal{G}(w)|^2,$$

$$\frac{d\Gamma_{B^-\to D^{0*}\ell^-\bar{\nu}}}{dw} = \frac{G_F^2 m_{D^*}^3}{4\pi^3}(m_B-m_{D^*})^2(w^2-1)^{1/2}|\eta_{EW}|^2|V_{cb}|^2\chi(w)|\mathcal{F}(w)|^2,$$

$v_P = p_P/m_P \qquad w = v_B \cdot v_{D^{(*)}} \qquad \eta_{EW} = 1.0066$



$$R(D) = \mathcal{B}(B \to D\tau\nu)/\mathcal{B}(B \to D\ell\nu) \qquad \text{with } \ell = e, \mu$$

$$R(D) = 0.300(8) \qquad R(D_s) = 0.314(6)$$

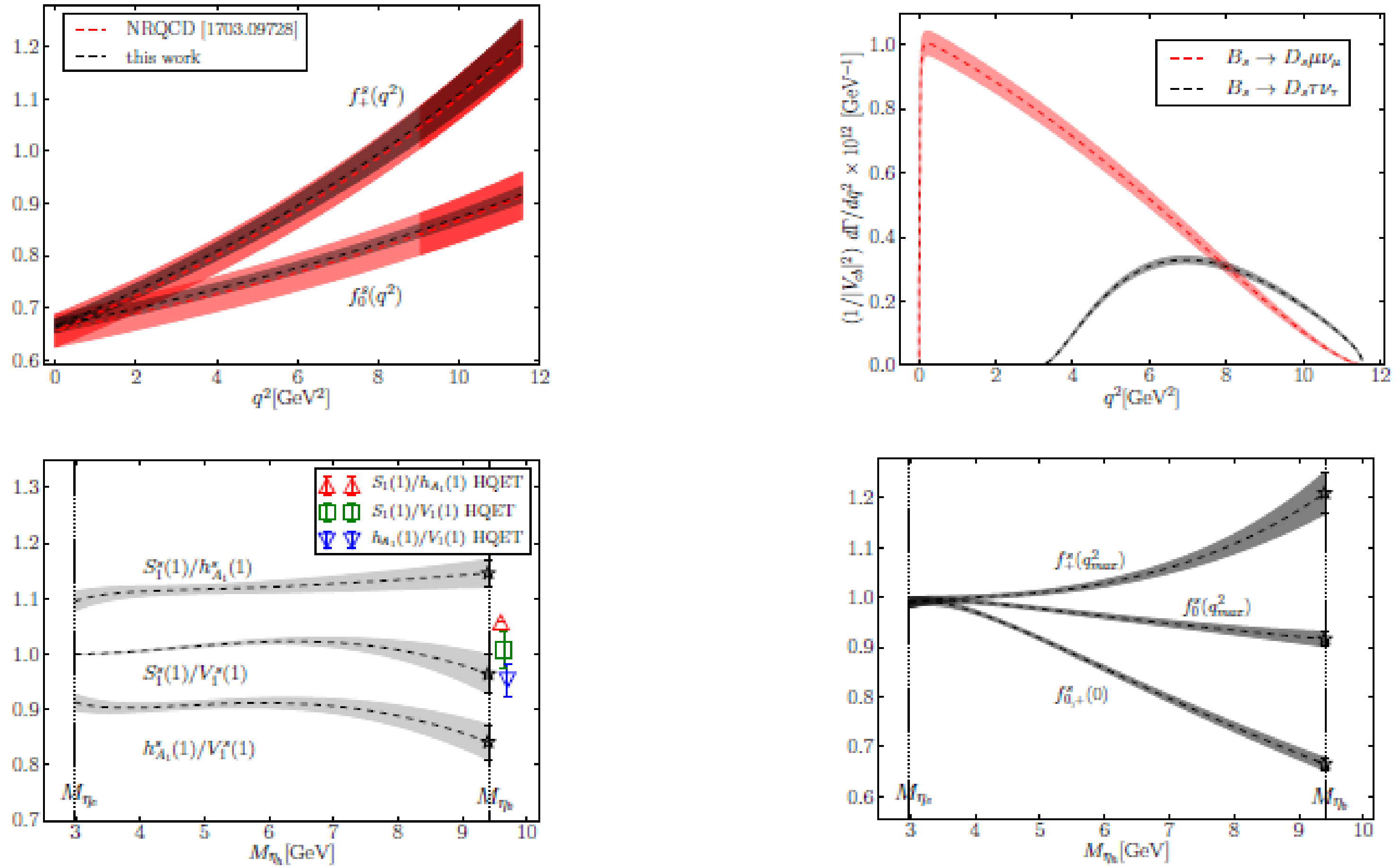
FLAG'19

MILC: Phys. Rev. D92 (2015) 034506,

HPQCD: Phys. Rev. D92 (2015) 054510

HPQCD $B_s \to D_s$

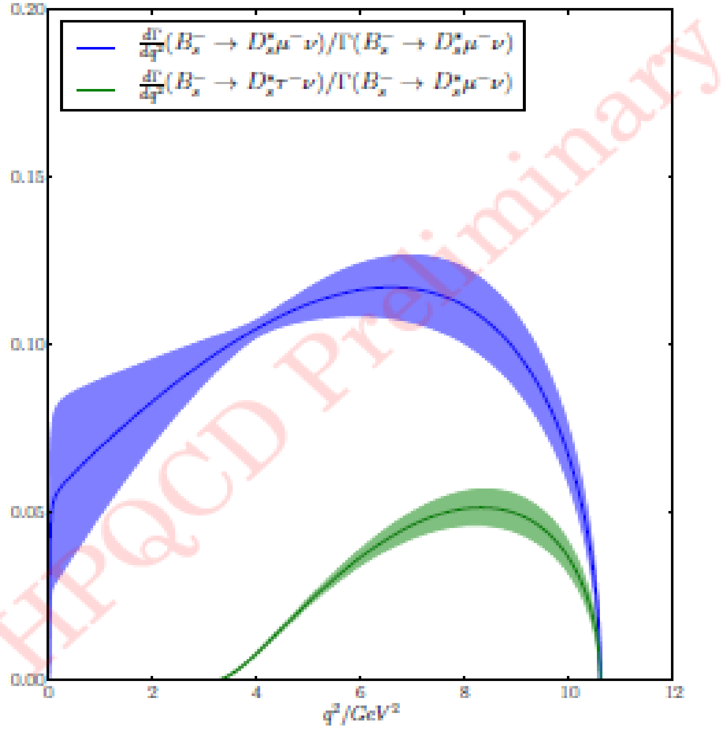
1906.00701



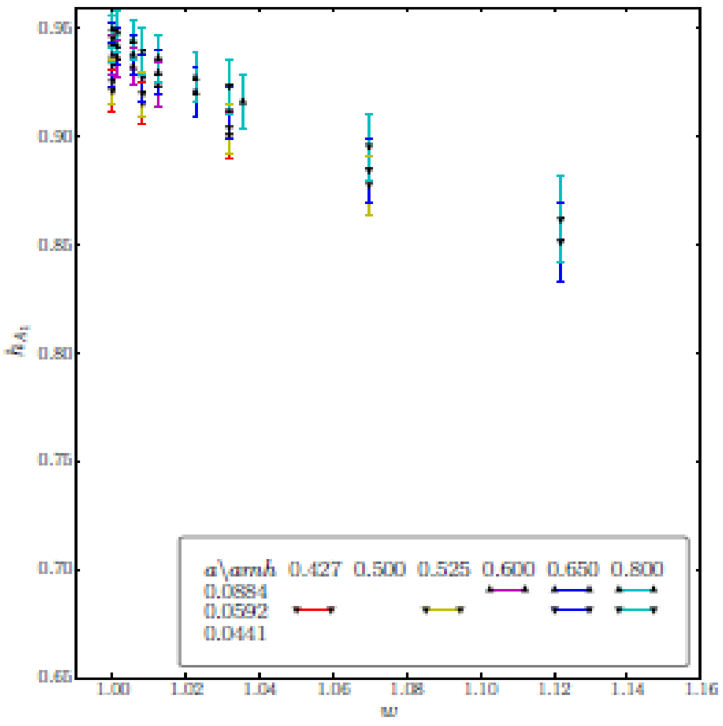
A. Lytle, Lat19

HPQCD $B_{(s)} \to D^*_{(s)}$

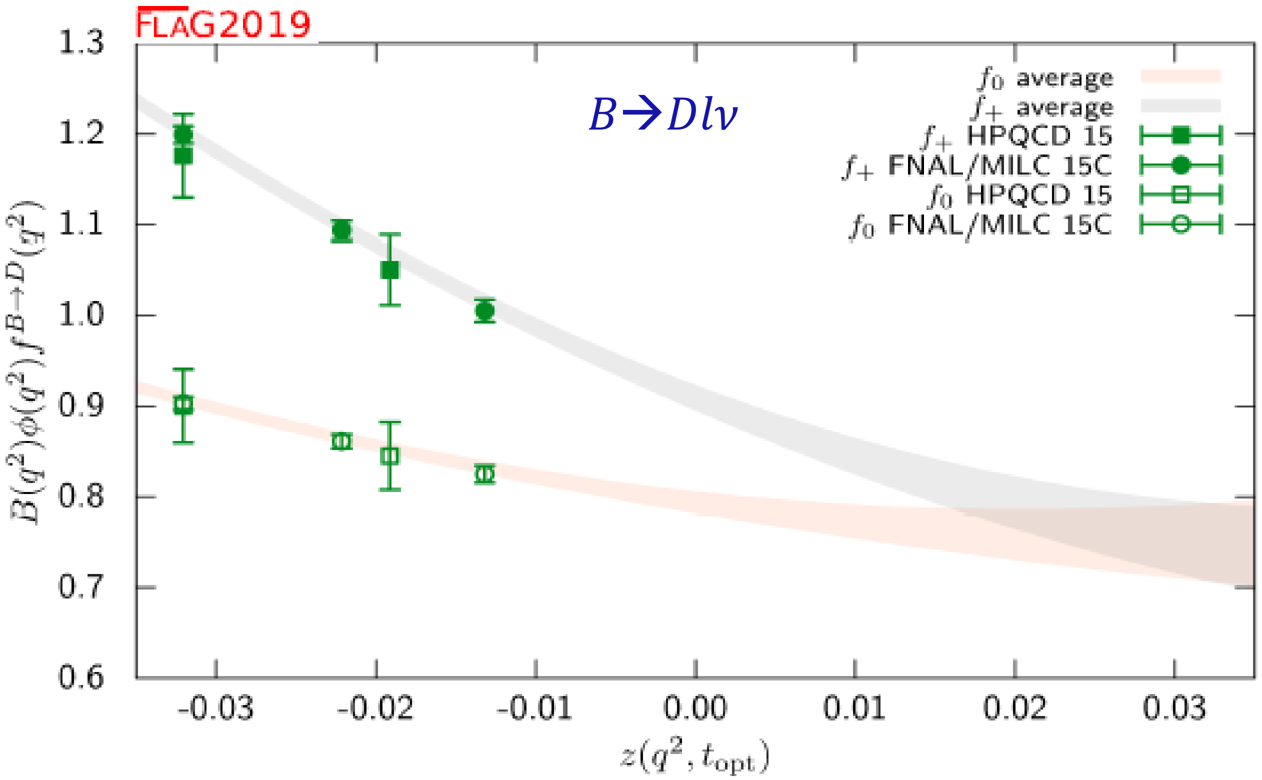
$B_s \to D_s^*$



$B \to D^*$



A. Lytle, Lat19



$$R(D) = \mathcal{B}(B \to D\tau\nu)/\mathcal{B}(B \to D\ell\nu) \qquad \text{with } \ell = e, \mu$$

$$R(D) = 0.300(8) \qquad R(D_s) = 0.314(6)$$

MILC: Phys. Rev. D92 (2015) 034506,

HPQCD: Phys. Rev. D92 (2015) 054510

Rare decay:

$B_s \to \mu^+\mu^-$

LHCb, CMS: Nature 522 (2015) 68

$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2) = 1.046(44)(15),$$
$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_\pi^2) = 1.054(47)(17)$$

MILC: Phys.Rev. D85 (2012) 114502

$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2) = 1.000(62)$$
$$f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_\pi^2) = 1.006(62)$$

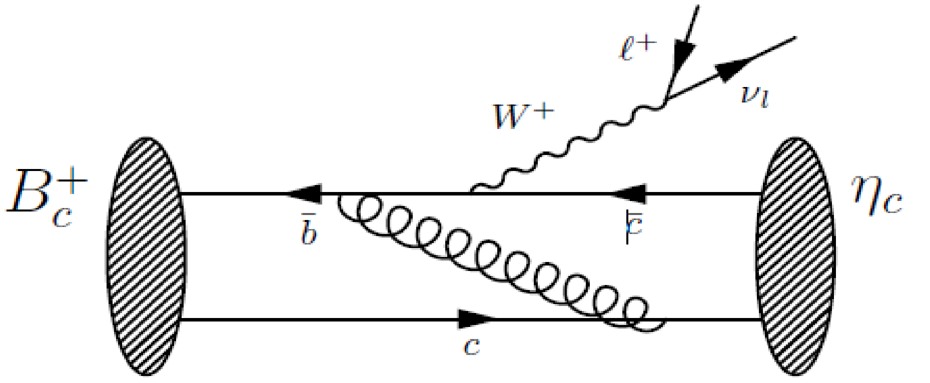
HPQCD: Phys. Rev. D95 (2017) 114506

$$\mathcal{R}(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\,\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\,\mu^+\nu_\mu)} = 0.71 \pm 0.17\,(\mathrm{stat}) \pm 0.18\,(\mathrm{syst}).$$

LHCb : Phys. Rev. Lett. 120 (2018) no.12, 121801

SM : 0.25-0.28

Form factors



- $B_C \to \eta_c l\nu$

$$\langle \eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M^2-m^2}{q^2}q^\mu$$

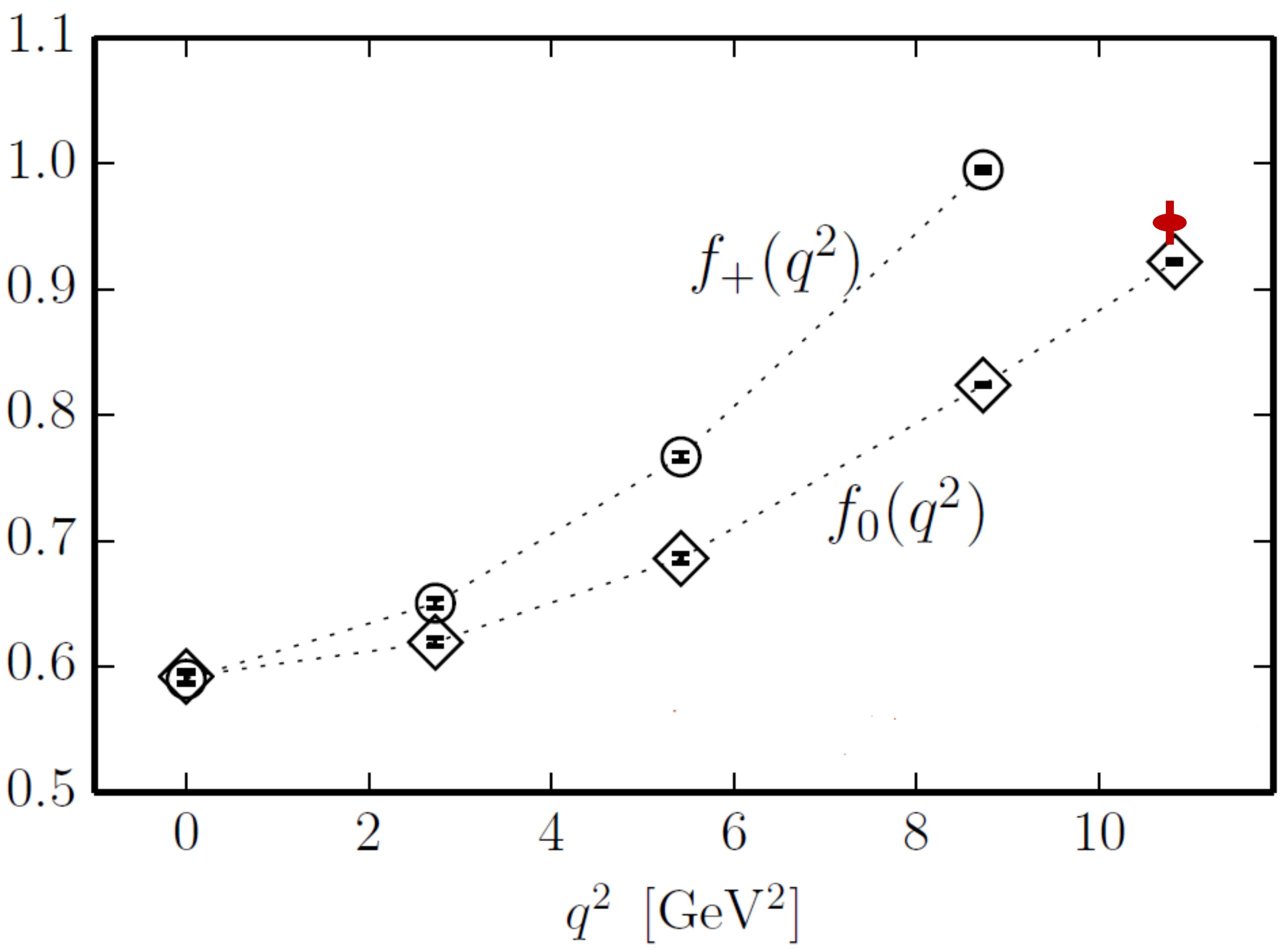
- $B_C \to J/\psi l\nu$

$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle = \frac{2i\varepsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon^*_\nu p_\rho P_\sigma V(q^2) - (M+m)\varepsilon^{*\mu}A_1(q^2) +$$
$$\frac{\varepsilon^*\cdot q}{M+m}(p+P)^\mu A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2)$$

$$q = P - p$$

$q^2_{\max}$ **: Outgoing hadron at rest**

$q^2 = 0$ **: Maximum recoil**



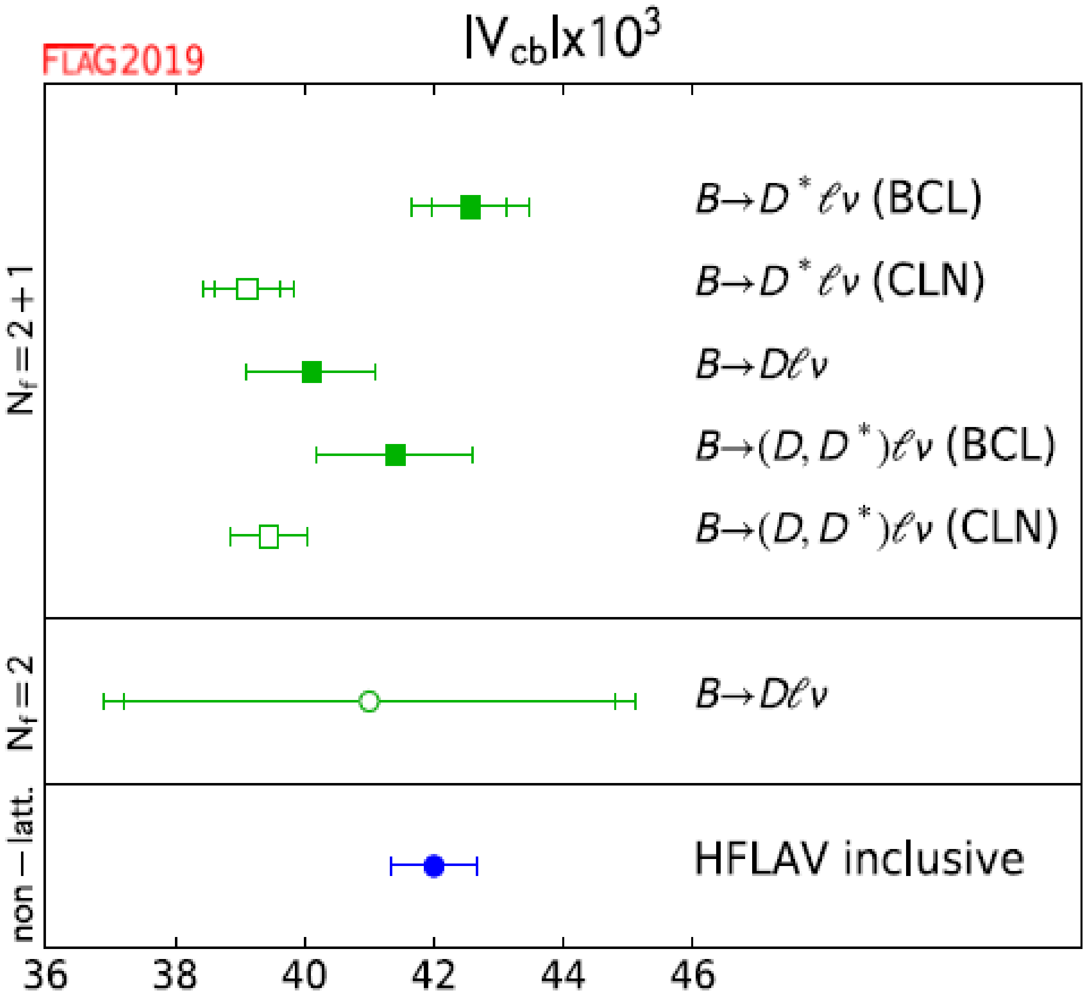
Colquhoun *et al* **HPQCD : 1611.01987**

A. Lytle : CKM2016

NM : Lattice 2017

$|V_{cb}|$ from FLAG

- Good agreement between the two decay channels for 2+1 flavor lattice form factors.
- Tension between inclusive and exclusive is not that bad.



Semileptonic form factors in baryon decays

$$\Lambda_c \to \Lambda \ell \nu$$

Alternate ways to get $|V_{cs}|$

$$\langle \Lambda | \bar{s} \gamma^{\mu} (1 - \gamma_5) c | \Lambda_c \rangle$$

$$\frac{\Gamma(\Lambda_c \to \Lambda e^+ \nu_e)}{|V_{cs}|^2} = 0.2007(71)(74)\ \mathrm{ps}^{-1}\,,$$

$$\frac{\Gamma(\Lambda_c \to \Lambda \mu^+ \nu_\mu)}{|V_{cs}|^2} = 0.1945(69)(72)\ \mathrm{ps}^{-1}\,.$$

S. Meinel, Phys. Rev. Lett. 118 (2017) 082001

$$\Lambda_b \to p \ell \nu$$

Alternate ways to get $|V_{bu}|$

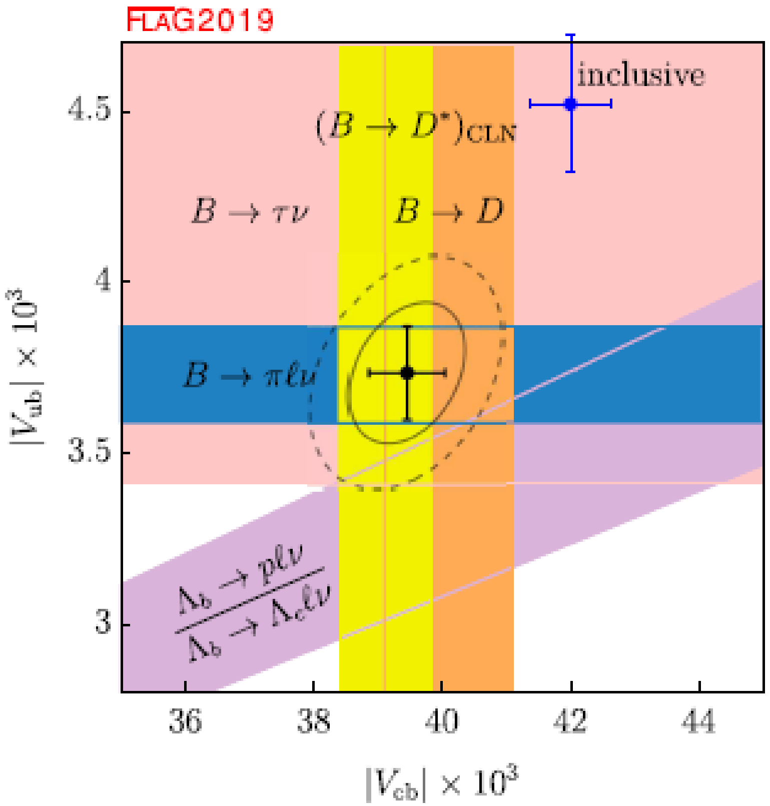
$$\Lambda_b \to \Lambda_c \ell \nu$$

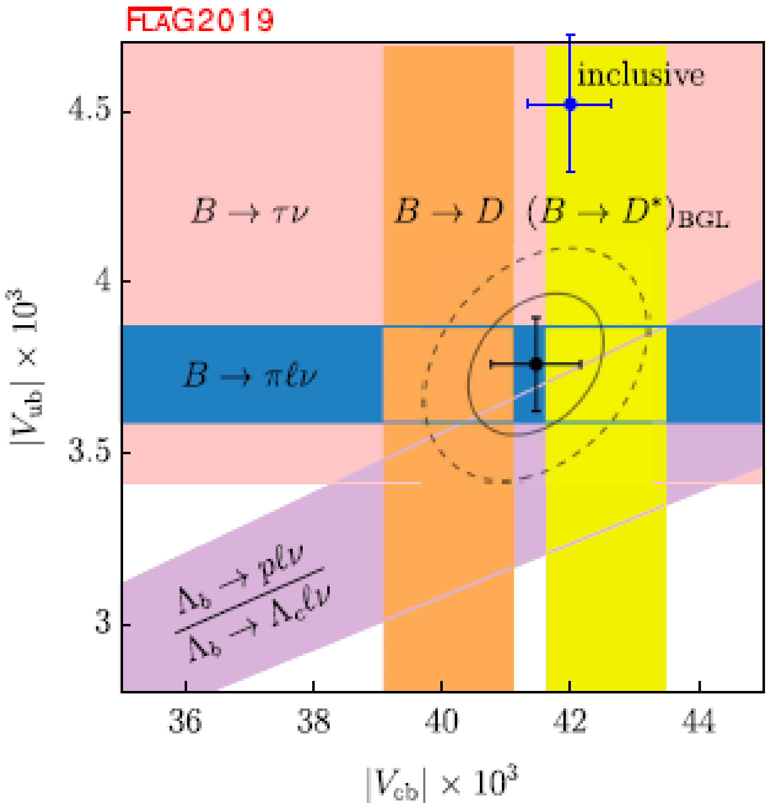
Alternate ways to get $|V_{bc}|$

Detmold et. al.: Phys. Rev. D92 (2015) 034503,

Possibility of exclusive determination of $|V_{ub}|/|V_{cb}|$

V_{ub} and V_{cb}



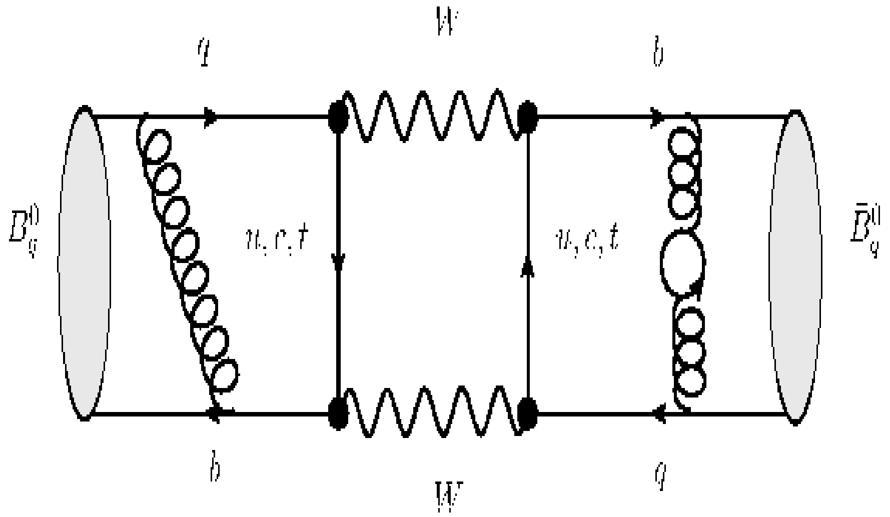


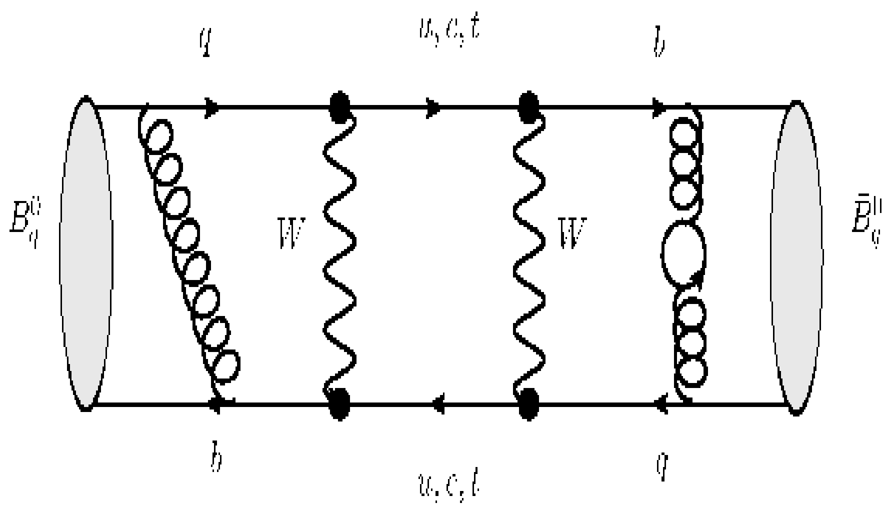
Using BGL rather than CNL for B→D* eliminates tension between inclusive and exclusive determinations of Vcb.
Tension for B→π remains!
Lambda decay not in fit to exclusive decays.

Third row: B meson mixing

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|B_d\rangle & \langle B_s|B_s\rangle & \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

B meson mixing





- B meson mixing is a loop level process
- Experiments can measure mass difference, lifetime difference for the two resulting eigenstates and also can measure a CP violating phase
- Short distance expansion of the loops results in effective weak Hamiltonian involving 4-quark operators
- GIM and loop suppression, so good place to look for BSM

Neutral B-meson mixing

$$\langle \bar{B}_q^0 | H_{eff}^{\Delta B=2} | B_q^0 \rangle \qquad H_{eff,BSM}^{\Delta B=2} = \sum_{q=d,s} \sum_{i=1}^{5} C_i Q_i^q$$

$$Q_1^q = [\bar{b}\gamma_\mu(1-\gamma_5)q]\,[\bar{b}\gamma_\mu(1-\gamma_5)q]$$

$$Q_2^q = [\bar{b}(1-\gamma_5)q]\,[\bar{b}(1-\gamma_5)q]\,,$$

$$Q_4^q = [\bar{b}(1-\gamma_5)q]\,[\bar{b}(1+\gamma_5)q]\,,$$

$$Q_3^q = \left[\bar{b}^\alpha(1-\gamma_5)q^\beta\right]\left[\bar{b}^\beta(1-\gamma_5)q^\alpha\right],$$

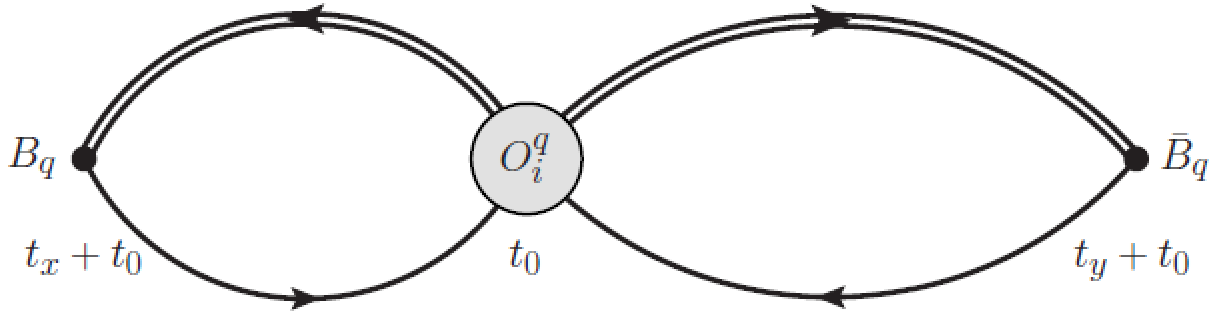
$$Q_5^q = \left[\bar{b}^\alpha(1-\gamma_5)q^\beta\right]\left[\bar{b}^\beta(1+\gamma_5)q^\alpha\right],$$

Bag Parameter: $$B_{B_q}(\mu) = \frac{\langle \bar{B}_q^0 | Q_R^q(\mu) | B_q^0 \rangle}{\frac{8}{3} f_B^2 m_B^2}$$

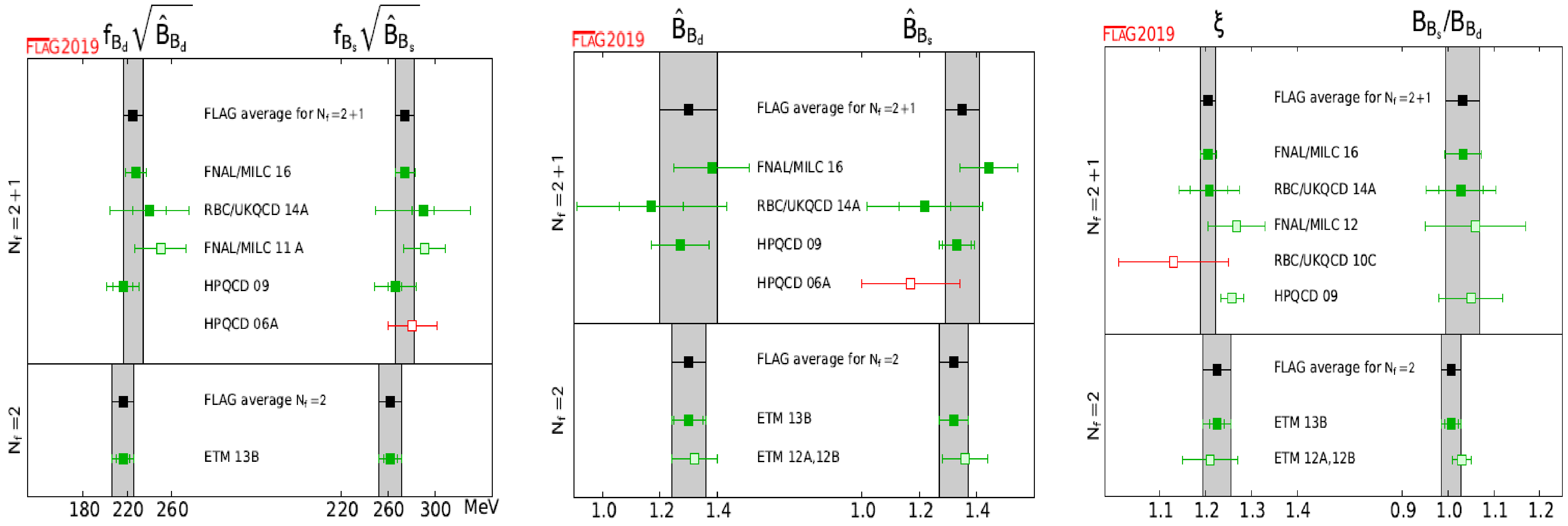
$$\hat{B}_{B_q} = \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \left\{1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2}\left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2}\right]\right\} B_{B_q}(\mu)$$

$$\Delta m_q = \frac{G_F^2 m_W^2 m_{B_q}}{6\pi^2} |\lambda_{tq}|^2 S_0(x_t)\eta_{2B} f_{B_q}^2 \hat{B}_{B_q} \qquad \lambda_{tq} = V_{tq}^* V_{tb}$$

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$



Neutral B-meson mixing



$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 225(9)\,\mathrm{MeV} \qquad f_{B_s}\sqrt{\hat{B}_{B_s}} = 274(8)\,\mathrm{MeV}$$

$$\hat{B}_{B_d} = 1.30(10) \qquad \hat{B}_{B_s} = 1.35(6)$$

$$\xi = 1.206(17) \qquad B_{B_s}/B_{B_d} = 1.032(38)$$

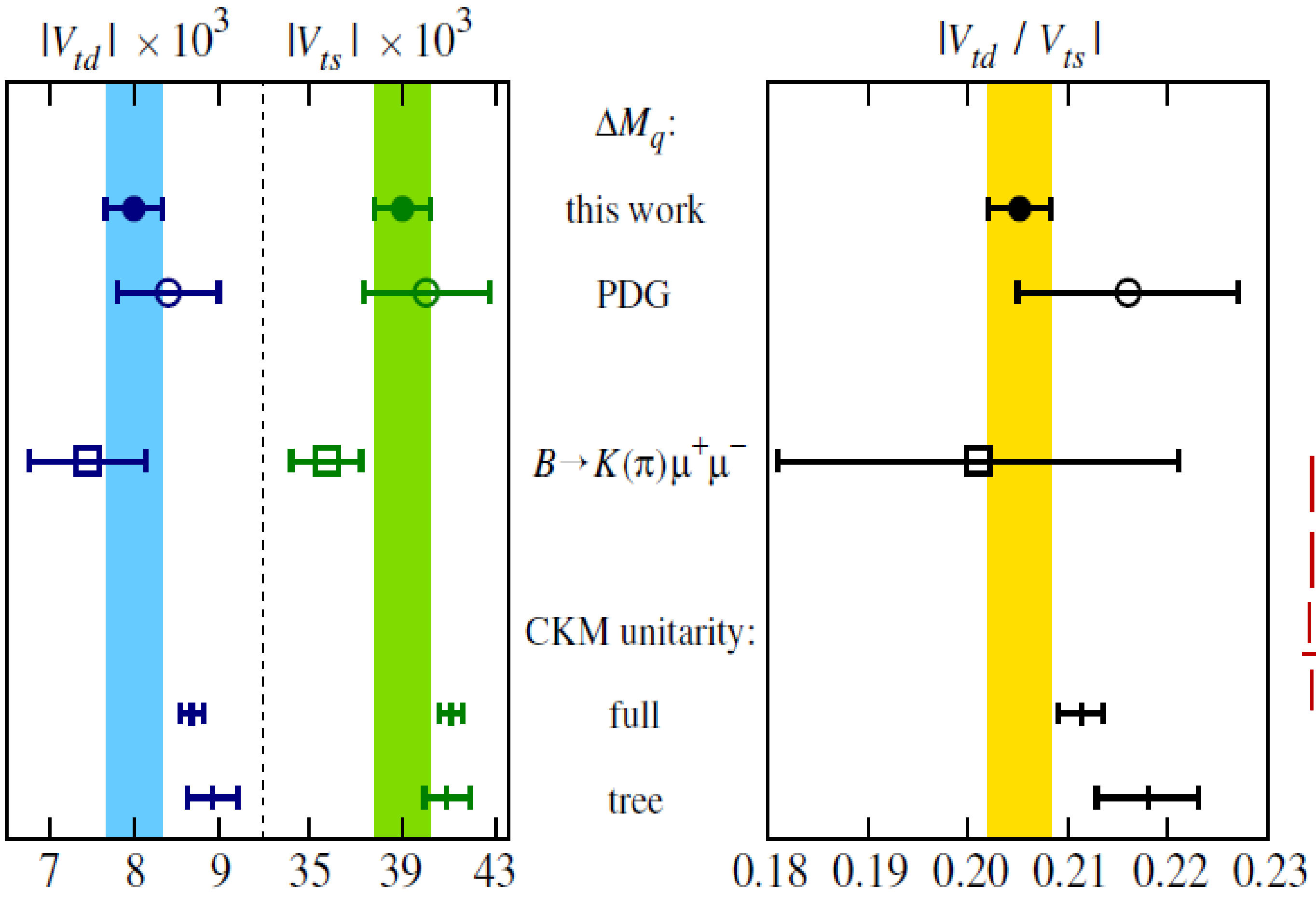
FNAL/MILC: Phys. Rev. D93 (2016) 113016,

RBC/UKQCD: Phys. Rev. D91 (2015) 114505,

HPQCD: Phys.Rev. D80 (2009) 014503,

Third row

Using experimental results on B,Bs mixings, MILC/FNAL reported :



Phys. Rev. D93 (2016) no.11, 113016

$|V_{td}| = 8.00(34)(8) \times 10^{-3}$ → **4.3%**

$|V_{ts}| = 39.0(1.2)(0.4) \times 10^{-3}$ → **3.2%**

$\frac{|V_{td}|}{|V_{ts}|} = 0.2052(31)(10)$ → **1.6%**

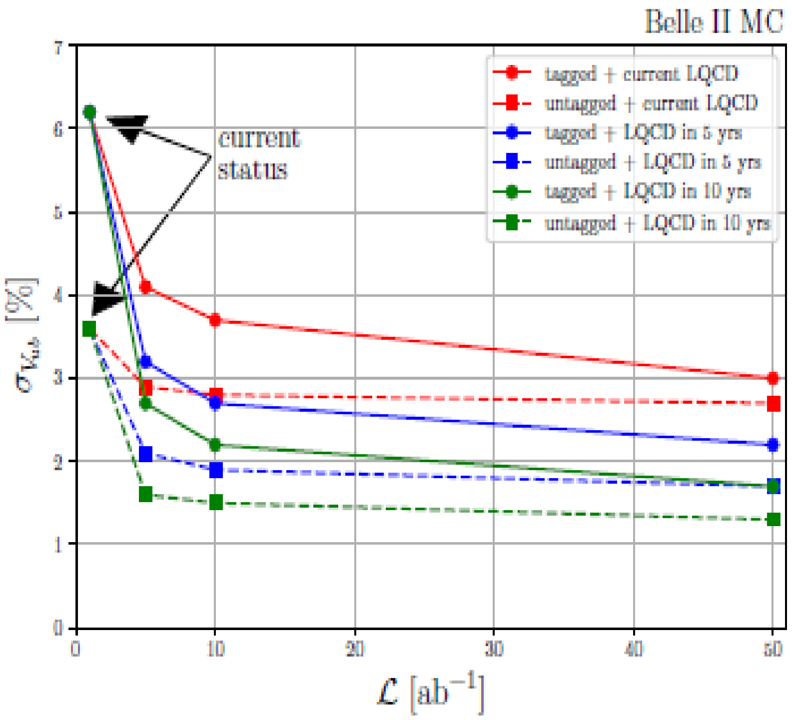
S.Gottlieb'Lat19

CKM Summary

Quantity	value	percentage error	Comment
$\|V_{ud}\|$	0.9737(16)	0.16	FLAG (with unitarity)
$\|V_{ud}\|$	0.9669(34)	0.35	FNAL/MILC (K_{l2} & K_{l3})
$\|V_{us}\|$	0.2249(7)	0.31	FLAG (with unitarity)
$\|V_{us}\|$	0.22333(61)	0.27	FNAL/MILC (K_{l2} & K_{l3})
$\|V_{cd}\|$	0.2219(43)	1.9	FLAG (2+1+1)
$\|V_{cs}\|$	1.002(14)	1.4	FLAG (2+1+1)
$\|V_{ub}\| \times 10^3$	3.76(14)	3.7	FLAG (BGL, combined)
$\|V_{cb}\| \times 10^3$	41.47(70)	1.7	FLAG (BGL, combined)
$\|V_{td}\| \times 10^3$	8.00(34)	4.3	FNAL/MILC
$\|V_{ts}\| \times 10^3$	39.0(1.3)	3.2	FNAL/MILC

S.Gottlieb'Lat19

Belle II prospects



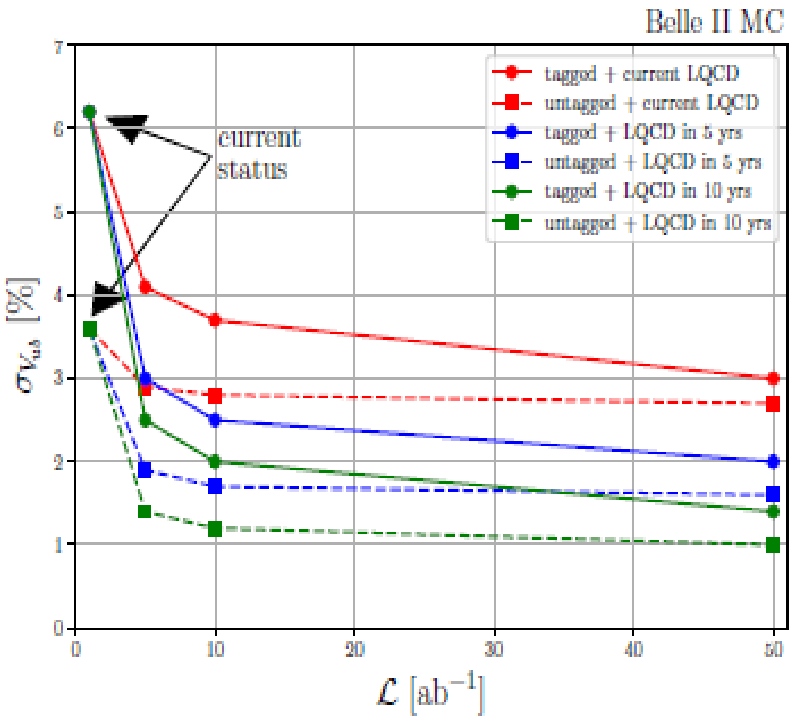


Fig. 87: Projections of the $|V_{ub}|$ uncertainty for various luminosity values and lattice-QCD error forecasts for $B \to \pi \ell \nu$ tagged and untagged modes. The figure on the left is obtained by using lattice forecasts with EM corrections and the figure on the right by forecasts without these corrections.

Conclusions

- Lattice QCD is playing a crucial role in determining decay constants and form factors of various hadrons and in turn helping in precise determination of the CKM matrix elements
  - A number of quantities are available to sub-percent accuracy.
  - Getting to the point where electromagnetic corrections important to lattice calculations
  - Expect to increase LQCD precision by factor of 3-5 over the next 5-10 years

- Heavy flavour physics is a precision tool to discover new physics.

  Lattice QCD calculations are absolutely necessary for this.

- Interplay between theory and experiments will provide more and more stringent test of the standard model of particle physics.

- BESIII, Belle II, and LHCb have a large role to play in the future of flavour physics