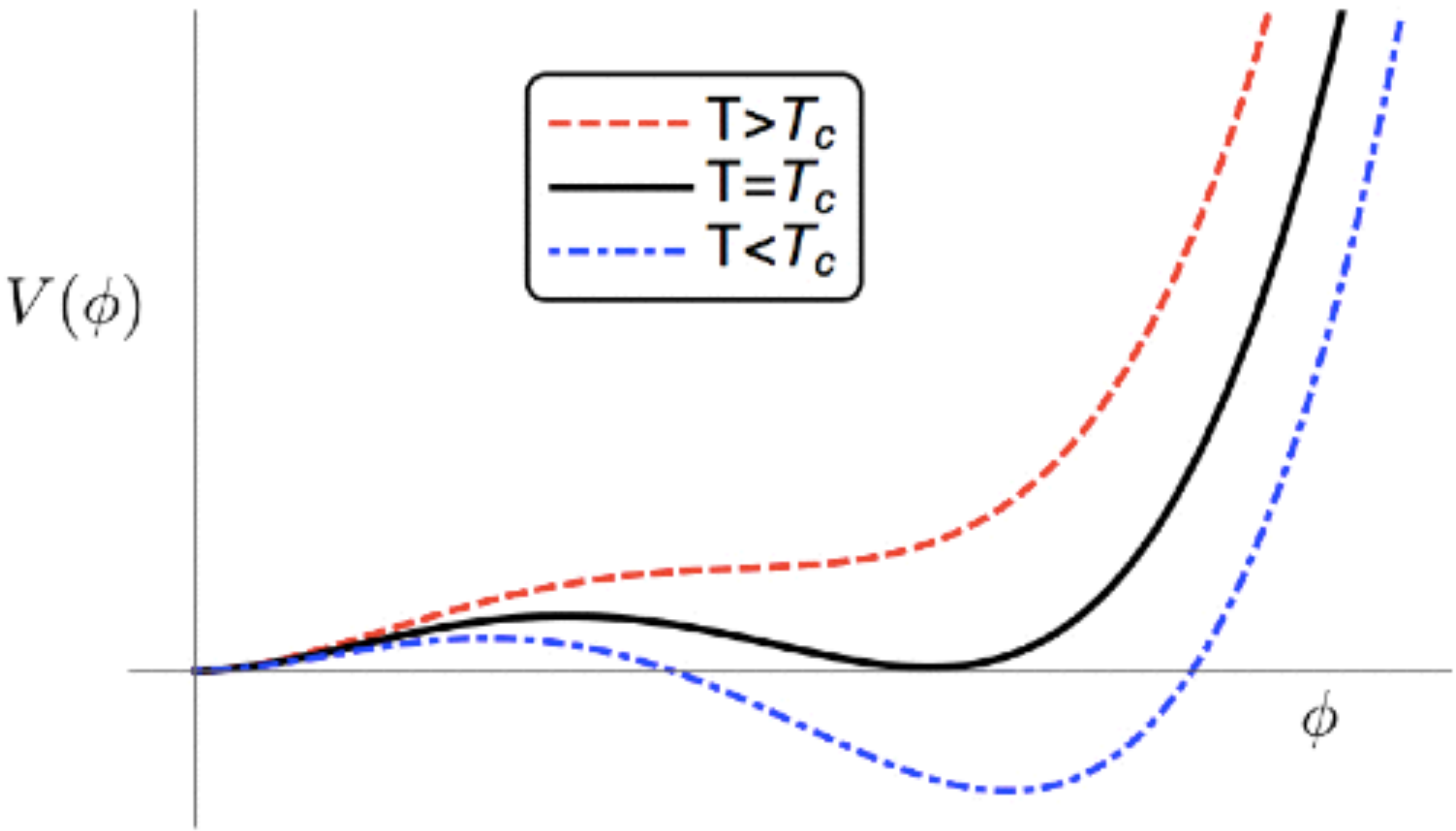
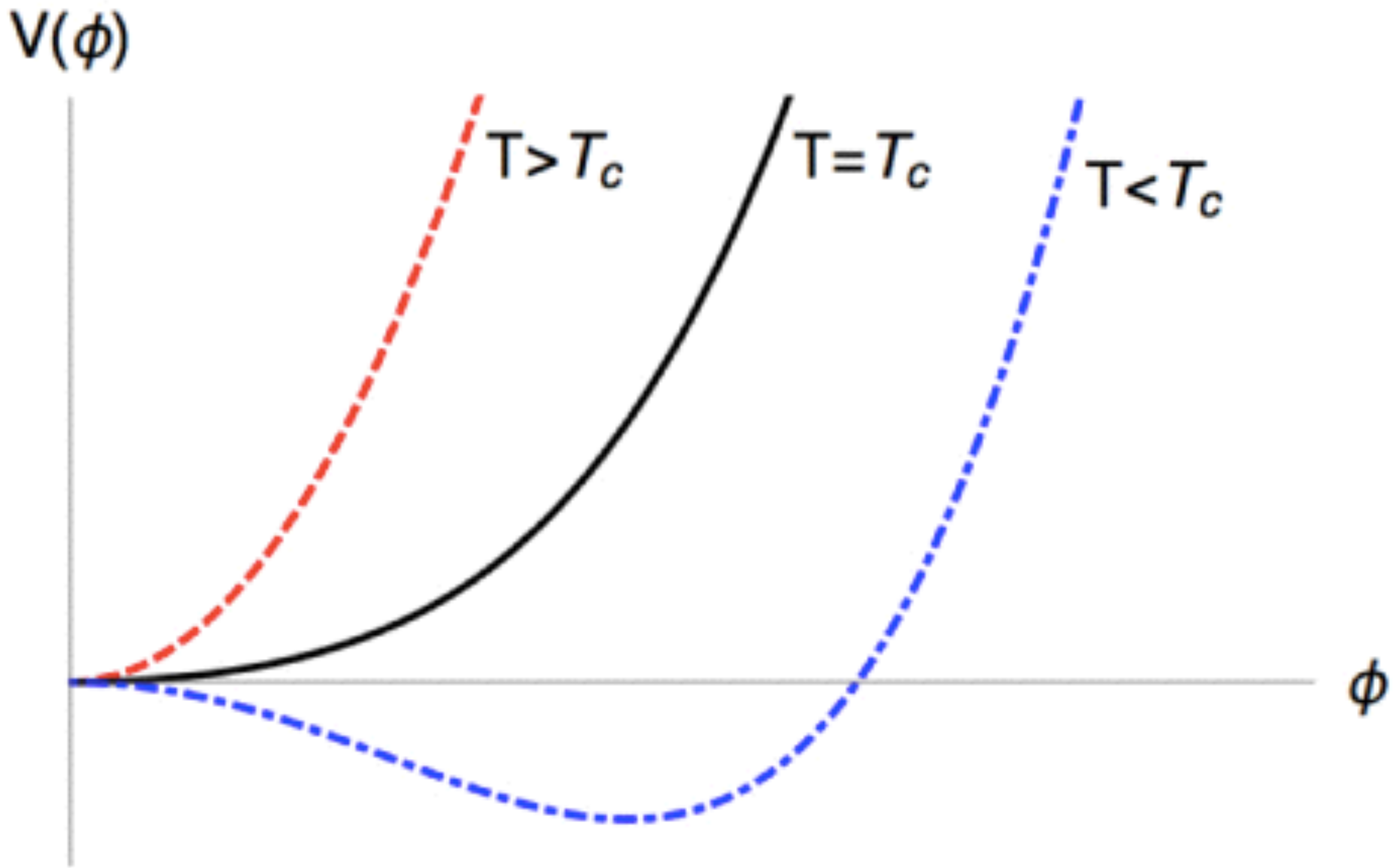
Phase transitions and stochastic gravitational waves

Subhendra Mohanty
Physical Research Laboratory

First order phase transitions

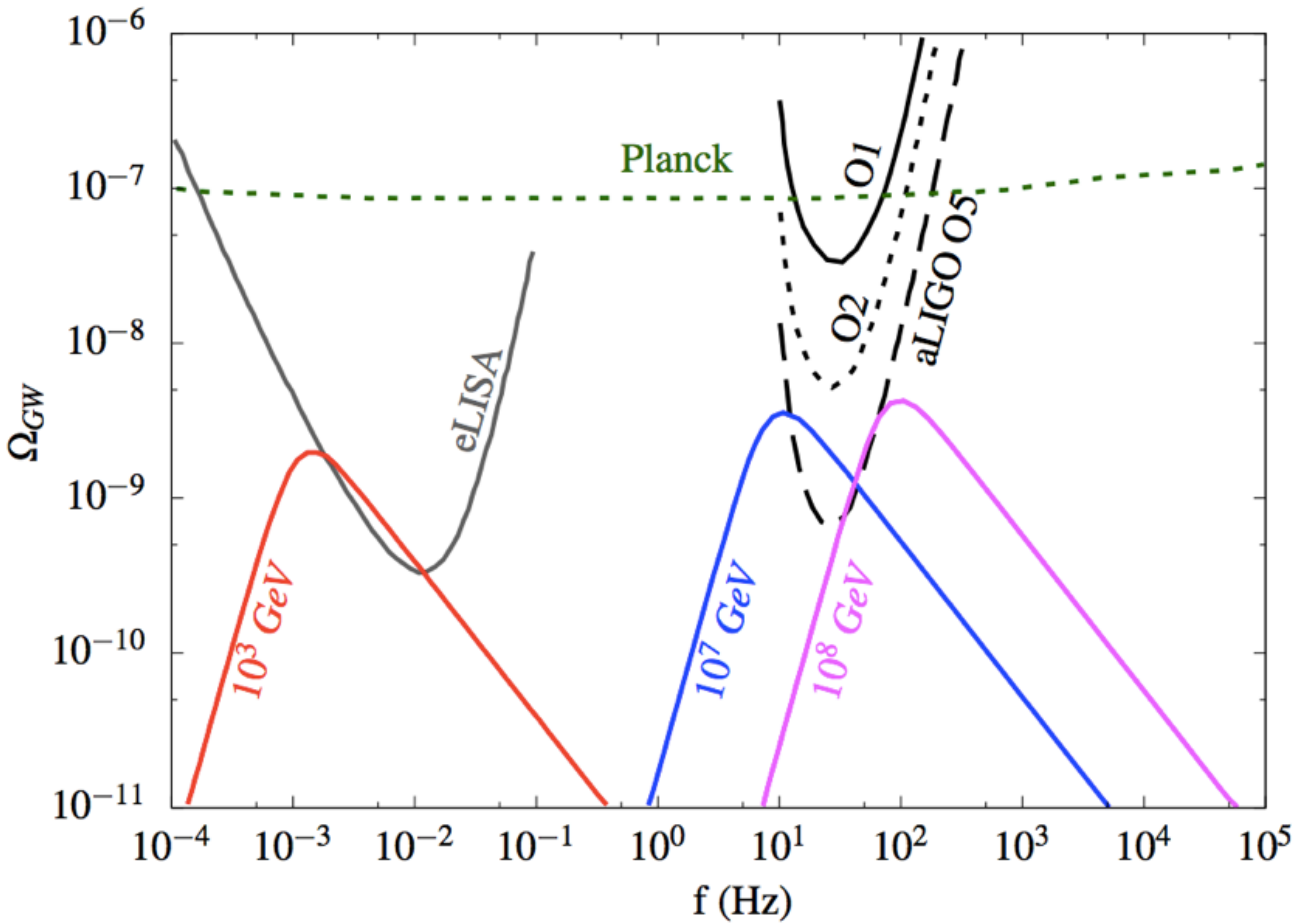


Second order phase transitions

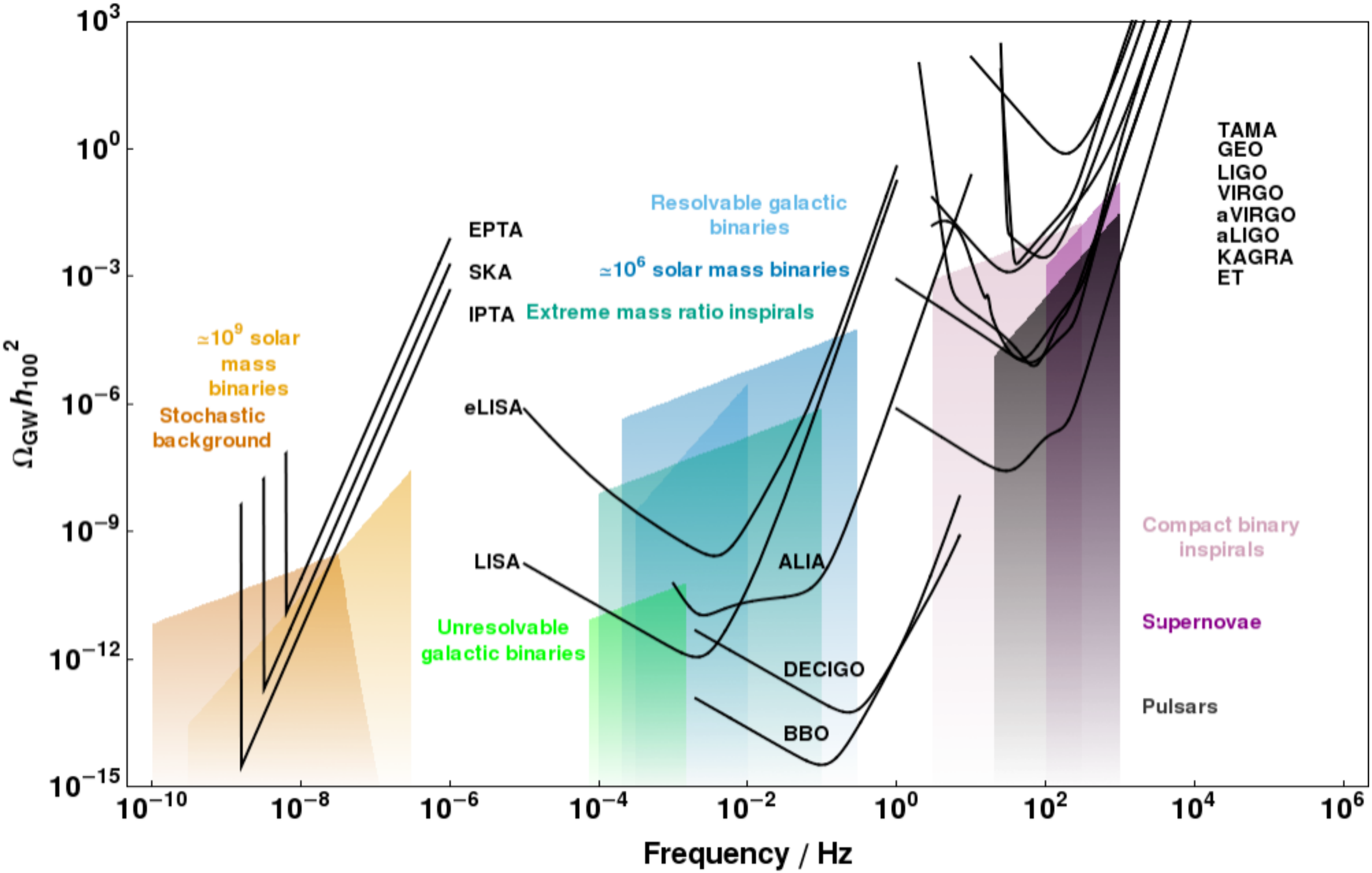


Stochastic gravitational waves are generated in FOPT

$$\Omega_{GW} = \frac{1}{\rho_{crit}} \frac{\langle \dot{h}_i(\mathbf{k},t)\dot{h}_i(\mathbf{k},t)\rangle}{8\pi G a(t)^2}$$



Planned GRW detectors



Effective potential at finite temperature

$$V(\phi_c,T) = V_0 + \sum_i V_{CW}(m_i) + \Delta V^{\beta}(m_i) - \frac{T}{12\pi}\sum_j n'_j \left[(\mathcal{M}_i^2)^{3/2} - (m_i^2)^{3/2}\right]$$

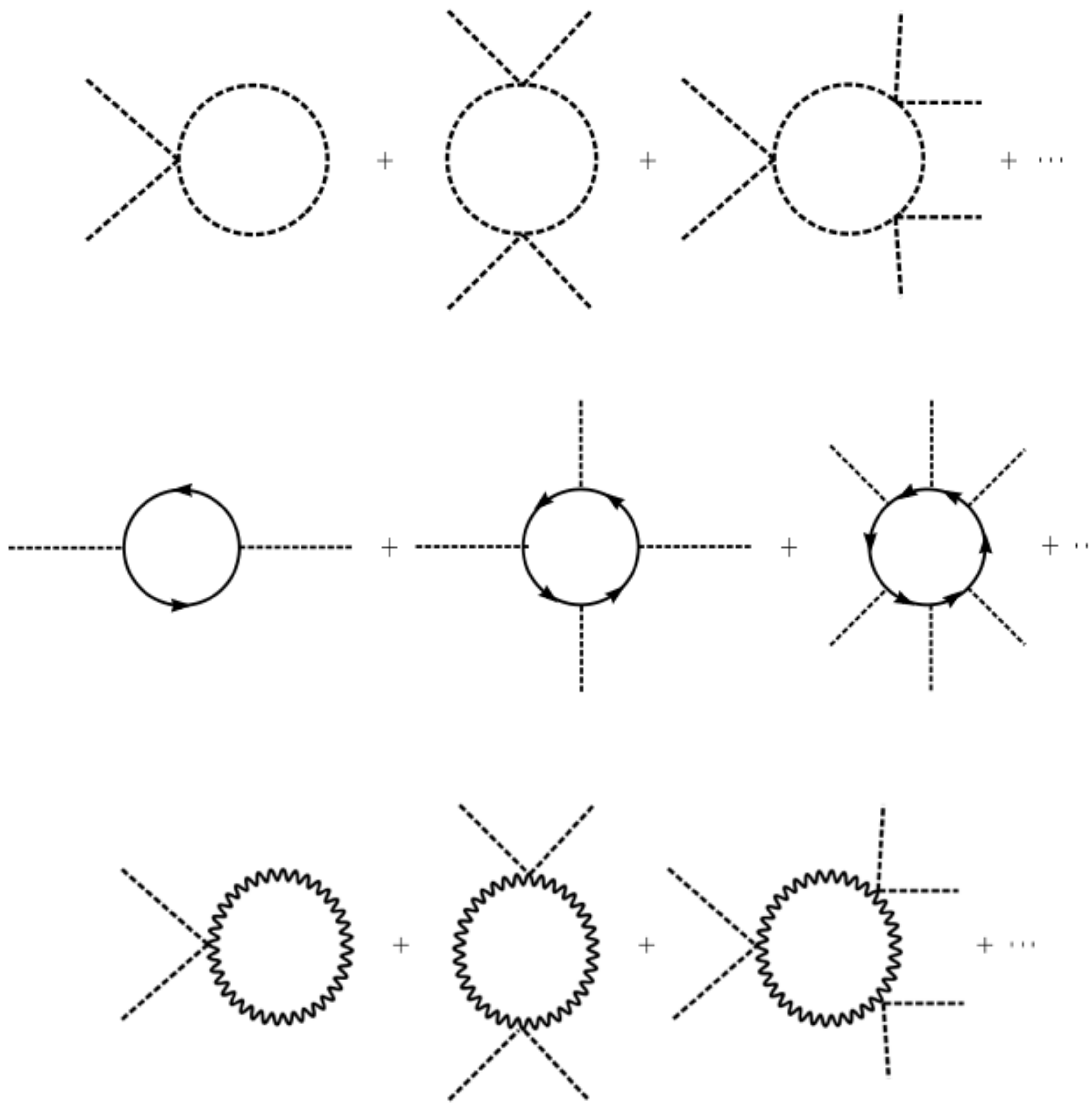
Tree level

One loop

One loop at non-zero T

Thermal masses
(ring diagrams)

One loop potential



Standard model Higgs potential

$$\Phi = \begin{pmatrix} \chi_1 + i\chi_2 \\ \frac{1}{\sqrt{2}}\left(\phi_c + h + i\chi_3\right) \end{pmatrix}$$

$$V_0(\Phi) = -\frac{\mu_h^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2$$

$$V_0(\phi_c) = -\frac{\mu_h^2}{2}\phi_c^2 + \frac{\lambda}{4}\phi_c^4$$

Higgs and "Goldstone boson" masses

$$m_h^2(\phi_c) = \frac{\partial^2 V_0(\Phi)}{(\partial h)^2}\Big|_{\Phi=\phi_c} = 3\lambda\phi_c^2 - \mu_h^2$$

$$m_{\chi_a}^2(\phi_c) = \frac{\partial^2 V_0(\Phi)}{(\partial \chi_a)^2}\Big|_{\Phi=\phi_c} = \lambda\phi_c^2 - \mu_h^2$$

Only at the minima of the potential

$$\phi_c = v = \sqrt{\frac{\mu_h^2}{\lambda}} = 246.22\ \text{GeV}$$

$$m_h^2(v) = 2\lambda v^2$$

$$m_{\chi_a}^2(v) = 0$$

Coleman-Weinberg potential for the SM Higgs

$$V_{CW}(\phi_c) = \frac{1}{64\pi^2} \sum_{i=h,\chi,t,W,Z} (-1)^{F_i} n_i \, m_i^4(\phi_c) \left(\log \frac{m_i^2(\phi_c)}{\mu^2} - c_i \right)$$

$$m_W^2(\phi_c) = \frac{g^2}{4}\phi_c^2 \qquad m_Z^2(\phi_c) = \frac{g^2 + g'^2}{2}\phi_c^2 \qquad m_t^2 = \frac{h_t^2}{2}\phi_c^2$$

Degrees of freedom

$$n_h = 1, \quad n_\chi = 3, \quad n_W = 6, \quad n_Z = 3 \quad \text{and} \quad n_t = 12$$

Finite temperature part

$$\Delta V^{\beta}(\phi_c) = \frac{T^4}{2\pi^2}\left[\sum_{i=W,Z,h,\chi} n_i J_B\left(\frac{m_i^2(\phi_c)}{T^2}\right) - n_t J_F\left(\frac{m_t^2(\phi_c)}{T^2}\right)\right]$$

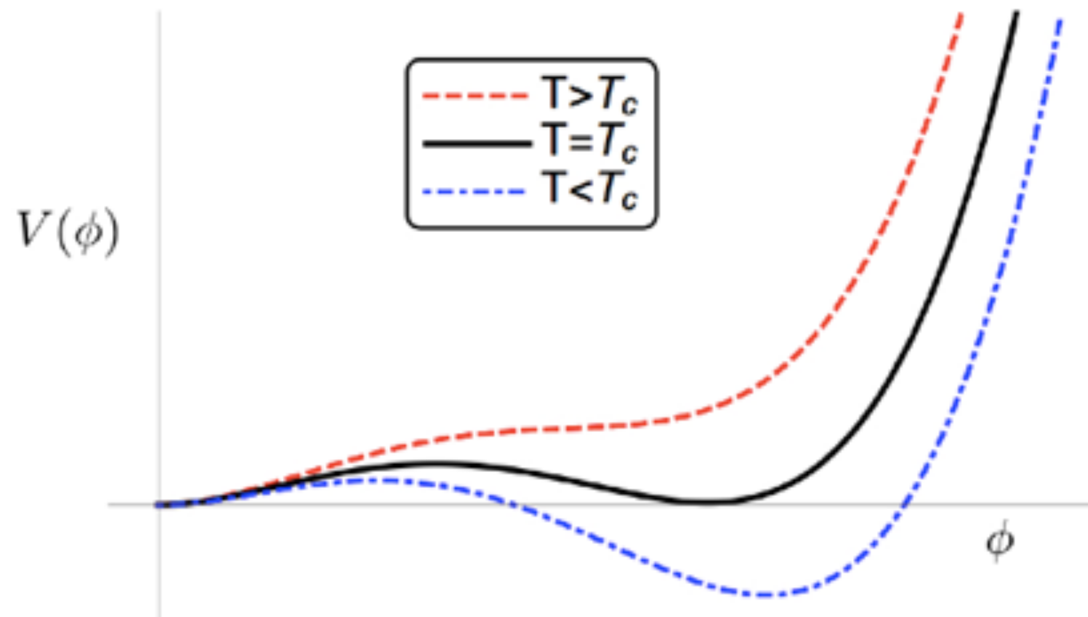
$$J_{B/F}(y^2) = \int_0^{\infty} dx\, x^2 \log\left(1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right)$$

$$\mathcal{M}_i{}^2 = m_i^2 + \Pi_i$$ Debye mass

$$\Pi_{GB}^{L} = \frac{11}{6} T^2\,\mathrm{diagonal}\,(g^2, g^2, g^2, g'^2)$$

$$\Pi_h = \Pi_\chi = T^2\left(\frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda\right)$$

Criteria for FOPT



$$\frac{\phi_{min}(T_c)}{T_c} = \frac{2m_W^3 + m_z^3}{\pi v m_h^2} = \left(\frac{42\,\mathrm{GeV}}{m_h}\right)^2$$

High temperature expansion of thermal potential

Bosons in the loop

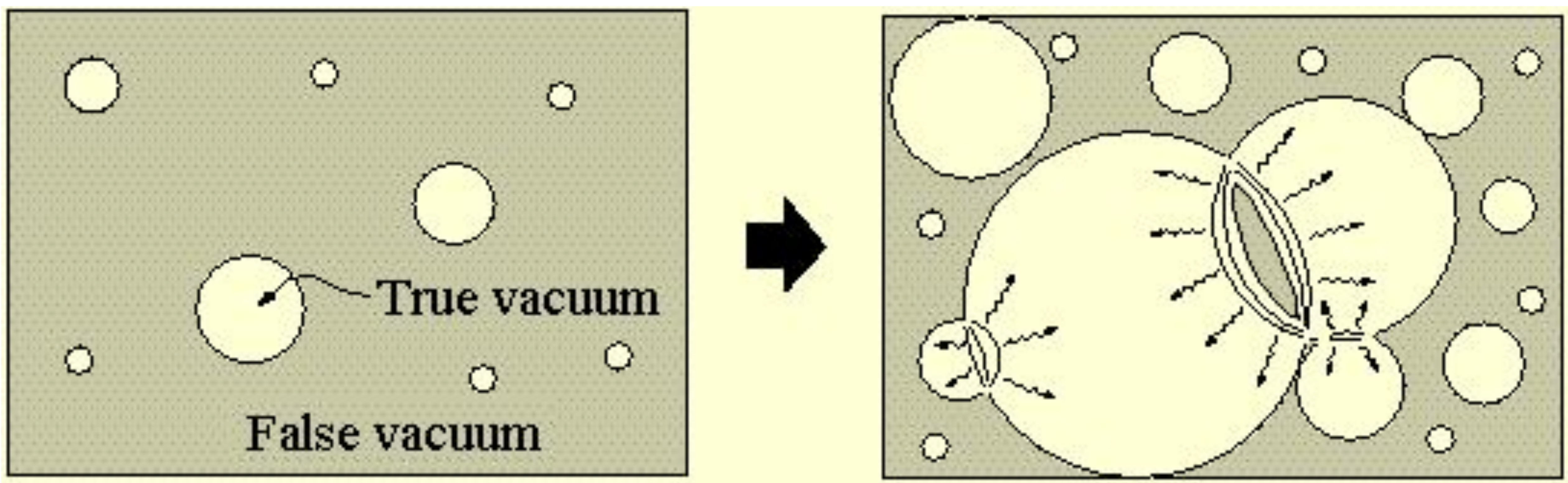
$$\Delta V_\phi^\beta = -\frac{\pi^2}{90}T^4 + \frac{1}{24}m^2T^2 - \frac{1}{12\pi}m^3T - \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{a_b T^2}\right) + \frac{1}{6\pi^2}\frac{m^6\,\xi(3)}{(4\pi)^4T^2} + O\left(\frac{m^8}{T^4}\right)$$

Fermions in the loop

$$\Delta V_F^\beta = \frac{7}{8}\frac{\pi^2}{90}T^4 - \frac{1}{48}M_f^2T^2 - \frac{M_f^4}{64\pi^2}\ln\left(\frac{M_f^2}{a_f^2T^2}\right) + \frac{7}{6\pi^2}\frac{M_f^6\,\xi(3)}{(4\pi)^4T^2}$$

Fermions in the loop don’t contribute a cubic term

Tunneling from false vacuum to true vacuum: bubble nucleation



Bubble nucleation rate:

$$\Gamma(t) = \Gamma_0(t)e^{-S(t)}$$

Inverse of time of phase transition:

$$\beta \equiv -\left.\frac{dS}{dt}\right|_{t=t_*}$$

In a strong first order phase transition

$$\frac{\beta}{H_*} \sim \ln\left(\frac{m_{\rm Pl}}{T_*}\right)$$

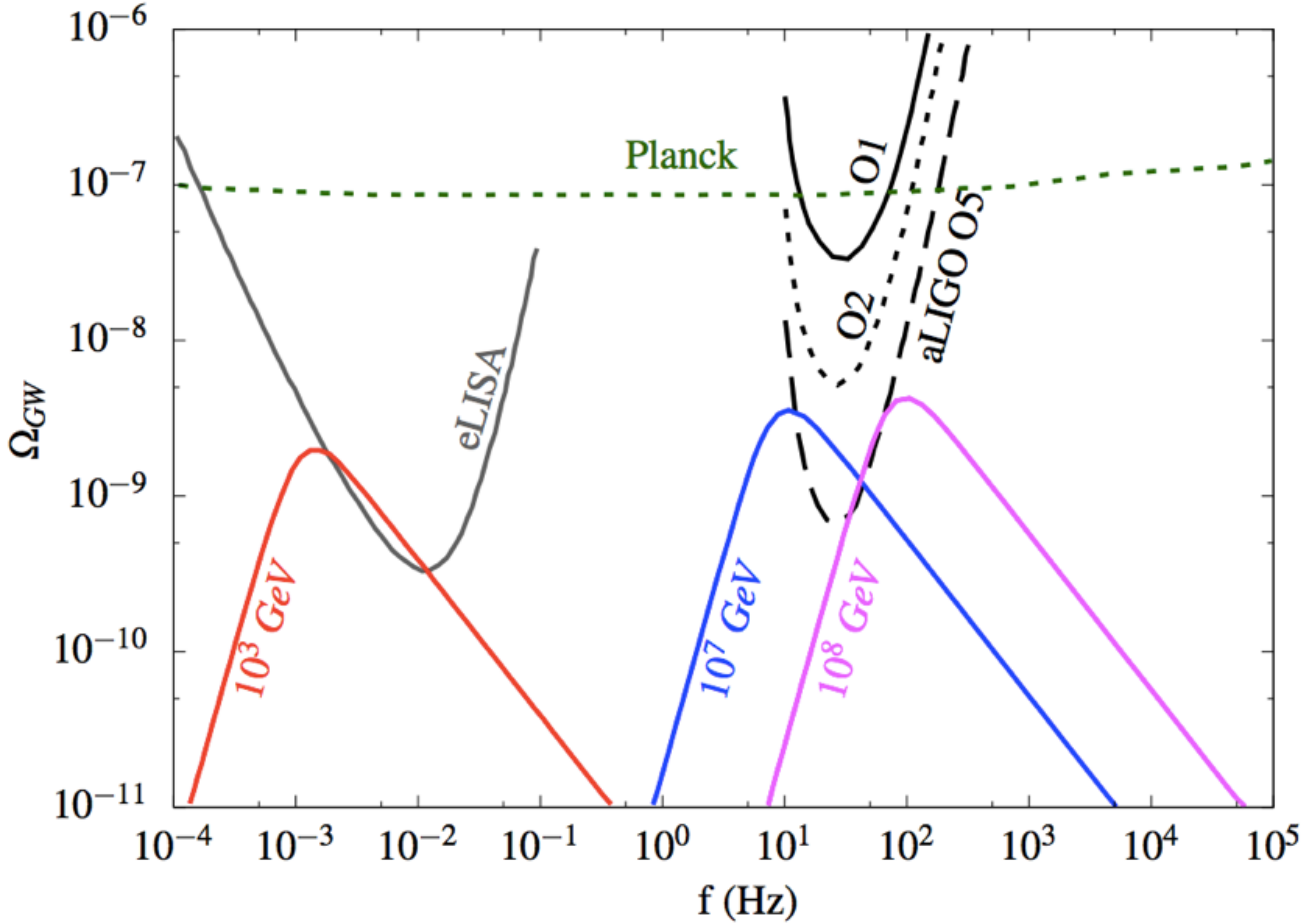
Ratio of vacuum every to radiation energy

$$\alpha \equiv \frac{\rho_{\text{vac}}}{\rho_*}$$

Peak frequency of GRW in the present

$$f = \frac{a_*}{a_0} f_* = \frac{a_*}{a_0} H_* \frac{f_*}{H_*} = \left(\frac{g_{0s}}{g_{*s}}\right)^{1/3} \left(\frac{T_0}{T_*}\right) H_* \frac{f_*}{H_*}$$

$$\frac{d\Omega_{GW}^B}{d \log k} \simeq \frac{2h^2}{3\pi^2} \Omega_{r0} \left(\frac{\mathcal{H}_*}{\beta}\right)^2 \Omega_{S*}^2 \, v^3 \, \frac{(k/\beta)^3}{1 + (k/\beta)^4}$$

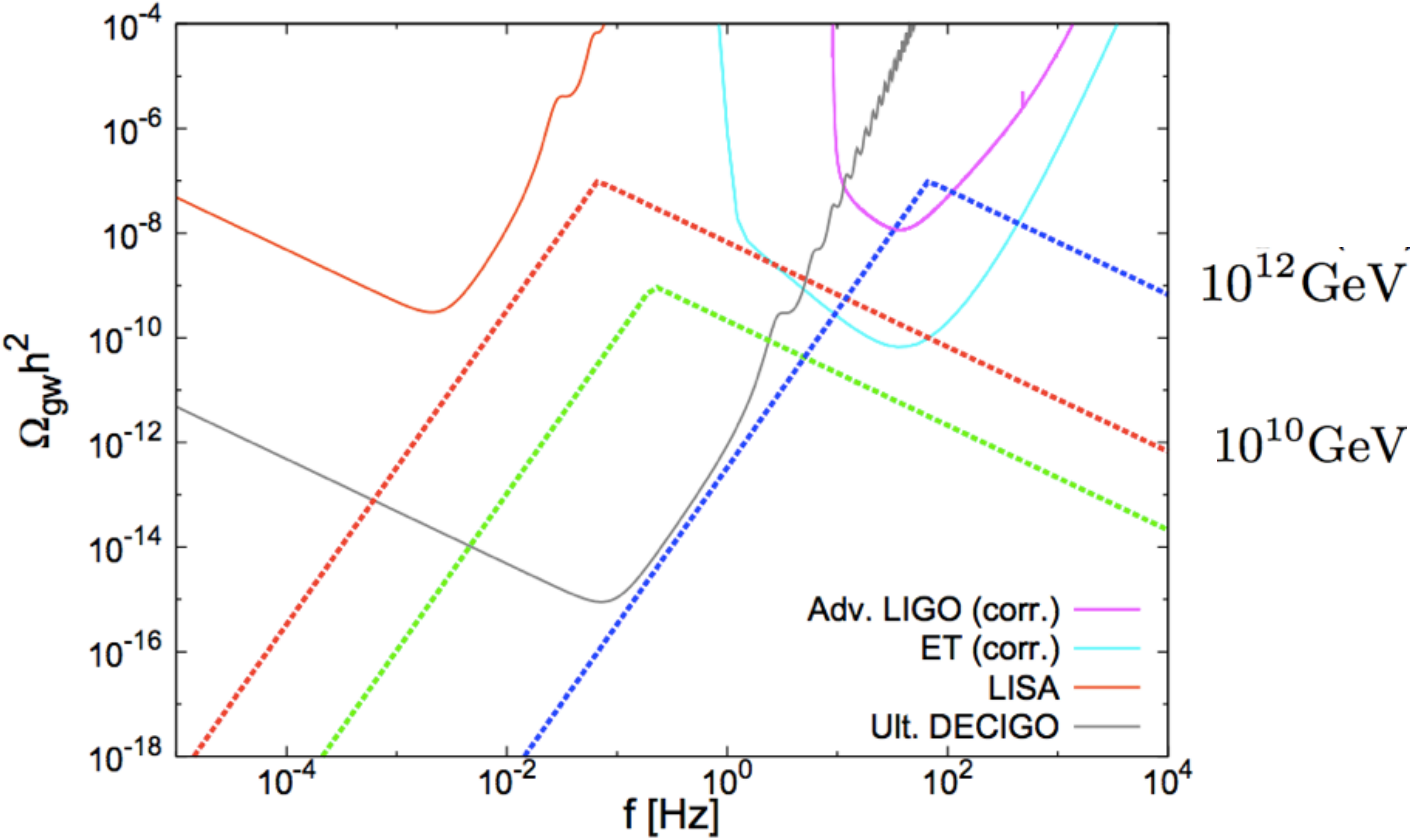


Planck
eLISA
O1
O2
aLIGO O5
10^3 GeV
10^7 GeV
10^8 GeV
Ω_{GW}
f (Hz)

On the estimation of gravitational wave spectrum from cosmic domain walls

Takashi Hiramatsu,[1,*] Masahiro Kawasaki,[2,3,†] and Ken'ichi Saikawa[4,‡]

1309.5001



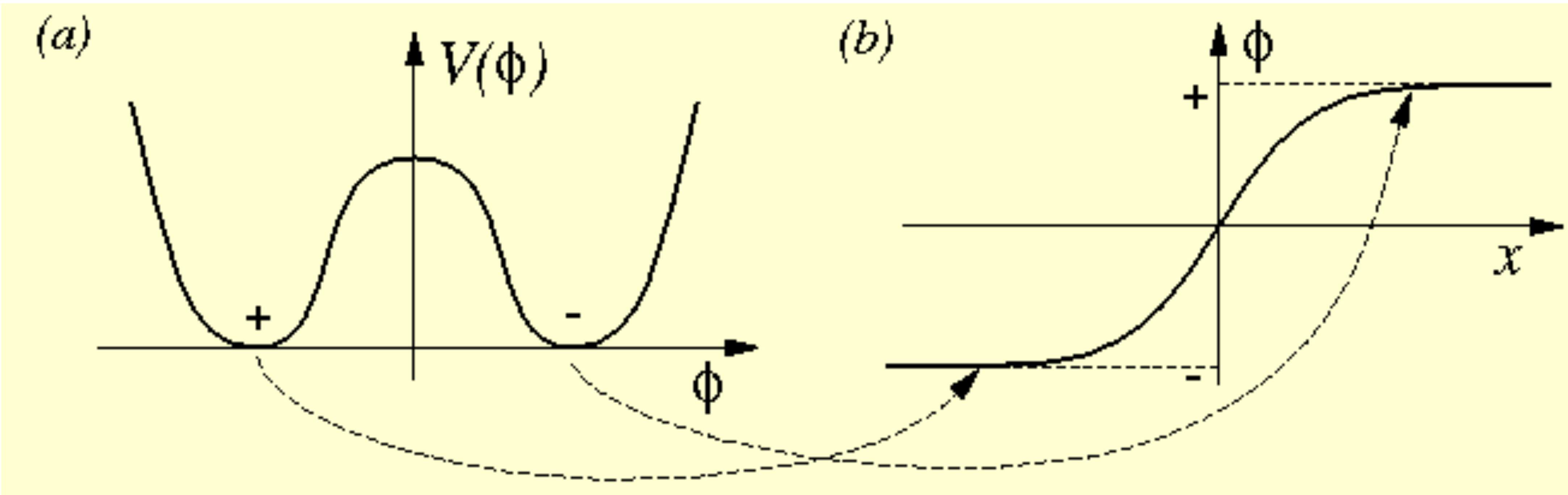
Z_2 symmetry $(\phi \to -\phi)$

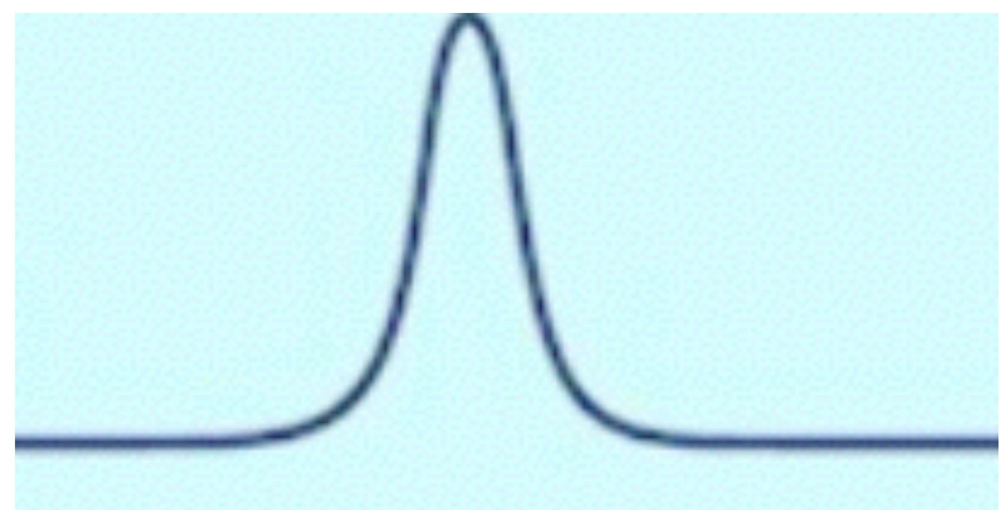
$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 \; + \; \lambda T^2 \phi^2/8$$

Z_2 symmetry spontaneously broken at T

$$T_c = 2\eta,$$

Domain wall





Thank You