Bottomonium Properties at T>0 from Lattice NRQCD

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Introduction: temporal correlators and spectral functions, EFT and potential models

Bottomonium correlators of point meson operators in NRQCD
S. Kim, PP, A. Rothkopf, PRD91 (2015) 054511
S. Kim, PP, A. Rothkopf, JHEP11(2018)088

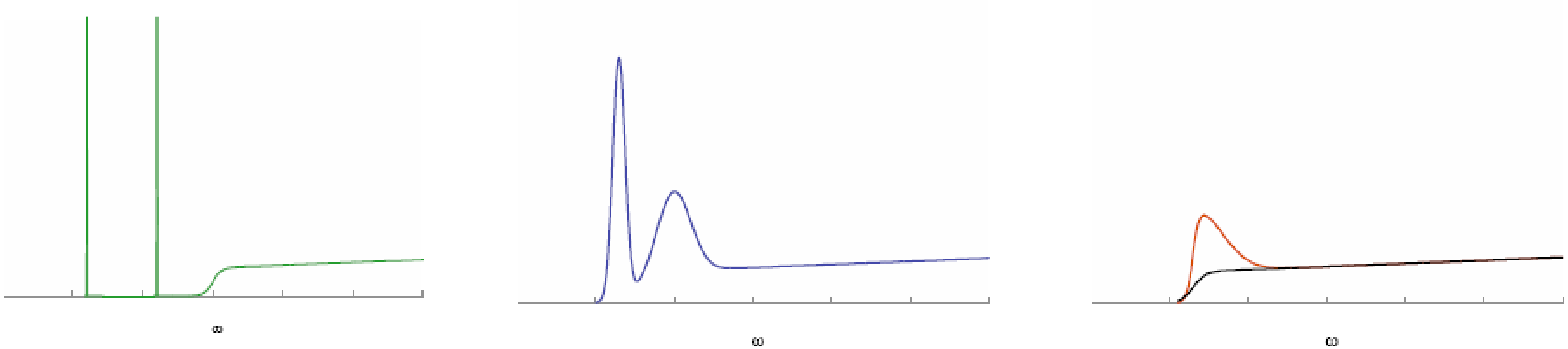
Bottomonium correlators from extended meson operators in NRQCD
Larsen, Meinel, Mukherjee, PP, arXiv:1910.07374

Meson Correlators and Spectral Functions

In-medium properties and/or dissolution of mesons are encoded in the spectral functions:

$$\rho(\omega,p,T) = \frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty} dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(0,0)]\rangle_T$$

Melting is see as progressive broadening and disappearance of the bound state peaks



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$D(\tau,p,T) = \int d^3x e^{ipx}\langle J(x,-i\tau)J(0,0)\rangle_T$$

$$D(\tau,p,T) = \int_0^{\infty} d\omega \rho(\omega,p,T)\frac{\cosh(\omega\cdot(\tau-1/(2T)))}{\sinh(\omega/(2T))}$$

MEM

σ(ω,p,T)

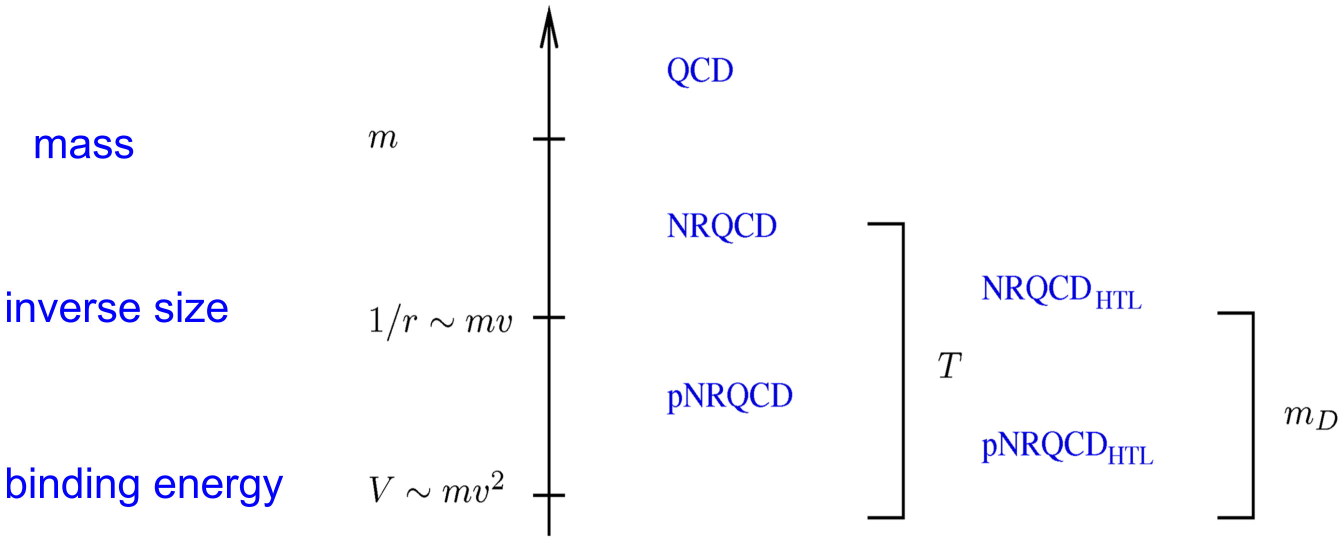
1S charmonium survives to *1.6*T_c ??

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Effective Field theory Approach for Heavy Quark Bound States and Potential Models

The heavy quark mass provides a hierarchy of different energy scales

Brambilla, Ghiglieri, PP, Vairo, PRD 78 (2008) 014017



The scale separation allows to construct sequence of effective field theories: NRQCD, pNRQCD

Potential model appears as the tree level approximation of pNRQCD and can be systematically improved in principle
Potential has an imaginary part at *T>0*

Why NRQCD ?

Quarkonia to a fairly good approximation are non-relativistic bound state

$$p_Q \sim M_Q v \ll M_Q$$

EFT approach: integrate the physics at scale of the heavy quark mass

NRQCD is the EFT at scale << M_Q

Heavy quark fields are non-relativistic Pauli spinors:

$$L_{NRQCD} = \psi^\dagger \left(D_\tau - \frac{D_i^2}{2M_Q} \right) \psi + \chi^\dagger \left(D_\tau + \frac{D_i^2}{2M_Q} \right) \chi + \ldots + \frac{1}{4} F_{\mu\nu}^2 + \bar{q} \gamma_\mu D_\mu q$$

Advantages:

- the large quark is not a problem for lattice calculations, lattice study of bottomonium is feasible (usually $a\, M_Q << 1$, which is challenging)
- The structure of the spectral function is simpler => more sensitivity to the bound state properties
- Quarkonium correlators are not periodic and can be studied at larger time extent ($=1/T$) => more sensitivity to bound state properties

NRQCD on the Lattice

Inverse lattice spacing provides a natural UV cutoff for NRQCD provided $a^{-1} \le 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$S_Q(x,\tau+a) = U_4^\dagger (1 - \frac{p^2}{2M_Q}\Delta\tau) S_Q(x,\tau),\ \Delta\tau = a/n$$

well behaved if $naM_Q < 3$

Davies, Thacker, PRD 45 (1992) 915

$$D(\tau) = \sum_x \langle O(x,\tau) S_Q(x,\tau) O^\dagger(0,0) S_Q^\dagger(x,\tau)\rangle_T,\quad O(^3S_1;x,\tau) = \sigma_i,\quad O(^3P_1;x,\tau) = \Delta_i\sigma_j - \Delta_j\sigma_i$$

Thacker, Lepage, PRD43 (1991) 196

The energy levels in NRQCD are related to meson masses by a constant lattice spacing dependent shift, e.g.

$$M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\text{shift}}(a)$$

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied by varying $a \leftrightarrow \beta = 10/g^2$ Bazavov et al, PRD85 (2012) 054503

$\Rightarrow 140\text{MeV} \le T \le 407\text{MeV}$

$2.759 \ge aM_b \ge 0.954$ (ok if $n = 2,\ 4$)
$0.757 \ge aM_c \ge 0.427$ (ok if $n \ge 8$)

Bayesian Reconstruction of Spectral Functions

$$D(\tau) = D(\mathbf{p}=0,\tau) = \sum_{\mathbf{x}} D(\mathbf{x},\tau) = \int_{-2M_q}^{\infty} d\omega\, e^{-\omega\tau}\, \rho(\omega)$$

Discretize the integral $D_i^\rho = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$ and find ρ_l

using Bayesian approach, i.e. maximizing

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I]$$

Likelihood:

$$P[D|\rho I] = \exp(-L) \qquad L[\rho] = \frac{1}{2}\sum_{ij}(D_i - D_i^\rho)C_{ij}(D_j - D_j^\rho)$$

Prior probability:

BR method: Burnier Rothkopf, PRL 111 (2013) 182003

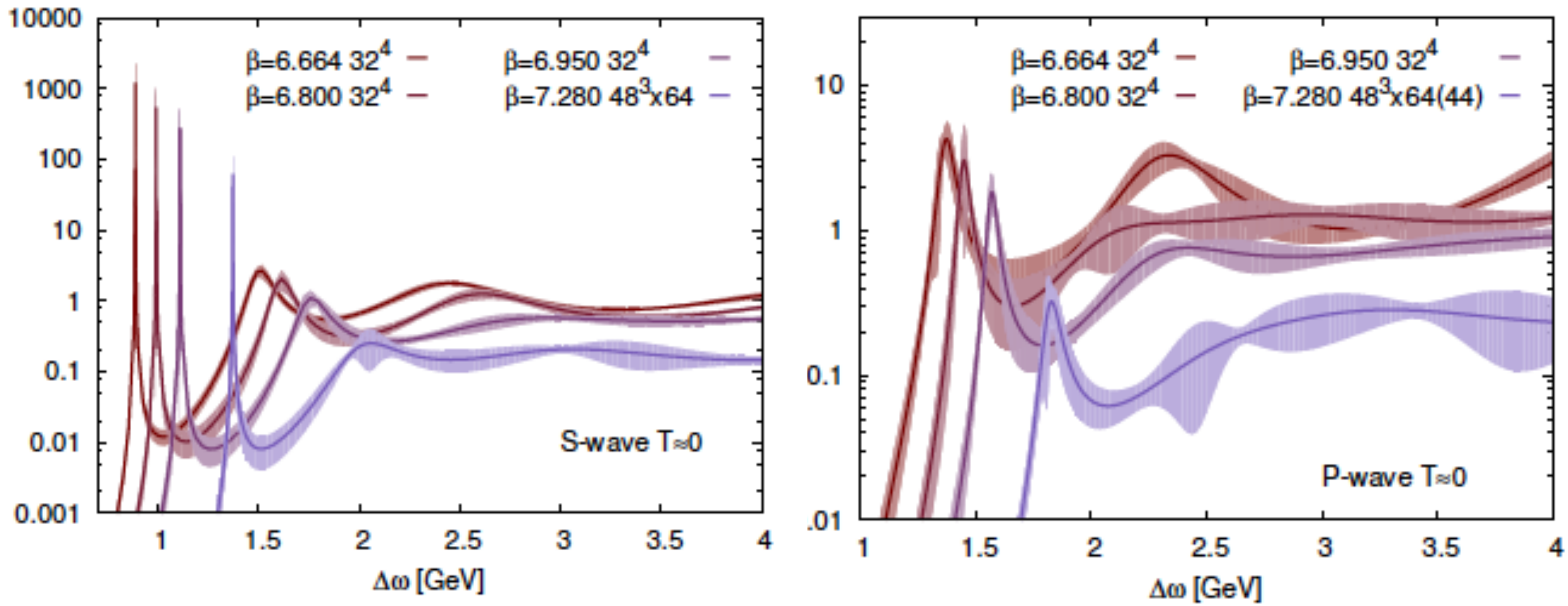
$$P[\rho|I] = \exp[S] \qquad S[\rho] = \alpha\sum_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)\Delta\omega_l.$$

MEM: $S[\rho] = \alpha \sum_l (\rho_l - m_l + \log[\rho_l/m_l])$

Smooth BR method (BR_l):

$$-\kappa \sum_l \left(\frac{d\rho}{d\omega}\right)_l \Delta\omega_l$$

Bottomonium Spectral Functions at T=0



Well resolved Υ ground state peak   Acceptable resolution for χ_b state

But excited states, Υ', Υ'', χ_b' cannot be resolved well

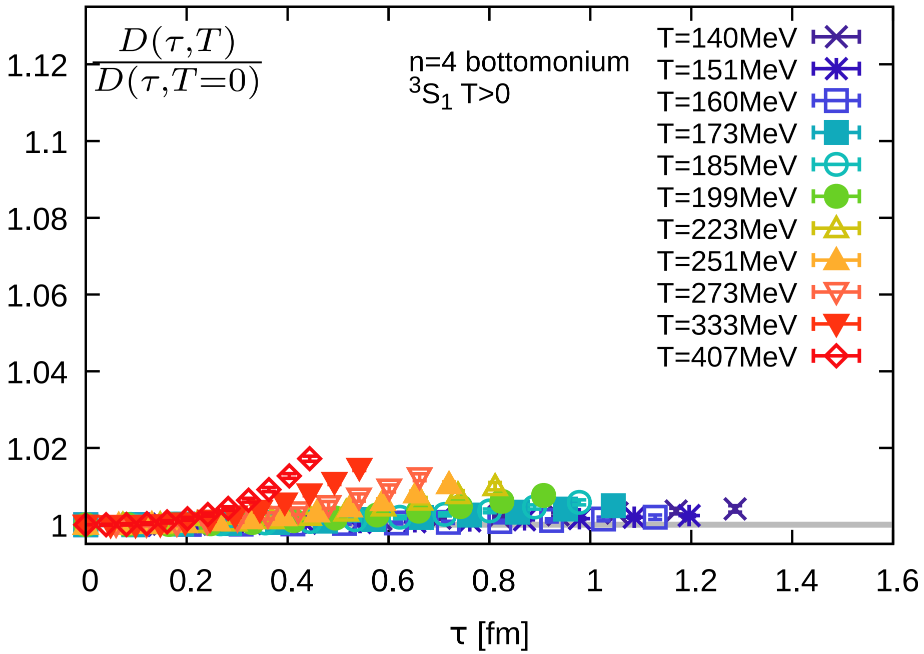
Define the NRQCD energy shift $C_{\rm shift}(a)$ by fixing the Υ peak to PDG

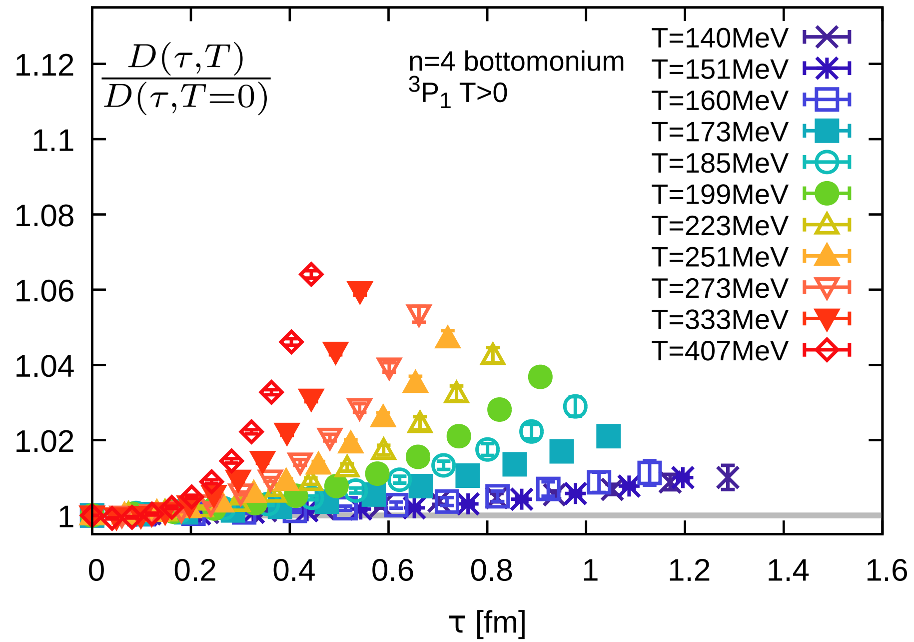
$$E_\Upsilon + C_{\rm shift}(a) = 9.46030 \text{ GeV}$$

$\Rightarrow$ prediction for mass of other states: η_b, χ_{b0}, χ_{b1}, h_b

reconstructed spectral functions show (ringing) artifacts at large ω

Temperature Dependence of the Bottomonium Correlators





change in Υ correlator $< 2\%$

change in χ_{b1} correlator $< 7\%$

$\Rightarrow$ hints for sequential melting pattern: stronger medium modification of χ_{b1} spectral function than for Υ spectral function

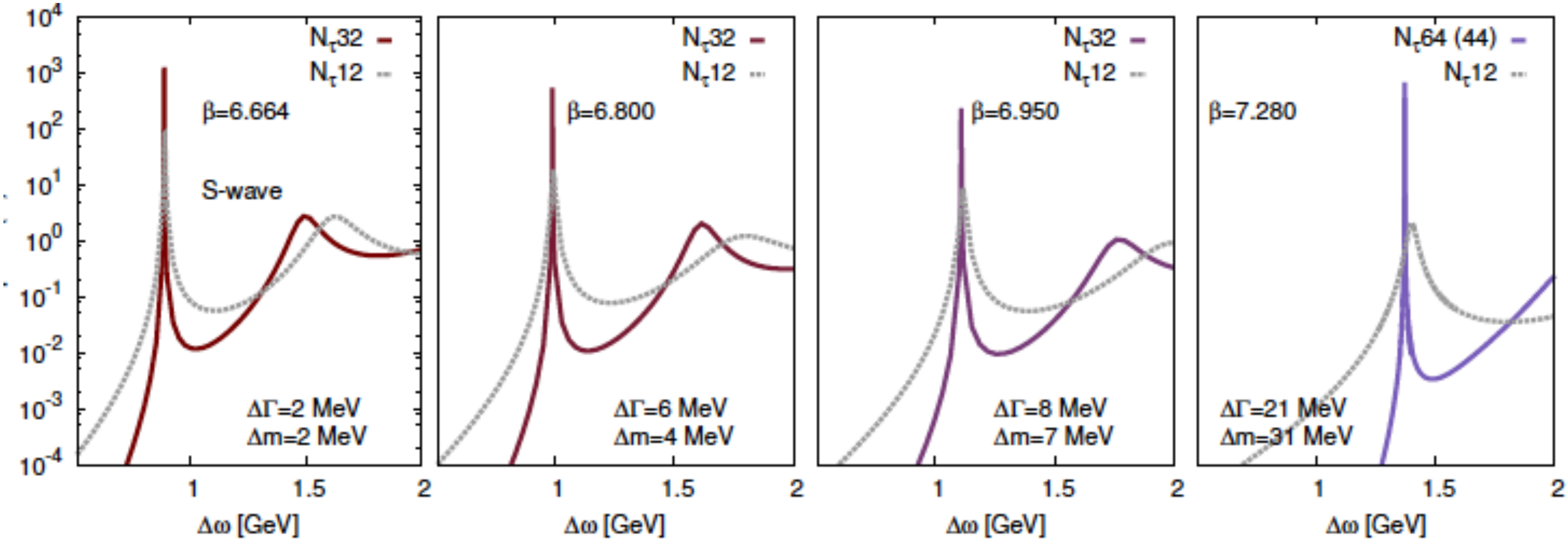
Reconstructing Spectral Functions at T>0

Two main problems:

1) $\tau < 1/T \Rightarrow$ limited temporal extent at high T

2) relatively small number of time slices ($N_\tau = 12$ in our study)

Study these effects at $T = 0$ by using only the first 12 data points:

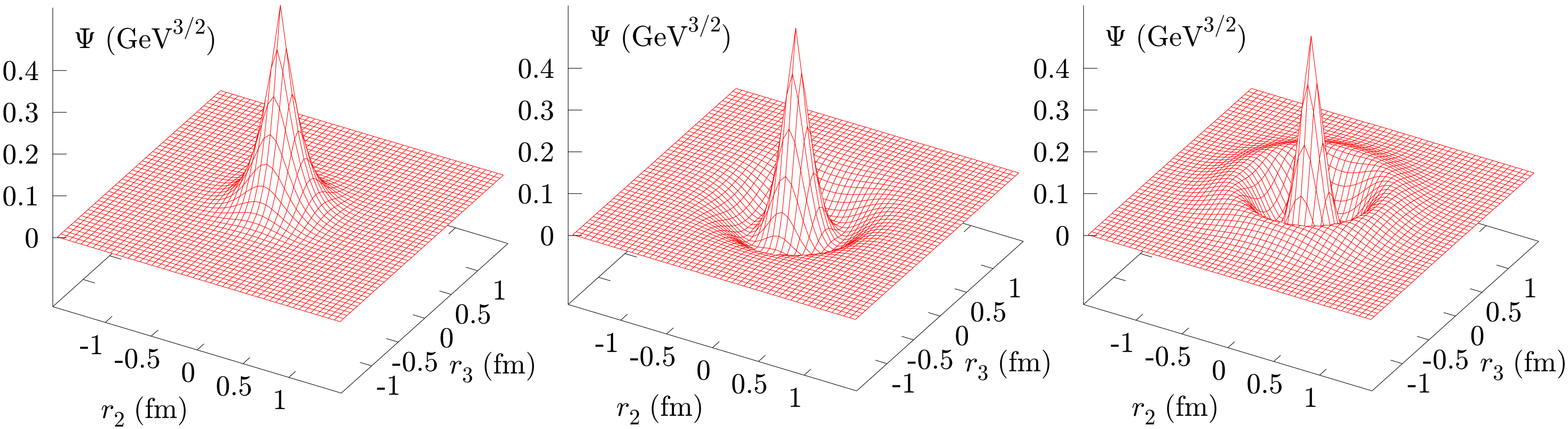


Decreasing $\tau_{max} = 1/T$ leads to broadening of the bound state peak

(to be taken into account in comparison $T = 0$ and $T > 0$ spectral functions)

Extended Meson Operators

$O_i(\mathbf{x},t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\bar{q}(\mathbf{x}+\mathbf{r},t)\Gamma q(\mathbf{x},t)\,.$ $\Psi_i(\mathbf{r})$ from potential model with Cornell potential



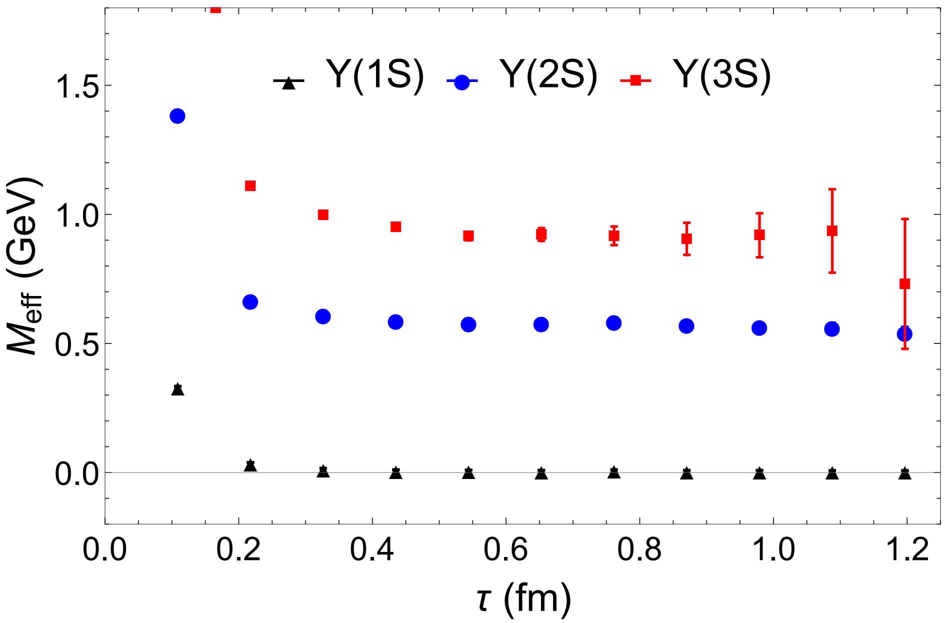
Good overlap with bottomonium states but

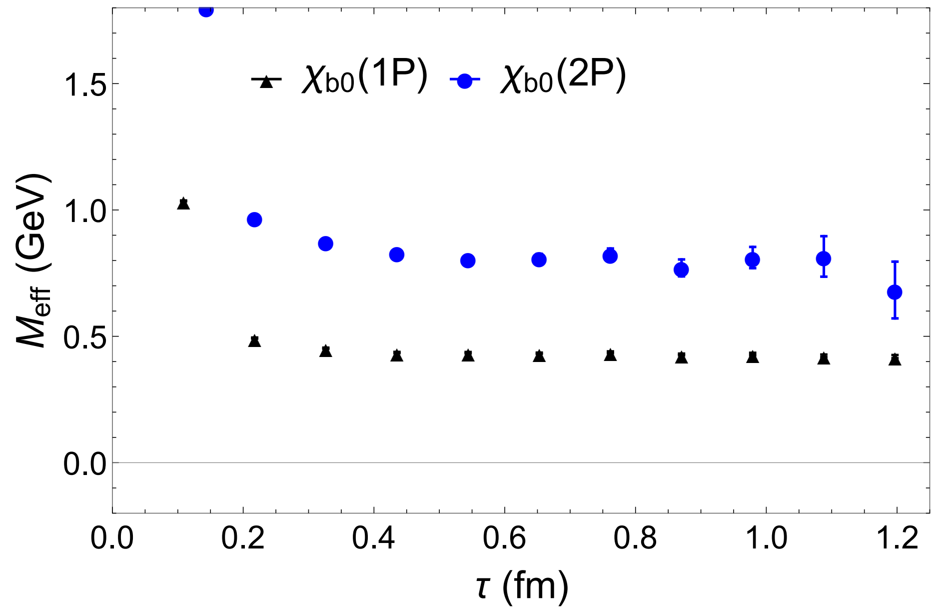
$G_{ij}(t) = \langle O_i(t)O_j^\dagger(0)\rangle \neq 0$ for $i \neq j$

$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j$ such that
$O_\alpha(t)\tilde{O}_\beta^\dagger(0)\rangle \propto \delta_{\alpha,\beta}$
$\Omega_{\alpha j}$ can be obtained as
$G_{ij}(t)\Omega_{\alpha j} = \lambda_\alpha(t,t_0)G_{ij}(t_0)\Omega_{\alpha j}.$

Correlators of Extended Meson Operators at T=0

$$aM_{\mathrm{eff}}(t) = \ln[C_\alpha(t)/C_\alpha(t+1)]$$





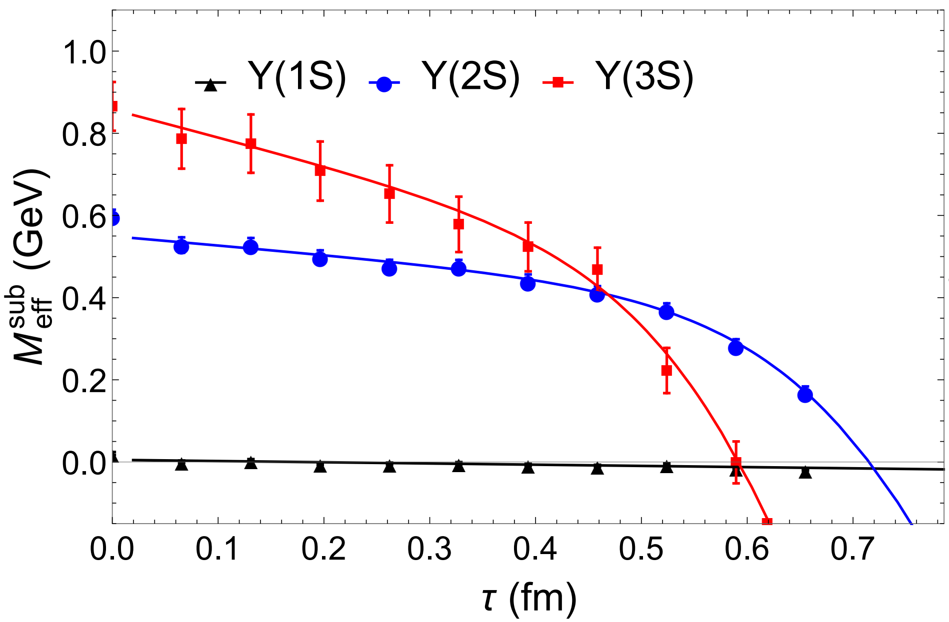
$$C_\alpha(\tau, T) = \int_{-\infty}^{\infty} d\omega \rho_\alpha(\omega, T) e^{-\omega\tau} \qquad \rho_\alpha(\omega, T) = \rho_\alpha^{\mathrm{med}}(\omega, T) + \rho_\alpha^{\mathrm{high}}(\omega)$$

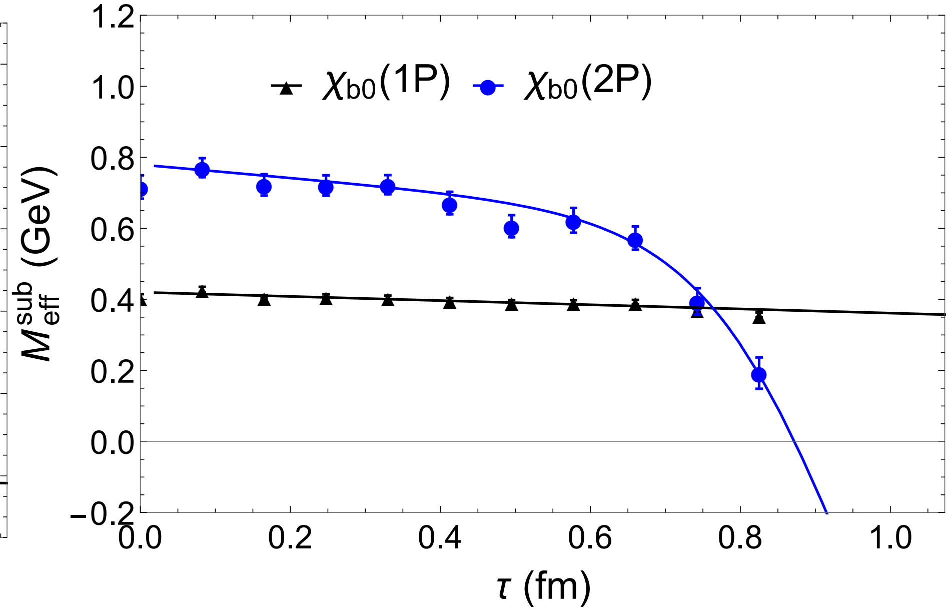
$$\rho_\alpha^{\mathrm{med}}(\omega, T=0) = A_\alpha \delta(\omega - M_\alpha) \Rightarrow C_\alpha(\tau, T=0) = A_\alpha e^{-M_\alpha \tau} + C_\alpha^{\mathrm{high}}(\tau)$$

Determine A_α,M_α from single exponential fit for $\tau > 0.6\mathrm{fm}$ and then $C_\alpha^{\mathrm{high}}(\tau)$

Correlators of Extended Meson Operators at T>0

$$C_\alpha^{\rm sub}(\tau,T) = C_\alpha(\tau,T) - C_\alpha^{\rm high}(\tau) \Rightarrow aM_{\rm eff}^{\rm sub}(\tau,T) = \ln\left(C_\alpha^{\rm sub}(\tau,T)/C_\alpha^{\rm sub}(\tau+a,T)\right)$$





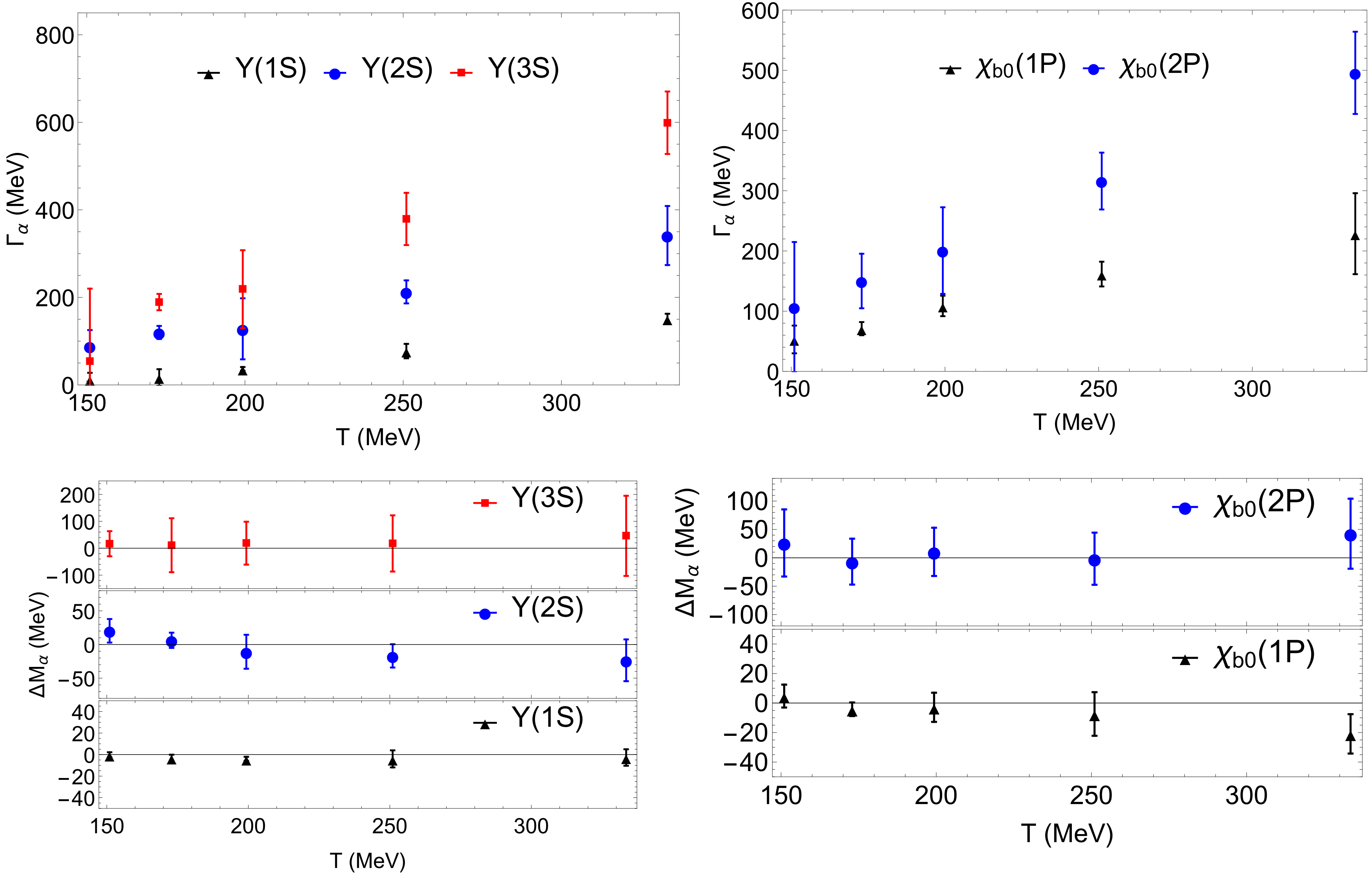
Fit $M_{\rm eff}^{\rm sub}(\tau,T)$ using a simple Ansatz:

$$\rho_\alpha^{\rm med}(\omega,T) = A_\alpha^{\rm cut}(T)\,\delta\left(\omega-\omega_\alpha^{\rm cut}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega-M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$$

Low energy tail

$$\Rightarrow M_\alpha(T), \Gamma_\alpha(T)$$

Thermal width and mass shift of bottomonium



Summary

- Bottomonium spectral functions of point meson operators are dominated by the continuum and therefore the corresponding Euclidean time correlation functions show small *T*-dependence:

$\Rightarrow$ Difficult to reconstruct in-medium bottomonium properties especially for P-states

$\Rightarrow$ No sensitivity to *2S* and *3S* bottomonium states

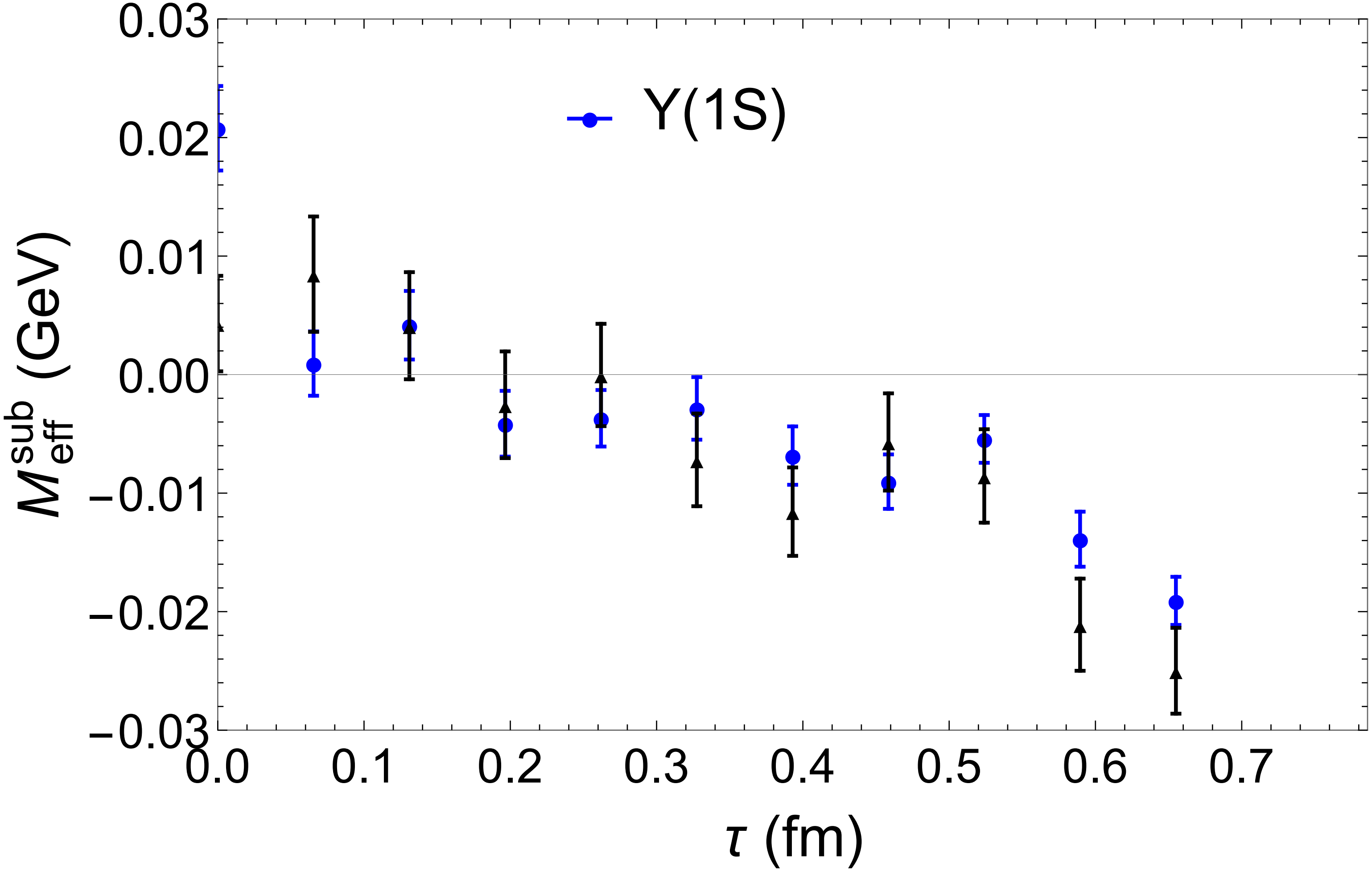
- Using appropriately chosen extended meson operators it is possible to reduce the relative contribution of continuum part of the spectral function and reconstruct the spectral functions for *1S*, *2S*, *3S*, *1P* and *2P* bottomonium states

The correlators of extended meson operators show significant *T*-dependence consistent with thermal broadening

Using simple form for the bottomonium spectral function we can extract the thermal widths of different bottomonium states and we see that

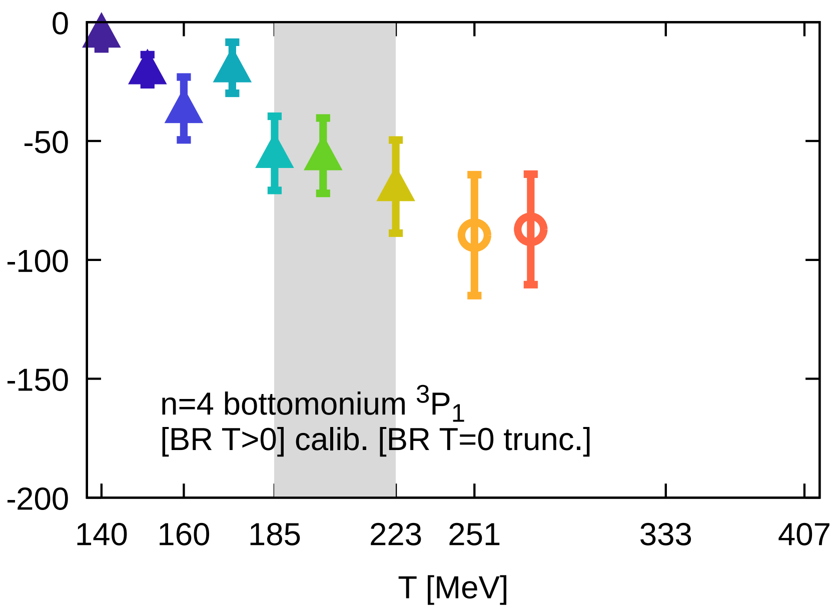
$$\Gamma_{3S}(T) > \Gamma_{2P}(T) > \Gamma_{2S}(T) > \Gamma_{1P}(T) > \Gamma_{1S}(T)$$

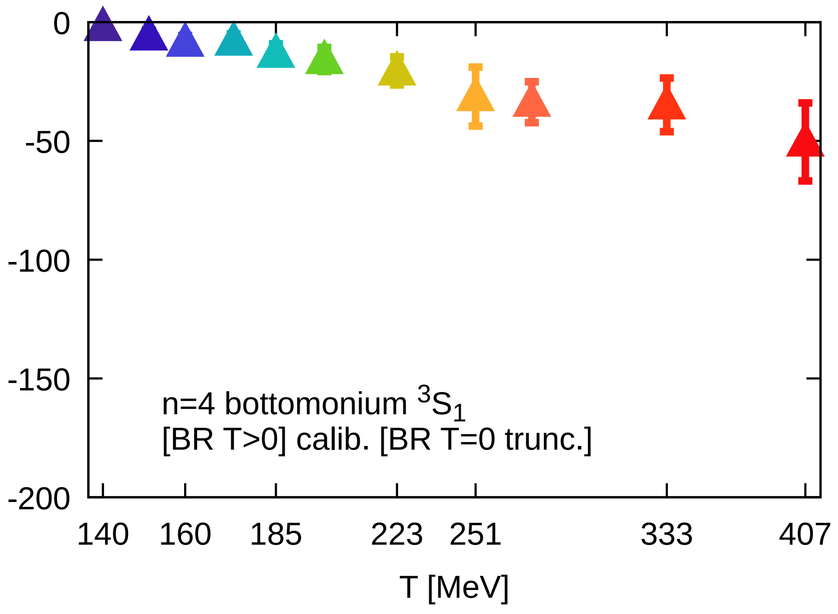
Backup slides



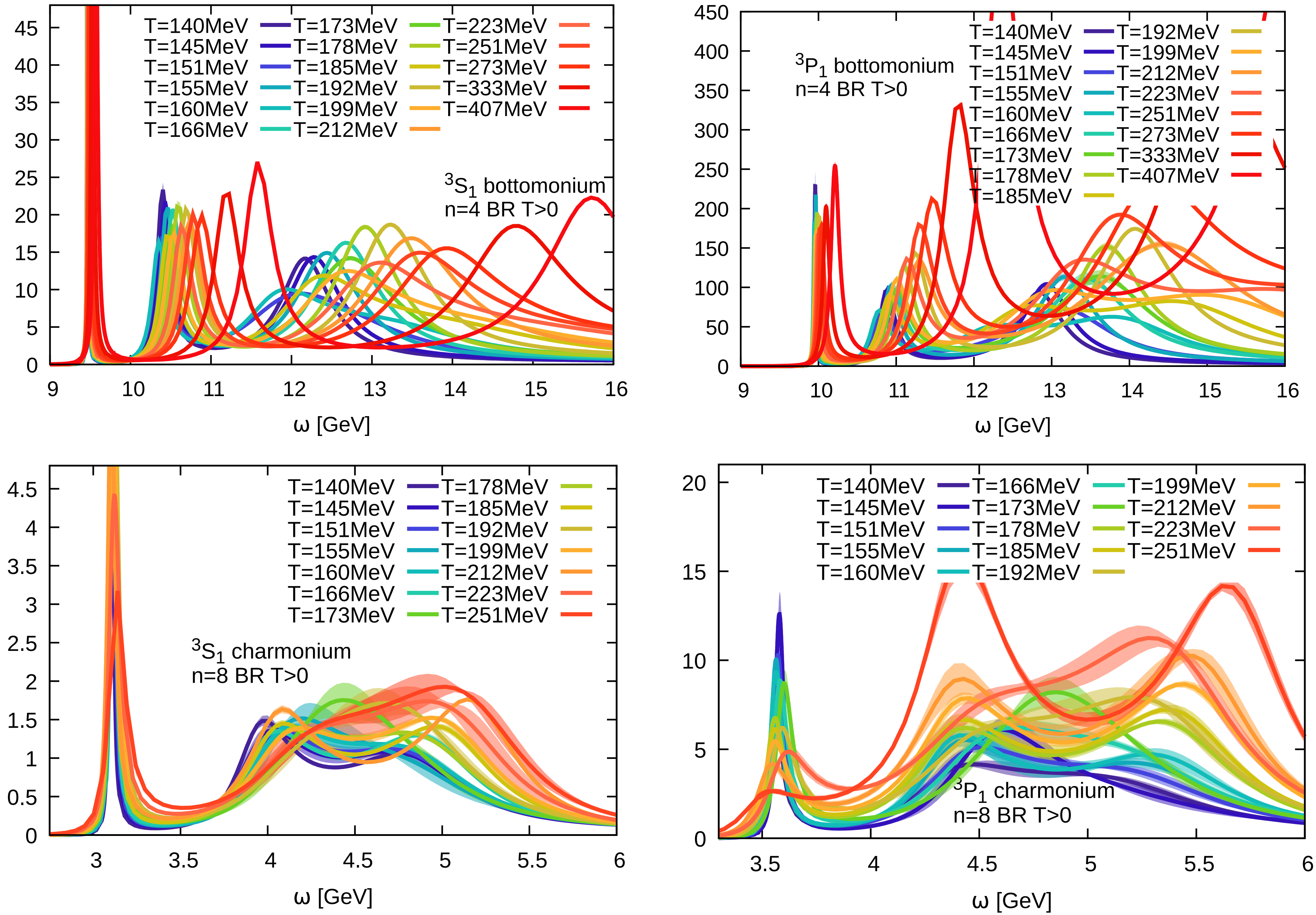
Υ(1S)
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$M_{\rm eff}^{\rm sub}$ (GeV)
τ (fm)

In Medium Bottomonium Mass Shifts From Bayesian analysis

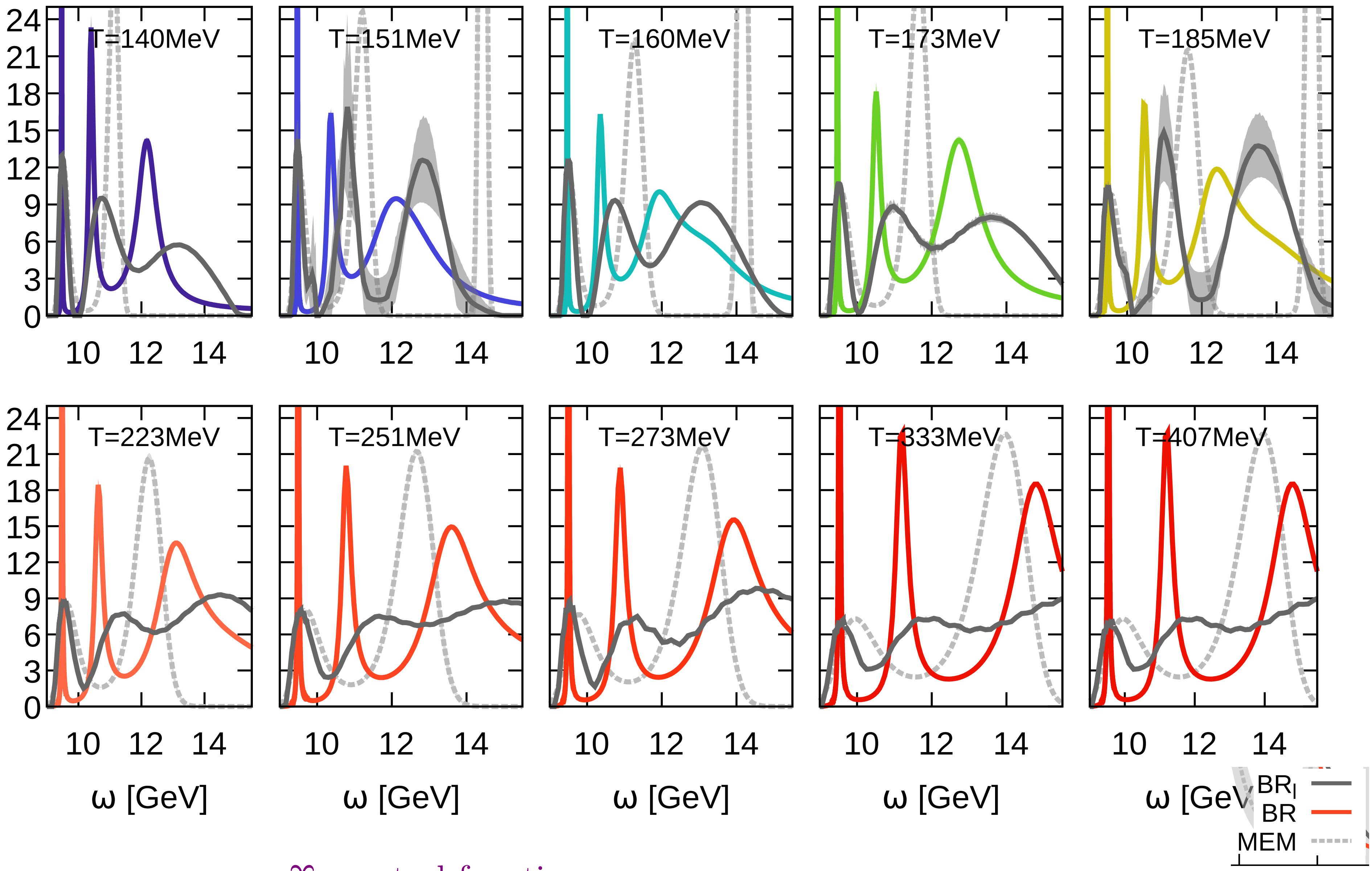




Quarkonium Spectral Functions at T>0

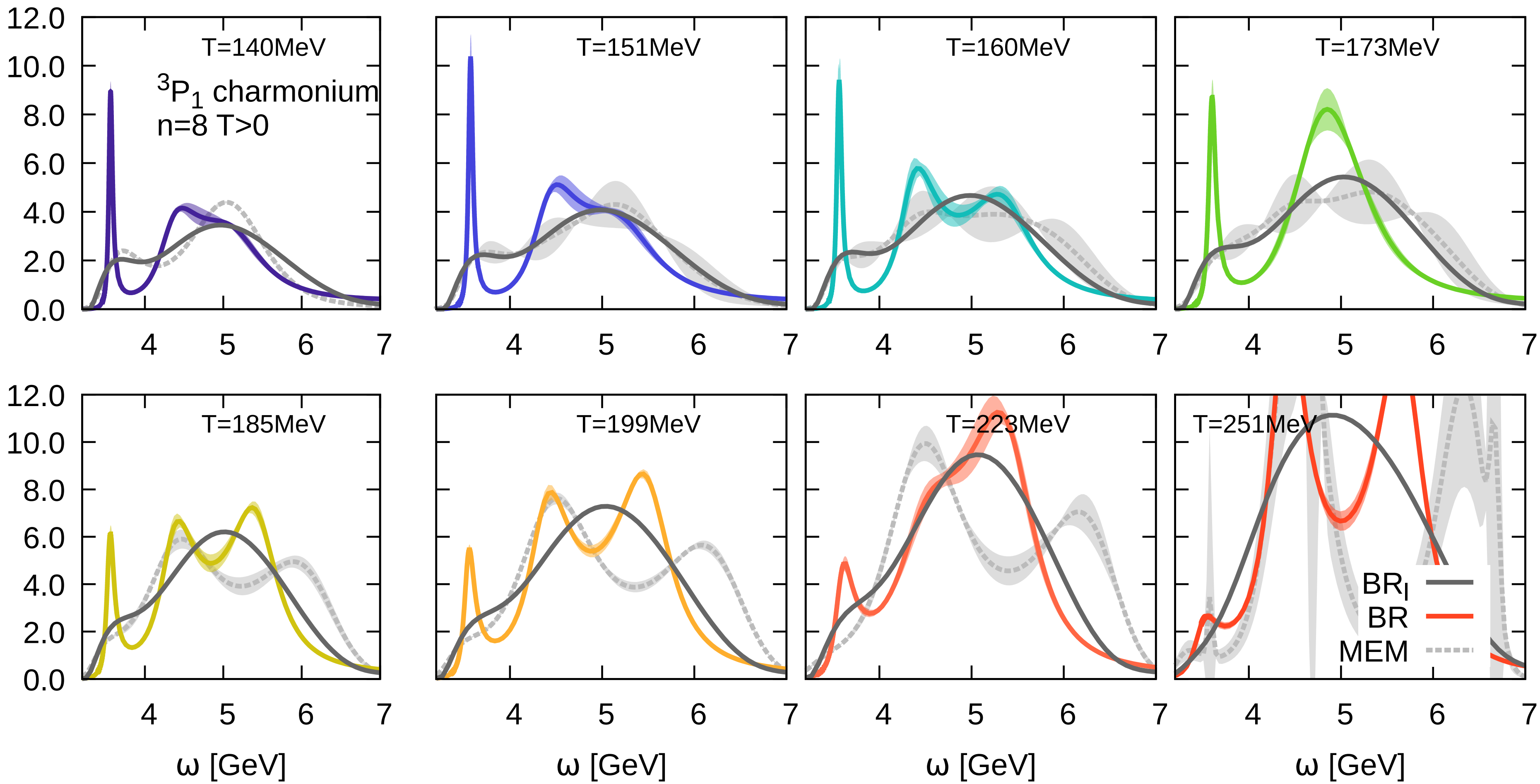


Bottomonium Spectral Functions with Different Methods



Υ spectral functions

Charmonium Spectral Functions with Different Methods



χ_c spectral functions