Heavy quark diffusion on the lattice

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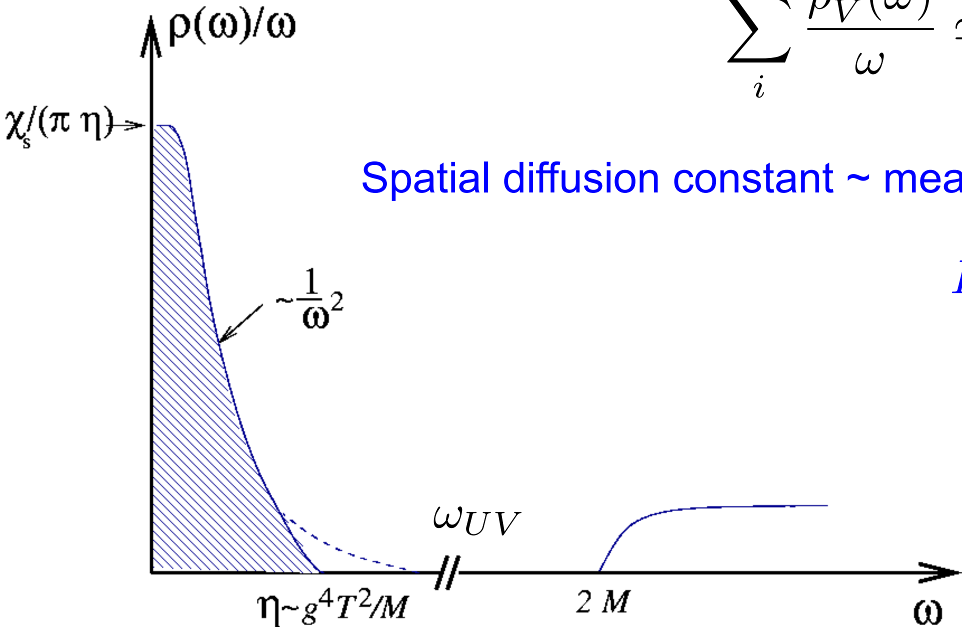
- Deconfinement and properties of QGP: lattice QCD vs. weak coupling
- Lattice determination of heavy quark diffusion in quenched approximation:
  a) electric field correlator method
  b) comments current-current correlators
- Charm correlations and fluctuations and charmed hadrons above T_c
- Summary

EMMI Rapid Reaction Task Force: Extraction of heavy flavor Transport coefficients in QCD mattrer , July 18-22, 2016

Current-current correlators and heavy quark diffusion

$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t} \int \mathrm{d}^3x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$



$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q D \frac{\eta^2}{\eta^2+\omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D}$$

Spatial diffusion constant ~ mean free path (weak coupling)

drag constant

$$D \sim \frac{1}{g^4 T}$$

Momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_2^q} \sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Bigg|_{\eta \ll \omega < \omega_{UV}}$$

$$\kappa^{(M)} = 2T^2/D$$

area under the peak ~ χ_2^q

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

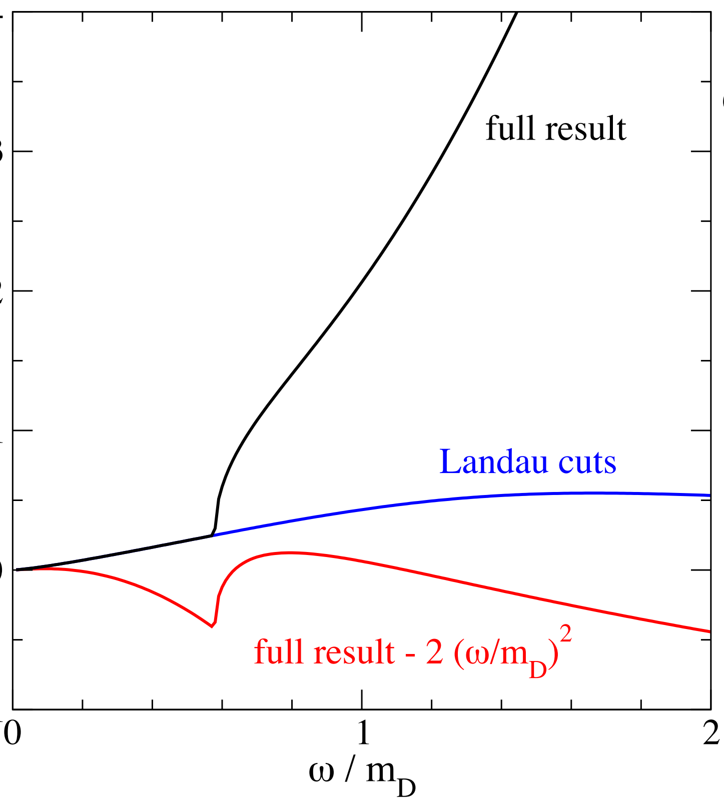
Current-current correlators in the heavy quark limit

$$\kappa = \frac{1}{3T}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_2^q}\int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\left\langle \frac{1}{2}\left\{\frac{\mathrm{d}\hat{J}^i(t,\vec{x})}{\mathrm{d}t},\frac{\mathrm{d}\hat{J}^i(t',\vec{0})}{\mathrm{d}t'}\right\}\right\rangle\right]$$

$$\frac{\mathrm{d}\hat{J}^i}{\mathrm{d}t} = \frac{1}{M}\left\{\hat{\phi}^\dagger gE^i\hat{\phi} - \hat{\theta}^\dagger gE^i\hat{\theta}\right\} + \mathcal{O}\left(\frac{1}{M^2}\right)$$

$$t \to i\tau$$

$$G_E(\tau) = \frac{1}{3\chi_2^q T}\sum_i\int \mathrm{d}^3x\left\langle\left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](\tau,\vec{x})\left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](0,\vec{0})\right\rangle$$



Integrate out ϕ, θ

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\,gE_i(\tau,\vec{0})\,U(\tau,0)\,gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle}$$

$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\tau-\frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

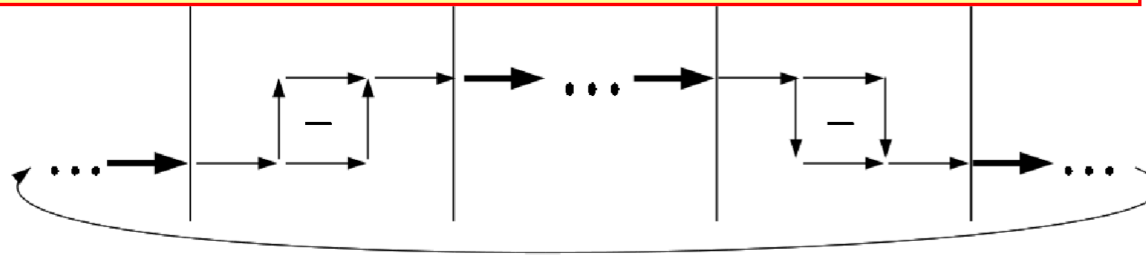
Transport coefficient ~ intercept of the spectral function not its width

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$

Caron-Huot, Laine, Moore, JHEP 0904 (2009) 053

Calculating the electric field strength correlator on the lattice

Straightforward to discretize by deforming the path of the Wilson lines to spatial direction

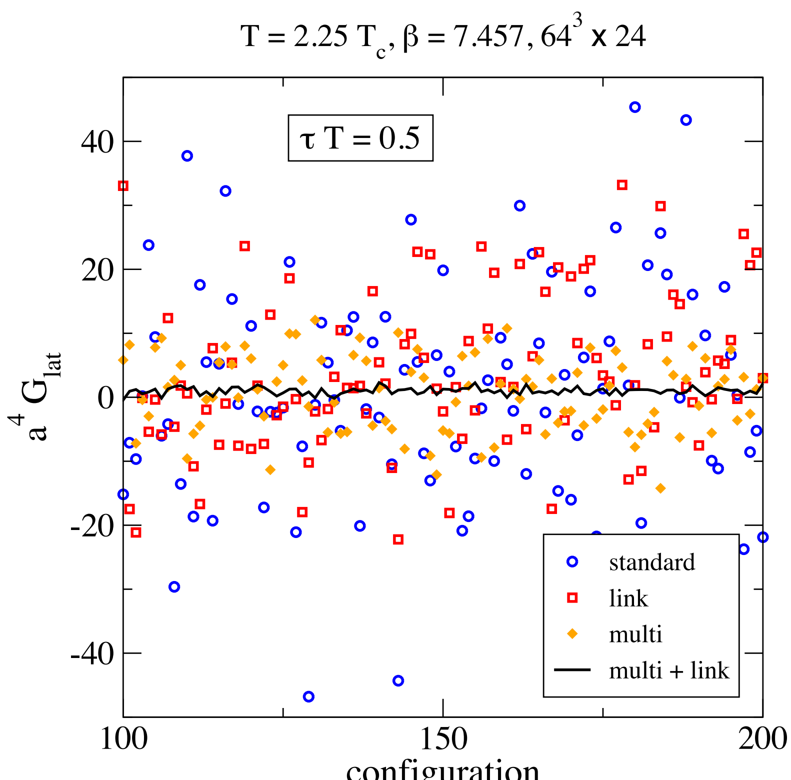


Challenge : MC noise

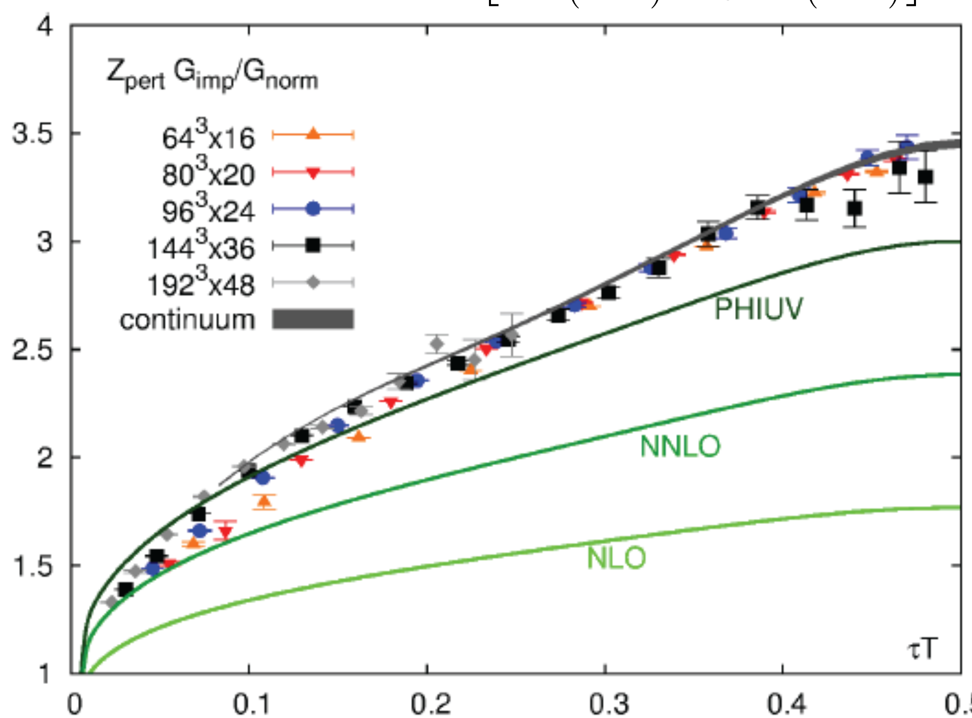
multilevel algorithm + link integration (only works for pure glue theory)

Luscher, Weisz, JHEP 0109 (2010), 010; First lattice calculation by Banerjee et al, PRD 85 (2012) 014510

Francis, Kaczmarek, Laine, et al, arXiv:1109.3941, arXiv:1311.3759, PRD 92 (2015) 116003



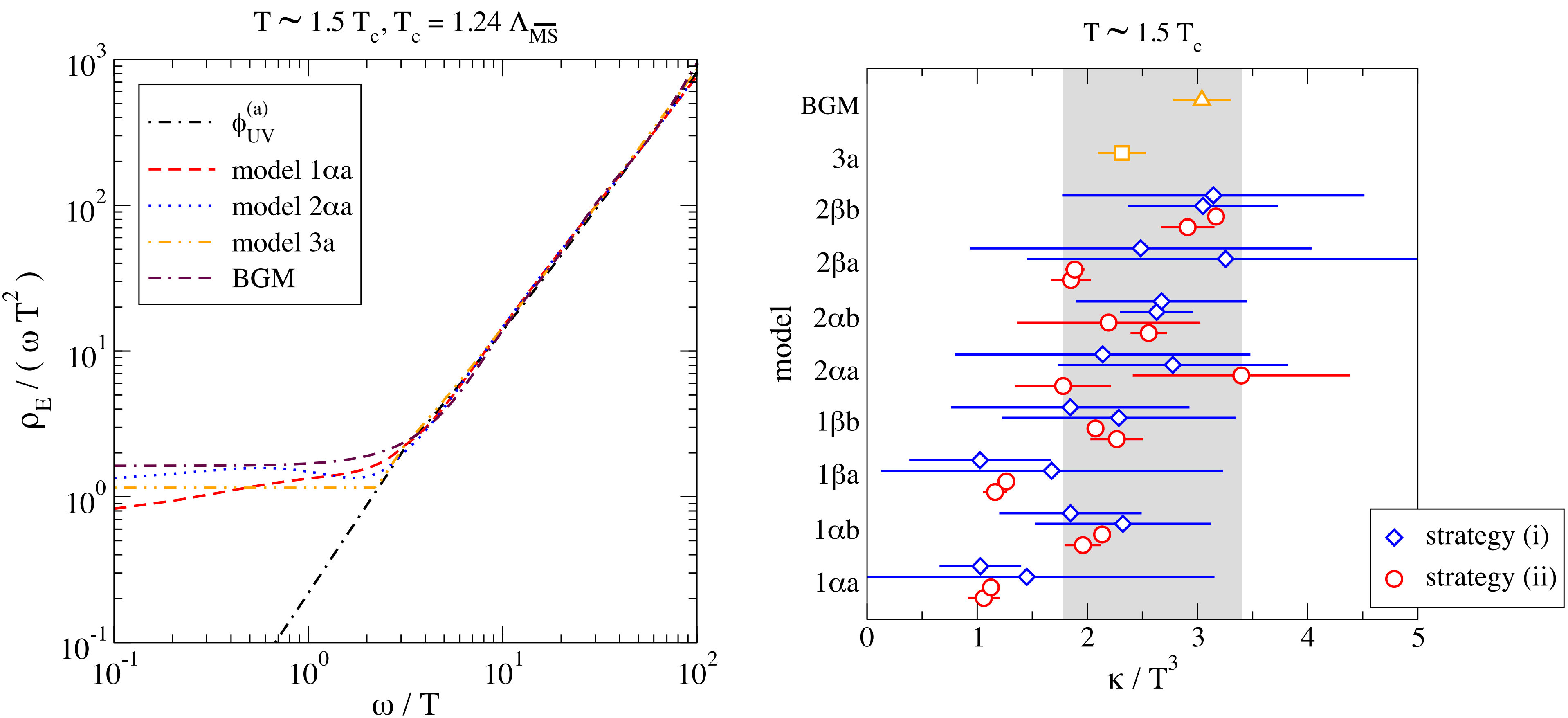
$$G_{norm}(\tau) = g^2 C_F\, \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right]$$



Extracting the spectral function and the diffusion constant

Fit the lattice using a forms of the spectral function constrained by low and high energy asymptotic behavior + corrections

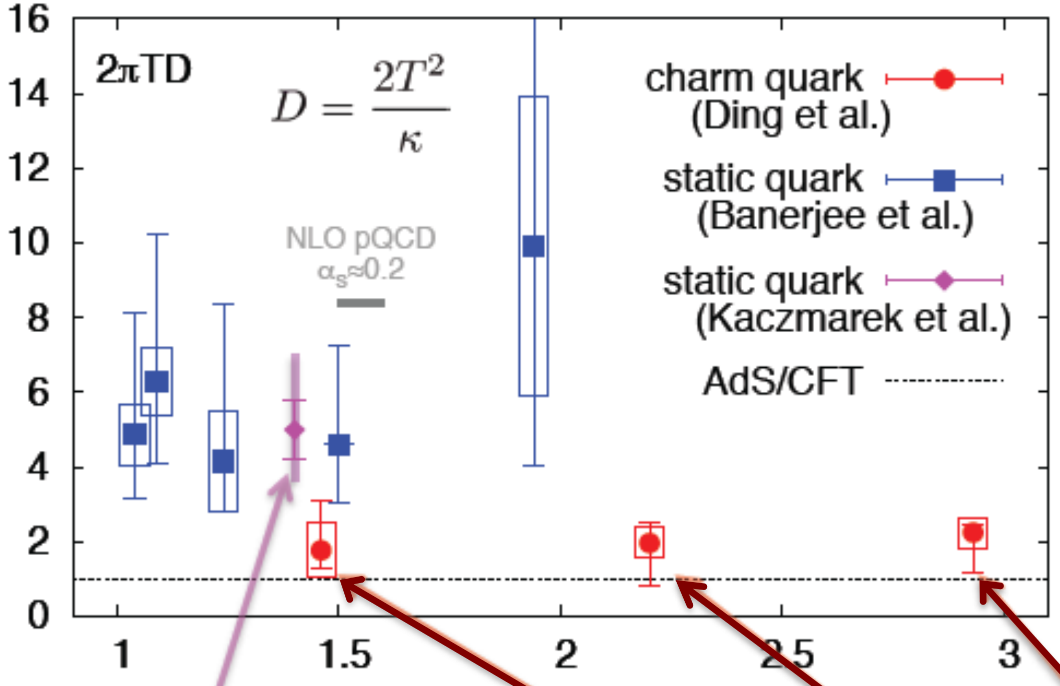
$$\rho^{low}(\omega) = \frac{\kappa\omega}{2T} \qquad \rho^{high}(\omega) = \frac{g^2(\mu_\omega)C_F}{6\pi}\omega^3, \mu_\omega = max(\omega, \pi T)$$



Francis, Kaczmarek, Laine, et al, PRD 92 (2015) 116003

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega) = (1.8 - 3.4)$$

Comparison with other lattice approaches



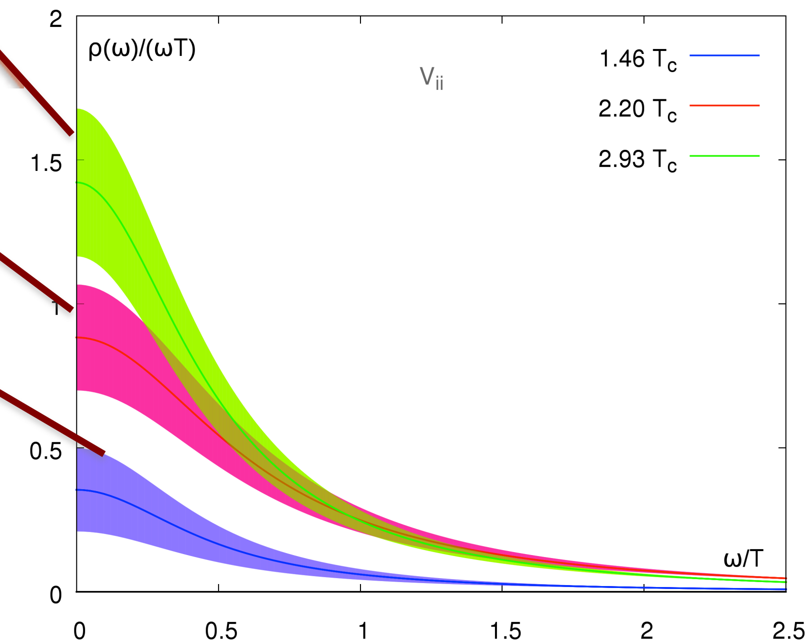
Electric correlator metho with multi-level algorigthm but no continuum limit

Banerjee et al, PRD 85 (2012) 014510

D is slightly smaller than the pQCD result

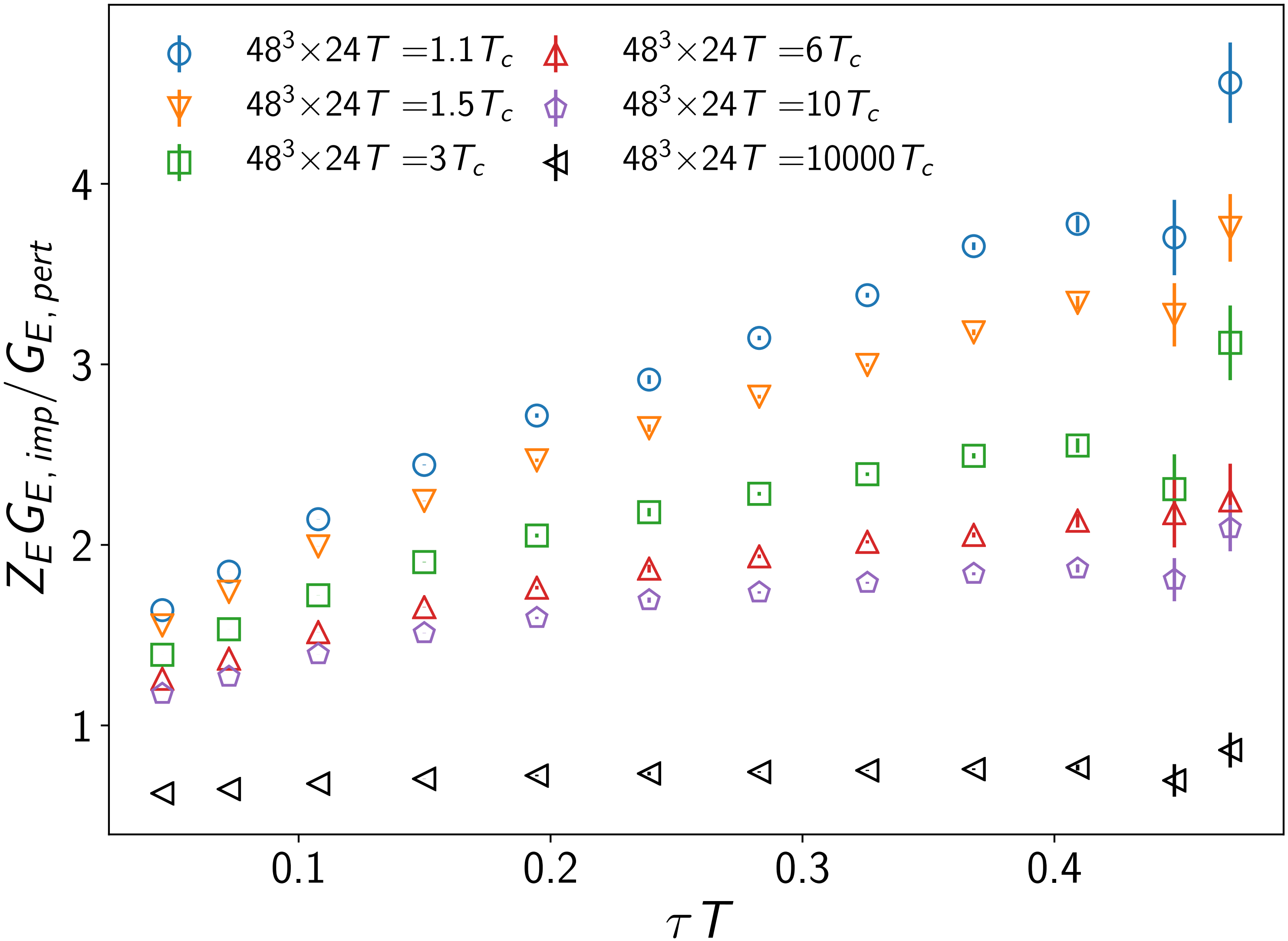
Determination from vector charmonium correlators
Ding et al, PRD 86 (2012) 014509

The width of the transport peak is dominated by Systematic effects, it too broad because of limited resolution => *D* is too small



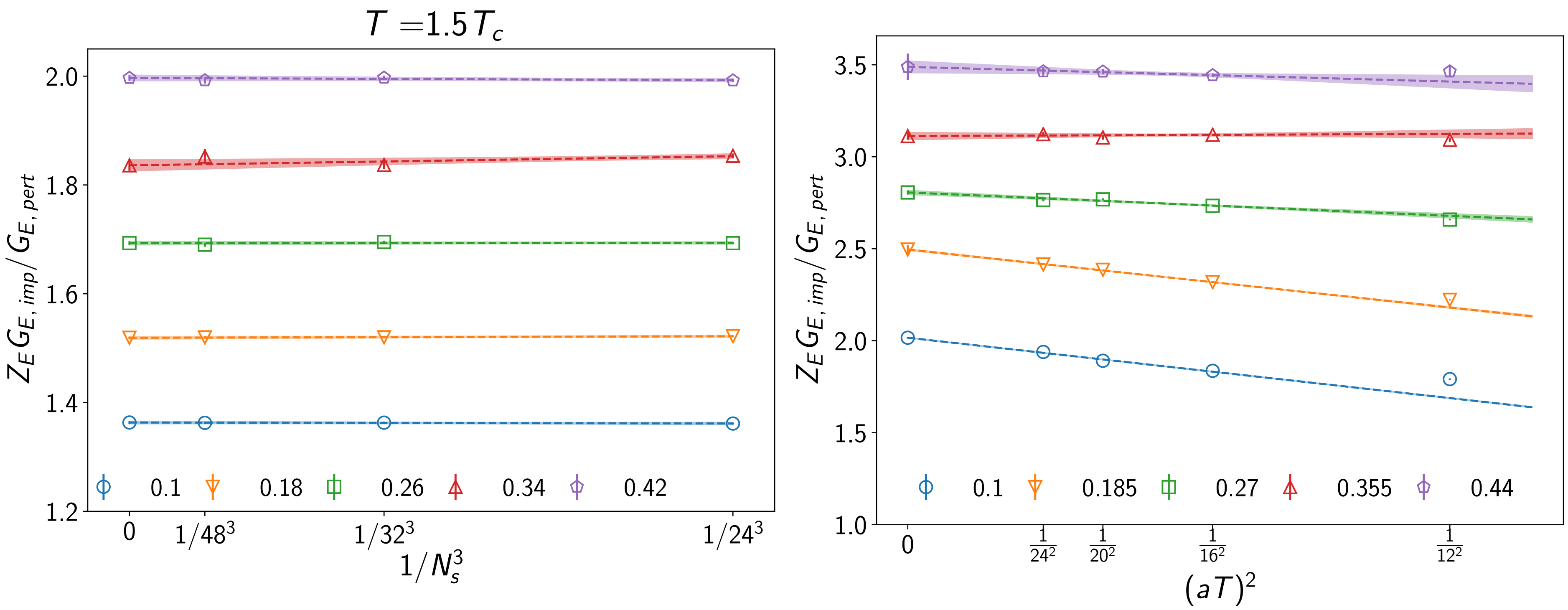
New lattice results in large T window

Brambilla, Leino, PP, Vairo, arXiv:1912.00689



Infinite volume and continuum extrapolations

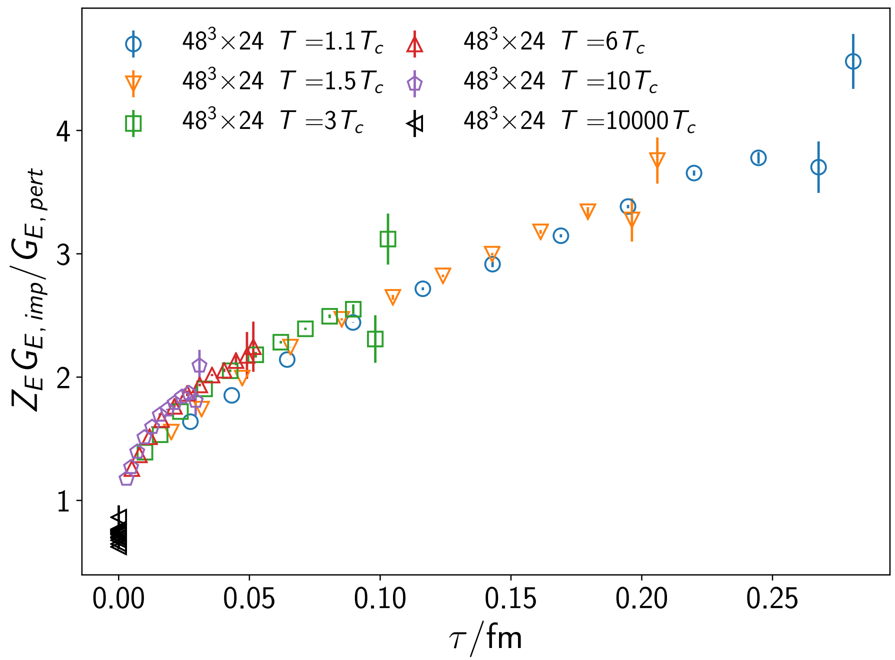
Brambilla, Leino, PP, Vairo, arXiv:1912.00689

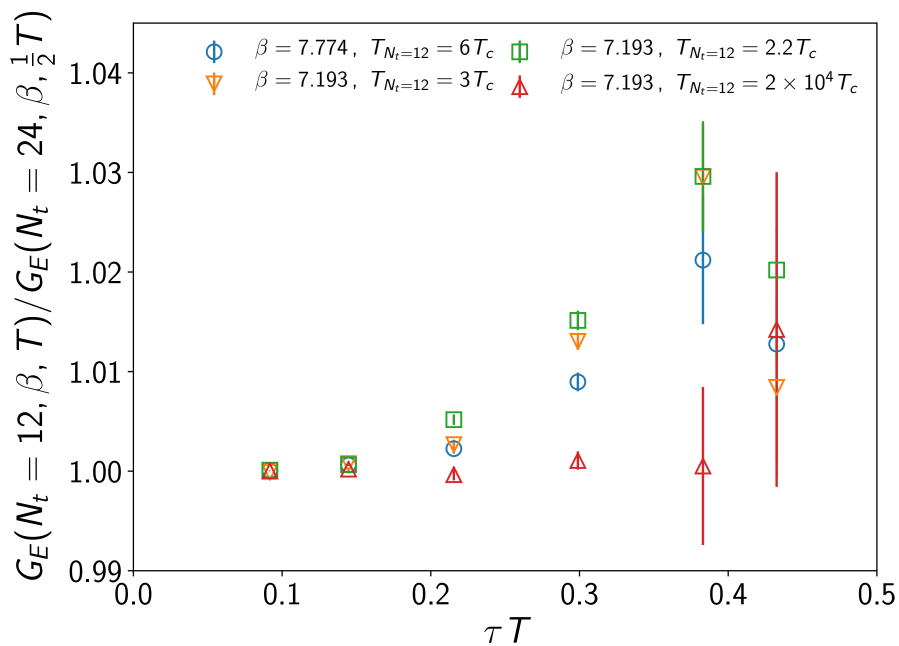


Temperature dependence of the electric correlator

Brambilla, Leino, PP, Vairo, arXiv:1912.00689

The temperature dependence is very small

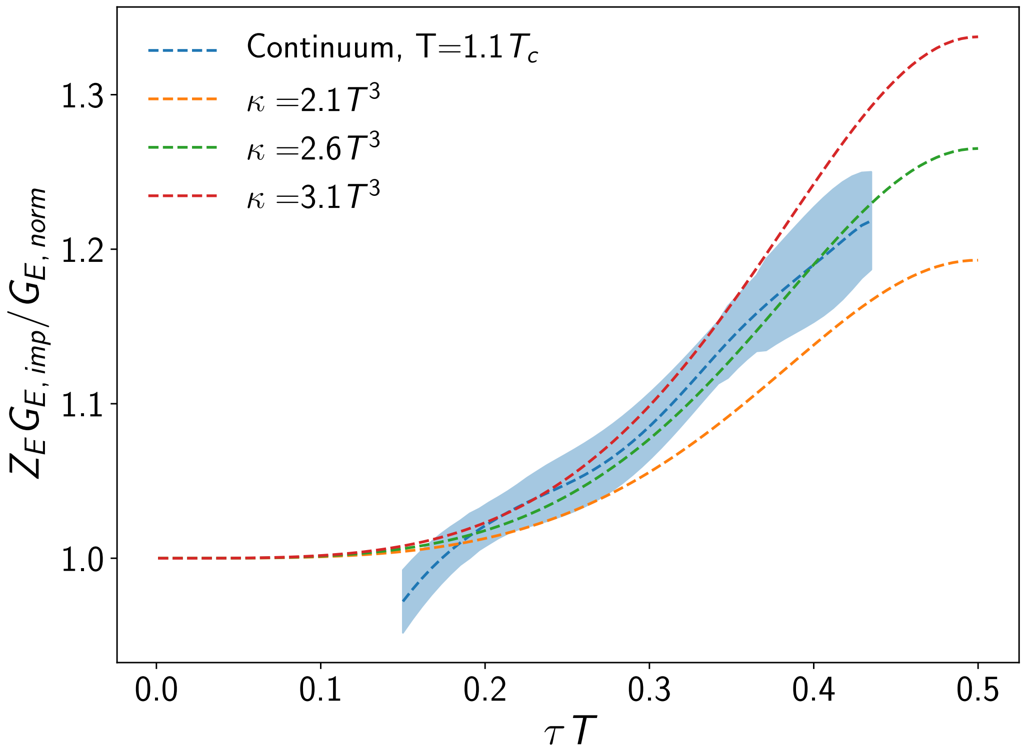




Fitting the electric correlator

Brambilla, Leino, PP, Vairo, arXiv:1912.00689

Take into account the running of the strong coupling constant and include the linear part at small energy



Preliminary estimates:

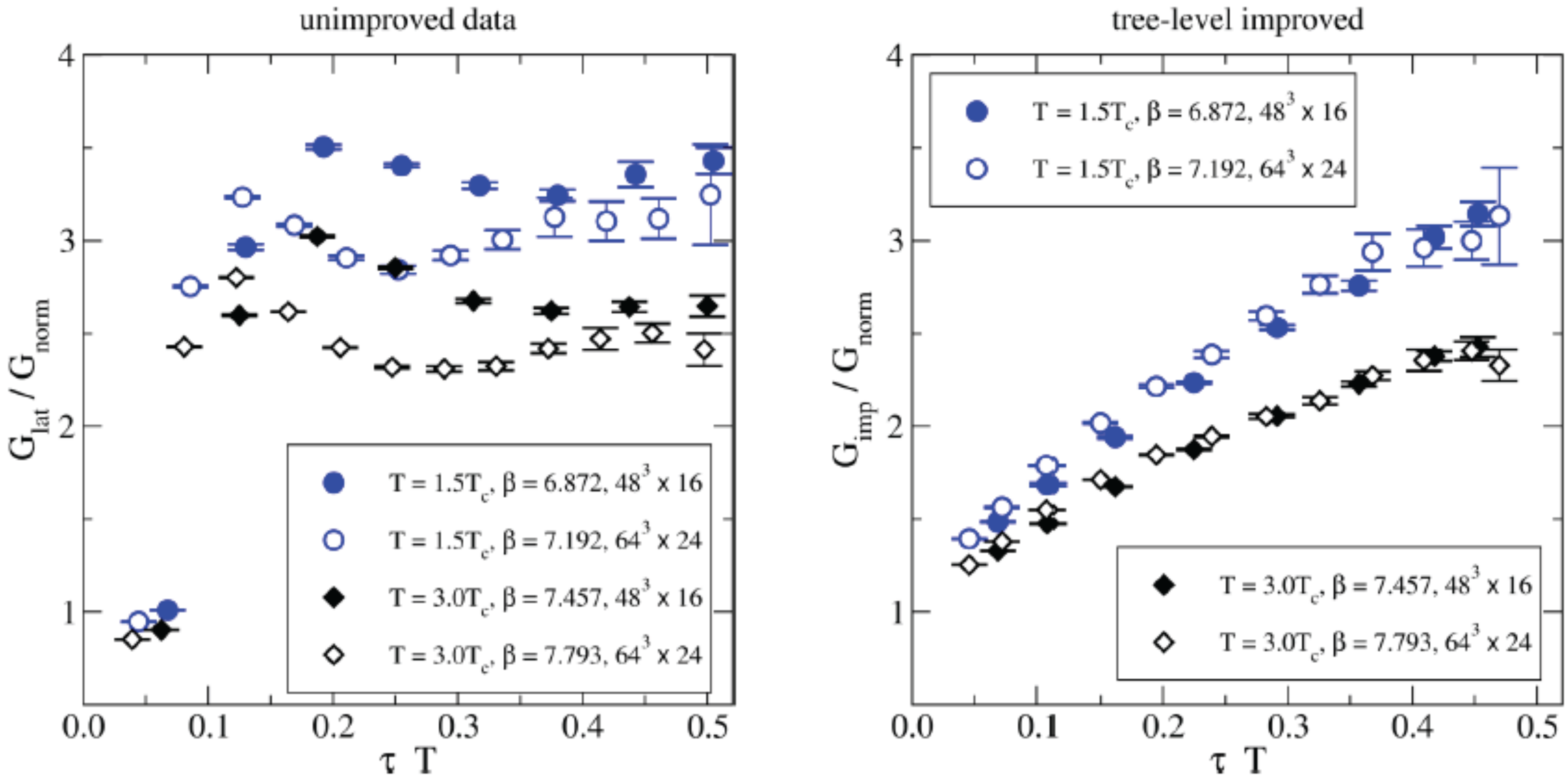
$$2.28 < \frac{\kappa}{T^3} < 3.57 \text{ for } \qquad T = 1.1T_c\,,$$
$$1.99 < \frac{\kappa}{T^3} < 2.69 \text{ for } \qquad T = 1.5T_c\,,$$
$$1.05 < \frac{\kappa}{T^3} < 2.26 \text{ for } \qquad T = 3T_c\,,$$
$$0 < \frac{\kappa}{T^3} < 1.5 \text{ for } \qquad T = 6T_c\,,$$
$$0 < \frac{\kappa}{T^3} < 0.91 \text{ for } \qquad T = 10T_c\,,$$
$$0 < \frac{\kappa}{T^3} < 0.39 \text{ for } \qquad T = 10^4T_c\,.$$

Summary

- Heavy quark diffusion coefficient can be determined in quenched approximation
- The T-dependence of the momentum heavy quark diffusion coefficient is
- The NLO correction to the heavy quark diffusion coefficient is large but the lattice results re not incompatible with the NLO perturbative result at high T
- It is not known how to do the calculations of heavy quark diffusion coefficient in QCD with dynamical quarks

Back-up:

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G^{\mathrm{LO}}_{\mathrm{cont}}(\overline{\tau T}) = G^{\mathrm{LO}}_{\mathrm{lat}}(\tau T)$$

From Kaczmarek