All Extremal Black holes are Primordial

Mostafizur Rahman

Centre for Theoretical Physics
Jamia Millia Islamia
New Delhi - 110025.

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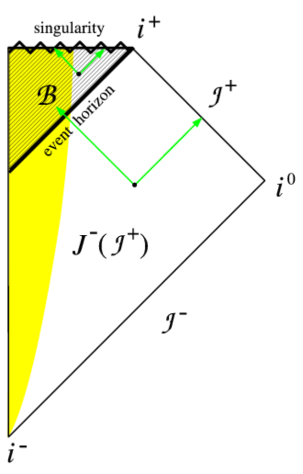
Black holes

Black hole spacetime

- Black hole region, $B = M - J^{-}(\Im^{+})$.
- Event horizon, $H = \partial B$.



Rotating Black holes

- The line element of Kerr black hole spacetime in Boyer-Lindquist coordinate (t, r, θ, ϕ) can be expressed as follows

$$ds^2 = -\frac{\Delta_r}{\Sigma}\left[dt - a\sin^2\theta\ d\phi\right]^2 + \Sigma\left[\frac{dr^2}{\Delta_r} + d\theta^2\right] + \frac{\sin^2\theta}{\Sigma}\left[adt - (r^2 + a^2)\ d\phi\right]^2$$

where,

$$\Delta_r(r) = r^2 + a^2 - 2Mr\,, \qquad \Sigma = r^2 + a^2\cos^2\theta\,,$$

- The position of the horizon is given by the solution of

$$\Delta_r(r) = r^2 + a^2 - 2Mr = 0$$

$$r\pm = M \pm \sqrt{M^2 - a^2}$$

- Extremal black holes correponds to $r_+ = r_-$ i.e. $a = M$

Throne limit

Throne limit

- When a black hole is formed due to collapse of a massive star, there should be an unavoidable presence of matter that accretes around the black hole.
- Throne studied the evolution of black holes in presence of accreting matters.
- When a black hole absorbs matter and radiation from the accreting disk, its mass and angular momenta evolves.
- Throne modeled the accreting matters as a very thin disk of gas in the equatorial plane.
- From the inner edge of the disk, gas particles are dumped into the black hole.
- The radius of this inner edge is given by the innermost stable circular orbit (ISCO) radius.

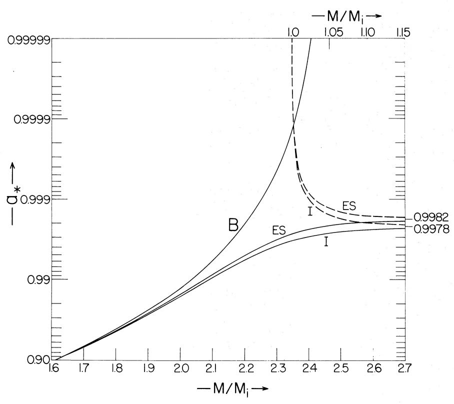
Throne limit

- If each of these particles carries a specific energy E_{ms} and angular momenta L_{ms}, the evolution of the black hole is governed by the following equation

$$\frac{da_*}{d\ln M} = \frac{1}{M}\frac{L_{ms}}{E_{ms}} - 2a_*$$

$$\frac{dM}{dM_0} = E_{ms}$$

[1]



[1] Kip Throne (1974)

Throne limit

- The other effect that contributes significantly in determining the final state of the black hole is capture of photons by the black hole.
- The BH's capture cross section is greater for the photons of negative angular momentum than the photon of positive angular momentum. As a result this effect decreases the angular momentum of the BH.
- Considering both of these two effects, Throne showed that it is impossible spin up the BH beyond $a_* \approx 0.998$.

Primordial black holes and Throne limit

- Depending on the mechanism of production of PBH at the end of inflation, there is no restriction on the initial spin of the black hole.
- Moreover, since the PBH does not come from the collapse of the star, it does not naccesarily evolve in a matter rich environment[2].
- So the PBH are not subject to the Throne limit.
- So the only way that a PBH can lose its angular momentum is via Hawking radiation.

[2] Alexandre Arbey, Jrmy Auffinger, Joseph Silk (2019)

Hawking Radiation

Hawking radiation

- Using semi-classical approximation, Hawking showed that black holes acts as perfect thermodynamic object.
- The expectation value $< n_s(E) >$ for the number of particles of a given species, emitted in a mode with energy E and angular momentum m as measured by an observer sitting at infinity , is given by

$$< n_{sm}(E) >= \frac{\Gamma_{sm}(E)}{\exp[\frac{E}{T_H}] \pm 1} \tag{1}$$

- The grey body factor $\Gamma_{sm}(E)$ is the probability for an outgoing mode of s species with energy E and angular momenta m to reach an asymptotic observer at infinity.

Greybody factor

- A wave like equation for radial and time variables usually has the Schrödinger-like form for stationary backgrounds

$$\frac{d^2\phi_E}{dr_*^2} + (E^2 - V(E,r))\phi_E = 0$$

- Now let ϕ_E be the solution of the equation which describes the scattering of an outgoing wave originating at $r_* \to -\infty$ (i.e., the outer black hole horizon). Then we must have

$$\phi_E \to T\ e^{-iEr_*}\,, \qquad r_* \to \infty$$

$$\phi_E \to e^{-iEr_*} + R\ e^{iEr_*}\,, \qquad r_* \to -\infty$$

- Then $\Gamma_{sj}(E) = T^2$.

Mass and Angular momentum loss due to Hawking radiation

- Mass and angular momentum loss of the BH due to Hawking radiation, is given by

$$-M^2\frac{dM}{dt} = f(M, a_*) = M^2\int_0^{\text{inf}}\sum_{dof}\frac{E}{2\pi}\frac{\Gamma_{sm}(E)}{\exp[\frac{E'}{T_H}]\pm 1}dE \tag{2}$$

$$-\frac{M}{a_*}\frac{dJ}{dt} = g(M, a_*) = \frac{M}{a_*}\int_0^{\text{inf}}\sum_{dof}\frac{m}{2\pi}\frac{\Gamma_{sm}(E)}{\exp[\frac{E'}{T_H}]\pm 1}dE \tag{3}$$

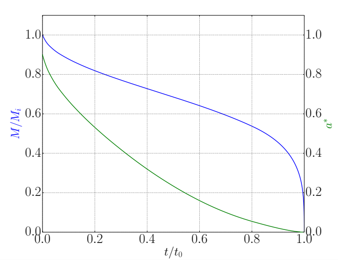
  here, $E' = E - m\Omega$.

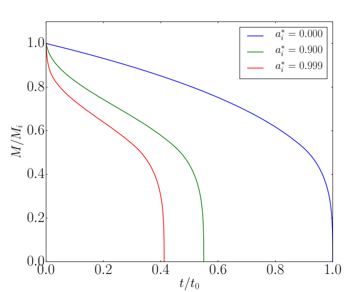
- The authors solved the above equation using the package *BlackHawk* starting from different initial mass and spin of the black hole.

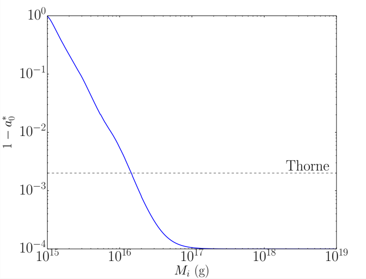
Results

Results

Alexandre Arbey, Jrmy Auffinger, Joseph Silk (2019)







Formation threshold of rotating primordial black holes

- PBHs are directly produced in the very early Universe by the gravitational collapse of large primordial density perturbations generated during inflation.
- For the time being ignore the mechanism for rotating PBH production and lets focus on the threshold criteria for PBH production in the radiation dominated era.
- A starting point is to assume that the overdense region, whose proper size is initially super Hubble and to approximate the region as part of the closed FLRW Universe by choosing a coordinate system where the energy density of the fluid is used as a clock, so that $t = const.$ surfaces coincide with $\rho = const.$ surfaces[3].
- The evolution of this overdense region is given by the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\bar{\rho}(1+\delta) - \frac{1}{a^2} \tag{4}$$

[3] Minxi Hea, Teruaki Suyamac (2019)

Formation threshold for rotating black holes

Formation threshold and Jeans length

- This region continues to grow in accordance with Friedmann equation.
- However, when the size of the overdense region becomes greater than Jeans length, gravity wins against pressure and the system undergoes gravitional collapse.
- Size of the region at the time of maximum expansion

$$\lambda_{max} = \frac{a_{max}}{a_{hc}} \frac{1}{H_{hc}} \tag{5}$$

H_{hc} is the horizon radius, a_{hc} is the scale factor at Hubble crossing.

- Gravitational collapse happens when

$$\lambda_{max} > R_J \tag{6}$$

- In this scenario, the dispersion relation takes the form

$$\omega^2 = c_s^2 k^2 - 8\pi G \bar{\rho}_{max}(1 + \delta_{max}) \tag{7}$$

c_s is the sound speed of the radiation.

- The Jeans length,

$$R_J = \frac{c_s}{\sqrt{8\pi G \bar{\rho}_{max}(1 + \delta_{max})}} \approx c_s a_{max} \tag{8}$$

- Then the criteria for PBH formation redues to

$$\frac{a_{max}}{a_{hc}} \frac{1}{H_{hc}} > c_s a_{max} \tag{9}$$

- In terms of density contrast

$$\delta_{hc} > \delta_{th} = c_s^2 \tag{10}$$

Rotating case

- Add small rotation in the overdense region. In this scenario, the dispersion relation takes the form

$$\omega^2 = c_s^2 k^2 + 4\Omega^2 - 8\pi G \bar{\rho}_{max}(1 + \delta_{max}) \tag{11}$$

  Ω is the angular velocity of the rotating system.

- The Jeans length,

$$R_J \approx \frac{c_s}{\sqrt{G\rho}}(1 + \frac{\Omega^2}{2\pi G\rho}) \tag{12}$$

- Then the criteria for PBH formation redues to

$$\frac{a_{max}}{a_{hc}} \frac{1}{H_{hc}} > \frac{c_s}{\sqrt{G\rho}}(1 + \frac{\Omega^2}{2\pi G\rho}) \tag{13}$$

- In terms of density contrast

$$\delta_{hc} > \delta_{th} = c_s^2(1 + \frac{25 c_s^2 a_*^2}{6(1 + c_s^2)^3}) \tag{14}$$