Predictions for $B_s \to \bar{K}^*\mu^+\mu^-$ in Non-Universal $\boldsymbol{Z'}$ Models

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Based on: A.K.Alok, A. Dighe, SG, D. Kumar: arXiv: 1912.02052

Plan of the Talk

- Correlating $b \to s\, \ell^+ \ell^-$ and $b \to d\, \ell^+ \ell^-$ decays.
- The Non-Universal Z' model.
- Constraints from b $\to s\ (d)\ \ell^+\ell^-$, $B - \bar{B}$ mixing, Neutrino Trident.
- Predictions for observables in $B_s \to \bar{K}^* \mu^+\mu^-$ decay.
- Summary

$b \to s\,\ell^+\ell^-$ and $b \to d\,\ell^+\ell^-$ Transitions

- A set of coherent deviations from SM in observables in $b \to s\,\ell^+\ell^-$ decays. Global fits point to possible New Physics scenarios in the form of V and A operators.

- It is interesting to look at implications of $b \to s\,\ell^+\ell^-$ measurements in other sectors like $b \to d\,\ell^+\ell^-$ sector.

- Possible in model-dependent way eg: in Z' models: Couplings constrained from $b \to s\,\ell^+\ell^-$, $b \to d\,\ell^+\ell^-$ decays, Mixing and Neutrino Trident. Look for imprints of Z'boson in other sectors.

- The $b \to d\,\ell^+\ell^-$ transitions gives rise to inclusive $\bar{B} \to X_d\mu^+\mu^-$decays and exclusive decays like $B^+ \to \pi^+\mu^+\mu^-$, $B_s \to \overline{K}^*\mu^+\mu^-$.

- LHCb measured differential branching ratio and CP Asymmetry in $B^+ \to \pi^+\mu^+\mu^-$ BR = $(1.83 \pm 0.24 \pm 0.05) \times 10^{-8}$. Recently LHCb reported evidence of $B_s \to \overline{K}^*\mu^+\mu^-$ decay : BR = $(2.9 \pm 1.1) \times 10^{-8}$.

The Z' Model and Constraints on Couplings

$$\Delta\mathcal{L}_{Z'} = J^\alpha Z'_\alpha$$

$$J^\alpha = g_L^{\mu\mu}\,\bar{L}\gamma^\alpha P_L L + g_R^{\mu\mu}\,\bar{L}\gamma^\alpha P_R\, L + g_L^{bd}\,\bar{Q}_1\gamma^\alpha P_L Q_3 + g_L^{bs}\,\bar{Q}_2\gamma^\alpha P_L Q_3 + h.c.$$

V. Barger, L. L. Everett, J. Jiang et. al '09

$$\mathcal{H}_{Z'}^{eff} = \frac{1}{2M_{Z'}^2} J_\alpha J^\alpha \quad \supset \quad \frac{g_L^{bs}}{M_{Z'}^2}\left(\bar{s}\gamma^\alpha P_L b\right)\left[\bar{\mu}\gamma_\alpha\left(g_L^{\mu\mu}P_L + g_R^{\mu\mu}P_R\right)\mu\right]$$

$$+\frac{g_L^{bd}}{M_{Z'}^2}\left(\bar{d}\gamma^\alpha P_L b\right)\left(\bar{\mu}\gamma_\alpha\left(g_L^{\mu\mu}P_L + g_R^{\mu\mu}P_R\right)\mu\right)$$

$$+\frac{\left(g_L^{bs}\right)^2}{2M_{Z'}^2}\left(\bar{s}\gamma^\alpha P_L b\right)\left(\bar{s}\gamma_\alpha P_L b\right) + \frac{\left(g_L^{bd}\right)^2}{2M_{Z'}^2}\left(\bar{d}\gamma^\alpha P_L b\right)\left(\bar{d}\gamma_\alpha P_L b\right)$$

$$+\frac{g_L^{\mu\mu}}{M_{Z'}^2}\left(\bar{\mu}\gamma^\alpha\left(g_L^{\mu\mu}P_L + g_R^{\mu\mu}P_R\right)\mu\right)\left(\bar{\nu}_\mu\gamma_\alpha P_L\nu_\mu\right)$$

$b \rightarrow s\ \mu\mu$

$b \rightarrow d\ \mu\mu$

$B_s - \bar{B}_s$ mixing

$\nu_\mu N \rightarrow \nu_\mu N\ \mu^+\mu^-$

$B_d - \bar{B}_d$ mixing

Constraints from
- $b \rightarrow s\ \ell^+\ell^-$ measurements.
- $B_s - \bar{B}_s$ mixing and $B_d - \bar{B}_d$ mixing.
- Neutrino Trident Production.

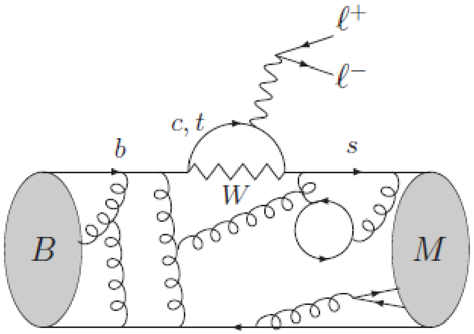
Effective Hamiltonian for b → $q\ l^+l^-$

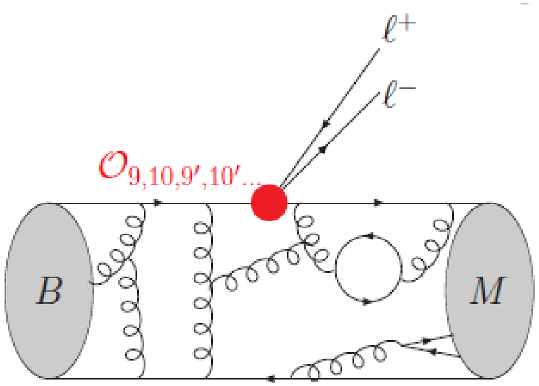
$$\mathcal{H}_{\rm eff}^{\rm SM} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tq}\left[\sum_{i=1}^{6,8} C_i^{bq}\mathcal{O}_i + C_7^{bq}\mathcal{O}_7 + C_9^{bq,{\rm SM}}\mathcal{O}_9 + C_{10}^{bq,{\rm SM}}\mathcal{O}_{10}\right].$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}[\bar{s}\sigma_{\mu\nu}(m_s P_L + m_b P_R)b]F^{\mu\nu}$$

$$\mathcal{O}_{9(10)} = \frac{\alpha_{\rm em}}{4\pi}(\bar{s}\gamma^\mu P_L b)(\overline{\mu}\gamma_\mu(\gamma_5)\mu)$$

$$C_7 = -0.29,$$
$$C_9^{\rm SM} = 4.1,\ C_{10}^{\rm SM} = -4.3$$





Addition of new Z’ boson modifies the Wilson coefficient:

$$C_i^{bq} = C_i^{bq,{\rm SM}} + C_i^{bq,{\rm NP}}$$

$$C_9^{bq,{\rm NP}} = -\frac{\pi}{\sqrt{2}G_F\alpha V_{tb}V_{tq}^*}\frac{g_L^{bq}(g_L^{\mu\mu}+g_R^{\mu\mu})}{M_{Z'}^2},$$

$$C_{10}^{bq,{\rm NP}} = \frac{\pi}{\sqrt{2}G_F\alpha V_{tb}V_{tq}^*}\frac{g_L^{bq}(g_L^{\mu\mu}-g_R^{\mu\mu})}{M_{Z'}^2}.$$

1D good fit scenario: $C_{10}^{bs,NP} = 0 : g_L^{\mu\mu} = g_R^{\mu\mu}$

$C_9^{bs,\,NP} = -C_{10}^{bs,NP} : g_R^{\mu\mu} = 0$

Anomalies in b → $s\, l^+ l^-$

① $R_K = BR(B^+ \to K^+\mu^+\mu^-)/\, BR(B^+ \to K^+e^+e^-)$:

Moriond 2019 $[1.1, 6] : 0.846^{+0.062}_{-0.054} \pm 0.016$: $\mathbf{2.5}\sigma$

Run I : $[1.1, 6] : 0.745^{+0.090}_{-0.074} \pm 0.036$ Run II : $0.928^{+0.089}_{-0.076} \pm 0.020$

② $R_K^* = BR(B^0 \to K^{*0}\mu^+\mu^-)/\, BR(B^0 \to K^{*0}e^+e^-)$: $\mathbf{2.4}\sigma$

$[1.1, 6] : 0.685^{+0.113}_{-0.069} \pm 0.047$ $[0.045, 1.1] : 0.66^{+0.110}_{-0.070} \pm 0.024$

Belle: First measurement of R_K^* in B^0 and B^+ decays.
[0.045 , 1.1] , [1.1, 6.0], [15.0 - 19.0] q^2 bin .

③ $B \to K^*\, \mu^+\mu^-$: Angular observable P_5' measured by ATLAS and LHCb in [4.0, 6.0] q^2 bin differ from SM by 3.3 σ.

CMS measurement in [4.3, 6.0] q^2 bin consistent with SM within 1σ .

Belle measurement in [4.3,8.68] q^2 bin differ by 2.6 σ.

④ $B_s^0 \to \varphi\, \mu^+\mu^-$: Measured value of Branching ratio by LHCb is smaller than SM by 3.7 σ.

$\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-) = (2.58^{+0.33}_{-0.31} \pm 0.08 \pm 0.19) \times 10^{-8}$, SM : $(4.81 \pm 0.56) \times 10^{-8}$

Possible New Physics solutions

- All decays showing discrepancies from the SM are induced by the quark level transition b → s . Indications of New Physics in the $b \rightarrow s\,\mu^{+}\mu^{-}$

- New Physics effects in $b \rightarrow s\,\mu^{+}\mu^{-}$ have been analyzed in a model-independent way using effective Hamiltonian with all possible Lorentz structures.
- Any large effects in the b → s sector, can only be due to new physics in the form of vector (V) and axial-vector operators (A).

Alok, Datta, Dighe et. al JHEP 121, 122,(2011)

- Several global fits confirm this and suggest New Physics in the form of V A operators.

B. Capdevilla, A. Crivellin et.al JHEP 1801 093
W. Altmannshofer, P. Strangl, D. Straub PRD 96, 055008;
M. Ciuchini, A. Coutinho et. al. EPJC 77, 688; G. D'Amico et. al 1704.05438v4

- Anomalies in R_K , R_K^{*} can be due to NP in $b \rightarrow s\,\mu^{+}\mu^{-}$ or $b \rightarrow s\,e^{+}e^{-}$.

Discrepancies in P_5' and B_s^{ϕ} can be due to NP in $b \rightarrow s\,\mu^{+}\mu^{-}$ only.
We assume NP in $b \rightarrow s\,\mu^{+}\mu^{-}$: Breaks Lepton Flavor Universality

Constraints from b → $s\, l^+ l^-$ data

A. K. Alok, A. Dighe, SG, D. Kumar: **JHEP 06 (2019) 089**

We consider 122 observables in $b \to s\,\mu\mu$ sector for global fit using Flavio:
Differential Branching ratio: ($B^0 \to K^0\mu^+\mu^-$), ($B^+ \to K^+\mu^+\mu^-$), (B→ $X_s\, \mu^+\mu^-$), ($B^+ \to K^{*+}\mu^+\mu^-$), ($B^0 \to K^{*0}\mu^+\mu^-$) in several q^2 bins.
Angular observables and differential BR of $B_s^0 \to \varphi\mu^+\mu^-$.
Branching ratio ($B_s \to \mu^+\mu^-$) .
Angular observables in differential q^2 bins for the decay $B^0 \to K^{*0}\mu^+\mu^-$.
CP violating observables in $B^0 \to K^{0*}\mu^+\mu^-$ decay by LHCb.

Post-Moriond: New (2014 + 2019) R_K: [1.1, 6] q^2 bin by LHCb, R_K^* [1.1,6] , [0.045,1.1] and [15,19] q^2 bins (Belle). P_5' measurement by LHCb, ATLAS, CMS and Belle.

M. Alguero et. al 1903.09578 ; M. Ciuchini et. al 1903.09632 ;
J. Aebischer et. al 1903.10434 ; G. D'Amico et. al 1704.05438v4
A. Arbey et. al 1904.08399;

A χ^2 fit is done by using CERN minimization code MINUIT :

$$\chi^2_{b\to s\mu\mu}(C_i, C_j) = \left(\mathcal{O}_{th}(C_i, C_j) - \mathcal{O}_{exp}\right)^T \mathcal{C}^{-1} \left(\mathcal{O}_{th}(C_i, C_j) - \mathcal{O}_{exp}\right).$$

The χ^2 function is minimized to get the best fit points.

Covariance matrix obtained as: $\mathcal{C} = \mathcal{C}_{\text{theory}} + \mathcal{C}_{\text{exp}}$. $\quad C_i = C_{9,10}^{bs,\text{NP}}$

Constraints from $B_s - \bar{B}_s$, $B_d - \bar{B}_d$ Mixing

$$M_{12}^q = \frac{1}{3} M_{B_q} f_{B_q}^2 \widehat{B}_{B_q} \left[N\, C_{\rm VLL}^{\rm SM} + \frac{\left(g_L^{bq}\right)^2}{2M_{Z'}^2} \right]$$

$$\Delta M_q = 2|M_{12}^q| = \Delta M_q^{\rm SM} \left| 1 + \frac{\left(g_L^{bq}\right)^2}{2\, N\, C_{\rm VLL}^{\rm SM}\, M_{Z'}^2} \right| .$$

We consider ΔM_d and the ratio M_R to minimize the theoretical uncertainties.

$$\chi^2_{\Delta M_d} = \left(\frac{\Delta M_d - \Delta M_d^{\rm exp,m}}{\sigma_{\Delta M_d}} \right)^2$$

$$\Delta M_d^{\rm exp} = (0.5065 \pm 0.0019)\,{\rm ps}^{-1}$$ (HFLAV ’19)

$$\Delta M_d^{\rm SM} = (0.547 \pm 0.046)\,{\rm ps}^{-1} .$$ (FLAG ’19)

$$M_R = \frac{\Delta M_d}{\Delta M_s} \, , \quad {\rm with} \quad M_R^{\rm SM} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}} \, ,$$

$$\chi^2_{M_R} = \left(\frac{M_R - M_R^{\rm exp,m}}{\sigma_{M_R}} \right)^2$$

$$\xi = 1.2014^{+0.0065}_{-0.0072} \, , \; M_R^{\rm SM} = 0.0297 \pm 0.0009 ,$$

$$M_R^{\rm exp} = 0.0285 \pm 0.0001$$ (HFLAV ’19)

Constraints from $B_s - \bar{B}_s$, $B_d - \bar{B}_d$ Mixing

Constraints on the phase of g_L^{bs} and g_L^{bd} :

$$\chi^2_{J/\Psi\phi} = \left(\frac{S_{J/\Psi\phi} - S^{\rm exp,m}_{J/\Psi\phi}}{\sigma_{J/\Psi\phi}}\right)^2$$

$$\chi^2_{J/\Psi K_S} = \left(\frac{S_{J/\Psi K_S} - S^{\rm exp,m}_{J/\Psi K_S}}{\sigma_{J/\Psi K_S}}\right)^2$$

$$S_{J/\Psi\phi} = -\frac{Im(M^s_{12})}{|M^s_{12}|}\ ,\ S_{J/\Psi K_S} = \frac{Im(M^d_{12})}{|M^d_{12}|}$$

$$S^{\rm exp}_{J/\Psi\phi} = 0.02 \pm 0.03$$

$$S^{\rm exp}_{J/\Psi K_S} = 0.69 \pm 0.02$$

PDG ‘18

Constraints from Neutrino Trident Production:

$$\chi^2_{\rm trident} = \left(\frac{\sigma/\sigma_{\rm SM} - (\sigma/\sigma_{\rm SM})^{\rm exp,m}}{0.28}\right)^2$$

$$\left(\frac{\sigma}{\sigma_{\rm SM}}\right)^{\rm exp} = 0.82 \pm 0.28$$

Constraint from $B^+ \to \pi^+\mu^+\,\mu^-$ and $B_d \to \mu^+\mu^-$ decays:

$$\chi^2_{B^+\to\pi\mu\mu} = \left(\frac{\mathcal{B}(B^+ \to \pi\mu\mu) - \mathcal{B}^{\rm exp,m}_{\pi\mu\mu}}{\sigma_{\mathcal{B}_{\pi\mu\mu}}}\right)^2$$

$$\mathcal{B}^{\rm exp}_{\pi\mu\mu} = (1.83 \pm 0.24) \times 10^{-8}$$

LHCb ’15,
J. J. Wang, R. M. Wang et. al ‘07

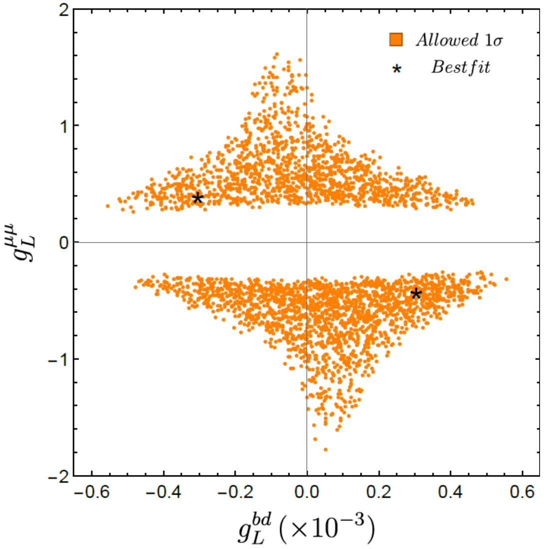
$$\chi^2_{B_d\to\mu\,\mu} = \left(\frac{\mathcal{B}(B_d \to \mu^+\,\mu^-) - \mathcal{B}^{\rm exp,m}_{B_d\to\mu\mu}}{\sigma_{\mathcal{B}_{B_d\to\mu\mu}}}\right)^2$$

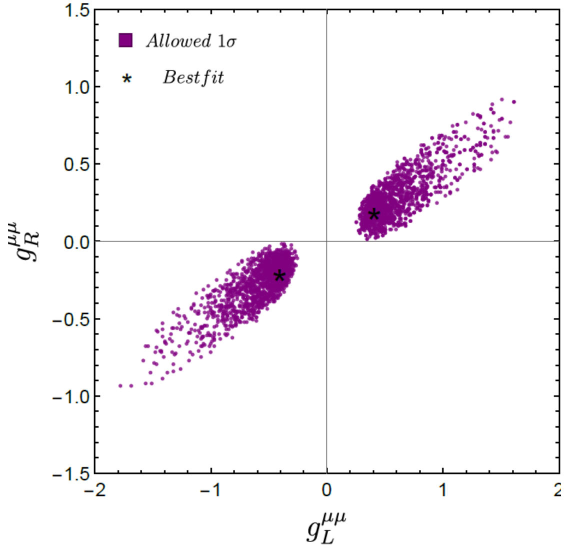
$$\mathcal{B}^{\rm exp}_{B_d\to\mu\mu} = (3.9 \pm 1.6) \times 10^{-10}$$

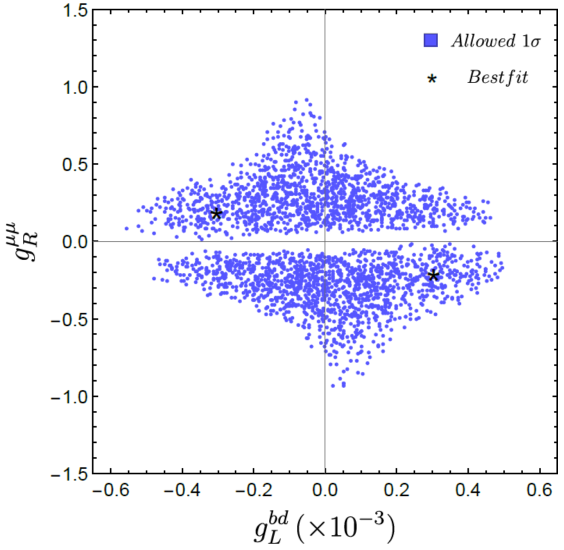
(HFLAV ’16)

Fit Results for Real Couplings

$$\chi^2_{tot} = \chi^2_{b\to s\mu\mu} + \chi^2_{\Delta M_d} + \chi^2_{M_R} + \chi^2_{J/\Psi\phi} + \chi^2_{J/\Psi K_S} + \chi^2_{trident} + \chi^2_{B^+\to\pi\,\mu\,\mu} + \chi^2_{B_d\to\mu\mu}\,.$$

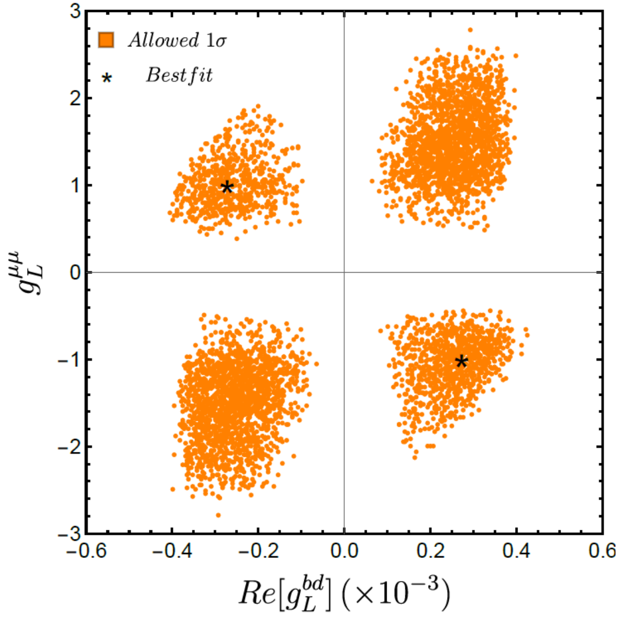


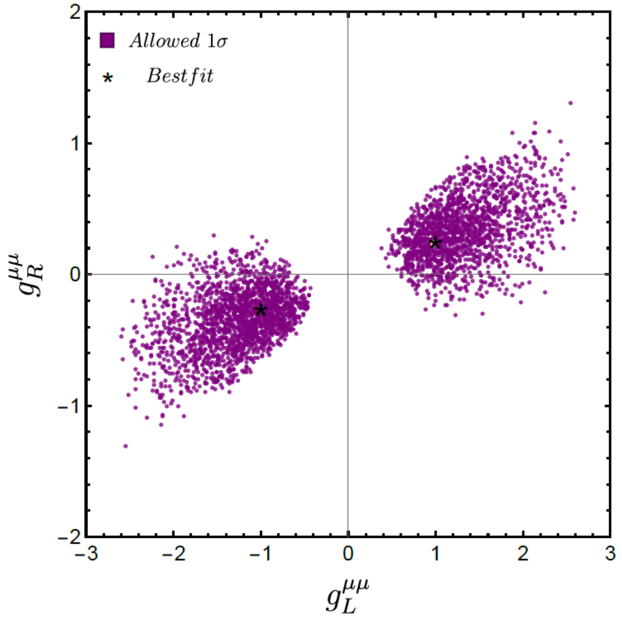


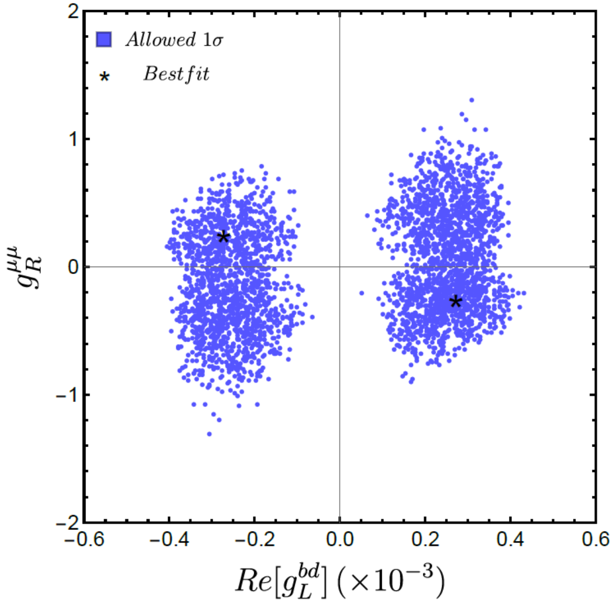


Fit Results for Complex Couplings

Couplings g_L^{bs} and g_L^{bd} are complex. $g_L^{\mu\mu}$ and $g_R^{\mu\mu}$ are real.







Predictions for $B_s \to \bar{K}^* \mu^+ \mu^-$ decays

$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{td}^* \Big\{ \Big[C_9^{bd} \langle \bar{K}^* | \bar{d} \gamma^\mu P_L b | B_s \rangle - \frac{2 m_b}{q^2} C_7^{bd} \langle \bar{K}^* | \bar{d}\, i \sigma^{\mu\nu} q_\nu P_R\, b | B_s \rangle \Big] (\bar{\mu} \gamma_\mu \mu) + C_{10}^{bd} \langle \bar{K}^* | \bar{d} \gamma^\mu P_L b | B_s \rangle (\bar{\mu} \gamma_\mu \gamma_5 \mu) \Big\}$$

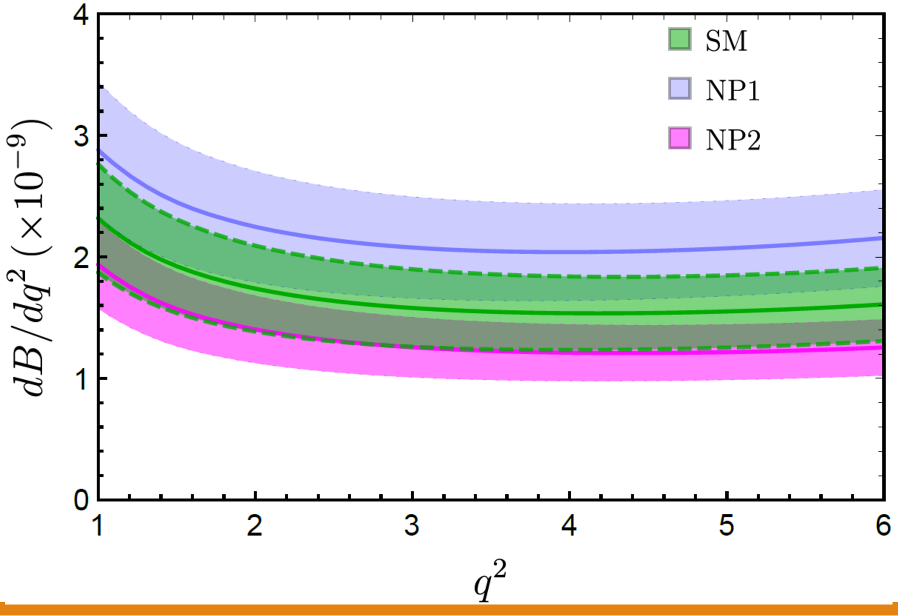
$$\frac{dB}{dq^2} = \tau_{B_s} \frac{d\Gamma}{dq^2} = \tau_{B_s} \frac{1}{4} (3\, I_1^c + 6 I_1^s - I_2^c - 2 I_2^s) \quad , \quad R_{K^*}^{(s)} = \frac{d\,\Gamma(B_s \to \bar{K}^* \mu^+ \mu^-)/d\,q^2}{d\,\Gamma(B_s \to \bar{K}^* e^+ e^-)/d\,q^2}$$

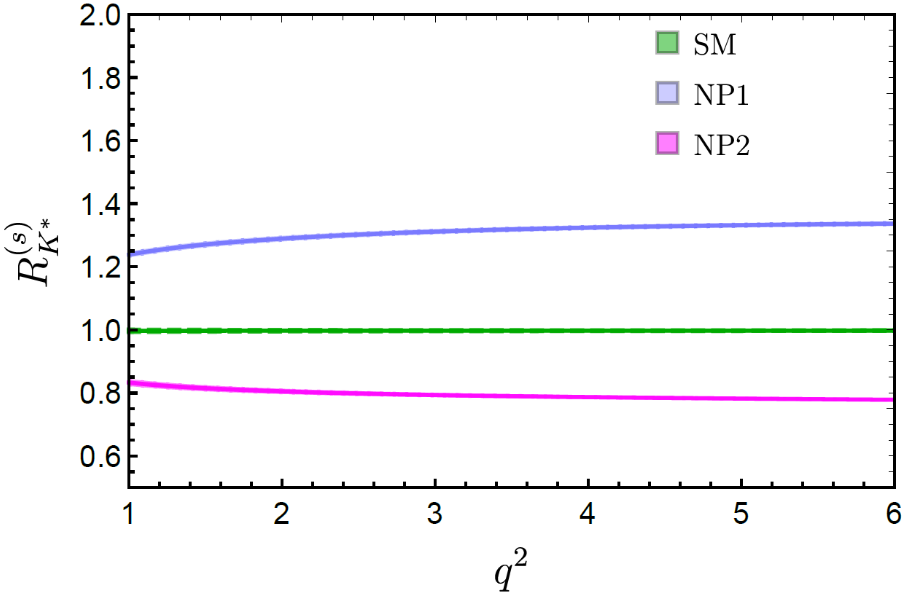
W. Altmannshofer, P. Ball, A. Bharucha, et. al '09

B. Kindra, N. Mahajan '18

NP1: $(C_9^{bd,NP}, C_{10}^{bd,NP})$ = (+ 0.98, - 0.17)

NP2 : $(C_9^{bd,NP}, C_{10}^{bd,NP})$ = (- 0.80, + 0.19)





Predictions for $B_s \to \bar{K}^* \mu^+ \mu^-$ decays: Real Couplings

$$A_{FB}(q^2) = \frac{1}{d\Gamma/dq^2}\left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\theta_l \frac{d^4\Gamma}{dq^2 d\cos\theta_l} = \frac{-3I_6^s}{3I_1^c + 6I_1^s - I_2^c - 2I_2^s}$$

$$F_L(q^2) = \frac{3I_1^c - I_2^c}{3I_1^c + 6I_1^s - I_2^c - 2I_2^s} \,.$$

W. Altmannshofer, P. Ball, A. Bharucha, et. al '09
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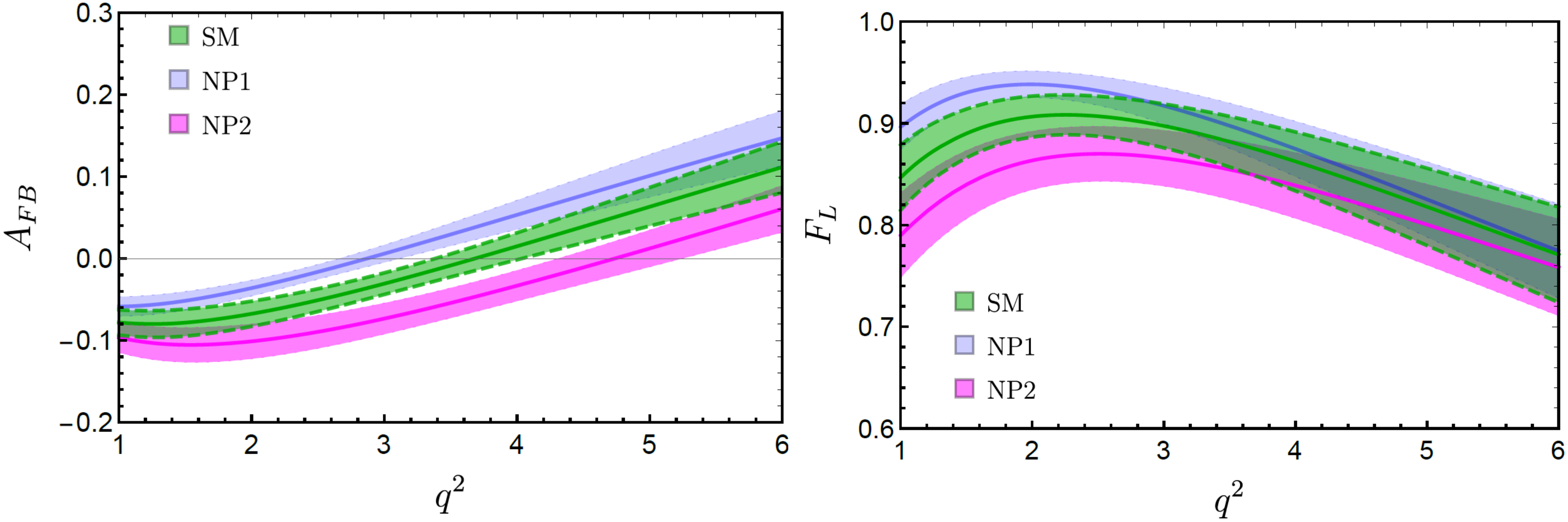
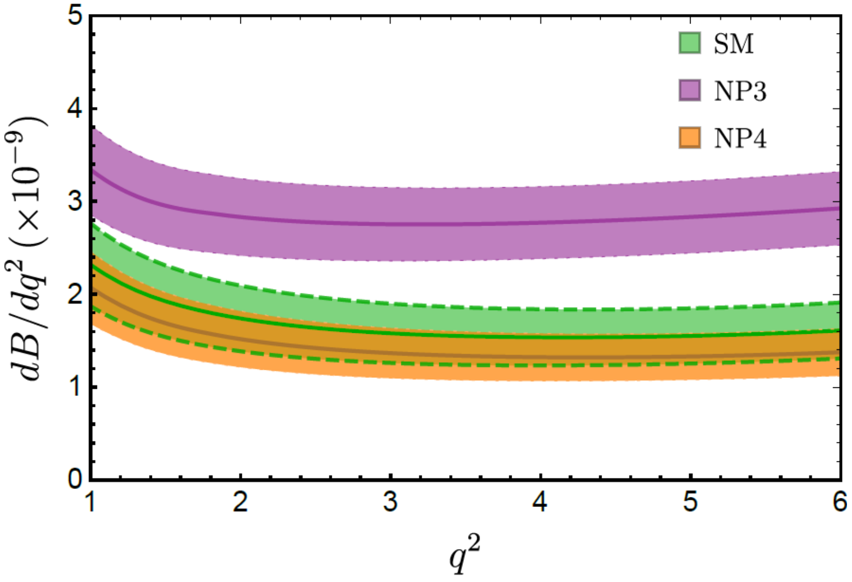
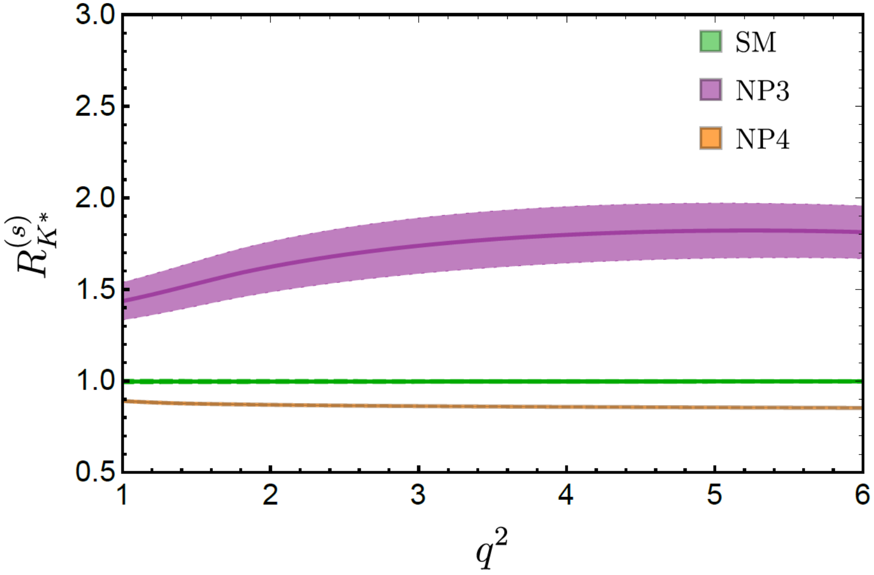
A. Bharucha, D. M. Straub and R. Zwicky ,15

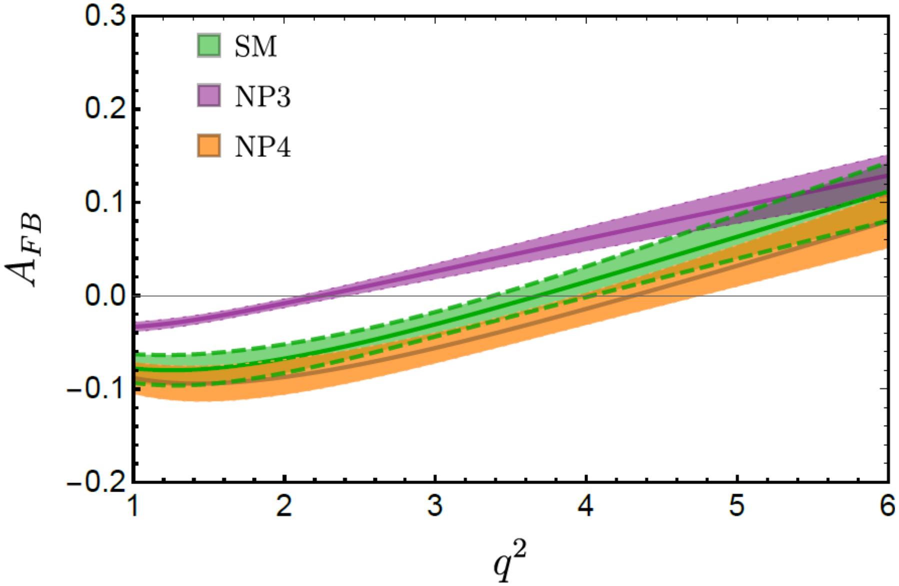


Predictions for $B_s \to \bar{K}^* \mu^+ \mu^-$ decays: Complex Couplings



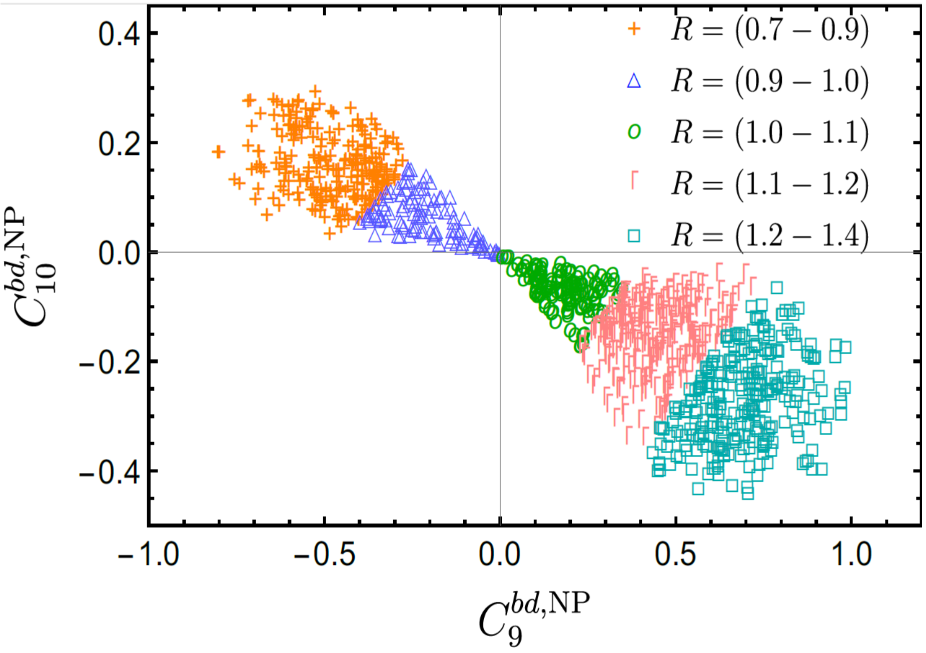


Scenario	NP3	NP4
$C_9^{bd,NP}$	$-1.4 + 4.9\,i$	$-0.6 + 0.8\,i$
$C_{10}^{bd,NP}$	$+0.7 - 2.3\,i$	$+0.2 - 0.2\,i$

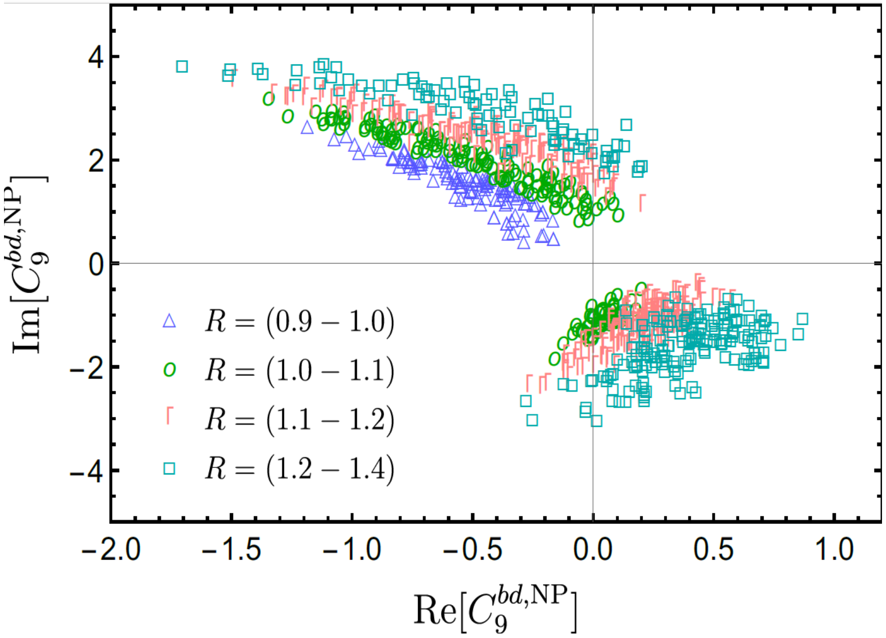


Predictions for Integrated $R_K^{*(s)}$

Real Couplings



Complex Couplings

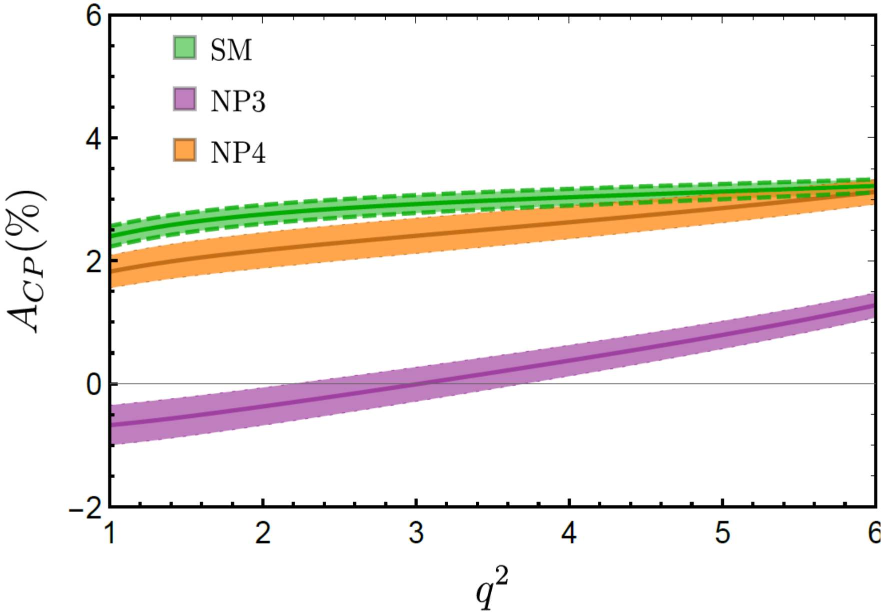


Predictions for CP asymmetry A_{CP}

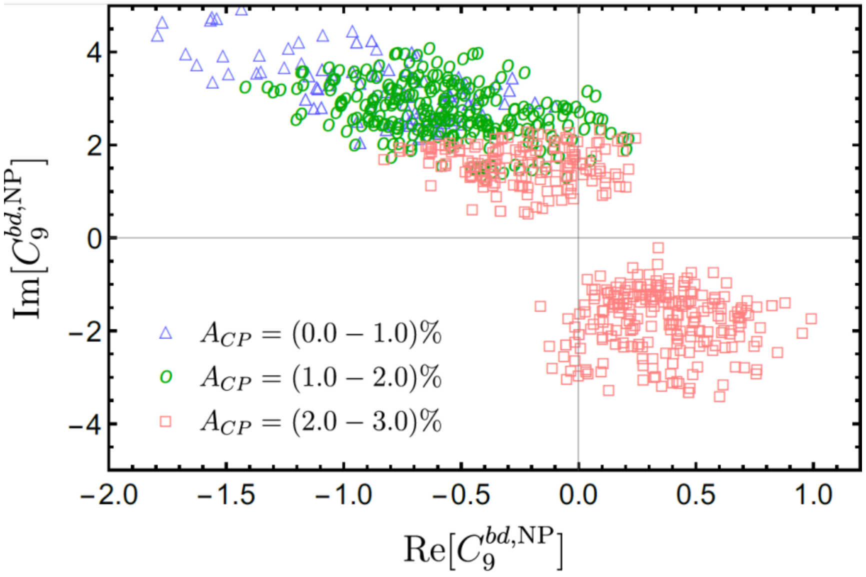
The direct CP asymmetry in $b \to d\,\mu^+\mu^-$ expected to be about an order of magnitude larger than $b \to s\,\mu^+\mu^-$. Within SM $A_{CP} \sim 2.5\ \%$: Within experimental reach.

$$A_{CP}(q^2) = \frac{dB/dq^2 - d\bar{B}/dq^2}{dB/dq^2 + d\bar{B}/dq^2}$$

Complex Couplings: $A_{CP}(q^2)$



Complex Couplings: Integrated A_{CP}



Summary

- Useful implications of constraints from $b \to s\, \ell^+ \ell^-$ and $b \to d\, \ell^+ \ell^-$ measurements, $B - \bar{B}$ mixing and Neutrino Trident on Couplings of Non-universal Z' models.

- Real Z'couplings: Large enhancement and suppression possible in LFU Ratio in $B_s \to \bar{K}^* \mu^+ \mu^-$ decays: $R_{K*}^{(s)}(q^2) \sim (0.8 – 1.2)$.

- Complex Couplings: About 50% enhancement possible in differential branching ratio.
  LFU Ratio $R_{K*}^{(s)}(q^2) \sim (0.8 – 1.8)$.
  Significant enhancement in $A_{FB}(q^2)$ with zero-crossing shifting towards very low q^2.
  Significant suppression in $A_{CP}(q^2)$ compared to the SM prediction of 2.5%.

- Integrated $R_{K*}^{(s)}$ and A_{CP} measurements with a precision 0.1 and 1% would help identify precise ranges of NP couplings $(C_9^{bd,NP}, C_{10}^{bd,NP})$.

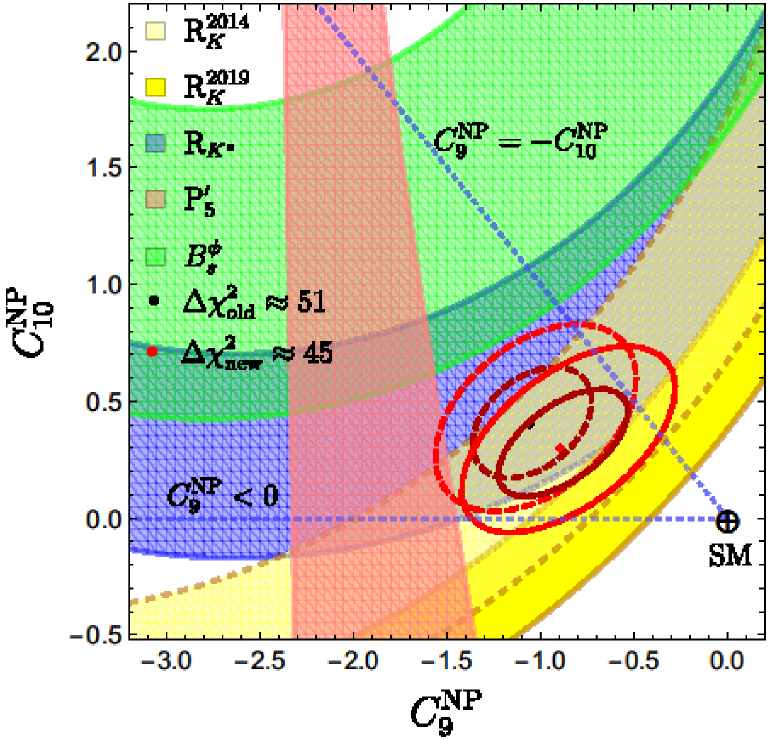
- $b \to d\, \ell^+ \ell^-$ decays important , CP Asymmetry can be measured.
- LHCb Run II : Full angular analysis of $B_s \to \bar{K}^* \mu^+ \mu^-$ possible .
  Lepton Universality Violation (LUV) Ratio can be measured with few % accuracy.

Thank you!

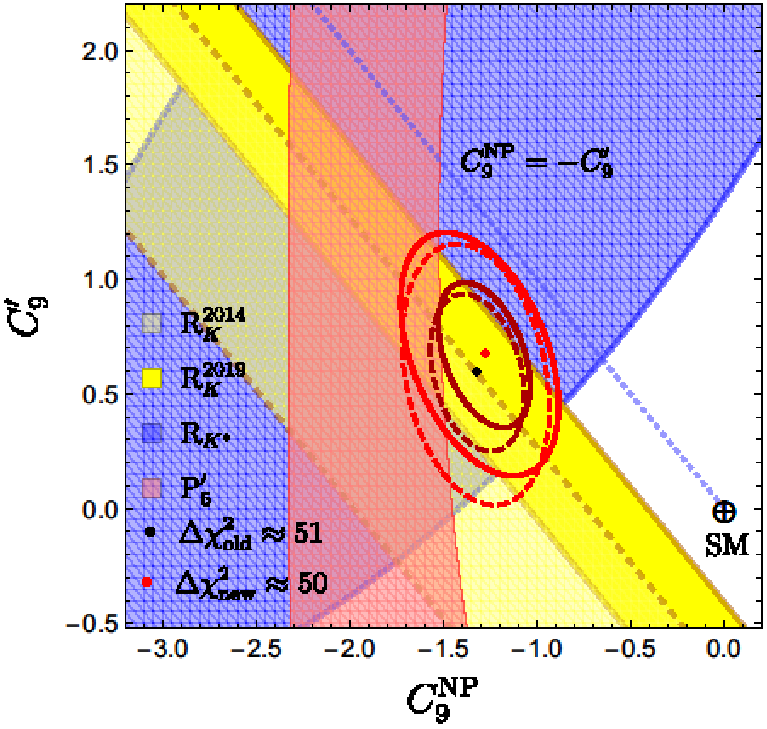
2 D fits before and after Moriond 2019 R_K update

1σ regions allowed from measurements of R_K [1.1, 6], R_K^* [1.1,6] , average of ATLAS and LHCb measurements of P_5' and B_s^ϕ .
Superimposed are 1σ and 2σ contours corresponding to global fit of all 122 observables: Before (dashed) and After Moriond (Solid).

$$R_K = R_{K^*} \approx 1 + 0.24\,(C_9^{\rm NP} - C_{10}^{\rm NP})$$



$$R_K \approx 1 + 0.24\,(C_9^{\rm NP} + C_9')$$
$$R_{K^*} \approx 1 + 0.24\,C_9^{\rm NP} - 0.17 C_9'$$



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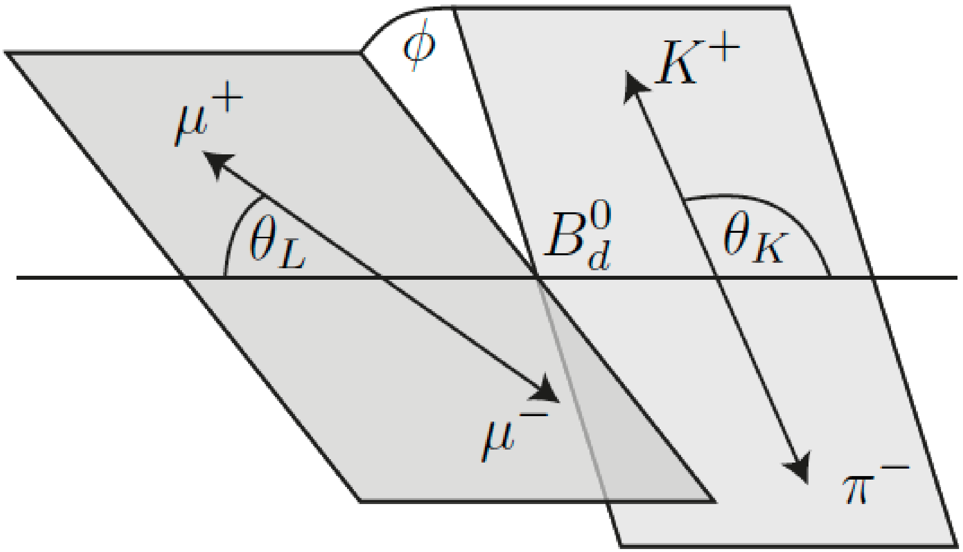
Angular Analysis



$$\begin{aligned}\frac{1}{\Gamma}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi} = \frac{9}{32\pi}\Big[&\frac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \frac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell \\ &- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + \frac{1}{2}(1-F_\mathrm{L})A_\mathrm{T}^{(2)}\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \\ &\sqrt{F_\mathrm{L}(1-F_\mathrm{L})}P_4'\sin 2\theta_K\sin 2\theta_\ell\cos\phi + \sqrt{F_\mathrm{L}(1-F_\mathrm{L})}P_5'\sin 2\theta_K\sin\theta_\ell\cos\phi + \\ &(1-F_\mathrm{L})A_{Re}^\mathrm{T}\sin^2\theta_K\cos\theta_\ell + \sqrt{F_\mathrm{L}(1-F_\mathrm{L})}P_6'\sin 2\theta_K\sin\theta_\ell\sin\phi + \\ &\sqrt{F_\mathrm{L}(1-F_\mathrm{L})}P_8'\sin 2\theta_K\sin 2\theta_\ell\sin\phi + (S/A)_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\ \Big]\end{aligned}$$

$$S_{3,6,9}/F_T = P_{1,2,3} \quad S_{4,5,7}/\sqrt{F_T F_L} = P'_{4,5,6} \quad P_2 = AFB/F_T$$

$$\frac{\sigma}{\sigma_{\rm SM}} = \frac{1}{1+(1+4s_W^2)^2}[\left(1+\frac{v^2 g_L^{\mu\mu}(g_L^{\mu\mu}-g_R^{\mu\mu})}{M_{Z'}^2}\right)^2 + \left(1+4s_W^2+\frac{v^2 g_L^{\mu\mu}(g_L^{\mu\mu}+g_R^{\mu\mu})}{M_{Z'}^2}\right)^2]$$

$$N = \frac{G_F^2 M_W^2}{16\pi^2}\left(V_{tb}V_{tq}^*\right)^2 \ \ , \ C_{\rm VLL}^{\rm SM} = \eta_B x_t\left[1+\frac{9}{1-x_t}-\frac{6}{(1-x_t)^2}-\frac{6x_t^2\ln x_t}{(1-x_t)^3}\right]$$