Heavy quark transport in a Polyakov loop plasma

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Outline

- Introduction
- Matrix model
- HQ transport
- Results
- Conclusion

Introduction

- Why?

  1. Interaction of HQ with the bulk medium modifies the spectra of open mesons[1]

- Important implications, $M \gg T$:

  1. No thermal production
  2. Relaxation time of HQ is larger than light quarks by a factor M/T
  3. Momentum transfer from the medium is small i.e., $Q \sim T$

- Interaction of HQ with the bulk medium is encoded in the transport coefficients

[1] Physics Letters B 747 (2015) 260–264

- LO perturbative calculations ($q/g\ Q \to q/g\ Q$) estimated very small drag coefficient and large thermalization time of HQ
- HTL resummed propagator in t-channel scattering estimates slightly large drag compared to pQCD
- Non-perturbative approaches:
  1. Quasi particle model[2]
  2. Resonance model[3]
  3. T-matrix model[4]

[2] PHYSICAL REVIEW C 96, 044905 (2017)
[3] PHYSICAL REVIEW C 71, 034907 (2005)
[4] Phys. Rev. Lett. 100, 192301 (2008)

- Resonance model

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_m - iG_s\left(\bar{q}\Phi_0^*\frac{1+\not{v}}{2}c - \bar{q}\gamma^5\Phi\frac{1+\not{v}}{2}c + h.c.\right)$$
$$- G_V\left(\bar{q}\gamma^\mu\Phi_\mu^*\frac{1+\not{v}}{2}c - \bar{q}\gamma^\mu\gamma^5\Phi_{1\mu}\frac{1+\not{v}}{2}c + h.c.\right)$$

- T-matrix model

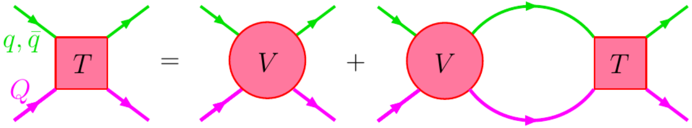

Figure: T-matrix model

- The region from $\phi \neq 0$ to $\phi = 1$ cannot be treated in pQCD

Matrix model [6]

- Constant background gauge field

$$A_\mu^{ab} = \delta_{\mu 0}\delta^{ab} Q^a/g$$

For $SU(3)$, $Q^a = 2\pi T(-q, 0, q)$.

$$\phi = \frac{1}{3}\left(1 + 2\cos 2\pi q\right).$$

- The background gauge field acts as an imaginary chemical potential[5]

$$f_a(E) = \frac{1}{e^{\beta(E - iQ^a)} + 1}, \qquad \tilde{f}_a(E) = \frac{1}{e^{\beta(E + iQ^a)} + 1},$$

$$f_{ab}(E) = \frac{1}{e^{\beta(E - i(Q^a - Q^b))} - 1}.$$

quark→ single index gluon→double index

[5] PHYSICAL REVIEW D 80, 036004 (2009)

[6] K. Fukushima, V. Skokov, Polyakov loop modeling for hot QCD

- Color averaged statistical distribution function

$$f_q(E) = \frac{1}{3}\sum_{a=1}^{3} f_a(E) = \frac{\phi e^{-\beta E} + 2\phi e^{-2\beta E} + e^{-3\beta E}}{1 + 3\phi e^{-\beta E} + 3\phi e^{-2\beta E} + e^{-3\beta E}}.$$

$$9f_g(E) = \frac{3}{e^{\beta E} - 1} + \frac{e^{\beta E}(6\phi - 2) - 4}{1 + e^{2\beta E} + e^{\beta E}(1 - 3\phi)} + \frac{e^{\beta E}(9\phi^2 - 6\phi - 1) - 2}{1 + e^{2\beta E} + e^{\beta E}(1 + 6\phi - 9\phi^2)}$$

- Deviation in distribution function from distribution function from $\phi = 0$ phase is proportional to ϕ for quark and ϕ^2 for gluon

- In a process with hard momentum transfer the quark/gluon propagator does not depend on q^a.

- For a process with soft momentum transfer the quark/gluon propagators are resummed and depends on q^a

$$D\mu\nu; abcd(K) = P^L_{\mu\nu}\frac{k^2}{K^2}D^L_{abcd}(K) + P^T_{\mu\nu}D^T_{abcd}(K)$$

$$D^L_{\mu\nu;abcd}(K) = \left(\frac{i}{K^2 - F}\right)_{abcd}$$

$$D^T_{\mu\nu;abcd}(K) = \left(\frac{i}{K^2 - G}\right)_{abcd}$$

- F and G have same structure as that of vanishing background field

$$F = -2m^2\left(1 - \frac{x}{2}\ln\left(\frac{x+1}{x-1}\right)\right),$$

$$G = m^2\left(x^2 + \frac{x(1-x^2)}{2}\ln\left(\frac{x+1}{x-1}\right)\right),$$

- Debye mass small compared to perturbative Debye mass

$$\begin{aligned}(m_D^2)_{abcd} &= \frac{g^2}{6}\Bigg[\delta_{ad}\delta_{bc}\left(\sum_{e=1}^{3}\left(\mathcal{D}(Q_{ae}) + \mathcal{D}(Q_{eb})\right) - N_f(\tilde{\mathcal{D}}(Q_a) + \tilde{\mathcal{D}}(Q_b))\right)\\ &- 2\delta_{ab}\delta_{cd}\left(\mathcal{D}(Q_{ac}) - \frac{N_f}{N}\left(\tilde{\mathcal{D}}(Q_a) + \tilde{\mathcal{D}}(Q_c)\right) + \frac{N_f}{N^2}\sum_{e=1}^{3}\tilde{\mathcal{D}}(Q_e)\right)\Bigg]\end{aligned}$$

$$\mathcal{D}(Q_a) = \frac{3}{\pi^2}\int_0^\infty dEE\left(\frac{1}{e^{\beta(E+iQ_a)}-1} + \frac{1}{e^{\beta(E-iQ_a)}-1}\right),$$

- Debye mass is real

Debye mass

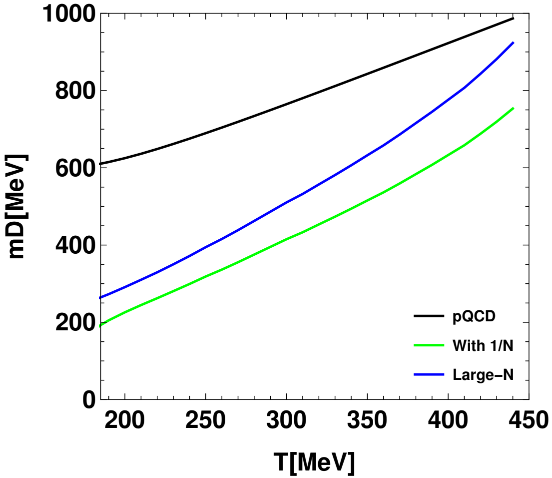

Figure: Debye mass as a function of T

Heavy quark transport

- The Brownian motion of HQ is characterized by the Fokker-Planck equation

$$\left[\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{E_p}\frac{\partial}{\partial \boldsymbol{x}}+\boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right]f_Q(\boldsymbol{p},\boldsymbol{x},t)=C[f_Q],$$

- The interaction of HQ with the bulk is encoded in the transport coefficients

$$C[f_Q]=\int d^3k[w(\boldsymbol{p}+\boldsymbol{k},\boldsymbol{k})f_Q(\boldsymbol{p}+\boldsymbol{k})-w(\boldsymbol{p},\boldsymbol{k})f_Q(\boldsymbol{p})],$$

$$\frac{\partial}{\partial t}f_Q(\boldsymbol{p},t)=\frac{\partial}{\partial p_i}\left(A_i(\boldsymbol{p})f_Q(\boldsymbol{p},t)+\frac{\partial}{\partial p_j}B_{ij}(\boldsymbol{p})f_Q(\boldsymbol{p},t)\right).$$

$$A_i(\boldsymbol{p})=A(\boldsymbol{p})p_i,$$

$$B_{ij}(\boldsymbol{p})=B_0(\boldsymbol{p})P_{ij}^{\parallel}+B_1(\boldsymbol{p})P_{ij}^{\perp},$$

- The drag and the momentum diffusion coefficients

$$A(\boldsymbol{p}) = \langle 1 \rangle - \frac{\langle \boldsymbol{p} \cdot \boldsymbol{p'} \rangle}{|\boldsymbol{p}|^2}$$

$$B_0(\boldsymbol{p}) = \frac{1}{4}\left(\langle |\boldsymbol{p'}|^2 \rangle - \frac{\langle (\boldsymbol{p} \cdot \boldsymbol{p'})^2 \rangle}{|\boldsymbol{p}|^2} \right)$$

- For $l^{a/ef} Q^c \to l^{b/gh} Q^d$ scattering

$$\begin{aligned} \langle X(\boldsymbol{p'}) \rangle &= \frac{1}{2E_p} \int_q \int_{p'} \int_{q'} \Big(\sum_{a,e} |\mathcal{M}_{qQ}|^2_{ab} f_a(q)(1 - f_e(q')) \\ &+ \sum_{e,f,g,h} |\mathcal{M}_{gQ}|^2_{efgh} f_{ef}(q)(1 + f_{gh}(q')) \Big) \\ &\times (2\pi)^4 \delta^4(p + q - p' - q') X(\boldsymbol{p'}) \end{aligned}$$

$$\int_q = \int \frac{d^3 q'}{(2\pi)^3 2E_{q'}}$$

- In the non-relativistic limit, $A(p) = \gamma =$const and $B_0(P) = D =$const

$$\langle \mathbf{p} \rangle = \mathbf{p}_0 \exp(-\gamma t)$$

$$\langle \mathbf{p}^2 \rangle - \langle \mathbf{p} \rangle^2 = \frac{3D}{\gamma}(1 - \exp(-\gamma t))$$

- At temperature T the drag and diffusion coefficient satisfy Dissipation fluctuation theorem

$$D = M\gamma T$$

- Scattering amplitude for $qQ \to qQ$ depends on the color of initial and final state particles

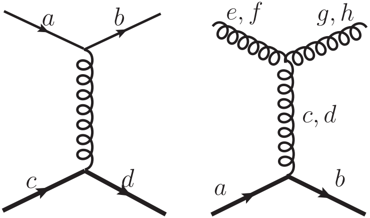

Figure: Coulomb scattering (left) of HQ (bold solid line) and light quark/antiquark (thin solid line). t-channel Compton scattering (right). The curly line represent a gluon.

$$|\mathcal{M}_{qQ}|^2_{ab} = \frac{g^4}{16N_c^2}\mathcal{P}^{jk}_{ab}\mathcal{P}^{ml}_{cd}\mathcal{P}^{j'k'}_{ba}\mathcal{P}^{m'l'}_{dc}\frac{(8(s-M^2)^2+8(u-M^2)^2+16M^2t)}{(t+(m_D^2)_{mljk})(t+(m_D^2)_{m'l'j'k'})}$$

$$\mathcal{P}^{ab}_{cd} = \delta^a_c\delta^b_d - \frac{1}{N}\delta^{ab}\delta_{cd}$$

- Scattering amplitude for $gQ \to gQ$ scattering depends on the color of initial/final quarks and gluons

$$|\mathcal{M}_t|^2_{efgh} = \frac{16g^4}{8N(N^2-1)}\mathcal{P}\left(\frac{-(M^2-s)(M^2-u)}{(t+(m_D^2)_{mlcd})t+(m_D^2)_{m'l'c'd'}}\right)$$

$$\mathcal{P} = \mathcal{P}^{ml}_{ab}\mathcal{P}^{l'm'}_{ba} f^{cd,ef,gh} f^{d'c',fe,hg}$$

$$f^{kl,mn,ab} = \frac{i}{\sqrt{2}}(\delta^{kn}\delta^{mb}\delta^{al} - \delta^{kb}\delta^{ml}\delta^{an})$$

- The color indices for initial/final light quark/gluon is summed with the distribution function

- Heavy quark drag

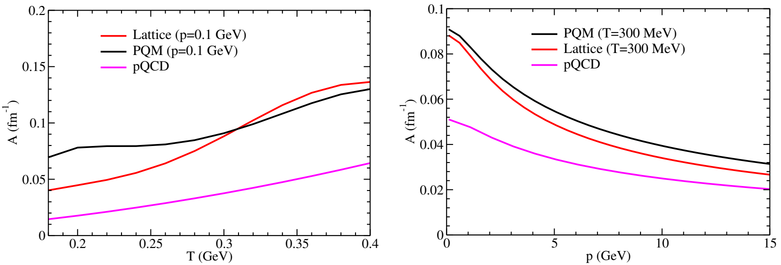

Figure: Drag coefficient as a function of temperature (left) and as a function of momentum (right).

- Viscosity modifies the thermal distribution function, $f = f^0 + \delta f$[7]
- δf is proportional to the spatial gradients i.e, $\Delta_\mu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\nu$
- For FD and BE distribution functions

$$\delta f = f(E)(1 \pm f(E))\left(\frac{\eta}{2sT^3} p^\mu p^\nu \nabla_{\langle \mu u_\nu \rangle} + \frac{\xi}{5T^3 s} p^\mu p^\nu \Delta_{\mu\nu}\Theta\right)$$

$$\nabla_{\langle \mu u_\nu \rangle} = \Delta_\mu u_\nu + \Delta_\nu u_\mu - \frac{2}{3}(g^{\mu\nu} - u^\mu u^\nu)\Theta$$

where $\Theta = \Delta_\mu u^\mu$

[7] PHYSICAL REVIEW C 68, 034913 (2003)

- Viscous corrections can be incorporated by modifying the thermal distribution function

$$f(E)_a = f(E)_a^0 + \frac{f(E)_a^0(1 - f(E)_a^0)}{T^3\tau}\left[\frac{\eta}{s}\left(-p_z^2 + \frac{p^2}{3}\right) + \frac{\xi}{s}\frac{p^2}{5}\right]$$

$$f(E)_{ab} = f(E)_{ab}^0 + \frac{f(E)_{ab}^0(1 + f(E)_{ab}^0)}{T^3\tau}\left[\frac{\eta}{s}\left(-p_z^2 + \frac{p^2}{3}\right) + \frac{\xi}{s}\frac{p^2}{5}\right]$$

- Diffusion coefficient

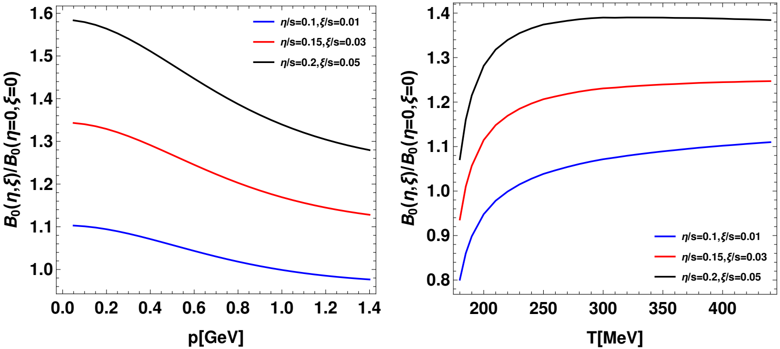

Figure: Diffusion coefficient as a function of momentum (left) and as a function of temperature (right).

- Drag coefficient

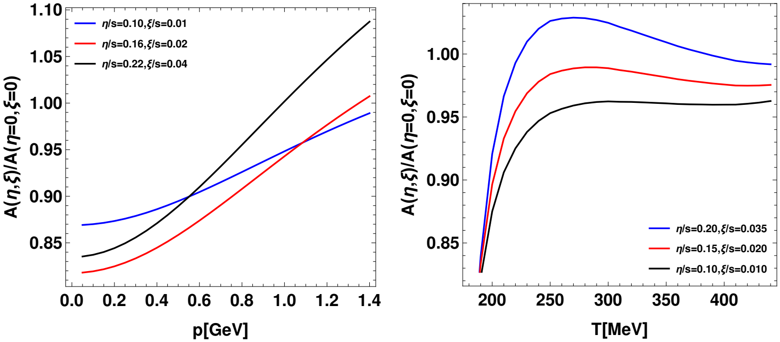

Figure: Drag coefficient as a function of momentum (left) and as a function of temperature (right).

Summary and conclusion

- Debye mass is small as compared to pQCD
- Drag and diffusion coefficients are enhanced with the inclusion of Polyakov loop
- Viscous effects enhances the diffusion and reduces the drag
- At high temperature, viscous effects enhances the drag coefficient
- For small value of η/s and ξ/s the viscous effects are somewhat weak

Future direction

- Radiative corrections to the drag and the diffusion coefficients may be important which can be incorporated in any other non-perturbative model
- It is interesting to see whether non-perturbative effects with radiative corrections can explain observed R_{AA} and v_2

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Thank You