Structure of kaon in light-cone quark model

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Ref: Satvir Kaur and Harleen Dahiya, Phys. Rev. D 100, 074008 (2019)

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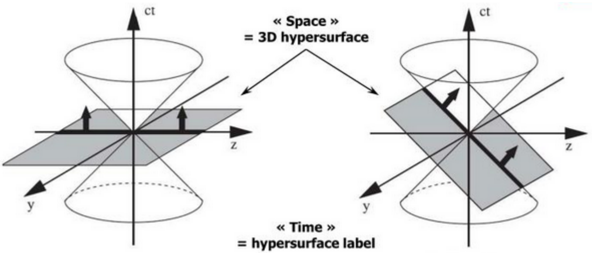
Overview

Light-cone dynamics



Equal t		Equal τ
p^0	$\Leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\Leftrightarrow$	$\vec{p}_\perp$
p^3	$\Leftrightarrow$	$p^+ = p^0 + p^3$

-P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).
- S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

- Energy-momentum dispersion relation:

In the instant form,
$$p^0 = \sqrt{\vec{p}^2 + m^2}.$$

In the front form,
$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}.$$

No square-root for the Hamiltonian in light front form.
Therefore, simplifes the dynamical structure.

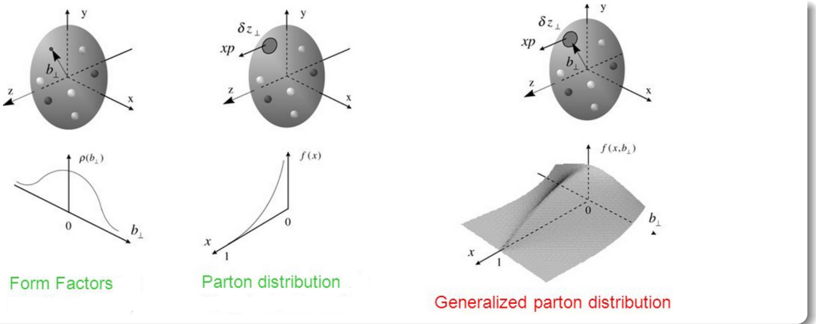
Light-front provides the wavefunctions (LFWFs) required to describe the structure and dynamics of hadrons in terms of their constituents (quarks and gluons).

3-D structure of hadron

- FFs and PDFs provide information to shape the physical picture of hadron.

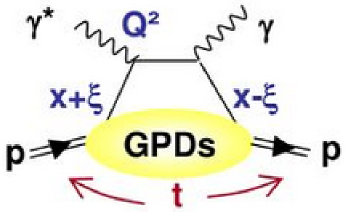
FFs and PDFs have deficiencies.

- FFs $\rightarrow$ No dynamical information on the constituents.
- PDFs $\rightarrow$ No knowledge of constituent's spatial locations and transverse motion.



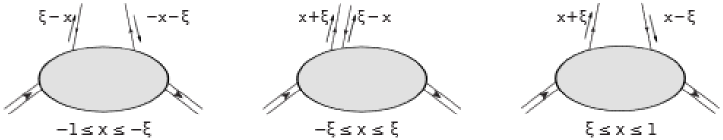
Generalized Parton Distributions (GPDs)

- GPDs encode information on the distribution of partons both in the transverse plane and in the longitudinal direction.



- $GPDs(x, \xi, t)$:
  - $x \pm \xi \rightarrow$ Longitudinal momentum fraction carried by active quark,
  - $t = \Delta^2 = (P' - P)^2 \rightarrow$ total momentum transferred.

- The distributions have the *support interval* $x \in [-1, 1]$



  - DGLAP region for anti-quark : $-1 < x < -\xi$,
  - ERBL region (quark anti-quark pair) : $-\xi < x < \xi$,
  - DGLAP region for quark : $\xi < x < 1$.

-M. Diehl, Phys. Rept. 388, 41 (2003).

- We restrict our calculations in DGLAP regions i.e. $\xi < x < 1$.

Light-cone quark model

- The mesonic light-cone Fock state wavefunctions are expanded as $|M\rangle = |q\bar{q}\rangle\psi_{q\bar{q}} + |q\bar{q}g\rangle\psi_{q\bar{q}g} + ...$
- The expansion of a kaon state in terms of its constituents eigenstates :

$$
\begin{aligned}
|M(P, S_z)\rangle &= \sum_{\lambda_1,\lambda_2} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} |x, \mathbf{k}_\perp, \lambda_1, \lambda_2\rangle \\
&\quad \psi_{S_z}^{\lambda_1,\lambda_2}(x, \mathbf{k}_\perp).
\end{aligned}
$$

- The light-cone wavefunctions $\psi_{S_z}^{\lambda_1,\lambda_2}(x, \mathbf{k}_\perp)$ can be defined for different combinations of helicities of quark and spectator antiquark in kaon as :

$$\begin{aligned}
\psi_0^{\uparrow,\uparrow}(x, \mathbf{k}_\perp) &= -\frac{1}{\sqrt{2}} \frac{k_1 - ik_2}{\sqrt{\mathbf{k}_\perp^2 + l^2}} \varphi(x, \mathbf{k}_\perp), \\
\psi_0^{\uparrow,\downarrow}(x, \mathbf{k}_\perp) &= \frac{1}{\sqrt{2}} \frac{(1-x)m_1 + xm_2}{\sqrt{\mathbf{k}_\perp^2 + l^2}} \varphi(x, \mathbf{k}_\perp), \\
\psi_0^{\downarrow,\uparrow}(x, \mathbf{k}_\perp) &= -\frac{1}{\sqrt{2}} \frac{(1-x)m_1 + xm_2}{\sqrt{\mathbf{k}_\perp^2 + l^2}} \varphi(x, \mathbf{k}_\perp), \\
\psi_0^{\downarrow,\downarrow}(x, \mathbf{k}_\perp) &= -\frac{1}{\sqrt{2}} \frac{k_1 + ik_2}{\sqrt{\mathbf{k}_\perp^2 + l^2}} \varphi(x, \mathbf{k}_\perp),
\end{aligned}$$

with

$$l^2 = (1-x)m_1^2 + xm_2^2 - x(1-x)(m_1 - m_2)^2.$$

-W. Qian and B. -Q. Ma, Phys. Rev. D 78, 074002 (2008).

- The momentum-space wavefunction $\varphi(x, \mathbf{k}_\perp)$:

$$\varphi(x, \mathbf{k}_\perp) = A \exp\left[-\frac{\frac{\mathbf{k}_\perp^2 + m_1^2}{x} + \frac{\mathbf{k}_\perp^2 + m_2^2}{1-x}}{8\beta^2} - \frac{(m_1^2 - m_2^2)^2}{8\beta^2 \left(\frac{\mathbf{k}_\perp^2 + m_1^2}{x} + \frac{\mathbf{k}_\perp^2 + m_2^2}{1-x} \right)} \right].$$

where we took the parameters as
mass of u-quark : $m_1 = 0.25\ GeV$,
mass of $\bar{s}$-quark : $m_2 = 0.5\ GeV$,
$\beta = 0.393\ GeV$.

-B. -W. Xiao, X. Qian, and B. -Q. Ma, Eur. Phys. J. A 15, 523 (2002).

Formalism

- *Definition of associated GPD for kaon* :

$$H_K(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \left\langle M(P'') \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| M(P') \right\rangle \Big|_{z^+=0,\mathbf{z}_\perp=\mathbf{0}_\perp}$$

$$(P'-P'')^2 = \Delta^2 = t$$

$$H_K(x,\xi,t) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\big[\psi_0^{*\uparrow,\uparrow}(x'',k'')\psi_0^{\uparrow,\uparrow}(x',k') + \psi_0^{*\uparrow,\downarrow}(x'',k'')\psi_0^{\uparrow,\downarrow}(x',k') + \psi_0^{*\downarrow,\uparrow}(x'',k'')\psi_0^{\downarrow,\uparrow}(x',k') + \psi_0^{*\downarrow,\downarrow}(x'',k'')\psi_0^{\downarrow,\downarrow}(x',k')\big].$$

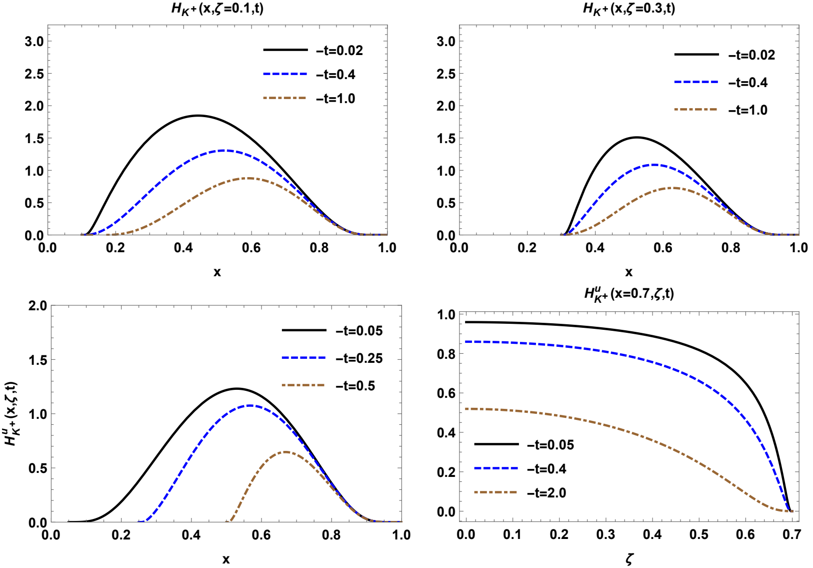
- Nonzero skewness (ξ) $\rightarrow$ relavant to experimental extraction of GPDs.

- The unpolarized kaon GPD for u-quark :

$$
\begin{aligned}
H^{(u)} &= \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\Bigg[\mathbf{k}_\perp^2 - \frac{(1-x)^2}{1-\xi^2}\frac{\boldsymbol{\Delta}_\perp^2}{4} - \frac{\xi(1-x)}{1-\xi^2}(k_x\Delta_x + k_y\Delta_y) \\
&\quad + \mathcal{M}'_u\mathcal{M}''_u\Bigg]\frac{\varphi_u^*(x'',\mathbf{k}''_\perp)\varphi_u(x',\mathbf{k}'_\perp)}{\sqrt{\mathbf{k}''^2_\perp + l''^2_u}\sqrt{\mathbf{k}'^2_\perp + l'^2_u}},
\end{aligned}
$$

with

$$
\begin{aligned}
\mathcal{M}'_u &= \frac{1-x}{1+\xi}m_1 + \frac{x+\xi}{1+\xi}m_2, \\
\mathcal{M}''_u &= \frac{1-x}{1-\xi}m_1 + \frac{x-\xi}{1-\xi}m_2, \\
l'^2_u &= \frac{1-x}{1+\xi}m_1^2 + \frac{x+\xi}{1+\xi}m_2^2 - \frac{(1-x)(x+\xi)}{(1+\xi)^2}(m_1-m_2)^2, \\
l''^2_u &= \frac{1-x}{1-\xi}m_1^2 + \frac{x-\xi}{1-\xi}m_2^2 - \frac{(1-x)(x-\xi)}{(1-\xi)^2}(m_1-m_2)^2.
\end{aligned}
$$


$H_{K^+}(x,\zeta=0.1,t)$
−t=0.02
−t=0.4
−t=1.0
x
$H_{K^+}(x,\zeta=0.3,t)$
−t=0.02
−t=0.4
−t=1.0
x
$H^u_{K^+}(x,\zeta,t)$
−t=0.05
−t=0.25
−t=0.5
x
$H^u_{K^+}(x=0.7,\zeta,t)$
−t=0.05
−t=0.4
−t=2.0
ζ

Quark Wigner distributions

- To understand the hadron structure more precisely, *the joint position and momentum distributions* i.e. the quantum analog to the classical phase-space distributions such as Wigner distributions were introduced.
- Wigner distributions were first introduced by E. Wigner in 1932.

-E. Wigner Phys. Rev. 70, 749 (1932)

- These distributions are the quasi-probabilistic distributions.
- In QCD, Wigner distributions were first introduced by Xiangdong Ji.

-X. -d. Ji, Phys. Rev. Lett. 91, 062001 (2003).

$$\rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} W^{[\Gamma]}(\vec{\Delta}_\perp, \vec{k}_\perp, x),$$

$$W^{[\Gamma]}(\vec{\Delta}_\perp, \vec{k}_\perp, x) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ip\cdot z} \langle P''| \bar{\psi}(-z/2)\Gamma\psi(z/2) |P'\rangle .$$

Here, Γ indicates the Dirac γ-matrix, specifically γ^+, $\gamma^+\gamma_5$, $i\sigma^{j+}\gamma_5$.

- The probabilistic densities in mixed space:

$$\int db_y dk_x \rho_{UX}(\mathbf{b}_\perp, \mathbf{k}_\perp) = \rho_{UX}(b_x, k_y),$$

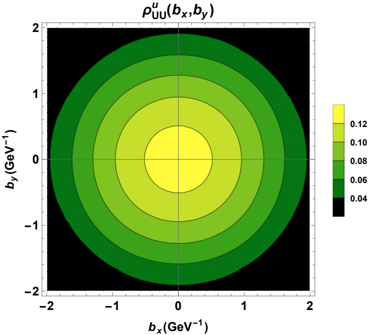
or

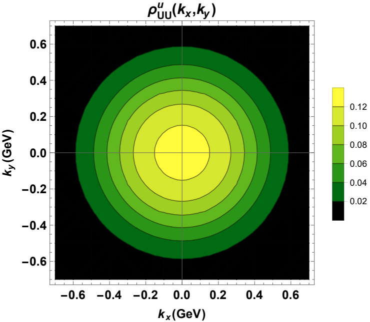
$$\int db_x dk_y \rho_{UX}(\mathbf{b}_\perp, \mathbf{k}_\perp) = \rho_{UX}(k_x, b_y).$$

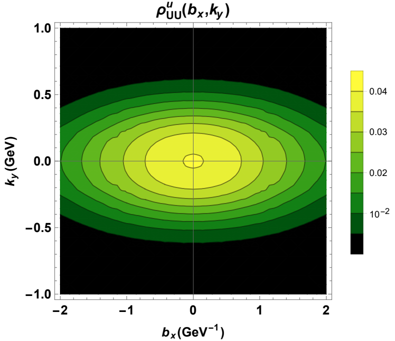
- For unpolarized quark in unpolarized kaon,

$$\begin{aligned}
\rho_{UU}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) &= \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x), \\
&= \frac{1}{16\pi^3} \int \frac{d\Delta_x d\Delta_y}{(2\pi)^2} \cos(\Delta_x b_x + \Delta_y b_y) \Big[\mathbf{k}_\perp^2 - (1-x)^2 \frac{\mathbf{\Delta}_\perp^2}{4} \\
&\quad + ((1-x)m_1 + x m_2)^2 \Big] \frac{\varphi_u^\dagger(x, \mathbf{k}''_\perp) \varphi_u(x, \mathbf{k}'_\perp)}{\sqrt{\mathbf{k}''^2_\perp + l_u^2} \sqrt{\mathbf{k}'^2_\perp + l_u^2}},
\end{aligned}$$

- For ρ_{UU} in transverse impact-parameter plane, we choose $k_\perp = 0.2\ GeV$ and in transverse momentum plane, $b_\perp = 0.4\ GeV^{-1}$.

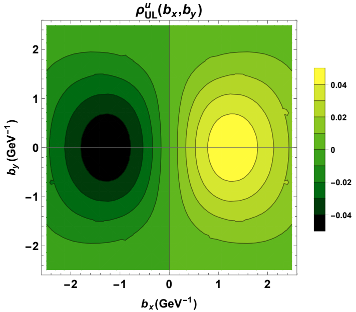

$\rho^u_{UU}(b_x,b_y)$
$b_y(\mathrm{GeV}^{-1})$
$b_x(\mathrm{GeV}^{-1})$
2
1
0
−1
−2
0.12
0.10
0.08
0.06
0.04

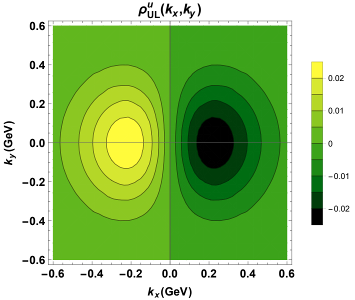

$\rho^u_{UU}(k_x,k_y)$
$k_y(\mathrm{GeV})$
$k_x(\mathrm{GeV})$
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.12
0.10
0.08
0.06
0.04
0.02

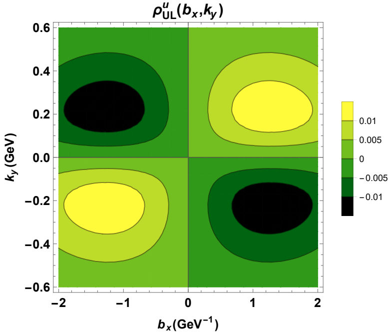

$\rho^u_{UU}(b_x,k_y)$
$k_y(\mathrm{GeV})$
$b_x(\mathrm{GeV}^{-1})$
1.0
0.5
0.0
−0.5
−1.0
−2
−1
0
1
2
0.04
0.03
0.02
10^{-2}

For the longitudinally-polarized quark in the unpolarized kaon, we have

$$\begin{aligned} \rho_{UL}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) &= \rho^{[\gamma^+\gamma_5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x), \\ &= \frac{1}{16\pi^3}\int \frac{d\Delta_x d\Delta_y}{(2\pi)^2} \sin(\Delta_x b_x + \Delta_y b_y)(1-x)(k_y\Delta_x - k_x\Delta_y) \\ &\times \frac{\varphi_u^\dagger(x, \mathbf{k}''_\perp)\varphi_u(x, \mathbf{k}'_\perp)}{\sqrt{\mathbf{k}''^2_\perp + l_u^2}\sqrt{\mathbf{k}'^2_\perp + l_u^2}}. \end{aligned}$$


$\rho^u_{UL}(b_x,b_y)$
$b_y(\mathrm{GeV}^{-1})$
$b_x(\mathrm{GeV}^{-1})$
0.04
0.02
0
−0.02
−0.04

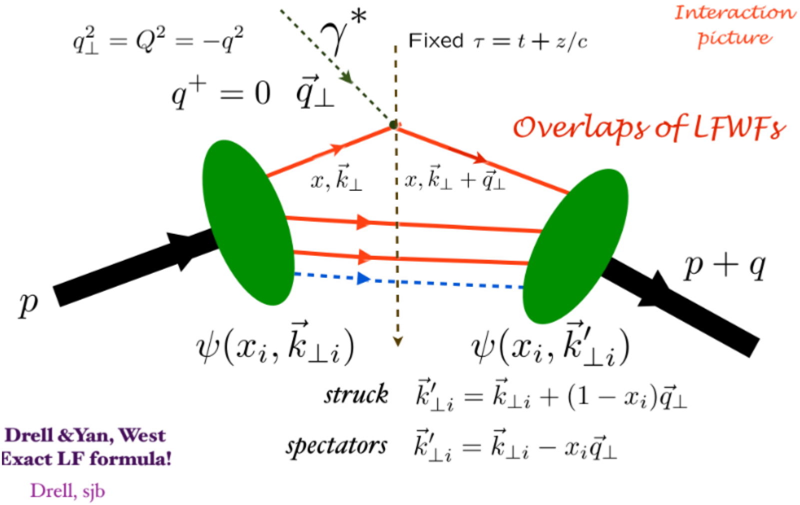

$\rho^u_{UL}(k_x,k_y)$
$k_y(\mathrm{GeV})$
$k_x(\mathrm{GeV})$
0.02
0.01
0
−0.01
−0.02


$\rho^u_{UL}(b_x,k_y)$
$k_y(\mathrm{GeV})$
$b_x(\mathrm{GeV}^{-1})$
0.01
0.005
0
−0.005
−0.01

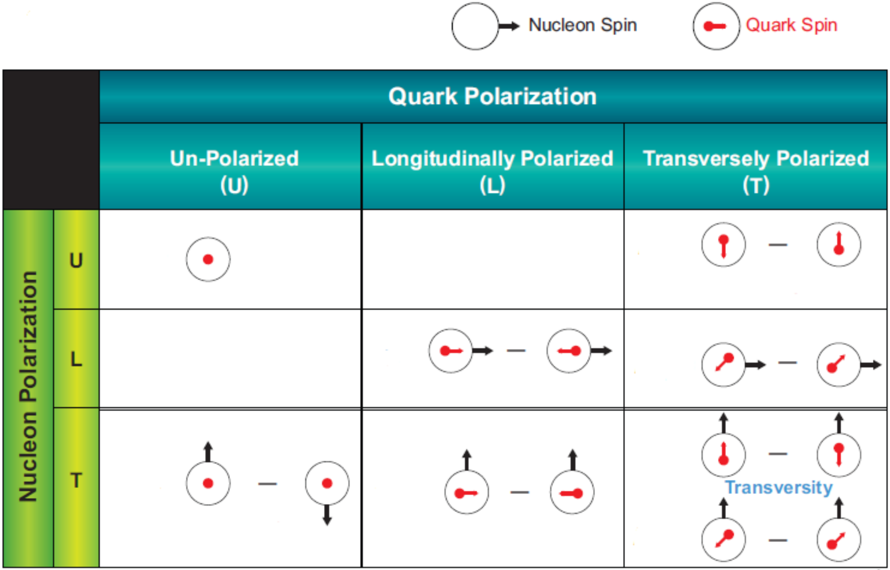
Conclusions

- To understand the 3-D structure of kaon, the GPDs plays an important role, while the Wigner distributions provide the phase-space distributions.
- A shift in the distribution peak is observed along higher magnitudes of x when there is an increase in the total momentum transfer to the kaon.
- If the momentum transfer along longitudinal direction is less, the spread is found to be maximum.
- We observe the circularly symmetric behaviour of unpolarized Wigner distribution and dipolar structure type distribution in case of unpolarized-longitudinal Wigner distribution.
- The probabilistic distributions are possible to extract from the Wigner distributions upon certain limits.


thank you



-By S. J. Brodsky


Nucleon Spin
Quark Spin
Quark Polarization
Un-Polarized (U)
Longitudinally Polarized (L)
Transversely Polarized (T)
Nucleon Polarization
U
L
T
Transversity