Ideal Relativistic Magnetohydrodynamic Simulations for QGP in Heavy-ion Collisions

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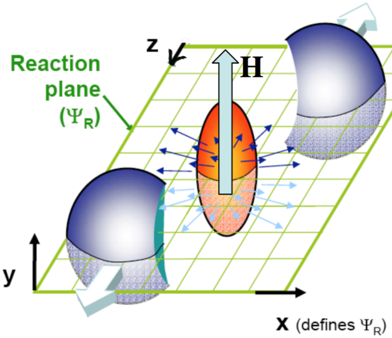
Outline of The Talk

1. Production and survival of magnetic field in relativistic heavy-ion collisions, and validity of ideal magneto-hydrodynamics approximations
2. RMHD equations and algorithm for solving them
3. RMHD simulations for relativistic heavy-ion collisions
4. Conclusions

Production of Magnetic Field in HIC



- No magnetic field in the case of central collisions. Fluctuations??
- Magnetic field generates in the non-central collisions along the y-axis at the center.
- Magnitude of magnetic field at the center can be $\sim 10^{15}$ Tesla ($\sim$ 0.1 GeV^2 $\sim 5m_\pi^2$) (10^4 times stronger than magnetic field of a magnetar).

Survival of Magnetic Field due to conducting plasma

- In the vacuum magnetic field decays immediately, while medium with a good conductivity protects magnetic field from decay.
- Medium forms at thermalization time $\tau_0 < 1$ fm in RHICE.
- Once the thermalized medium forms, evolution of the magnetized medium should be governed by relativistic magnetohydrodynamic (MHD) equations.
- Ideal MHD approximation(frozen flux) is not a very good approximation for QGP produce in RHICE as *diffusion time* ($\tau \simeq L^2\sigma/4$, $\sigma \simeq 0.04T^*$) is smaller than 1 fm, while whole plasma evolves upto 5-10 fm time.
- However, for simplicity, we consider QGP in RHICE as an ideal MHD fluid, in which conductivity at each spacetime point is considered to be infinite.

*Y. Yin, Phys. Rev. C 90, 044903 (2014); H.-T. Ding et al., Phys. Rev. D 83, 034504 (2011).

Ideal Relativistic Magneto-hydrodynamics Equations

The basic equations of relativistic magnetohydrodynamics:

- Energy-momentum tensor for e.m. field in medium,

$$T^{\alpha\beta}_{em} = \frac{1}{4\pi}[F^{\alpha}_{\gamma} G^{\beta\gamma} - \frac{1}{4} F_{\gamma\delta} G^{\gamma\delta} \eta^{\alpha\beta}]. \quad (1)$$

  $F^{\alpha\beta}$: Field-strength tensor, $G^{\alpha\beta}$: EM induction tensor.

- Energy-momentum tensor for matter part of ideal fluid,

$$T^{\alpha\beta}_{pl} = (\epsilon + P) u^{\alpha} u^{\beta} + P \eta^{\alpha\beta}. \quad (2)$$

  Here $\eta^{\alpha\beta}$ = diag(-1,1,1,1).

- Energy-momentum tensor for a pefect fluid interacting with e.m. field,

$$T^{\alpha\beta} = T^{\alpha\beta}_{pl} + T^{\alpha\beta}_{em}, \quad (3)$$

  where

$$\partial_{\alpha} T^{\alpha\beta} = 0. \quad (4)$$

Ideal Relativistic Magneto-hydrodynamics Equations

Ideal RMHD Equations

- Energy-momentum conservation equation

$$\partial_\alpha \left((\epsilon + p_g + |b|^2) u^\alpha u^\beta - b^\alpha b^\beta + (p_g + \frac{|b|^2}{2}) \eta^{\alpha\beta} \right) = 0 \quad (5)$$

- Maxwell's equations

$$\partial_\alpha (u^\alpha b^\beta - u^\beta b^\alpha) = 0 \quad (6)$$

- Four-vector b^α is related with the magnetic field and fluid velocity as,

$$b^\alpha = \gamma \left(\vec{v}.\vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v}.\vec{B}) \right) \quad (7)$$

- Total pressure of the fluid, $p = p_g + \frac{|\vec{B}|^2}{2\gamma^2} + \frac{(\vec{v}.\vec{B})^2}{2}$.

Ref.: A. Mignone and G. Bodo, Mon. Not. R. Astron. Soc.368, 1040 (2006).

Algorithm for solving RMHD Equations

- For computational purpose, the RMHD equations can be conveniently put in the following conservational form*,

$$\frac{\partial \mathbf{U}}{\partial t} + \sum_k \frac{\partial \mathbf{F}^k(\mathbf{U})}{\partial x^k} = 0, \tag{8}$$

where vector $\mathbf{U}$ is a collection of conservative variables,

$$\mathbf{U} = (m_x, m_y, m_z, B_x, B_y, B_z, E).$$

- m_k is the momentum density along k-th direction (using $p_g = \epsilon/3$),

$$m_k = (\frac{4}{3}\epsilon\gamma^2 + B^2)v_k - (\vec{v}.\vec{B})B_k. \tag{9}$$

- The total energy density,

$$E = \frac{4}{3}\epsilon\gamma^2 - p_g + \frac{B^2}{2} + \frac{v^2B^2 - (\vec{v}.\vec{B})^2}{2}. \tag{10}$$

*Ref.: A. Mignone and G. Bodo, Mon. Not. R. Astron. Soc.368, 1040 (2006).

Algorithm for solving RMHD Equations

- $\mathbf{F}^k$ are the fluxes along the $x^k = (x, y, z)$ directions,

$$\mathbf{F}^x(\mathbf{U}) = \begin{bmatrix} m_x v_x - B_x \frac{b_x}{\gamma} + p \\ m_y v_x - B_x \frac{b_y}{\gamma} \\ m_z v_x - B_x \frac{b_z}{\gamma} \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ m_x \end{bmatrix}$$

- $\mathbf{F}^{y,z}(\mathbf{U})$ are similarly defined by appropriate change of indices.

Algorithm for solving RMHD Equations

- U evolve with time following the conservation equation.
- Independent variables, $V = (\vec{v}, p_g, \vec{B})$, are required when computing the fluxes.
- To recover V from U, define : $W = \frac{4}{3}\epsilon\gamma^2$ and $S = \vec{m}.\vec{B}$,

$$E = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|\vec{B}|^2 - \frac{S^2}{2W^2} \tag{11}$$

$$|\vec{m}|^2 = (W + |\vec{B}|^2)^2\left(1 - \frac{1}{\gamma^2}\right) - \frac{S^2}{W^2}(2W + |\vec{B}|^2) \tag{12}$$

- In the beginning of each time step, $\vec{m}$, $\vec{B}$ and S are known. γ in terms of W (only unknown) is,

$$\gamma = \left(1 - \frac{S^2(2W + |\vec{B}|^2) + |\vec{m}|^2 W^2}{(W + |\vec{B}|^2)^2 W^2}\right)^{-\frac{1}{2}} \tag{13}$$

Algorithm for solving RMHD Equations

- From EoS,

$$p_g(W) = \frac{W}{4\gamma^2} \tag{14}$$

- Unknown quantity W can be found out from,

$$f(W) = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|\vec{B}|^2 - \frac{S^2}{2W^2} - E = 0 \tag{15}$$

- This equation is solved using Newton-Raphson method to get W.
- Once W has been computed, one can get back γ and p_g . Velocities can be found by expression of m_k,

$$v_k = \frac{1}{(W + |\vec{B}|^2)}\left(m_k + \frac{S}{W}B_k\right) \tag{16}$$

Simulation details

- We have performed (3+1)-d simulation for low energy collisions with $\sqrt{s} = 20$ GeV and with Cu nuclei on $200 \times 200 \times 200$ lattice with 0.1 fm lattice spacing.
- Because of computational limitations we have taken radius of copper nuclei as 4.0 fm with skin width 0.4 fm.
- Optical Glauber and Glauber Monte-Carlo like initial energy density are used for the simulations.
- We have taken EOS of ideal relativistic gas $p_g = \rho/3$ and zero chemical potential for simplicity.
- Initial central temperature is set to be ~ 180 MeV.

Simulation details

- Magnetic field is produced by considering two oppositely moving, uniformly charged, spheres and by taking appropriately Lorentz γ factor for their motion. The initial magnetic field profile is calculated at the thermalization time τ_0 of the system.
- We use Leap-Frog 2nd order method to solve ideal RMHD equations numerically in (3+1)-d.
- Initial fluid velocity in the transverse plane is taken to be zero.
- We have taken longitudinal velocity profile $\propto z$ with suitable maximum velocity at the edge of the plasma.
- We have performed our calculations in the central rapidity bin.

Flow Study by Fourier Analysis of fluid momentum distributions in the Transverse Plane

- The Fourier analysis of the azimuthal distribution function,

$$r(\phi) = \frac{\delta P(\phi)}{\overline{P}} = \frac{P(\phi) - \overline{P}}{\overline{P}} = \sum_n \Big(a_n \cos(n\phi) + b_n \sin(n\phi) \Big) \quad (17)$$

where, $a_n = \frac{1}{\pi} \int_0^{2\pi} r(\phi) \cos(n\phi)\, d\phi, \quad b_n = \frac{1}{\pi} \int_0^{2\pi} r(\phi) \sin(n\phi)\, d\phi.$

- The flow is characterized by the magnitude of flow coefficients, $v_n = \sqrt{a_n^2 + b_n^2}$ and by direction of flow ψ_n $(0 \leq \psi_n < 2\pi/n)$, where, $a_n = v_n \cos(n\psi_n)$ and $b_n = v_n \sin(n\psi_n)$.

The Azimuthal Distribution Function

$$r(\phi) = v_0 + \sum_{n=1}^{\infty} v_n \cos[n(\phi - \psi_n)] \quad (18)$$

Speed of sound in ideal MHD fluid

- Evolution of small perturbations (from the equilibrium value) in energy density, velocity and magnetic field in ideal MHD fluid provide three sound velocities for plane wave solution of perturbations with wave vector $\vec{k}$,

1. $\vec{k} \| \vec{B} \Rightarrow$ MHD equations gives magnetosonic wave with velocity,

$$c_{\|}^2 = c_s^2 \tag{19}$$

2. $\vec{k} \perp \vec{B} \Rightarrow$ MHD equations gives magnetosonic wave with velocity,

$$c_{\perp}^2 = c_s^2 + v_A^2 \tag{20}$$

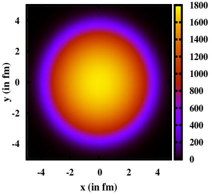
3. $\vec{k} \| \vec{B} \perp \vec{v} \Rightarrow$ transverse wave(*Alfvén* wave) moves with velocity v_A.

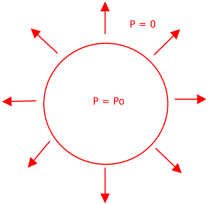
$$c_s = \left(\frac{\partial p_g}{\partial \epsilon}\right)^{1/2}, v_A \sim \left(\frac{B_0^2}{8\pi\epsilon}\right)^{1/2} \tag{21}$$

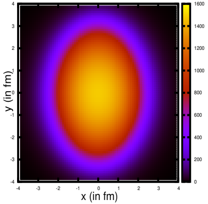
$$v_x = c_s^2 \left(\frac{xt}{\sigma_x^2} \right), \qquad (22)$$

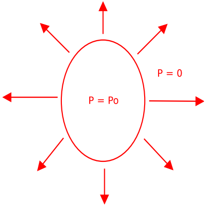
$$v_y = c_s^2 \left(\frac{yt}{\sigma_y^2} \right). \qquad (23)$$

(J-Y Ollitrault, Eur. J. Phys. 29 (2008) 275-302.)

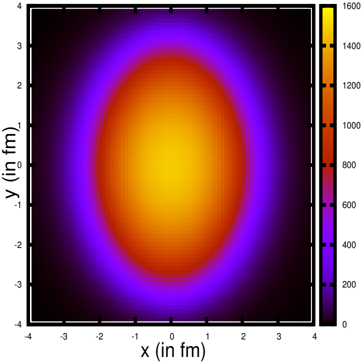


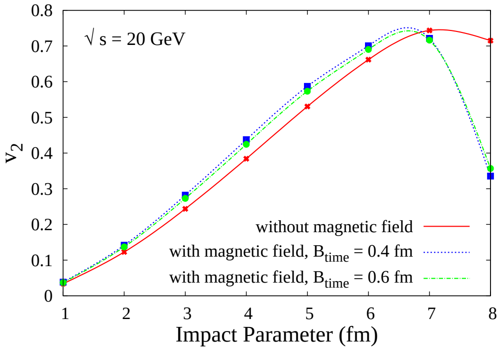






Ideal Relativistic Magnetohydrodynamics Simulation result :

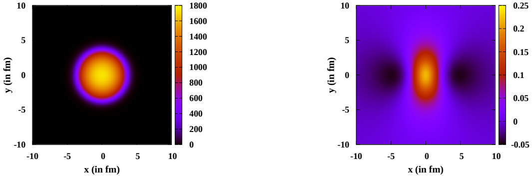




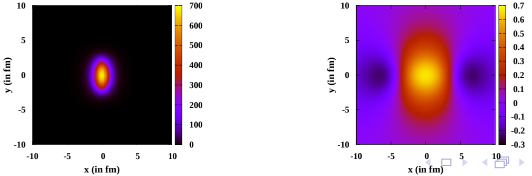
At low impact parameter, magnetic field is well inside the plasma region, hence argument of sound speed holds true.

At high impact parameter, extension of magnetic field much outside the plasma region. Therefore according to the Lenz's law, magnetic field opposes the expansion of the conducting fluid in the x-direction.

$b = 1\ fm$:

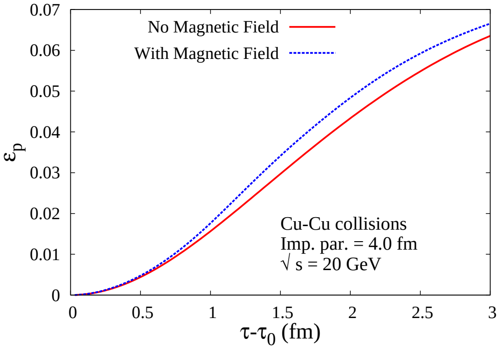


$b = 7\ fm$:



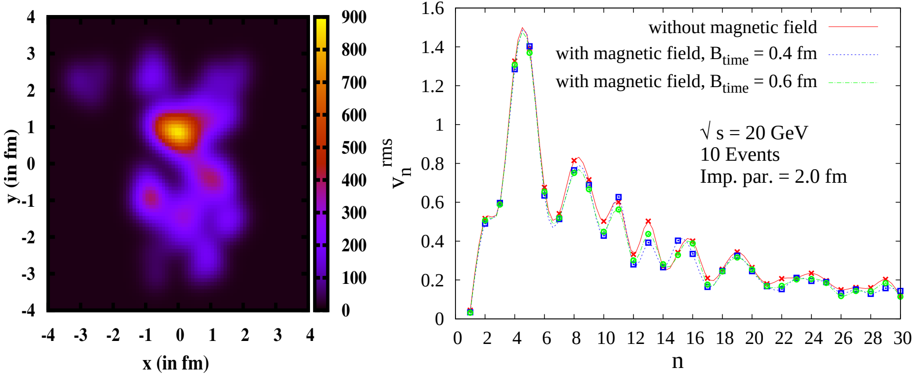
ϵ_p Plot

$$\epsilon_p = \frac{T_{pl}^{xx} - T_{pl}^{yy}}{T_{pl}^{xx} + T_{pl}^{yy}} = \frac{v_x^2 - v_y^2}{v_x^2 + v_y^2 + \frac{2p_g}{(\epsilon+p_g)\gamma^2}} \tag{24}$$

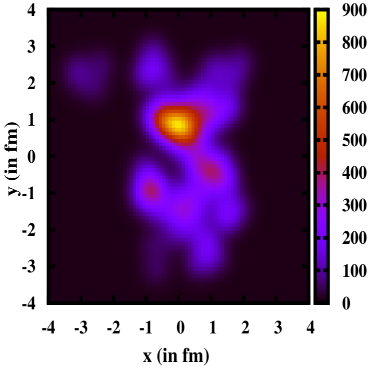


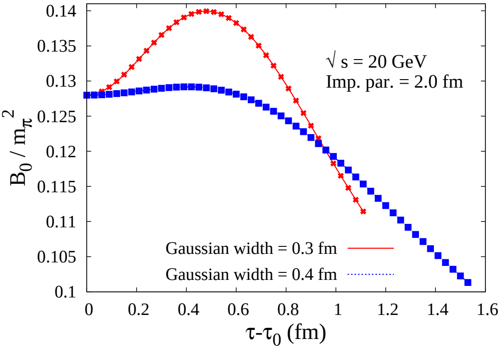
$$v_2 = \frac{T_{pl}^{0x} - T_{pl}^{0y}}{T_{pl}^{0x} + T_{pl}^{0y}} = \frac{v_x - v_y}{v_x + v_y} \tag{25}$$

Power spectrum



Enhancement of magnetic field due to fluctuation





- Magnetic flux can get reorganized due to evolution of density fluctuations. This can lead to enhancement of magnetic field at some spacetime points inside fluid.
- Local vortices(turbulent dynamics) in QGP $\Rightarrow$ Dynamo effect; local enhancement of magnetic field.

Conclusions

- Magnetic field in the fluid can change the ellipic flow depending upon the impact parameter of the collision. It can be very important in the study of viscosity of QGP and can provide signal of the presence of magnetic field.

- We have found that magnetic field can be temporarily enhanced in the fluid due to the evolution of fluctuations.

Thank You !!