Universal Physics of Nuclear Few-body systems

in Pionless Effective Theory

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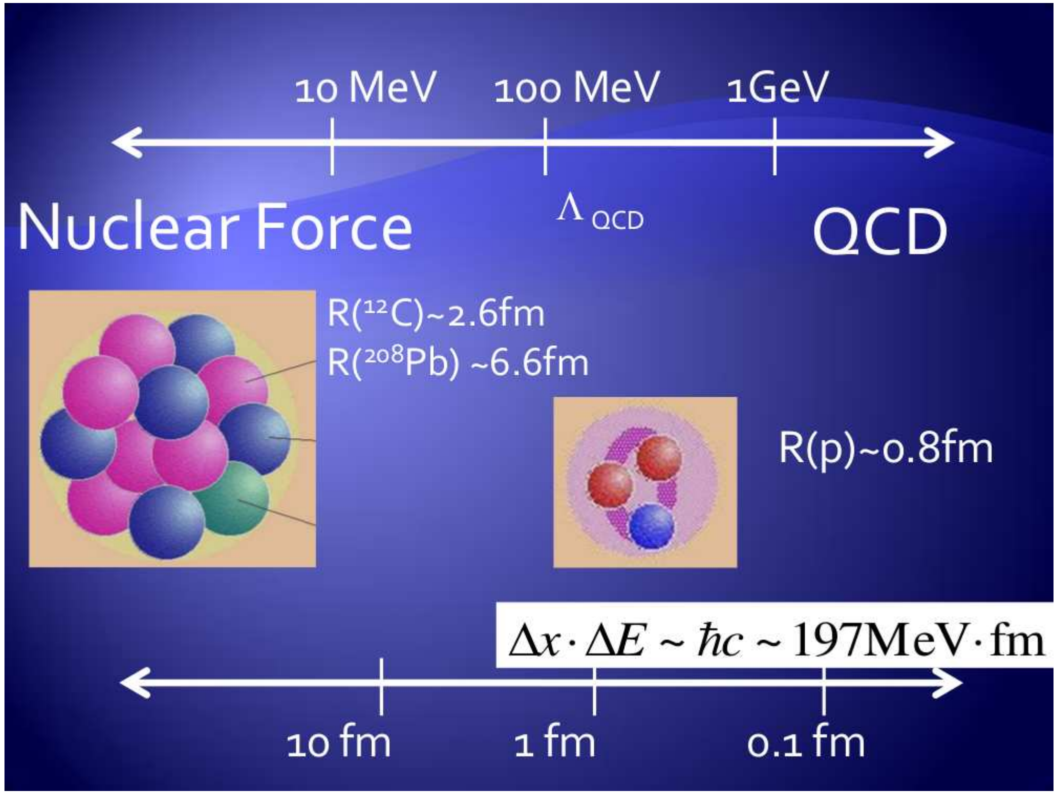
Outline

- **Introduction & Motivation:**
  - Study of Few-Body Systems of light nuclei
  - Search and Prediction of exotic bound states using Universality
    $\hookrightarrow$ e.g., Exotic bound states of n-rich **Hypernuclei**
- **Universal physics in 3-body clusters:**
  - Two-body sector
  - Three-body sector
- **Framework:** Low-energy Effective Field Theory
  $\hookrightarrow$ **TOY MODEL:** System of 3 identical interacting bosons
- **Few Results:** Feasibility of Efimov-like bound states in $nn\Lambda$

QCD and Strong Nuclear Force

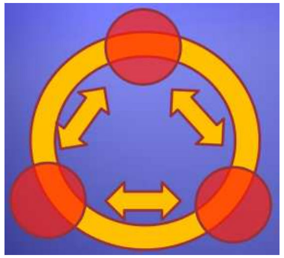
- **ASYMPTOTIC FREEDOM:** QCD becomes highly non-pertubative below $\lesssim 1$ GeV
- How strong force binds nucleons and other baryons within nuclei?
- Focus on the Low-energy or long distance scales $\gtrsim 1$ fm



Effective QCD & dynamics of light nuclei

☞ Analytical Approaches?

- **QCD** $\rightarrow$ True theory of underlying structure & dynamics of elementary hadrons
- Straightforward field theoretical extensions are not viable for systems with $A \geq 3$
  QM Few-body Problem !
- **Classical 3-body Problem?** Very difficult to solve analytically
  Classical $\geq$ 4-body Problem? No general analytical solns. with **non-linear** ints.



- No general analytical solution of **non-linear** multi-hadron dynamics in QCD

☞ Approximate / Semi-analytic / Numerical Methodologies:

- **Mean-field & Variational approaches** $\rightarrow$ statistically large $A \rightarrow \infty$ (continuum matter)
- **Potential Models** $\rightarrow$ small A; not very systematic, i.e., lack **universality**
- **Cluster EFTs** $\rightarrow$ $A \gtrsim$ 2,3,4, ...; simple yet powerful, systematic, model independent & **universal**

Exotics in Nuclear Spectroscopy

☞ **Color-singlet hadrons:**

- **Stable** hadronic bound states: *Mesons* ($q\bar{q}$) and *Baryons* (qqq)
- QCD does not restrict other **exotic metastable color-singlet** configurations

☞ **Compact multi-quark / -gluon Exotics** $\rightarrow$ Quark Model picture

Tetraquarks ($qq\bar{q}\bar{q}$), *Pentaquarks* ($qqqq\bar{q}$), *Dibaryons* ($qqqqq$), ...

Quarkonia ($q\bar{q} - Q\bar{Q},\ q\bar{Q} - Q\bar{q},\ qQ - \bar{q}\bar{Q},\ ...$) ; $Q = c, b, t$

Glueballs ($gg,\ ggg,\ ...$), *Hybrids* ($Q\bar{Q}g,\ ...$)

☞ **Meson Exotics** $\rightarrow$ "Molecular" nature to contrast with *tetraquark* structure

$f_0(500)$ or "σ" ($\pi\pi$), $K(800)$ or "κ" (πK), $f_0(980)$ & $a_0(980)$ ($K\bar{K}$), ...

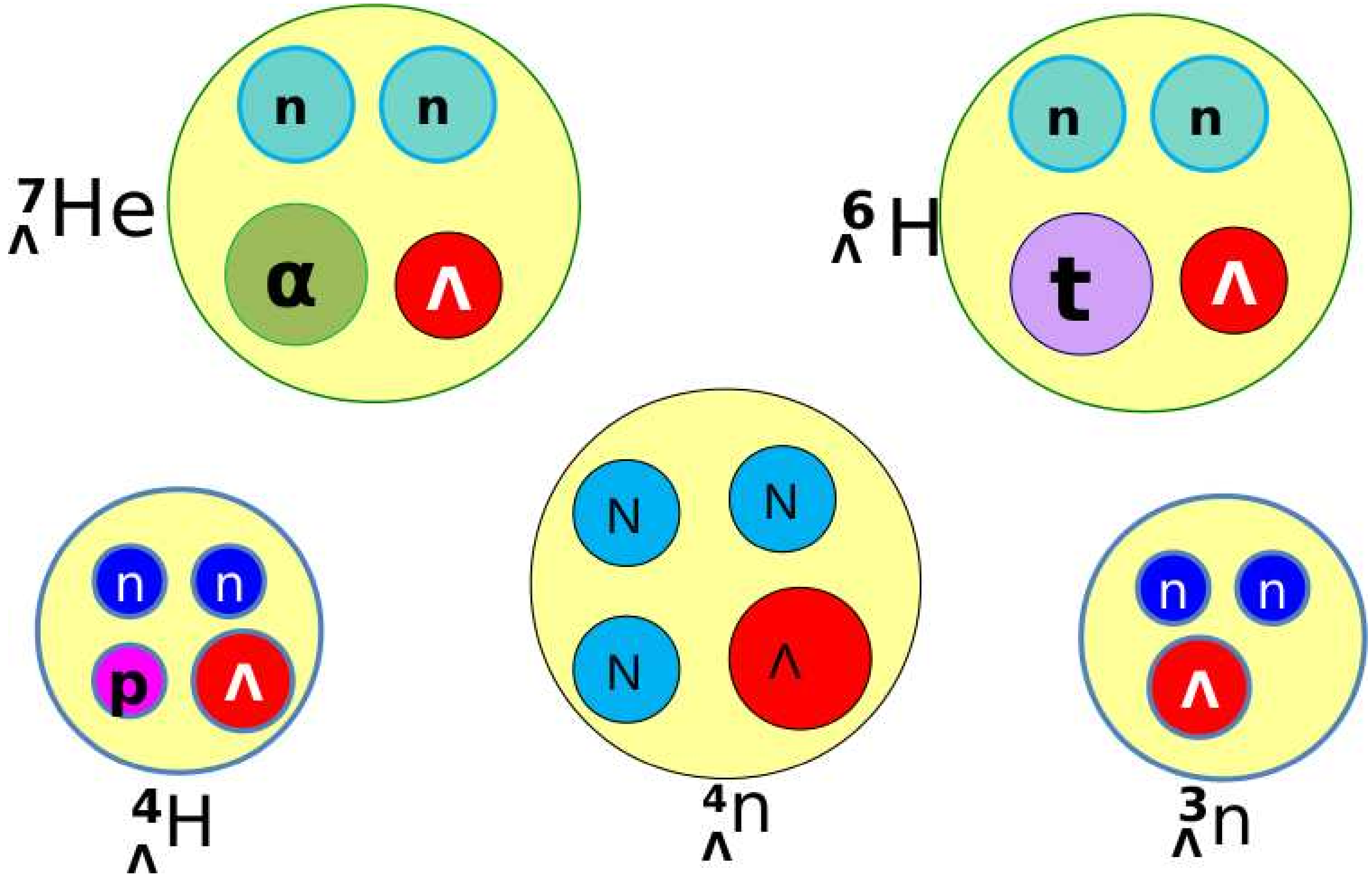
☞ **Exotic Nuclei** $\rightarrow$ "*Halo*" Clusters of "Molecular" nature

Neutron-rich states: *Tetra-neutron* ($^4n \rightarrow nnnn$), ...

Hypernucler states: *Hypertriton* ($^3_\Lambda$H$\rightarrow \Lambda d$), ...

Recently suggested n-rich Λ-Hypernuclei

☞ **Single-Λ systems:** $^{6}_{\Lambda}$H, $^{4}_{\Lambda}$H, $^{7}_{\Lambda}$He, $^{4}_{\Lambda}$n, $^{3}_{\Lambda}$n, ...??

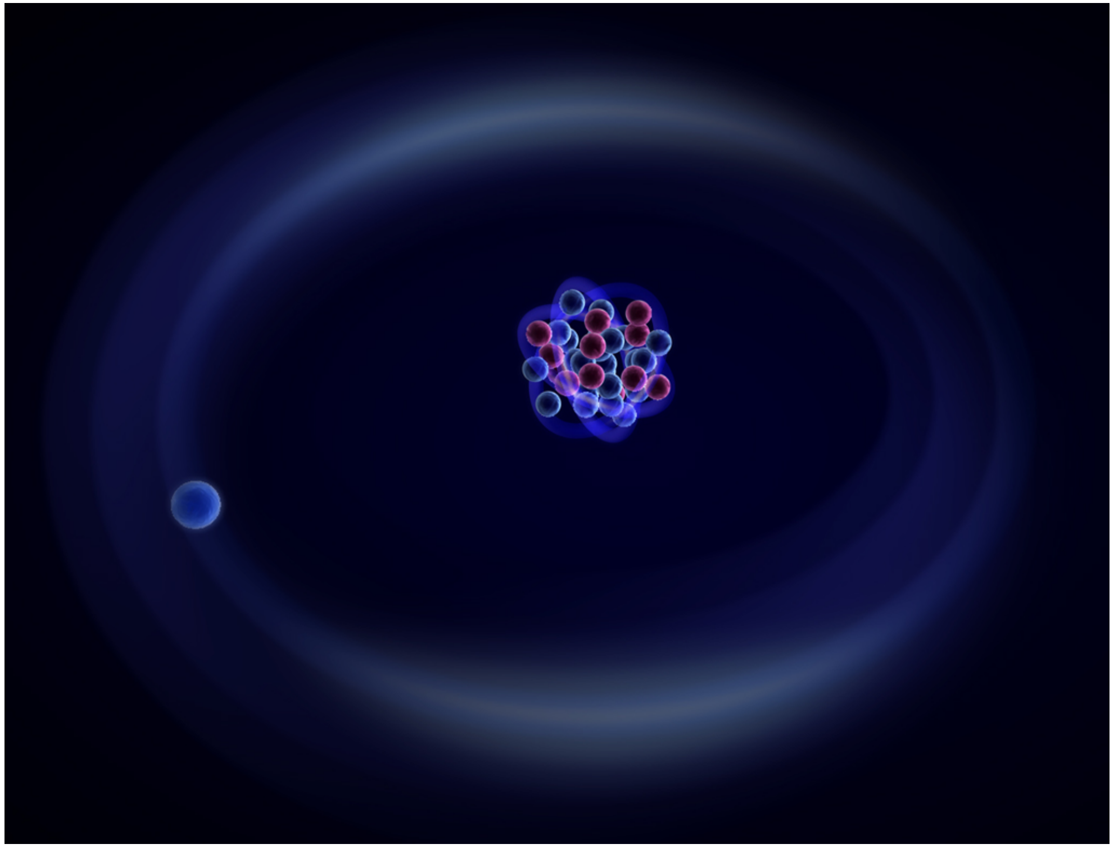


☞ **Double-Λ systems:** $^{4}_{\Lambda\Lambda}$H, $^{6}_{\Lambda\Lambda}$He, $^{5}_{\Lambda\Lambda}$H, $^{5}_{\Lambda\Lambda}$He, $^{10}_{\Lambda\Lambda}$Be, $^{13}_{\Lambda\Lambda}$B, ...??

What are Neutron-rich Halo Clusters?

☞ Loosely bound hadronic cluster with **tightly bound compact cores** with one to few **loosely bound neutrons** (near n-drip-lines) orbiting far away from the cores

e.g., famous ones $\rightarrow$ ^{6}He, ^{11}Li



^{37}Mg one-neutron halo (credit: Ken-icheiro Yoneda, RIKEN, Japan)

Two-body Universality

☞ **Universality** of n-rich Halo/Cluster states:

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)
P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

↪ Clusters of very different compositions behaving similarly at low-energies

↪ Similar long-range properties insensitive to short-distance details

↪ Suggests natural description in low-energy EFT framework

☞ $^{\not\pi}$**EFT** $\rightarrow$ Model independent approach & distinct scale separation:

$$l_{\rm long} \gg \frac{1}{\Lambda_{\rm Cut-off} \approx m_\pi} \gtrsim l_{\rm natural/short} \sim r_0 \Big\} \text{ 2-body int. range}$$

↪ ideal for study of s-wave **threshold states** $\rightarrow k_{\rm typical} \rightsquigarrow 0$

☞ Existence of **shallow 2-body bound-states** whenever $a_0/r_0 >> 1$

$$B_2^{thr} = -\frac{k^2_{\rm typical}}{2\mu_{\rm reduced}} = \lim_{a_0\to\infty} \frac{-1}{2\mu_{\rm reduced}a_0^2}\left[1+\mathcal{O}\left(\frac{r_0}{a_0}\right)\right] \rightarrow 0$$

☞ Involves *unnatural scalings* & *fine-tuning effects* among low-energy parameters

$\hookrightarrow$ e.g., *unnaturally* large $|a_0| \gg r_0 \rightarrow l_{\rm natural/short} = \{\lambda_\pi = m_\pi^{-1}, l_{vdW}, \ldots\}$

☞ Short-distance 2-body interactions $\longrightarrow$ *Van der Waals* universality class

$$V_{L.Jones}(NN) \sim V_{vdW}(\text{atomic}) \overset{r\to\infty}{\sim} -\kappa_{\rm const.}/r^6 \;\; ; \;\; l_{vdW} = \frac{1}{2}(2\mu_{\rm reduced}\kappa_{\rm const.})^{1/4}$$

Nuclear	$n - n$	$l_\pi \sim 1/m_\pi = 1.4$ fm	$\lvert a_0\rvert = 23.7$ fm	Fine-tuned
Atomic	^{4}He $-$ ^{4}He	$l_{vdW} = 10.2\, a_{\rm Bohr}$	$\lvert a_0\rvert = 189\, a_{\rm Bohr}$	Fine-tuned
Atomic	^{3}He $-$ ^{3}He	$l_{vdW} = 13.7\, a_{\rm Bohr}$	$\lvert a_0\rvert = 33\, a_{\rm Bohr}$	Natural

☞ **Fine-tuning $\not\pi$EFT Power Counting:** *Kaplan, Savage, Wise, U. van Kolck*

- non-perturbative resummation to all orders in $k_{\rm typical}|a_0| \gg 1$
- perturbative fixed order expansion in $Q \equiv$ **E**ffective **R**ange **E**xpansion *H. Bethe*

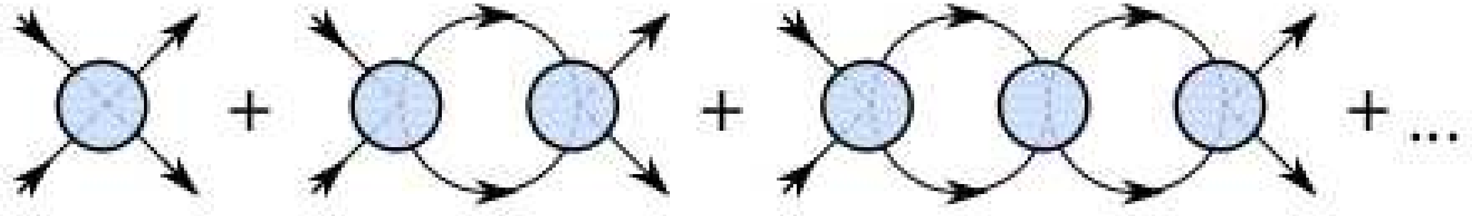
$$Q = k_{\rm typical}\, r_0 \sim k_{\rm typical}/\Lambda_{\rm Cut-off} \ll 1$$

Toy Model: Identical Bosons

☞ Most general **Non-relativistic Effective** Lagrangian for two-body sector:

symmetries: P,C,T, Galilean Inv.

$$\mathcal{L}_{2-\text{body}} = \psi^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2M} \right] \psi + \sum_{m+n=0,2,4,\ldots} g^{(2)}_{m+n} \left[(\psi^\dagger \overleftrightarrow{\nabla}^m \psi)(\psi^\dagger \overleftrightarrow{\nabla}^n \psi) \right] + \cdots$$



☞ **Unitarization:** Resum $N^{\rm th}$ order derivatively coupled tree vertices $\mathcal{V}_N^{\rm tree} \sim \mathcal{O}\left(Q^N\right)$

Q-Counting: $Q = \frac{k_{\rm typical}}{\Lambda_{\rm Cut-off} \sim m_\pi} \ll 1$; $g_j^{(2)}(\mu) \sim Q^{-j-1}$; $\mathcal{I}_{\rm loop}(\mu) \sim \mu \sim Q$

☞ **RG signature:**

$\hookrightarrow$ Existence of a **non-trivial UV fixed point** in $\{g_0^{(2)}, \cdots, g_N^{(2)}\}$ space

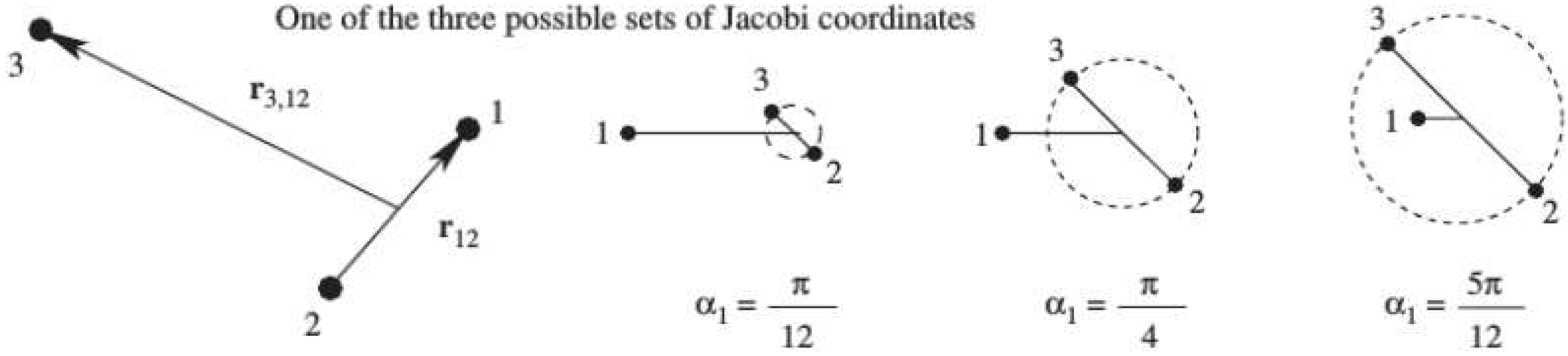
$\hookrightarrow$ **Unitary Limit:** Two-body interactions becoming resonant for $a_0 \to \pm\infty$

Three-body Universality: Interacting 3-boson system

- **Minimal Criterion:** at least two particle pairs with very large scattering lengths
  $\hookrightarrow$ system becomes approximately scale invariant and resonant

- **Hyperspherical representation:** Hyper-radius R and Hyper-angles α_k

One of the three possible sets of Jacobi coordinates

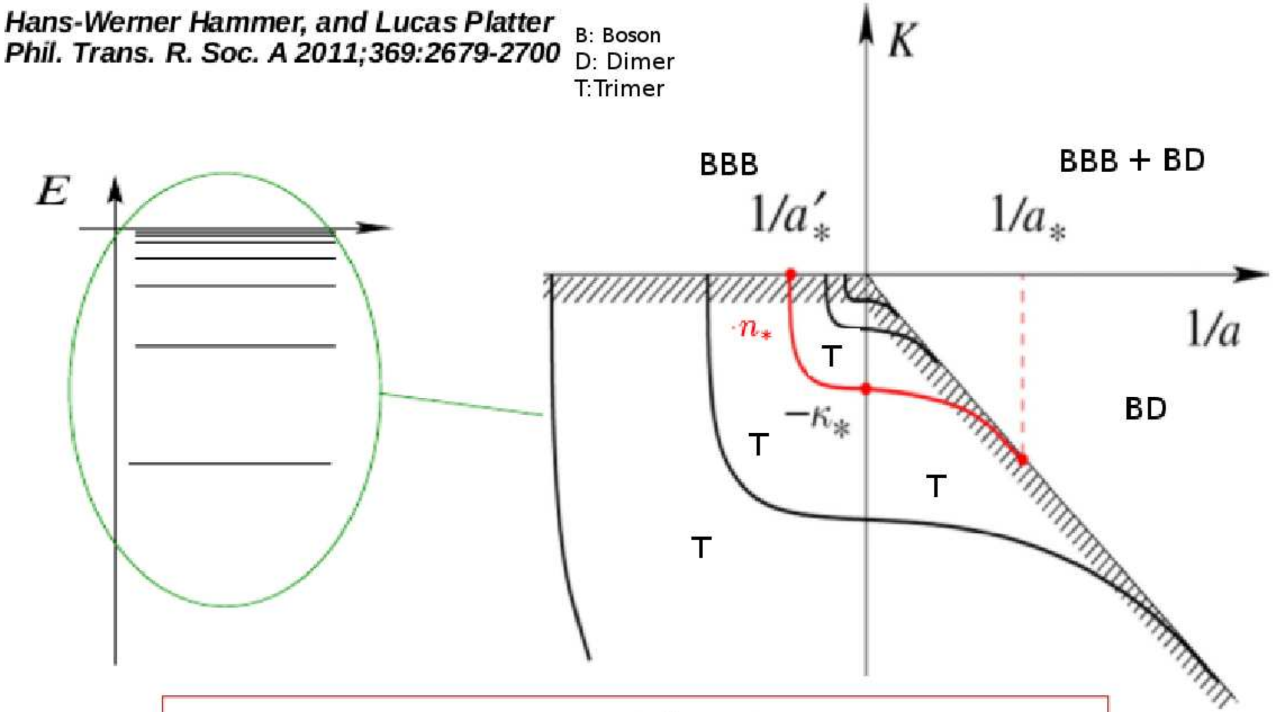


E. Braaten, H.-W. Hammer / Physics Reports 428 (2006) 259–390

- **Radial Faddeev Eqns.** in co-ordinate representation with suitable asymptotic boundary conditions yields an *attractive* $1/R^2$ potential *V. Efimov*, Phys. Lett. B33, 563 (1970)

$$\frac{1}{2\mu_{\rm reduced}a_0^2} \le |B_3^{(n)}| \sim \left(e^{-2\pi/s_0}\right)^{n-n_*}\frac{\kappa_*^2}{2\mu_{\rm reduced}} \le \frac{1}{2\mu_{\rm reduced}r_0^2}\ ;\ n = 0, \ldots, n_{max}$$

Efimov Effect: Energy Spectrum of 3-boson system



$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a \to 0} 515.035\ldots$$

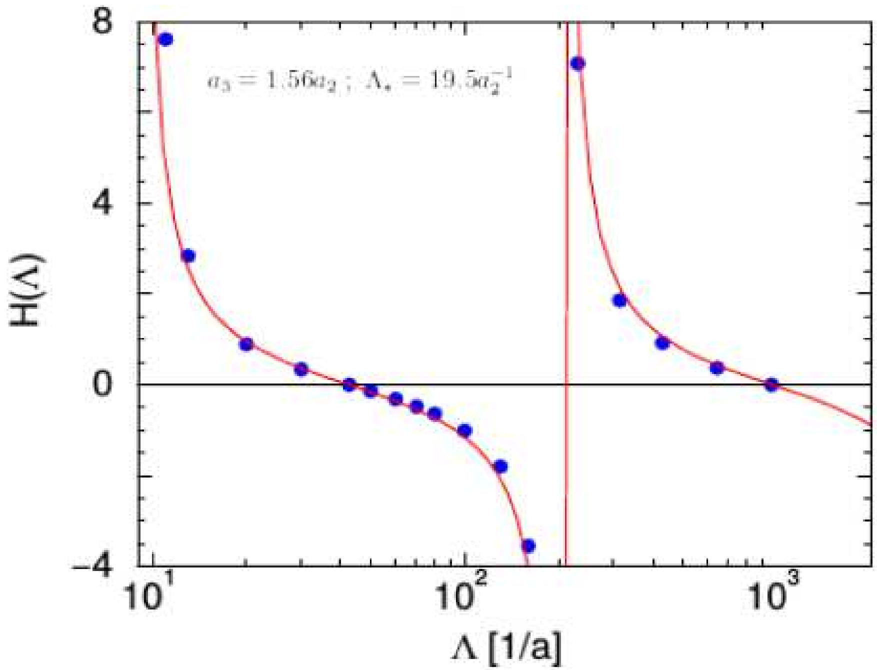
☞ **Unitary Limit:** Infinitely many arbitrarily shallow 3-body bound states emerging from zero-energy threshold

☞ **Universality:** Only depends on Ratio of masses & Total Spin and Isospin

RG Limit Cycle: Interacting 3-boson system

- **RG signature:** Onset of a **IR Limit Cycle** behavior in the 3-body coupling $H(\Lambda)$
- Breakdown of *continuous scale invariance* $\rightsquigarrow$ *discrete scaling symmetry*
- Momentum regulator (Λ) dependence in three-body coupling H:

$$\mathcal{L}_{3-\mathrm{body}} \supset \frac{H(\Lambda)}{\Lambda^4}(\psi^\dagger\psi)^3 + \cdots$$



$$\frac{\mathrm{doll}^{(n)}}{\mathrm{doll}^{(n+1)}} \approx e^{2\pi/s_0} \approx 515.035\ldots$$

with $\quad s_0 = 1.00624$

P.F. Bedaque et al. / Nuclear Physics A 646 (1999) 444–466

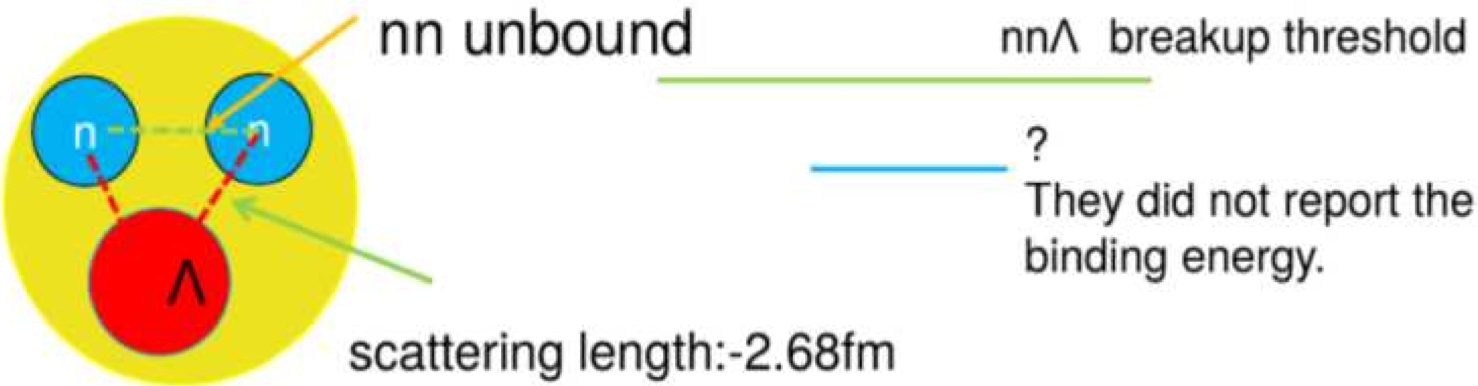
- **RG Limit Cycle:** Periodic self-similar repetition as in *Matryoshka dolls*

$J = 1/2\,,\, I = 1\ \&\ S = -1\ nn\Lambda$ Hypernucleus??

PHYSICAL REVIEW C 88, 041001(R) (2013)

Search for evidence of ${}^{3}_{\Lambda}n$ by observing $d + \pi^-$ and $t + \pi^-$ final states in the reaction of ${}^{6}\mathrm{Li} + {}^{12}\mathrm{C}$ at 2A GeV

C. Rappold,[1,2,*] E. Kim,[1,3] T. R. Saito,[1,4,5,†] O. Bertini,[1,4] S. Bianchin,[1] V. Bozkurt,[1,6] M. Kavatsyuk,[7] Y. Ma,[1,4] F. Maas,[1,4,5] S. Minami,[1] D. Nakajima,[1,8] B. Özel-Tashenov,[1] K. Yoshida,[1,5,9] P. Achenbach,[4] S. Ajimura,[10] T. Aumann,[1,11] C. Ayerbe Gayoso,[4] H. C. Bhang,[3] C. Caesar,[1,11] S. Erturk,[6] T. Fukuda,[12] B. Göküzüm,[1,6] E. Guliev,[7] J. Hoffmann,[1] G. Ickert,[1] Z. S. Ketenci,[6] D. Khaneft,[1,4] M. Kim,[3] S. Kim,[3] K. Koch,[1] N. Kurz,[1] A. Le Fèvre,[1,13] Y. Mizoi,[12] L. Nungesser,[4] W. Ott,[1] J. Pochodzalla,[4] A. Sakaguchi,[9] C. J. Schmidt,[1] M. Sekimoto,[14] H. Simon,[1] T. Takahashi,[14] G. J. Tambave,[7] H. Tamura,[15] W. Trautmann,[1] S. Voltz,[1] and C. J. Yoon[3]
(HypHI Collaboration)



Formation recently suggested by the experimental studies by the HypHI Collaboration (2013)

$nn\Lambda$ Searches and Predictions

☞ **Currently still a controversial topic:**

- *No concrete* experimental data other than rough bounds by **HypHI**
- **Supported in Lattice QCD:** *Beane et. al. NPLQCD Collaboration (2013)*
- **Inconsistent with Potential Models:** *Barnea, Gall, Contessi, Garcilazo, et al....*
  Models with $\Lambda N - \Sigma N$ mixing fail to reproduce $n\Lambda$ scattering length, ${}^{3}_{\Lambda}$H binding energy, and the ground and first excited states energies of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He.
- Expect more definitive exp. results in future from **ALICE, FAIR,..**

☞ First **EFT** analysis to investigate possible formation of Efimov-like three-body bound states *Ando et al., PRC* **92** *(2015) 024325*

$nn\Lambda$ @ LO in $\not\pi$EFT

- **RG Limit cycle signature:** Qualitative investigation of Efimov-like bound states
- **$\not\pi$EFT:** Pions are integrated out, $\Lambda_H \sim m_\pi$
- **Avoid coupled-channel analysis:** $\Sigma^{0,\pm}$-hyperons not explicitly incorporated

$$\delta M = M_\Sigma - M_\Lambda \simeq 80\ \mathrm{MeV} \sim \Lambda_{\mathrm{Hard}} = m_\pi \gg B_{nn\Lambda} \lesssim 1\ \mathrm{MeV}$$

- **2-body input Parameters in EFT:**
  - nn-Singlet Scat. length: $a_{nn} = -18.5$ fm (exp.) *Teramond & Gabioud*
  - $n\Lambda$-Singlet Scat. length: $a_{s(n\Lambda)} \xrightarrow{\text{isospin}} {}^1a_{p\Lambda} = -2.90$ fm χEFT *Polinder et. al.*
  - $n\Lambda$-Triplet Scat. length: $a_{t(n\Lambda)} \xrightarrow{\text{isospin}} {}^3a_{p\Lambda} = -1.60$ fm χEFT *Polinder et. al.*
- **3-body input:** (no datum currently) Rough estimate given from **HypHI Coll.**

  $\hookrightarrow B_{nn\Lambda} \sim 0.01 - 1.1\ \mathrm{MeV}$

1-Body Sector Lagrangian

☞ Explicit degrees of freedom:

- Neutron field N
- Λ-Hyperon field Λ

$$\mathcal{L}_n = N^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2M_n} \right] N + \text{higher derivatives}$$

$$\mathcal{L}_\Lambda = \Lambda^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2M_\Lambda} \right] \Lambda + \text{higher derivatives}$$

☞ 4-velocity vector which is chosen as $v^\mu = (1, \mathbf{0})$

Quasiparticle Formalism: 2-Body Sector

Beane, Badaque, Hammer, Ubi. vanKolck, Birse, et al

☞ **Cluster fields** or **Dibaryons** (non-dynamical auxiliary d.o.f.s):

- Spin-Singlet nn-Dibaryon field $\to s_{(nn)}$
- Spin-Singlet $n\Lambda$-Dibaryon field $\to s_{(n\Lambda)}$
- Spin-Triplet $n\Lambda$-Dibaryon field $\to t_{(n\Lambda)}$

☞ **Unitarization:** Improves convergence in the vicinity of bound states

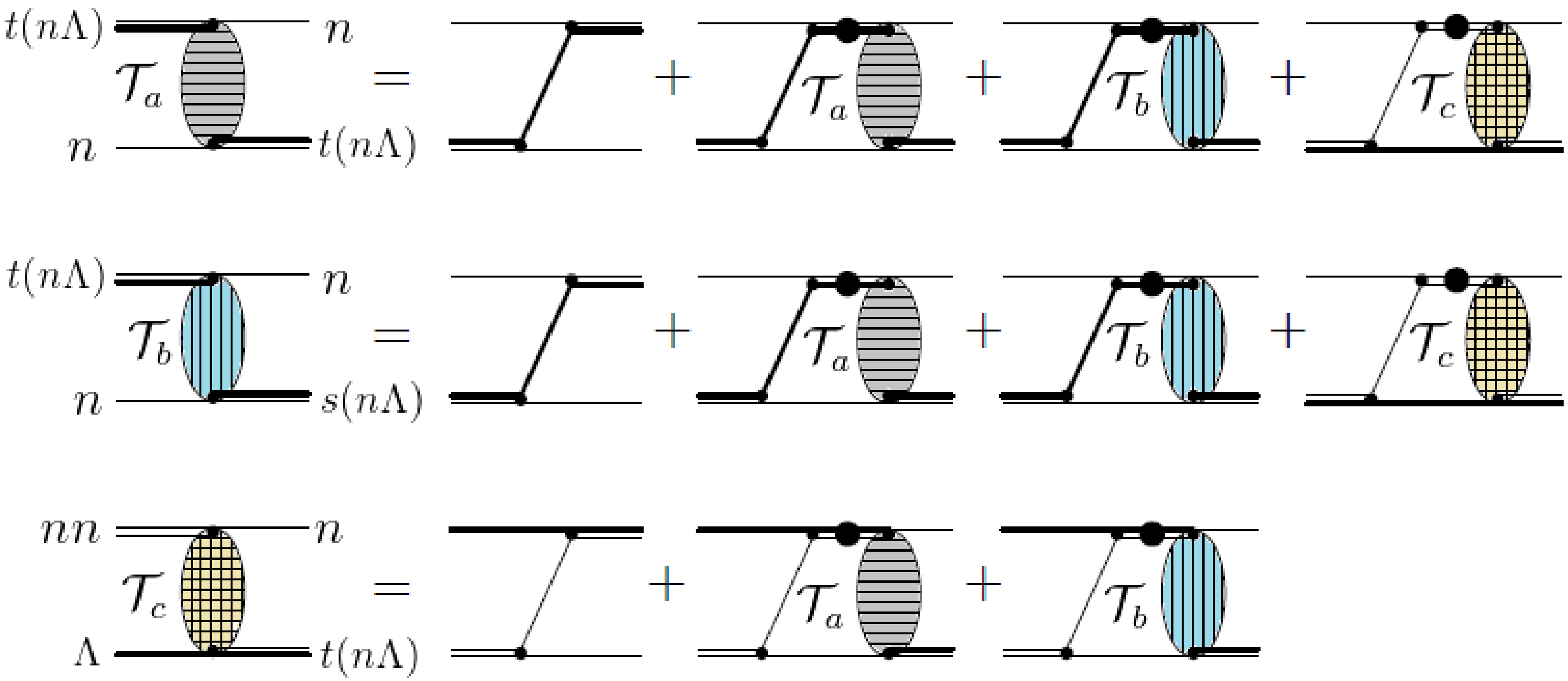
$$\mathcal{L}_{s(nn)} = -s^\dagger_{(nn)}\left[iv\cdot\partial + \frac{(v\cdot\partial)^2-\partial^2}{4M_n}\right]s_{(nn)} - y_{s(nn)}\left[s^\dagger_{(nn)}\left(N^T P^{(^1S_0)}_{(nn)} N\right) + \text{h.c.}\right] + \cdots$$

$$\mathcal{L}_{s(n\Lambda)} = -s^\dagger_{(n\Lambda)}\left[iv\cdot\partial + \frac{(v\cdot\partial)^2-\partial^2}{2(M_n+M_\Lambda)}\right]s_{(n\Lambda)} - y_{s(n\Lambda)}\left[s^\dagger_{(n\Lambda)}\left(N^T P^{(^1S_0)}_{(n\Lambda)} \Lambda\right) + \text{h.c.}\right] + \cdots$$

$$\mathcal{L}_{t(n\Lambda)} = -\sum_{j=1}^{3} t^\dagger_{(n\Lambda)j}\left[iv\cdot\partial + \frac{(v\cdot\partial)^2-\partial^2}{2(M_n+M_\Lambda)}\right]t_{(n\Lambda)j} - y_{t(n\Lambda)}\left[\sum_{j=1}^{3} t^\dagger_{(n\Lambda)j}\left(N^T P^{(^3S_1)}_{(n\Lambda)j} \Lambda\right) + \text{h.c.}\right] + \cdots$$

Faddeev-type Integral Eqns. in momentum space

- These **STM** *(Skornyakov, Ter-Martirosyan)* **Eqns.** must be **numerically solved** to obtain possible energy eigenvalues
- If Efimov effect is manifest then the integral equations are ill-defined in the UV limit
- Loops need to regulated with cut-off Λ dependent **3-body counter-terms** from additional 3-body Lagrangian $\mathcal{L}_{3-\mathrm{body}}$ (counter-terms not shown)



3-body Lagrangian (Part)

☞ **EFT TENET**: Introduce non-derivatively coupled contact terms with 3-body coupling $g(\Lambda)$ in the elastic channel, e.g.,

$$\boxed{n + t(n\Lambda) \to n + t(n\Lambda)}$$

Promoted to Leading Order in EFT

$$\mathcal{L}_{3-\text{body}} \supset \overbrace{-\frac{1}{6} M_\Lambda y^2_{t(n\Lambda)} \frac{g(\Lambda)}{\Lambda^2} t^\dagger_{(n\Lambda)i} N^\dagger \sigma_i \sigma_j N t_{(n\Lambda)j} + \cdots}$$

$+$ neglect sub-leading derivative operators in all channels

☞ **3-body input:** $B_{nn\Lambda} \lesssim 1$ MeV can be used to fix the coupling $g(\Lambda)$.

Asymptotic Analysis

☞ Investigate asymptotic behavior of integral equations to determine possible **Efimov-effect in unitary limit**

$$p \sim \Lambda \gg B_{nn\Lambda}, k \sim 1/a_{nn}, 1/a_{t(n\Lambda)}, 1/a_{s(n\Lambda)}$$

☞ Channel amplitudes obey simple power-law behavior: $\mathcal{T}_{a,b,c}(p) \sim p^{s-1}$

$\hookrightarrow$ integral equations decouple

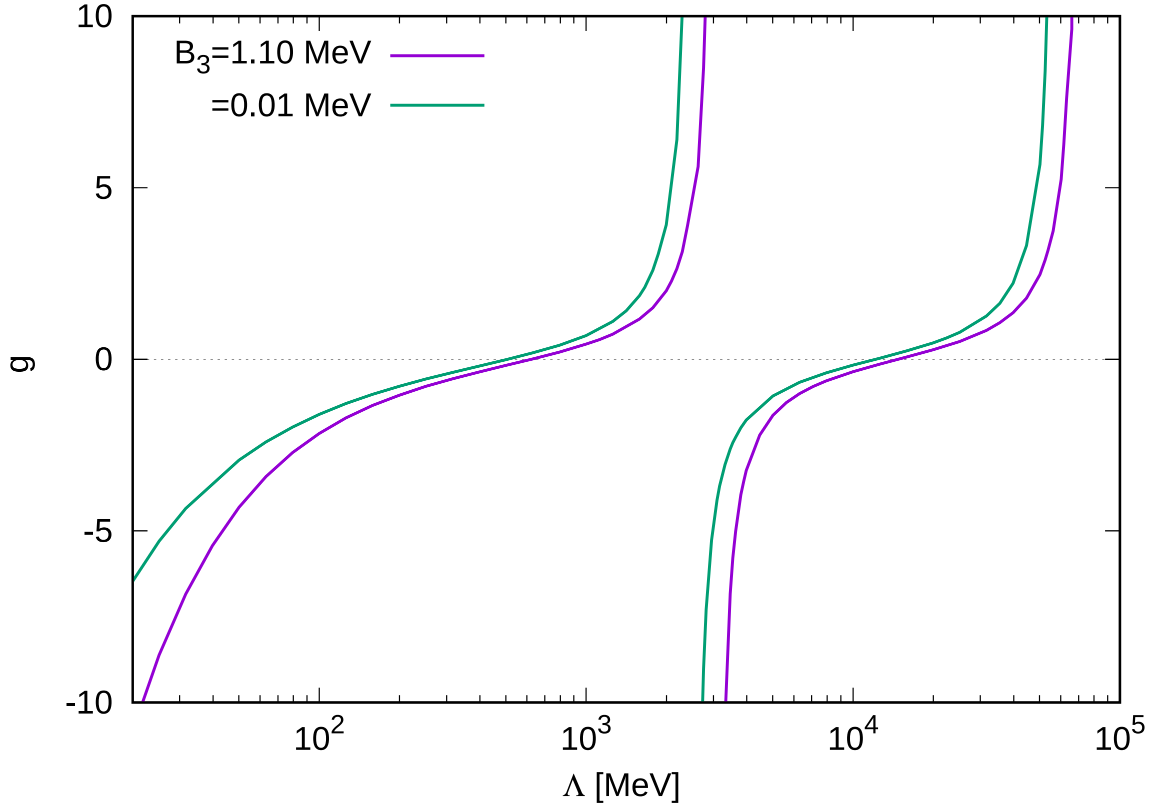
☞ **Asymptotic expansion** followed by **Mellin transformations:**

$$\left(\frac{M_\Lambda}{2\pi\mu_{n\Lambda}C_1}\right)\left[\frac{2\pi}{s}\frac{\sin\left[s\sin^{-1}\frac{\mu_{n\Lambda}}{M_\Lambda}\right]}{\cos[\pi s/2]}\right] + \left(\frac{M_n}{\pi^2\mu_{n\Lambda}C_1C_2}\right)\left[\frac{2\pi}{s}\frac{\sin\left[s\cot^{-1}\sqrt{\frac{2M_n-\mu_{n\Lambda}}{\mu_{n\Lambda}}}\right]}{\cos[\pi s/2]}\right]^2 = 1$$

$$C_1 = \sqrt{\frac{\mu_{n\Lambda}}{\mu_{n(n\Lambda)}}} \quad , \quad C_2 = \sqrt{\frac{M_n}{\mu_{\Lambda(nn)}}}$$

$$\mu_{n\Lambda} = \frac{M_n M_\Lambda}{M_n + M_\Lambda} \quad , \quad \mu_{n(n\Lambda)} = \frac{M_n(M_n + M_\Lambda)}{2M_n + M_\Lambda} \quad , \quad \mu_{\Lambda(nn)} = \frac{2M_n M_\Lambda}{2M_n + M_\Lambda}$$

Limit Cycle RG of 3-Body Coupling for $nn\Lambda$



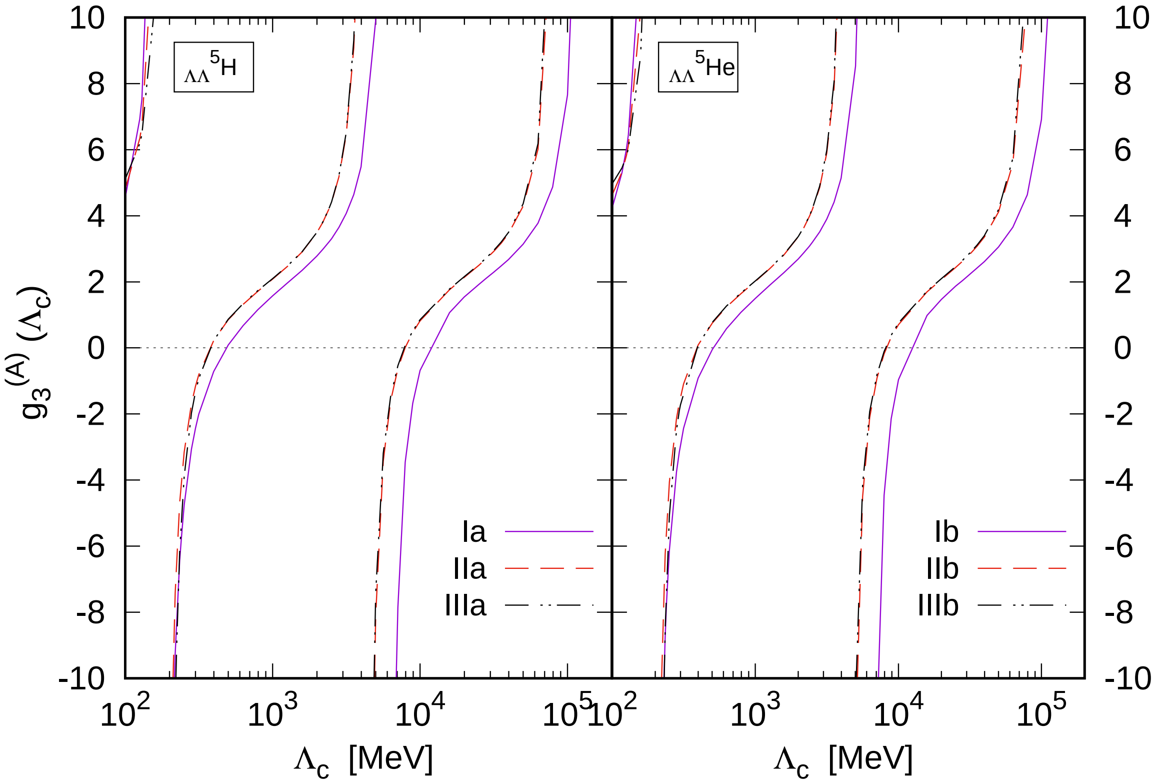
☞ **Efimov Effect:** Solving for s yields imaginary values $\rightarrow$ **Limit Cycle**

$$s = \pm i s_0 \quad ; \quad s_0 = 1.00760...$$

☞ **Cyclic Periodicity:** $g(\Lambda_n) = g(\Lambda_1 \sim 500 - 600\ \mathrm{MeV})$ with

$$\Lambda_n = \Lambda_1\, e^{n\pi/s_0}$$

Limit Cycle RG in ($^{5}_{\Lambda\Lambda}$H , $^{5}_{\Lambda\Lambda}$He) *G. Meher et al*



☞ **Efimov Effect:** Solving for s yields imaginary values $\rightarrow$ **Limit Cycle**

$$s = \pm i s_0 \quad ; \quad s_0 \approx 1.0351...$$

☞ **Cyclic Periodicity:** $g(\Lambda_n) = g(\Lambda_1 \sim 300 - 400\ \mathrm{MeV})$ with

$$\Lambda_n = \Lambda_1\, e^{n\pi/s_0}$$