

Neutron Star Cooling via Axion Emission

Avik Paul

Saha Institute of Nuclear Physics, HBNI, Kolkata

Collaborator

Debasish Majumdar

Saha Institute of Nuclear Physics, HBNI, Kolkata

WHEPP 2019

IIT Guwahati

December 7, 2019

Axion

$$\mathcal{L}_{new} = \frac{\bar{\theta} g^2}{32\pi^2} G_{\mu\nu}^{a\dagger} \tilde{G}_a^{\mu\nu}. \quad (1)$$

where, $\bar{\theta} = \theta_{QCD} + \arg \det (M)$

- Neutron electric dipole moment is expressed as

$$|d_n| \sim 10^{-16} \bar{\theta} e \text{ cm}, \quad (2)$$

- The experimental limit is $|d_n| < 6.3 \times 10^{-26} e \text{ cm}$ and from this the upper bound for $\bar{\theta}$ is obtained as $|\bar{\theta}| \lesssim 10^{-9}$.

Axion

- Axions are introduced to circumvent the strong CP problem that refers to the presence of CP violation term in QCD Lagrangian.
- The axion which is a pseudo Nambu-Goldstone boson that arises out of the Peccei-Quinn (PQ) solution where an anomalous chiral symmetry $U(1)_A$ is introduced and which is spontaneously broken at the PQ energy scale.
- The PQ solution of strong CP problem results in the prediction of new particle namely axion.

Axion Models

- **KSVZ Model:** A new heavy quark field introduced in this model. In this model axions couple only to the photons and hadrons but do not couple to leptons at tree level.
- **DFSZ Model:** Two Higgs doublet extension of the SM. In this model axions couple to the charged leptons in addition to the hadrons and photons.

Cooling of neutron star through Axion emission

- **Neutron star:** A neutron star with a typical radius of 10-12 km and generally having a mass range of 1-2 solar mass (M_{\odot}) is created as an aftermath of a massive supernova explosion.
- Neutron stars may undergo a cooling process through photon and neutrino emission.
- In this work we study the rate of neutron star cooling if it cools via axion emission in addition to the emission of photon and neutrino.
- Neutron star can emit Axions via nucleon nucleon bremsstrahlung processes.

Nucleon Nucleon Axion Bremsstrahlung

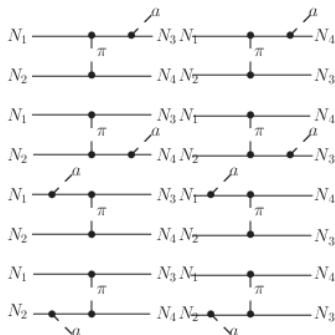


Figure 1: The direct and exchange diagrams for nucleon,nucleon Axion bremsstrahlung

- **Matrix Element for $N + N \rightarrow N + N + a$:** The interaction Hamiltonian axion-nucleon interaction can be written as

$$\mathcal{H}_{int} = -\frac{C_N}{2f_a} \bar{\psi}_N \gamma_\mu \gamma_5 \psi_N \partial^\mu a \quad (3)$$

- For the processes, $nn \rightarrow nn + a$ and $pp \rightarrow pp + a$, (“pure processes”) the spin-summed squared matrix element is of the form

$$\sum_{spins} |\mathcal{M}|_{NN}^2 = \frac{16(4\pi)^3 \alpha_\pi^2 \alpha_a}{3m_N^2} \left[\left(\frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\pi^2} \right)^2 + \left(\frac{l^2}{l^2 + m_\pi^2} \right)^2 + \frac{\mathbf{k}^2 l^2 - 3(\mathbf{k} \cdot \mathbf{l})^2}{(\mathbf{k}^2 + m_\pi^2)(l^2 + m_\pi^2)} \right] \quad (4)$$

where $\alpha_a \equiv (C_N m_N / f_a)^2 / 4\pi = g_{aN}^2 / 4\pi$, $g_{aN} = (C_N m_N / f_a)$,
 $(N = p, n)$, $\alpha_\pi \equiv (f^2 m_N / m_\pi)^2 / 4\pi \approx 17$, $f \approx 1.05$, $\mathbf{k} = \mathbf{p}_2 - \mathbf{p}_4$ and
 $\mathbf{l} = \mathbf{p}_2 - \mathbf{p}_3$.

- For the “mixed” process $np \rightarrow np + a$, spin-summed squared matrix element can be written as

$$\begin{aligned} \sum_{spins} |\mathcal{M}|_{np}^2 = & \frac{256\pi^2 \alpha_\pi^2}{3m_N^2} \frac{(g_{an} + g_{ap})^2}{4} \left[2 \left(\frac{l^2}{l^2 + m_\pi^2} \right)^2 - \frac{4(\mathbf{k} \cdot \mathbf{l})^2}{(\mathbf{k}^2 + m_\pi^2)(l^2 + m_\pi^2)} \right] \\ & + \frac{256\pi^2 \alpha_\pi^2}{3m_N^2} \frac{(g_{an}^2 + g_{ap}^2)}{2} \left[\left(\frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\pi^2} \right)^2 + 2 \left(\frac{l^2}{l^2 + m_\pi^2} \right)^2 \right. \\ & \left. + 2 \frac{\mathbf{k}^2 l^2 - (\mathbf{k} \cdot \mathbf{l})^2}{(\mathbf{k}^2 + m_\pi^2)(l^2 + m_\pi^2)} \right]. \end{aligned} \quad (5)$$

- **Energy Loss Rate Expression:** The Axion energy-loss rate per unit volume is given by

$$Q_a = \int \frac{d^3 \mathbf{K}_a}{2\omega_a (2\pi)^3} \omega_a \int \prod_{i=1}^4 \frac{d^3 \mathbf{P}_i}{2E_i (2\pi)^3} f_1 f_2 (1 - f_3) (1 - f_4) \times (2\pi)^4 \delta^4 (P_1 + P_2 - P_3 - P_4 - K_a) S \sum_{spins} |\mathcal{M}|_{NN}^2 \quad (6)$$

- **Nondegenerate Limit:** $T \lesssim 10$ MeV, $\rho_B \approx 10^{12}$ g/cm³
- Nucleon Fermi energy

$$\epsilon_{F,N} = \frac{p_{F,N}^2}{2m_N} = \frac{(3\pi^2 Y_N n_B)^{2/3}}{2m_N} \ll T \quad (7)$$

- **Degenerate Limit:** $T \approx 100$ KeV, $\rho_B \approx 2\rho_{nuc} \approx 5.6 \times 10^{14}$ g/cm³

$$\epsilon_{F,N} \gg T \quad (8)$$

Nondegenerate Limit

- For simplicity, we first neglect the pion mass contribution in Eqs. 4 and 5. With this approximation the squared matrix element reduces to

$$\sum_{spins} |\mathcal{M}|_{NN}^2 = \frac{256\pi^2 \alpha_\pi^2}{3m_N^2} \tilde{g}_{NN}^2 \quad (9)$$

$$\tilde{g}_{NN}^2 \equiv \begin{cases} g_{an}^2(3 - \beta) & nn \rightarrow nn + a \\ g_{ap}^2(3 - \beta) & pp \rightarrow pp + a \\ \left(\frac{g_{an} + g_{ap}}{2}\right)^2 (2 - 4\beta/3) + \frac{g_{an}^2 + g_{ap}^2}{2} (5 - 2\beta/3) & np \rightarrow np + a. \end{cases} \quad (10)$$

- Here, $\tilde{g}_{NN} = \tilde{C}_N m_N / f_a$ is an effective coupling and $\beta \equiv 3\langle(\hat{\mathbf{k}} \cdot \hat{\mathbf{l}})^2\rangle = 1.3078$.
- The Axion-nucleon coupling

$$|\tilde{C}_N| = \begin{cases} 0.013 & nn \rightarrow nn + a \\ 0.442 & pp \rightarrow pp + a \\ 0.495 & np \rightarrow np + a. \end{cases} \quad (11)$$

- The total energy loss rate per unit volume as

$$\begin{aligned} Q_a^{ND} &= \frac{\xi(T)}{280} \frac{n_B^2 T^{7/2}}{m_N^{5/2} \pi^{7/2}} \left(Y_n^2 \sum_{spins} |\mathcal{M}|_{nn}^2 + Y_p^2 \sum_{spins} |\mathcal{M}|_{pp}^2 + Y_n Y_p \sum_{spins} |\mathcal{M}|_{np}^2 \right) \\ &= \frac{32}{105} \xi(T) \frac{\alpha_\pi^2 n_B^2 T^{7/2}}{m_N^{9/2} \pi^{3/2}} \left(Y_n^2 \tilde{g}_{nn}^2 + Y_p^2 \tilde{g}_{pp}^2 + 4Y_n Y_p \tilde{g}_{np}^2 \right) \\ &= \frac{32}{105} \xi(T) \frac{\alpha_\pi^2 n_B^2 T^{7/2}}{m_N^{9/2} \pi^{3/2}} g_{ND}^2 \end{aligned} \quad (12)$$

- With $Y_p \approx 0.1$, $Y_n \approx 0.9$, $\xi(T) \approx 0.5$, using Eqs. 10, 11 and

$$m_a = 6\mu\text{eV} \left(\frac{10^{12}\text{GeV}}{f_a} \right) \text{ relation}$$

$$g_{ND} = 4.71 \times 10^{-8} \left(\frac{m_a}{\text{eV}} \right) \quad \text{and} \quad C_N^{ND} = 0.30. \quad (13)$$

- In this nondegenerate case the Axion energy loss rate per unit volume reduces to

$$Q_a^{ND} = 2.90166 \times 10^{31} \text{ erg cm}^{-3} \text{ yr}^{-1} T_9^{3.5} \rho_{12}^2 m_{\text{eV}}^2 \quad (14)$$

where $m_{\text{eV}} \equiv m_a/\text{eV}$ and $T_9 \equiv T/10^9$ and $\rho_{12} \equiv \frac{\rho}{10^{12}\text{g/cm}^3} = \left(\frac{n_B}{m_N} \right)$.

Degenerate Limit

- In the degenerate limit, the squared matrix elements simplifies to

$$\sum_{spins} |\mathcal{M}|_{NN}^2 = \frac{256\pi^2 \alpha_\pi^2}{m_N^2} \tilde{g}_{NN}^2 \quad \text{here } \beta = 0. \quad (15)$$

- With

$$\tilde{g}_{NN}^2 \equiv \begin{cases} g_{an}^2 & nn \rightarrow nn + a \\ g_{ap}^2 & pp \rightarrow pp + a \\ (g_{an}^2 + g_{ap}^2 + (g_{an}g_{ap})/3) & np \rightarrow np + a. \end{cases} \quad (16)$$

-

$$|\tilde{C}_N| = \begin{cases} 0.01 & nn \rightarrow nn + a \\ 0.34 & pp \rightarrow pp + a \\ 0.338 & np \rightarrow np + a. \end{cases} \quad (17)$$

- In this case the integration in Eq. 6 can be simplified analytically without neglecting the pion masses. Here the pionic contribution $F(u)$ is given by

$$F(u) = 1 - \frac{5u}{6} \arctan\left(\frac{2}{u}\right) + \frac{u^2}{3(u^2 + 4)} + \frac{u^2}{6\sqrt{2u^2 + 4}} \times \arctan\left(\frac{2\sqrt{2u^2 + 4}}{u^2}\right) \quad (18)$$

where $u = m_\pi/p_{F,N}$.

- With $u \approx 0.32Y_N^{-1/3}$ and consider $\rho_B \approx 2\rho_{nuc}$, $F(u)$ can be replaced by $F(Y_N)$ and the total emission rate is given as

$$\begin{aligned} Q_a^D &= \frac{31}{967680} \left(\frac{3n_B}{\pi}\right)^{1/3} T^6 \left(Y_n^{1/3} F(Y_n) \sum_{spins} |\mathcal{M}|_{nn}^2 + Y_p^{1/3} F(Y_p) \sum_{spins} |\mathcal{M}|_{pp}^2 \right. \\ &\quad \left. + 4Y_{np}^{1/3} F(Y_{np}) \sum_{spins} |\mathcal{M}|_{np}^2\right) \\ &= \frac{31\pi^{5/3} (3n_B)^{1/3} \alpha_\pi^2 T^6}{3780m_N^2} \left(Y_n^{1/3} F(Y_n) \bar{g}_{an}^2 + Y_p^{1/3} F(Y_p) \bar{g}_{ap}^2 + Y_{np}^{1/3} F(Y_{np}) \bar{g}_{np}^2\right) \\ &= \frac{31\pi^{5/3} (3n_B)^{1/3} \alpha_\pi^2 T^6}{3780m_N^2} g_D^2, \end{aligned} \quad (19)$$

- The effective nucleon fraction for the mixed processes are given by

$$Y_{np}^{1/3} = \frac{1}{2\sqrt{2}}(Y_n^{2/3} + Y_p^{2/3})^{1/2} \left[2 - \frac{|Y_n^{2/3} - Y_p^{2/3}|}{Y_n^{2/3} + Y_p^{2/3}} \right] \quad (20)$$

- Using the values of nucleon number fractions $Y_p = 0.01$, $Y_n = 0.99$ and $Y_{np} = 0.06$, we can calculate $F(Y_n) \approx 0.64$, $F(Y_p) \approx 0.12$ and $F(Y_{np}) \approx 0.31$ for the pure and mixed processes. Also using the above values one can attain the effective coupling constants g_D and C_N^D for degenerate case as

$$g_D = 2.04 \times 10^{-8} \left(\frac{m_a}{eV} \right) \quad \text{and} \quad C_N^D = 0.13. \quad (21)$$

- In this Degenerate case using Eqs. 19 and 21 the Axion emission rate per unit volume is obtained as

$$Q_a^D = 4.84244 \times 10^{30} \text{ erg cm}^{-3} \text{ yr}^{-1} T_9^6 m_{eV}^2 \left(\frac{\rho_{NS}}{\rho_{nuc}} \right)^{1/3}. \quad (22)$$

Neutron star cooling

- In Newtonian formulation the energy balance equation for the neutron star is given by

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\nu(T) - L_a(T) - L_\gamma(T_e). \quad (23)$$

- The photon luminosity is given by the Stefan-Boltzmann law

$$L_\gamma = 4\pi \sigma R^2 T_e^4 \quad (24)$$

- Various neutrino processes are involved in the cooling of neutron stars. Dominant neutrino emitting processes are direct Urca processes and modified Urca processes.
- The other neutrino emitting processes are electron-positron pair annihilation, plasmon decay, electron synchrotron, photoneutrino emission, electron-nucleus bremsstrahlung, cooper pairing of neutrons, neutron-neutron bremsstrahlung, neutron-nucleus bremsstrahlung.

Results

- We adopt APR equation of state for our work.
- We use the NSCool code for calculating the neutrino and axion luminosities.

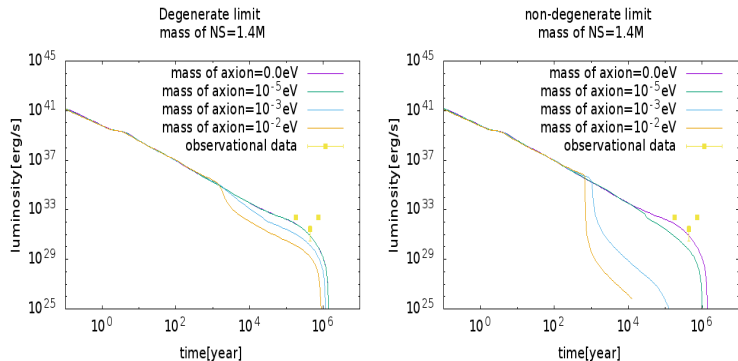


Figure 2: Luminosity (L) vs time (t) graph for both degenerate (left panel) and non-degenerate (right panel) limits with $M = 1.4M_{\odot}$ and $m_a = 0\text{eV}, 10^{-5}\text{eV}, 10^{-3}\text{eV}, 10^{-2}\text{eV}$ (from top to bottom). The observational data for three pulsars PSR B0656+14, Geminga and PSR B1055-52 are shown by dots with error bars from left to right.

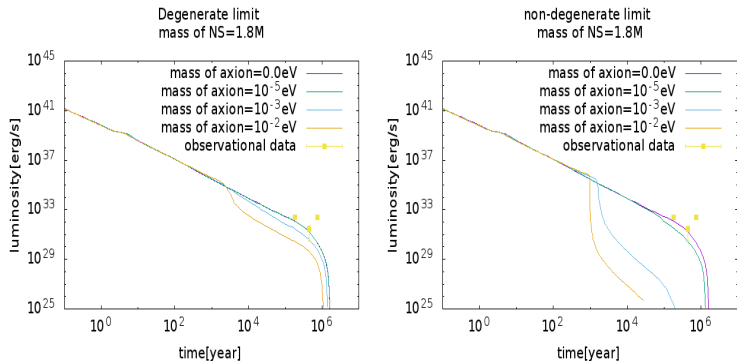


Figure 3: Same as in Figure 2 but for $M = 1.8M_{\odot}$

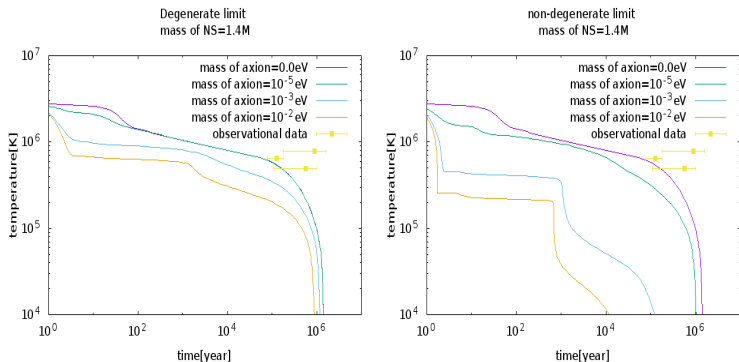


Figure 4: Surface temperature (T) vs time (t) graph for both degenerate (left panel) and non-degenerate (right panel) limits with $M = 1.4M_{\odot}$ and $m_a = 0\text{eV}, 10^{-5}\text{eV}, 10^{-3}\text{eV}, 10^{-2}\text{eV}$ (from top to bottom). The observational data for three pulsars PSR B0656+14, Geminga and PSR B1055-52 are shown by dots with error bars from left to right.

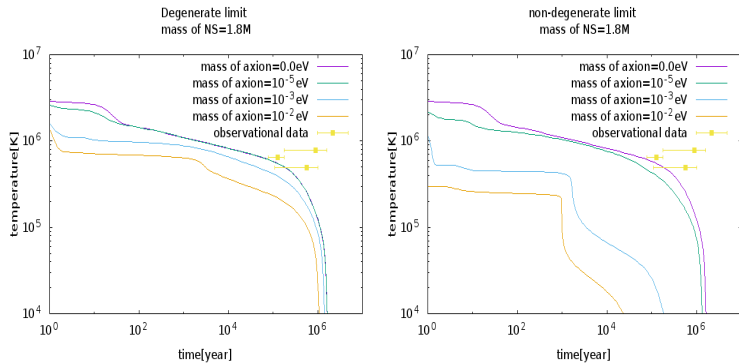


Figure 5: Same as in Figure 4 but for $M = 1.8M_{\odot}$

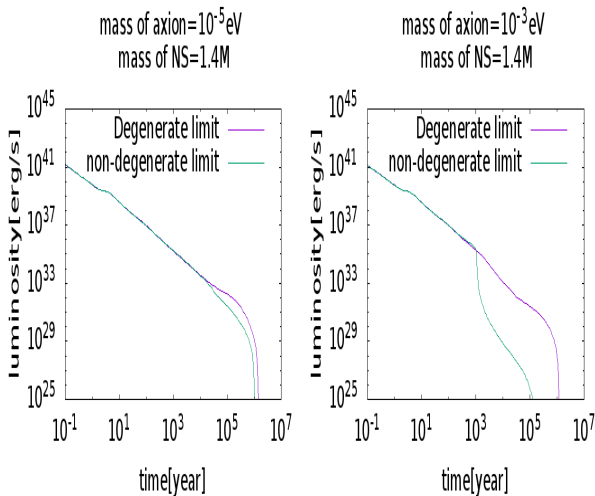


Figure 6: Comparison between degenerate and non-degenerate limits for L vs t plot with $m_a = 10^{-5}$ eV (left panel) and $m_a = 10^{-3}$ eV (right panel).

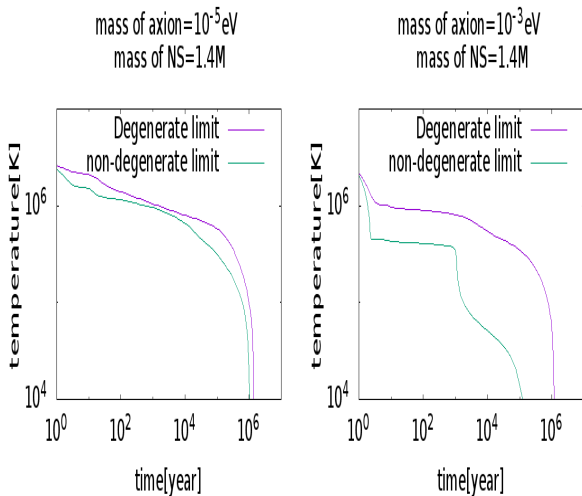


Figure 7: Comparison between degenerate and non-degenerate limits for T vs t plot with $m_a = 10^{-5} \text{ eV}$ (left panel) and $m_a = 10^{-3} \text{ eV}$ (right panel).

- From these analyses we derive a bound on axion masses $m_a \leq 10^{-3} \text{eV}$ which implies that the axion decay constant $f_a \geq 0.6 \times 10^{10} \text{GeV}$.
- We also find that the cooling patterns for degenerate and non-degenerate cases are different. This difference increases with the mass of the emitted axion.

Summary

- In this work we have explored the effect of axion emission on the cooling of neutron stars. For the axion emission we consider nucleon-nucleon axion bremsstrahlung process. We have made our analysis for the stars considering degenerate and non-degenerate limits.
- We demonstrate our results by calculating the variations of luminosity and temperature of neutron stars with time.
- We adopt APR equation of state for our work.
- We have demonstrated this by considering three axion masses namely 10^{-5}eV , 10^{-3}eV , 10^{-2}eV and the neutron star masses of $1.4M_{\odot}$, $1.8M_{\odot}$.
- For comparison with observational results we used luminosity and temperature of three pulsars namely PSR B0656+14, Geminga and PSR B1055-52.
- Finally we derive a bound on axion masses $m_a \leq 10^{-3}\text{eV}$ which implies that the axion decay constant $f_a \geq 0.6 \times 10^{10}\text{GeV}$.

DM Admixed EOS

Cooling of Dark-Matter Admixed Neutron Stars with density-dependent Equation of State

Sajad A. Bhat  and Avik Paul *Astroparticle Physics and Cosmology Division,
Saha Institute of Nuclear Physics, HBNI
1/AF Bidhannagar, Kolkata 700064, India*

arXiv:1905.12483v2 [hep-ph] 4 Jun 2019

Abstract

We propose a dark-matter (DM) admixed density-dependent equation of state where the fermionic DM interacts with the nucleons via Higgs portal. Presence of DM can hardly influence the particle distribution inside neutron star (NS) but can significantly affect the structure as well as equation of state (EOS) of NS. Introduction of DM inside NS softens the equation of state. We explored the effect of variation of DM mass and DM fermi momentum on the NS EOS. Moreover, DM-Higgs coupling is constrained using dark matter direct detection experiments. Then, we studied cooling of normal NSs using APR and DD2 EOSs and DM admixed NSs using dark-matter modified DD2 with varying DM mass and fermi momentum. We have done our analysis by considering different NS masses. Also DM mass and DM fermi momentum are varied for fixed NS mass and DM-Higgs coupling. We calculated the variations of luminosity and temperature of NS with time for all EOSs considered in our work and then compared our calculations with the observed astronomical cooling data of three pulsars namely PSR B0656+14, Geminga and PSR B1055-52. It is found that APR EOS agrees well with the pulsar data for lighter and medium mass NSs but cooling is very fast for heavier NS. For DM admixed DD2 EOS, we found that in case of medium and heavier mass NSs, all chosen DM masses and fermi momenta agree well with the observational data but for lower mass NSs, all DM fermi momenta and high DM masses barely agree with the observations. Furthermore, only lower DM mass agrees with observations in case of lighter NSs. Cooling becomes faster as compared to normal NSs in case of increasing DM mass and fermi momenta. It is inferred from the calculations that if low mass super cold NSs are observed in future that may support the fact that heavier WIMP can be present inside neutron stars.

Thank You

QCD Lagrangian

- The most general renormalisable QCD Lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_{\mu\nu}^{a\dagger}G_{\mu\nu}^a + \sum_A \bar{q}_A(i\gamma^\mu D_\mu - m_A)q_A. \quad (25)$$

- The field strength tensor is given by

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - gf_{abc}G_\mu^b G_\nu^c \quad (26)$$

- The covariant derivative has the form

$$D_\mu = \partial_\mu - ig\lambda_a G_\mu^a \quad (27)$$

- One can add a gauge invariant and renormalisable term in the Lagrangian as

$$(\text{constant}) \times \tilde{G}_{\mu\nu}^{a\dagger} \tilde{G}_a^{\mu\nu} = (\text{constant}) \times \tilde{G}_{\mu\nu}^{\bar{a}} \tilde{G}_a^{\mu\nu}, \quad (28)$$

where $\tilde{G}_{\mu\nu}^a = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}G_a^{\lambda\rho}$ is the dual to the field-strength tensor.

- It can be shown that

$$\tilde{G}_{\mu\nu}^{\bar{a}} \tilde{G}_a^{\mu\nu} = G_{\mu\nu}^{\bar{a}} G_a^{\mu\nu} = G_{\mu\nu}^{a\dagger} G_a^{\mu\nu}. \quad (29)$$

- It is also possible to consider a term like

$$\mathcal{L}_{new} = (\text{constant}) \times G_{\mu\nu}^{\bar{a}} \tilde{G}_a^{\mu\nu} \quad (30)$$

- It can be shown that

$$G_{\mu\nu}^{\bar{a}} \tilde{G}_a^{\mu\nu} = \partial_\mu K_\mu, \quad (31)$$

where,

$$K^\mu = 2\epsilon^{\mu\nu\lambda\rho} (G_\nu^a \partial_\lambda G_\rho^{\bar{a}} - \frac{1}{3} g f_{bc\bar{a}} G_\nu^a G_\lambda^b G_\rho^c), \quad (32)$$

or,

$$K^\mu = \epsilon^{\mu\nu\lambda\rho} G_\nu^a (G_{\lambda\rho}^{\bar{a}} + \frac{1}{3} g f_{bc\bar{a}} G_\lambda^b G_\rho^c). \quad (33)$$

- Taking surface integral of Eq. 31, and using Eq. 33, we get

$$\int d^4x G_{\mu\nu}^{a\dagger} \tilde{G}_a^{\mu\nu} = \int dS_\mu K^\mu \quad (34)$$

or,

$$\int d^4x G_{\mu\nu}^{a\dagger} \tilde{G}_a^{\mu\nu} = \frac{1}{3} g \epsilon^{\mu\nu\lambda\rho} f_{bc\bar{a}} \int dS_\mu G_\nu^a G_\lambda^b G_\rho^c = \frac{32\pi^2 n}{g^2}. \quad (35)$$

QCD Vacuum

- A gauge invariant QCD vacuum state can be represented as

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta} |n\rangle. \quad (36)$$

- The θ parameter of Eq. 36 is an eigenstate of the Hamiltonian and it can be proved by considering a matrix

$$\langle\theta' | e^{-iHt} | \theta\rangle = \sum_{n,n'} e^{i(n'\theta' - n\theta)} \langle n' | e^{-iHt} | n\rangle. \quad (37)$$

- Putting $n' = n + k$ and simplifying

$$\begin{aligned} \langle\theta' | e^{-iHt} | \theta\rangle &= \sum_{n,k} e^{in(\theta' - \theta)} e^{ik\theta'} \langle k | e^{-iHt} | 0\rangle \\ &= \delta(\theta' - \theta) \sum_k e^{ik\theta} \langle k | e^{-iHt} | 0\rangle. \end{aligned} \quad (38)$$

- Now, writing the matrix element of the time-evolution operator

$$\langle k | e^{-iHt} | 0 \rangle = \int \mathcal{D}\Psi \int \mathcal{D}\bar{\Psi} \int \mathcal{D}G_\mu^a \exp \left(i \int d^4x \mathcal{L}_{QCD} \right) \quad (39)$$

- Using Eqs. 35 and 39 into Eq., we obtain

$$\langle \theta' | e^{-iHt} | \theta \rangle = \int \mathcal{D}\Psi \int \mathcal{D}\bar{\Psi} \int \mathcal{D}G_\mu^a \exp \left(i \int d^4x (\mathcal{L}_{QCD} + \frac{\theta g^2}{32\pi^2} G_{\mu\nu}^{a\dagger} \tilde{G}_a^{\mu\nu}) \right) \quad (40)$$

- The effective value of the θ parameter is given by

$$\theta_{eff} = \bar{\theta} = \theta_{QCD} + \arg \det (M) \quad (41)$$

where M is quark Mass Matrix.

- So, finally we get an extra vacuum term that adds to the QCD Lagrangian given by

$$\mathcal{L}_{new} = \frac{\bar{\theta}_{QCD} g^2}{32\pi^2} G_{\mu\nu}^{a\dagger} \tilde{G}_a^{\mu\nu}. \quad (42)$$

The Strong CP Problem

- The $\bar{\theta}$ term violates P and CP invariance, so the QCD Lagrangian contains a CP violating term.
- An electric dipole moment for the neutron is the observed consequence of QCD or Strong CP violation. Neutron electric dipole moment is expressed as

$$|d_n| \sim 10^{-16} \bar{\theta} e \text{ cm} , \quad (43)$$

- The experimental limit is $|d_n| < 6.3 \times 10^{-26} e \text{ cm}$ and from this the upper bound for $\bar{\theta}$ is obtained as $|\bar{\theta}| \lesssim 10^{-9}$.
- There is no natural reason to expect dimensionless parameter $\bar{\theta}$ to be this small. CP violation occurs in the Standard Model by considering the quark masses to be complex and thus the natural value of $\bar{\theta}$ is expected to be of order 1 (between zero to 2π).
- This is the **Strong CP Problem**.

Peccei-Quinn Solution and the Axion

- The Peccei-Quinn (PQ) solution of strong CP problem results in the prediction of a new particle namely Axion.
- In the PQ mechanism, an additional scalar S is introduced into the theory which respects an additional $U(1)$ ($U(1)_{PQ}$) symmetry.

$$\mathcal{L} = \partial_\mu S^* \partial^\mu S - V(S) \quad (44)$$

- where $S(x) = \eta(x) e^{ia(x)/v_a}$ and $v_a = \langle 0 | S | 0 \rangle$.
- Under this symmetry the Axion field “ $a(x)$ ” translates as $a \rightarrow a + \alpha v_a$. The Lagrangian is now written as

$$\begin{aligned} \mathcal{L}_{QCD,tot} &= \mathcal{L}_{QCD} + \mathcal{L}_{\bar{\theta}} + \mathcal{L}_a \\ &= \mathcal{L}_{QCD} + \frac{\bar{\theta} g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a}{v_a} \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \end{aligned} \quad (45)$$

- Now the effective potential is of the form

$$V_{eff} \sim \cos\left(\bar{\theta} + \frac{a}{v_a}\right). \quad (46)$$

- Minimising this potential with respect to $\langle a \rangle$ gives the PQ solution as

$$\langle a \rangle = -v_a \bar{\theta}. \quad (47)$$

- Due to the periodicity of $\bar{\theta}$, $\langle a \rangle$ is periodic with $\langle a \rangle = 2\pi k v_a$
- Now, from Eq. 46 the $\bar{\theta}$ term can be eliminated by substituting the physical Axion field as $a_{phys} = a - \langle a \rangle = a + v_a \bar{\theta}$.
- Therefore, $\frac{g^2}{32\pi^2} (\bar{\theta} + \frac{a}{v_a}) = \frac{g^2}{32\pi^2} \frac{a_{phys}}{v_a}$.
- The effective potential for the Axion field has a minimum at $\langle a \rangle = -v_a \bar{\theta}$.

$$\left\langle \frac{\partial V_{eff}}{\partial a} \right\rangle = \frac{1}{v_a} \frac{g^2}{32\pi^2} \left\langle G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \right\rangle \Big|_{\langle a \rangle = -v_a \bar{\theta}} = 0 \quad (48)$$

where $\langle a \rangle = \langle 0 | a | 0 \rangle$.

- Since at the minimum, the $\bar{\theta}$ term is cancelled out, this gives a dynamical solution to the strong CP problem.
- Expanding V_{eff} at the minimum gives the Axion a mass

$$\left\langle \frac{\partial^2 V_{eff}}{\partial a^2} \right\rangle = m_a^2 = \frac{1}{v_a} \frac{g^2}{32\pi^2} \frac{\partial}{\partial a} \left\langle G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \right\rangle \Big|_{\langle a \rangle = -v_a \bar{\theta}}. \quad (49)$$

Solving CP Problem

- In the PQ solution, $\bar{\theta}$ is promoted from a parameter to a dynamical variable.
- This variable relaxes to the minimum of its potential and hence is small.
- Non-perturbative effects which make QCD, $\bar{\theta}$ dependent, results in a potential for the Axion.
- Although initially massless, this potential causes the Axion to acquire a mass and relax to the CP-conserving minimum.
- Hence the strong CP problem is solved.

Axion Models

- The Visible Axion Model:** Predicts an Axion with a mass of the electroweak (EW) scale. The model was quickly ruled out, because the Axion did not show up in experimental data and is replaced by the “invisible” Axion models, which predict a very light Axion.
- The Invisible Axion Model:** The scale of f_a is not taken to be that of weak scale v . The dynamical adjustment of the strong CP angle, $\bar{\theta} \rightarrow 0$ works for any scale for f_a . If $f_a \gg v$, the resulting Axions are very light ($m_a \sim \frac{1}{f_a}$), very weakly coupled (coupling $\sim \frac{1}{f_a}$) and very long lived ($\tau(a \rightarrow 2\gamma) \sim f_a^5$). Thus these Axions are, apparently, invisible.
- There are two types of invisible axion models – the KSVZ model and the DFSZ model. In the KSVZ model, the axion couples only to the photons and hadrons, while in the DFSZ model the axion couples to the charged leptons as well.