Lattice QCD for RHIC and EIC

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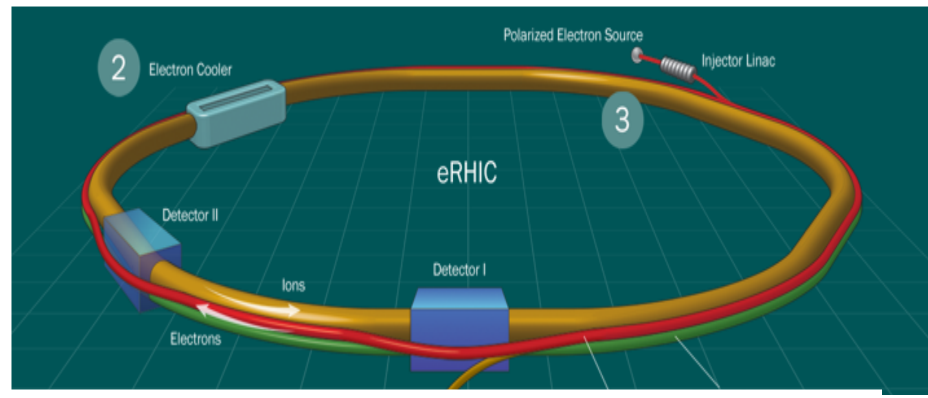
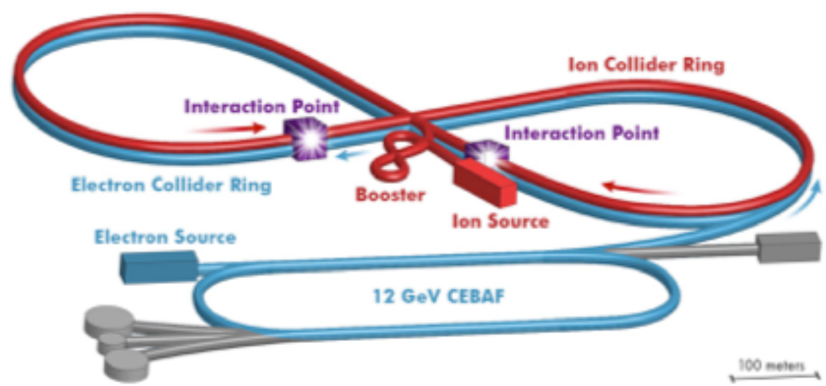
What can lattice QCD tell us about properties of hot and dense matter and the internal structure of hadrons ?





WHEPP, Guwahati, December 9, 2019

Symmetries of QCD in the vacuum at high T

$T \gg \Lambda_{QCD}$:

- Chiral symmetry : $m_{u,d} \ll \Lambda$

$SU_A(2)$ rotation $\psi \to e^{i\phi T^a \gamma_5}\psi$ $\psi_{L,R} \to e^{i\phi_{L,R}T^a}\psi_{L,R}$

$$\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R\rangle + \langle\bar{\psi}_R\psi_L\rangle \neq 0$$

$\langle\bar{\psi}\psi\rangle = 0$

restored

spontaneous symmetry breaking or Nambu-Goldstone symmetry realization

2008

hadrons with opposite parity have very different masses,
interactions between hadrons are weak at low *E*

- Axial or *U_A(1)* symmetry: invariance $\psi \to e^{i\phi\gamma_5}\psi$

Effectively restored ?

is broken by anomaly (ABJ) : $\langle\partial^\mu j^a_\mu\rangle = -\frac{\alpha_s}{4\pi}\langle\epsilon^{\alpha\beta\gamma\delta}F^a_{\alpha\beta}F^a_{\gamma\delta}\rangle$

topology

η´ meson mass, *π*-a_0 mass difference

- Center (Z3) symmetry : invariance under global gauge transformation

$\langle L\rangle \neq 0$

broken

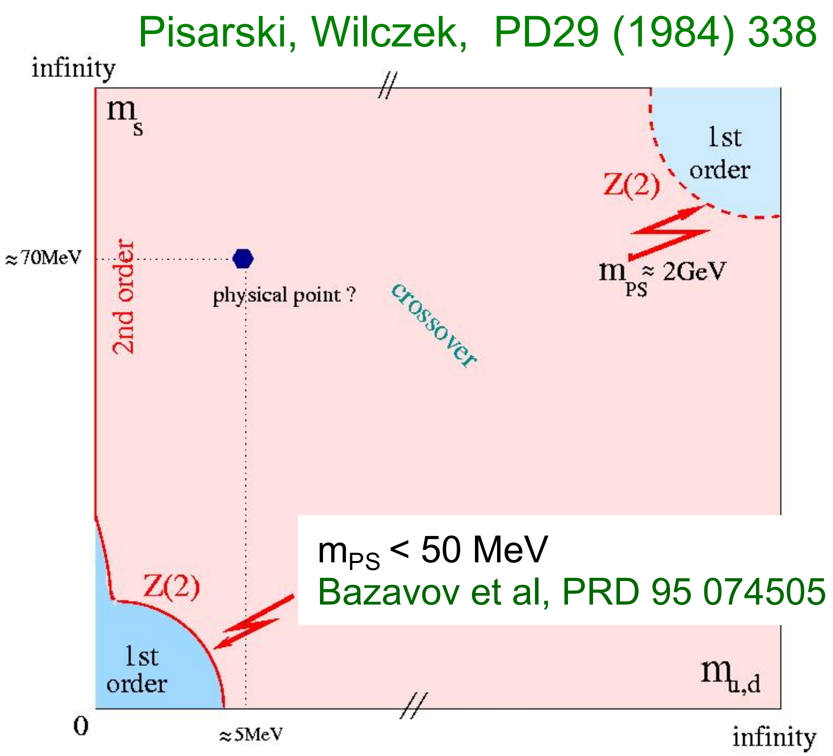
$$A_\mu(0,\mathbf{x}) = e^{i2\pi N/3}A_\mu(1/T,\mathbf{x}),\ \ N = 1,2,3$$

Exact symmetry for infinitely heavy quarks and the order parameter is the Expectation value of the Polyakov loop:

$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T}d\tau A_0(\tau,\vec{x})}$$

$\langle L\rangle = 0$

QCD phase diagram as function of the quark mass



For very large quark masses there is a 1st order deconfining phase transition

Chiral transition:

- For vanishing *u,d* -quark masses the Chiral transition is either 1st order or 2nd order phase transition

- For physical quark masses the transition is an analytic crossover

Aoki et al, Nature 443 (2006) 675
HotQCD, PRL 113 (2014) 082001

Evidence for 2nd order transition in the chiral limit
=> universal properties of QCD transition:

$SU_A(2) \sim O(4)$
relation to spin models

transition is a crossover
for physical quark masses

Finite Temperature QCD and its Lattice Formulation

$$\langle O \rangle = \mathrm{Tr} O e^{-\beta H - \mu N}$$

$\beta = 1/T$

evolution operator in imaginary time

$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0,\mathbf{x}) = A_\mu(\beta,\mathbf{x}) \quad \psi(0,\mathbf{x}) = -\psi(\beta,\mathbf{x})$$

Integral over functions → Lattice → integral with very large (but finite) dimension (> 1000000)

$\mu = 0$ Monte-Carlo Methods

$$\langle O \rangle = \int \prod_x dU_\mu(x) O(det M[U,m,\mu]) e^{-\sum_x S_G[U(x)]}, \quad U_\mu(x) = e^{igaA_\mu(x)}$$

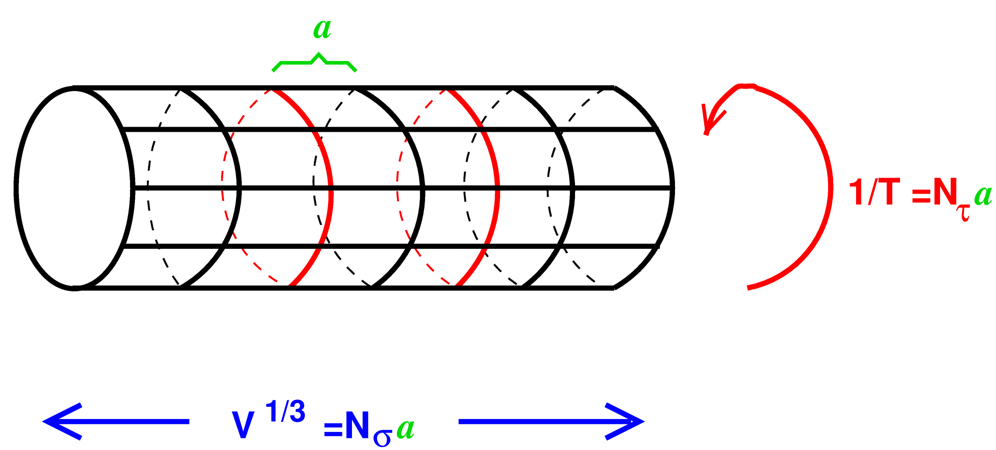
$\mu \neq 0$: $det M(U,m,\mu)$ complex → sign problem (Taylor expansion in μ)

continuum limit $a \to 0$

$N_\tau \to \infty$, N_σ/N_τ fixed

Costs : $\sim m^{-1}$

$\sim a^{-7} \sim N_\tau^7$



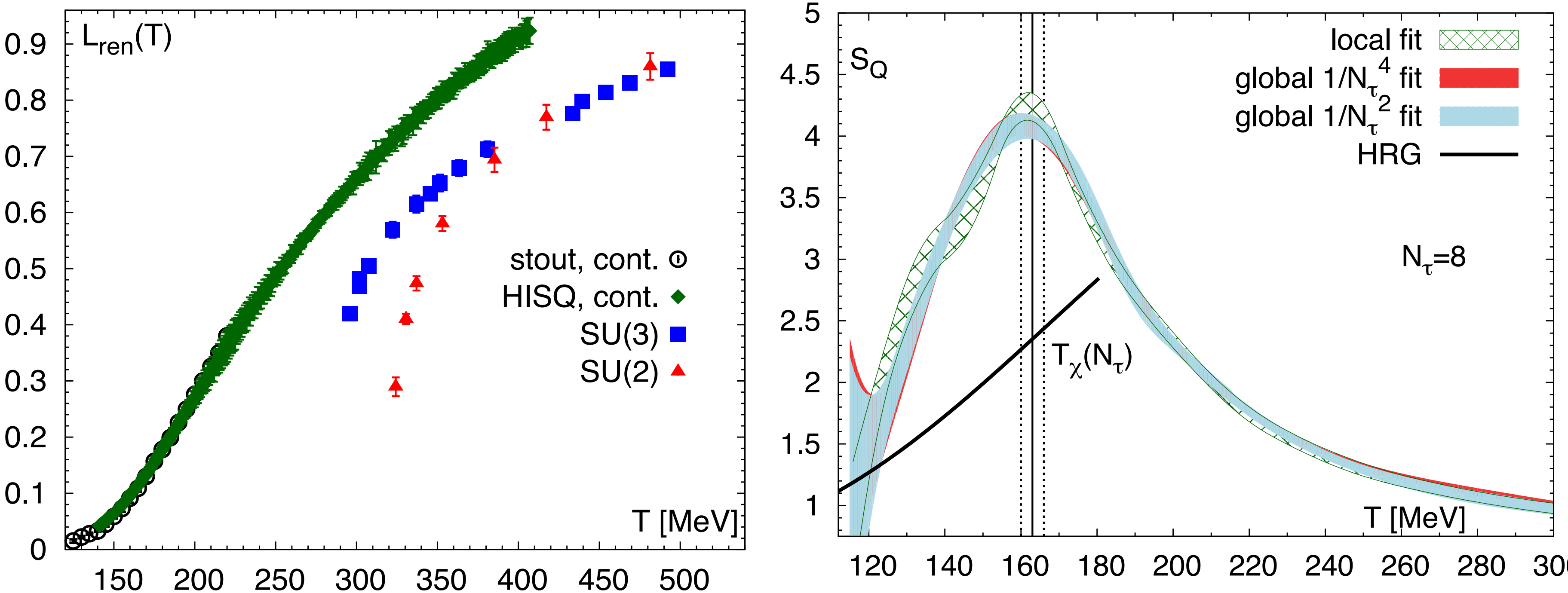
Deconfinement transition in QCD

Renormalized Polyakov loop is an order parameter for deconfinement transition in pure gluo-dynamics

$L_{ren} = \exp(-F_Q(T)/T)$ Entropy of a static quark: $S_Q = -\frac{\partial F_Q}{\partial T}$

TUMQCD, PRD 93 (2016) 114502

$\Rightarrow$ define T_{deconf} from S_Q

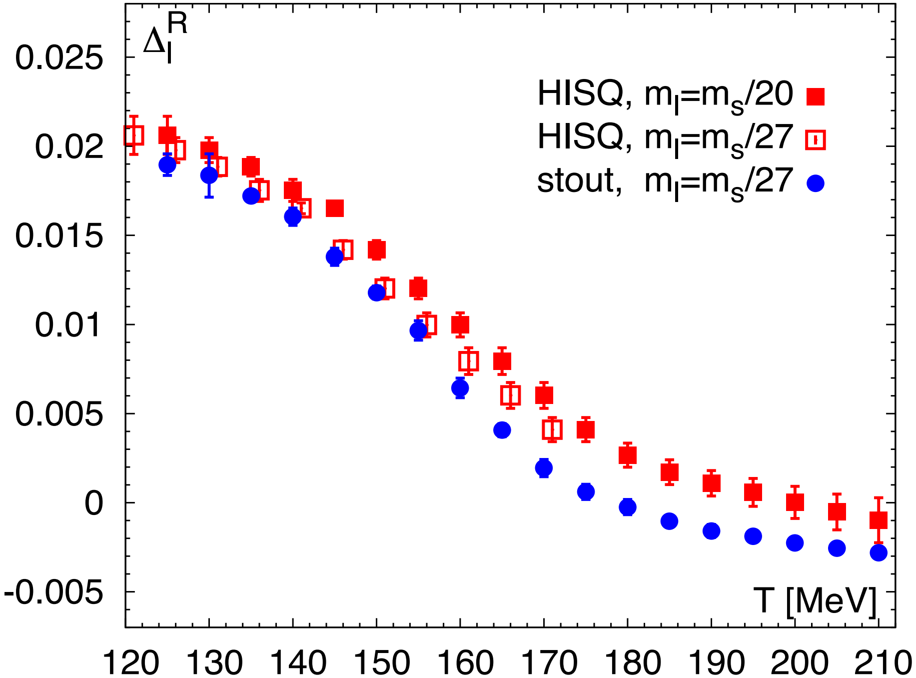


No apparent remnants of the deconfinement phase transition in QCD

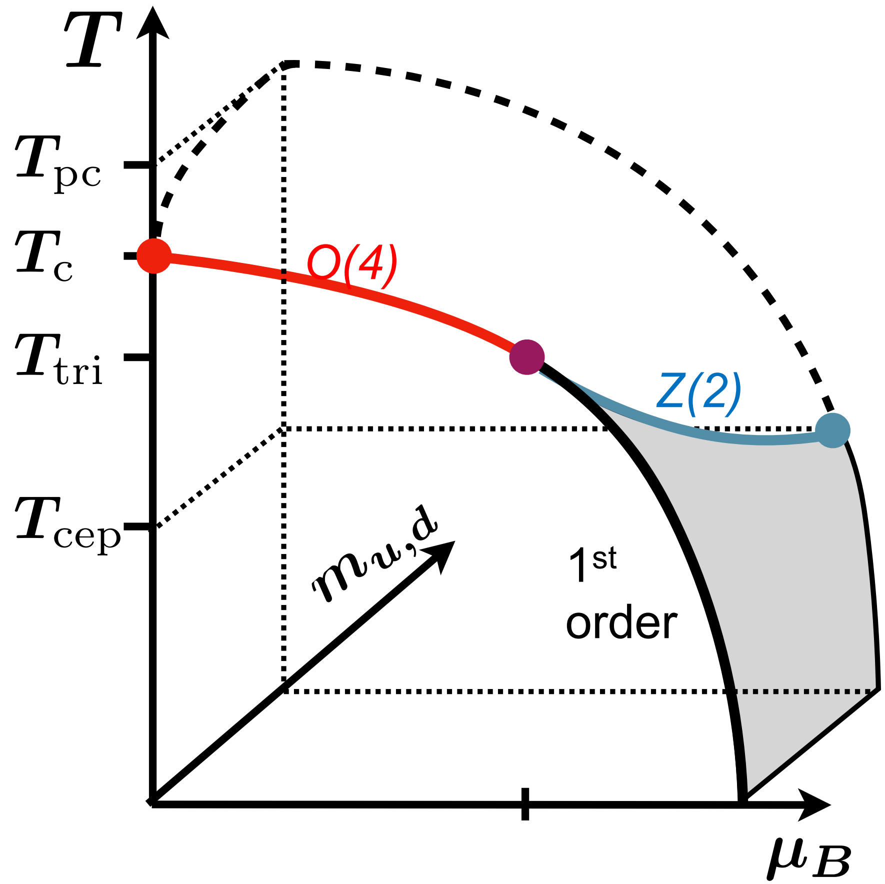
Chiral and deconfinement crossover temperatures agree

Chiral transition transition in QCD

$$\langle\bar\psi\psi\rangle \Rightarrow \Delta_l^R(T) =$$
$$= m_s r_1^4(\langle\bar\psi\psi\rangle_T - \langle\bar\psi\psi\rangle_{T=0}) + d,$$
$$d = m_s r_1^4 \langle\bar\psi\psi\rangle_{T=0}^{m_q=0}, \quad r_1 = 0.3106\text{fm}$$



$$T_c = (154 \pm 8 \pm 1(scale))\text{MeV}$$



Phase transition for $m_{u,d} \to 0$

Bazavov et al (HotQCD), PRD85 (2012) 054503;
Bazavov et al, PRD 87(2013)094505,
Borsányi et al, JHEP 1009 (2010) 073

O(N) scaling and the chiral transition temperature

$SU(2)_V \otimes SU(2)_A \sim O(4)$ governed by universal *O(4)* scaling

For sufficiently small m_l and in the vicinity of the transition temperature:

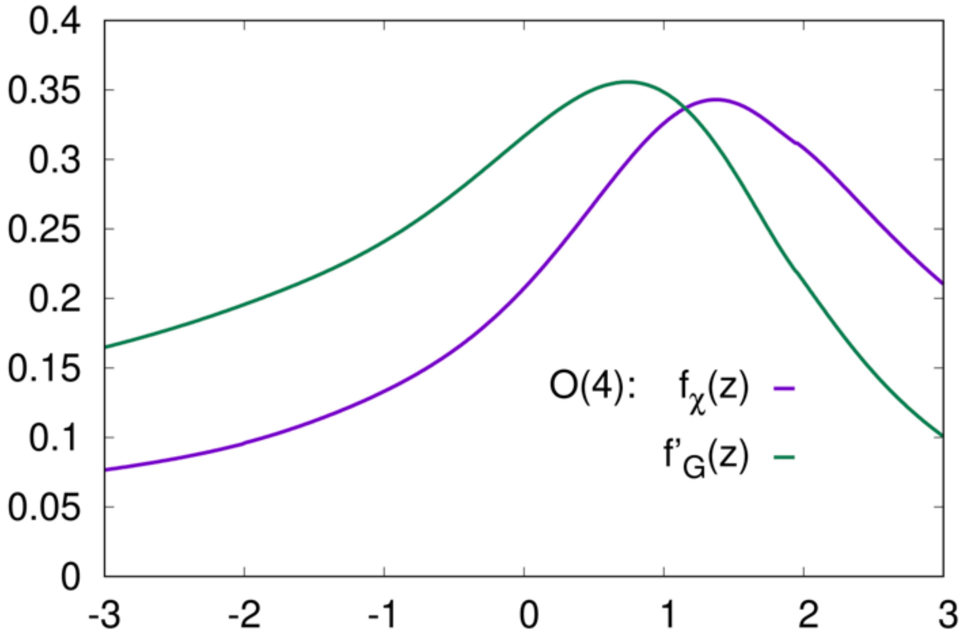
$$f(T, m_l) = -\frac{T}{V}\ln Z = f_{reg}(T, m_l) + f_s(t,h),\ t = \frac{1}{t_0}\left(\frac{T - T_c^0}{T_c^0} + \kappa\frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

$$\langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f \qquad M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\ z = t/h^{1/\beta\delta}$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the susceptibilities

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta - 1} \longrightarrow T_{m,l} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \longrightarrow T_{t,l}$$



$= T_c^0$ in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0} h^{1/\delta-1} f_\chi(z) + reg.\right)$$

universal scaling function has a peak at $z = z_p$

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0 \frac{z_p}{z_0} H^{1/(\beta\delta)} + \ldots$$

The chiral cross-over temperature for physical masses

Chiral order parameter:

$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle\bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle\bar{s}s\rangle\right] \qquad \langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f$$

and the corresponding susceptibilities:

$$\chi^\Sigma = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma \qquad \chi = \frac{m_s^2}{f_K^4}\left[\langle(\bar{u}u + \bar{d}d)^2\rangle - (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)^2\right]$$

For non-zero chemical potential we use Taylor expansion

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \quad \chi(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad C_0^\Sigma = \Sigma, \quad C_0^\chi = \chi$$

Derivatives in μ_X^2 are similar to derivatives in T $\qquad$ *e.g.* $\partial_T C_0^\chi \sim C_2^\chi$

$\Rightarrow$ the following quantities will peak at T_c

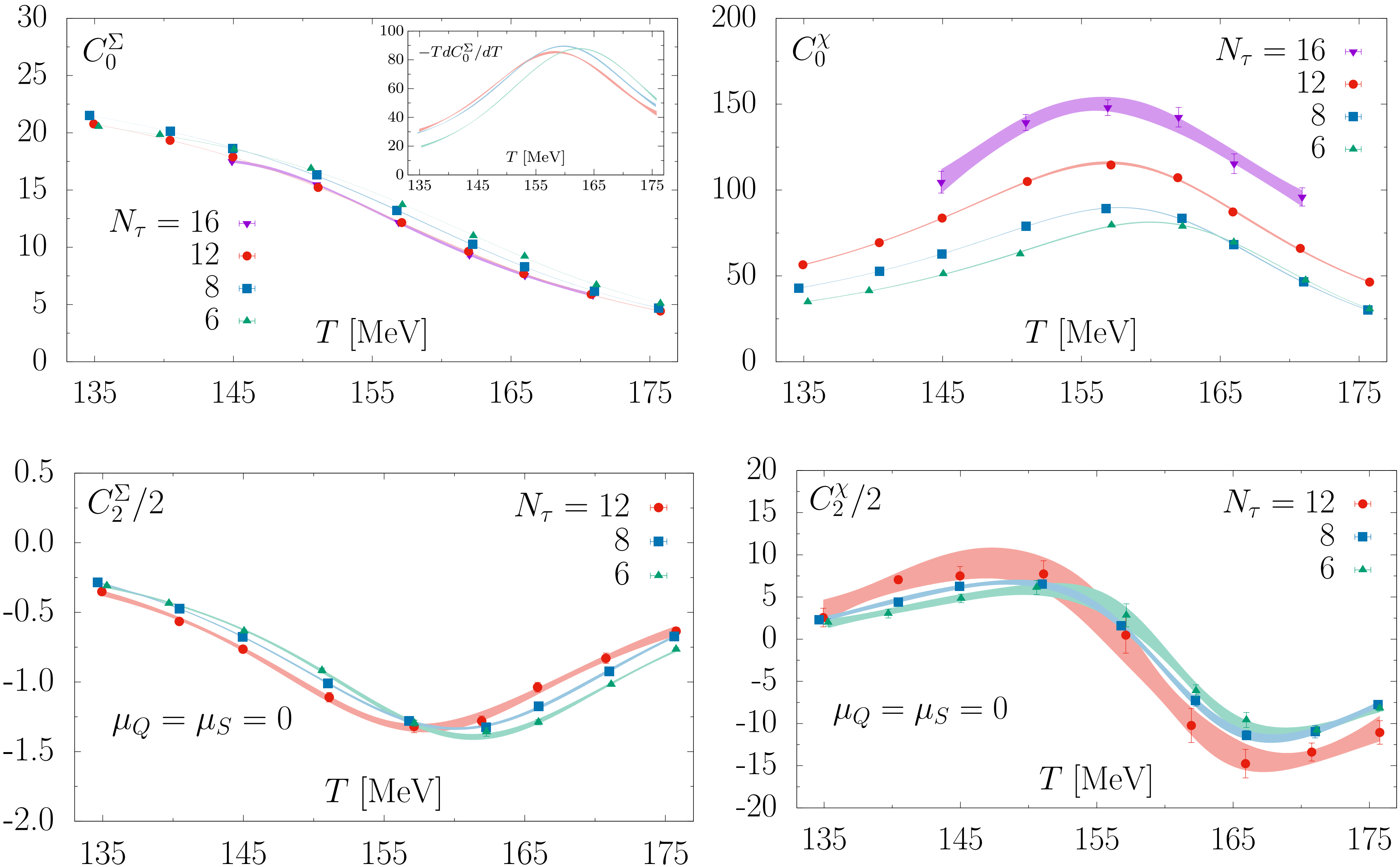
$\chi^\Sigma,\ C_0^\chi(T) \sim \chi_{l,m} \qquad \partial_T C_0^\Sigma,\ C_2^\Sigma(T) \sim \chi_{t,m}$ $\qquad$ HotQCD, PLB 795 (2019) 15

5 different definitions of T^{pc}:

$$\partial_T C_0^\chi = 0,\ \partial_T C_0^\Sigma = 0\ ,C_2^\chi = 0 \qquad \partial_T^2 C_0^\Sigma = 0,\ \partial_T C_2^\Sigma = 0$$

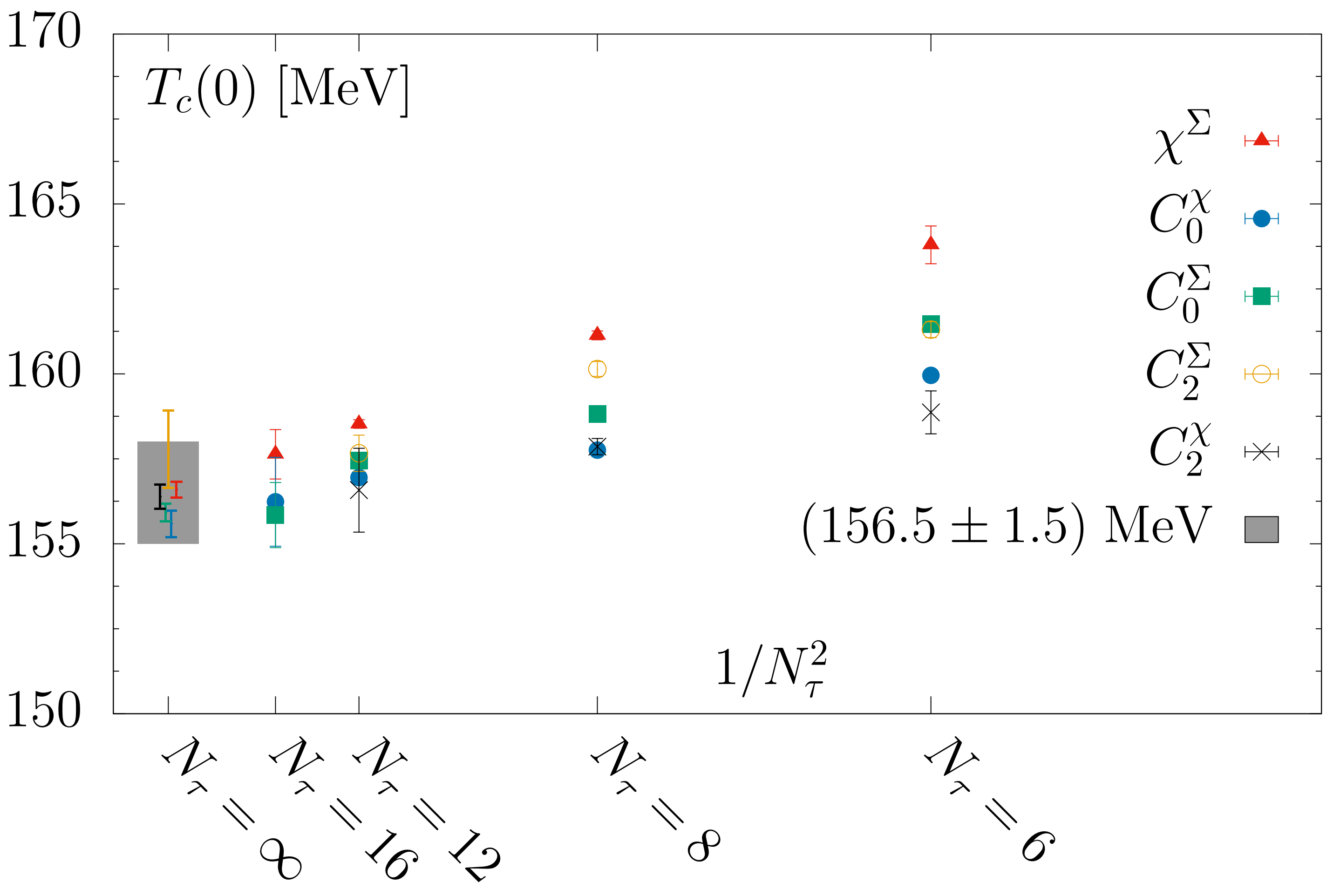
The 5 different T_c values reduce to $T_{l,m}$ and $T_{l,t}$ if regular part is zero

Lattice calculations based on 100K - 500 K configurations, $N_\tau = 6 - 12$, and 4K configurations for $N_\tau = 16$



HotQCD, PLB 795 (2019) 15

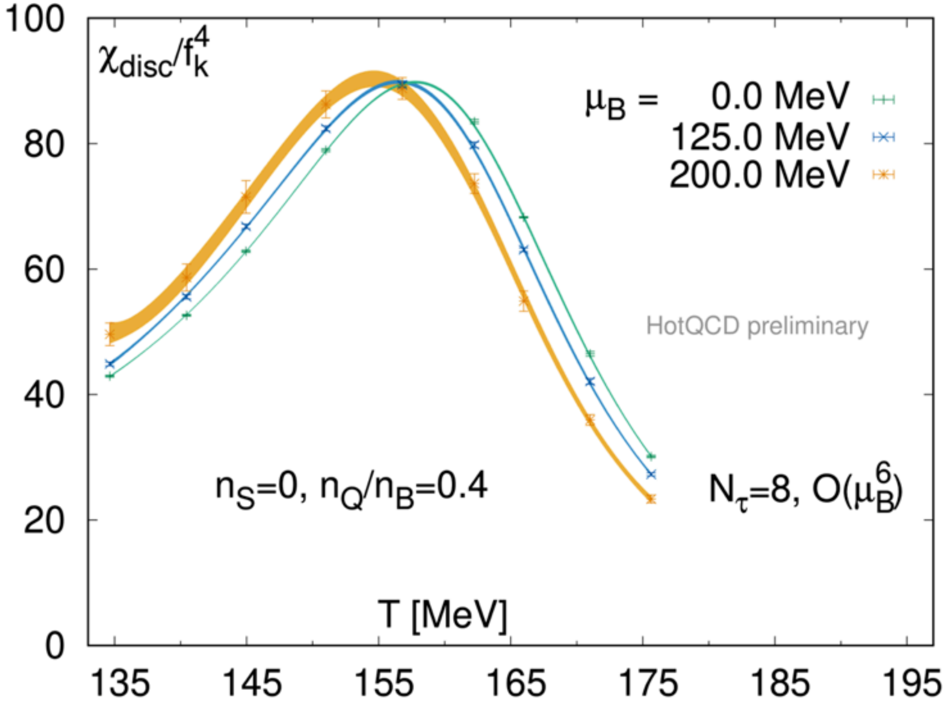
Different definitions of T_c surprisingly agree in the continuum limit and we for zero chemical potential we get $T_c = 156 \pm 1.5$ MeV

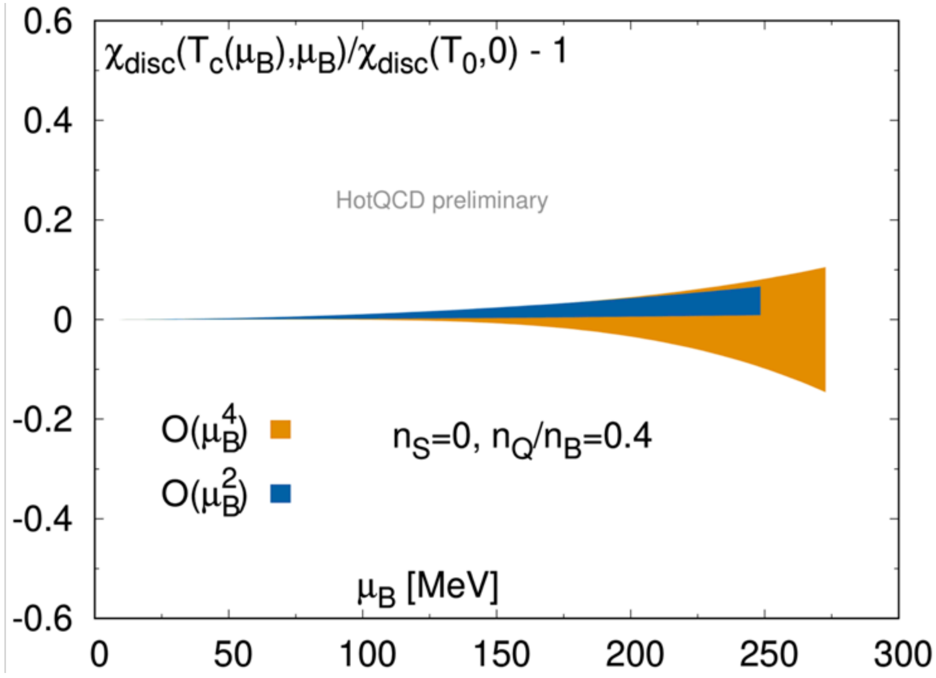


HotQCD, PLB 795 (2019) 15

The chiral susceptibility at baryon density non-zero density

Conditions in heavy ion collisions: $n_B > 0$, $n_S = 0$, $n_Q = 0.4 n_B$ (for Au, Pb)

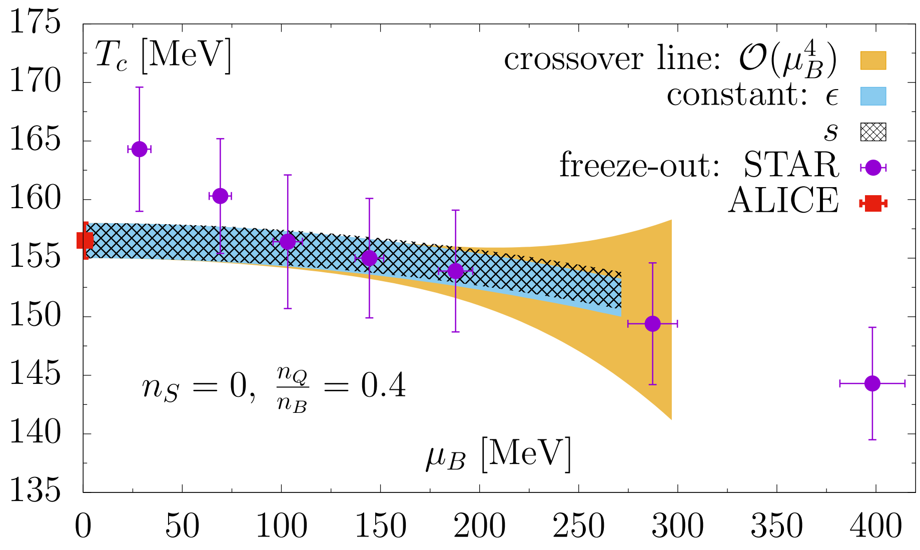




little change in peak-height & width with increasing baryon chemical potential: no indication of a stronger transition becoming stronger

The chiral cross-over temperature at non-zero density

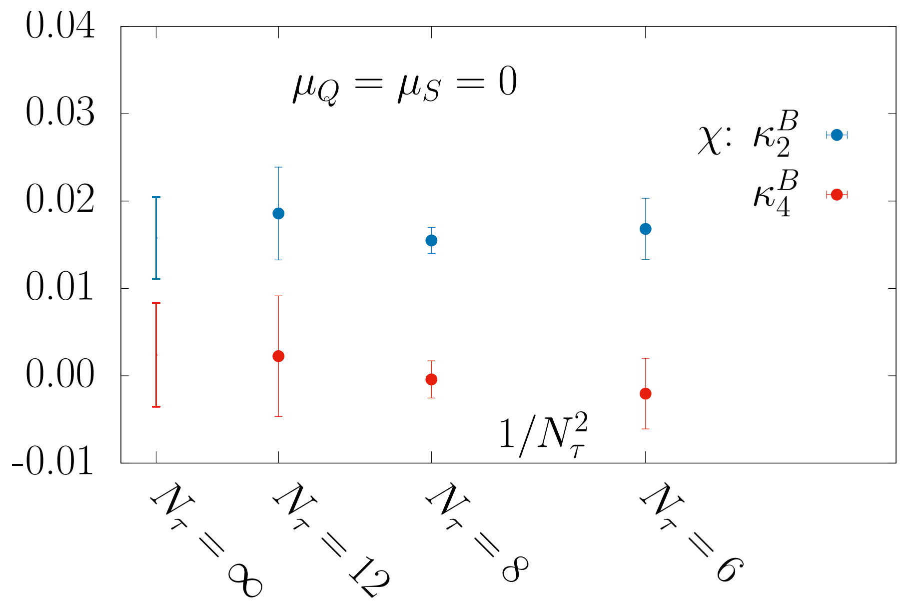
$$T_c(\mu_B) = T_c(0)\left[1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4\right]$$



The freeze-out condition corresponding to constant energy density or constant entropy density agrees with the crossover line within errors

The μ_B dependence of T_c is small

$$\kappa_2^{B,\chi} \simeq \kappa_2^{B,\Sigma}$$

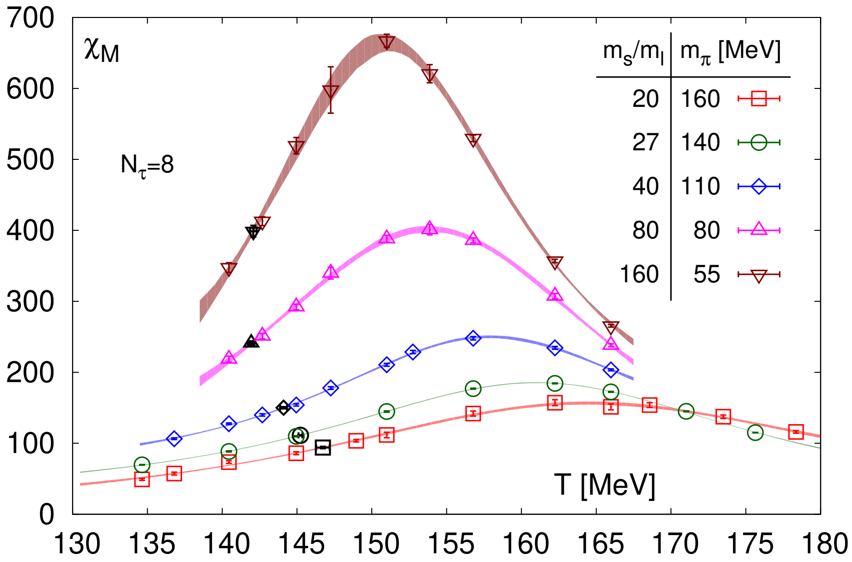


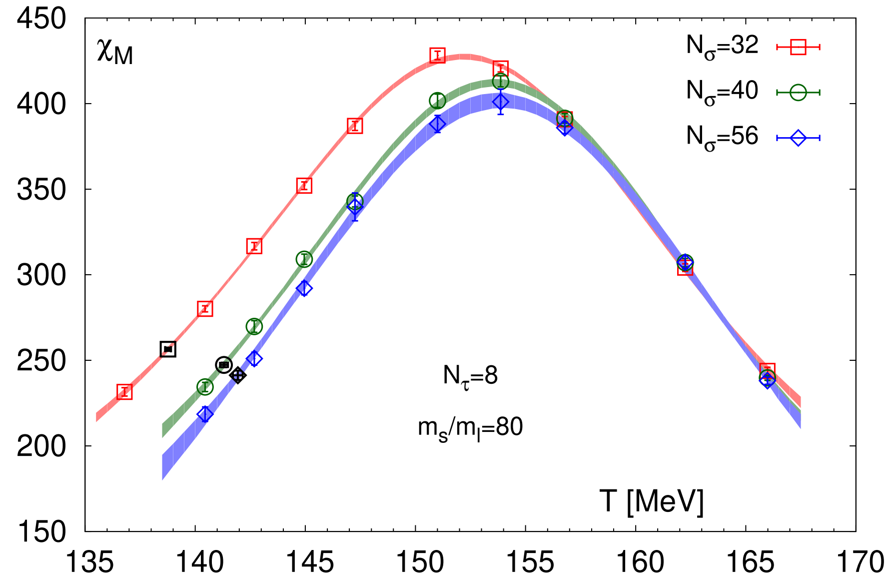
HotQCD, PLB 795 (2019) 15

Chiral phase transition in 2+1 flavor QCD

What is the nature of the chiral transition in 2+1 flavor QCD for fixed m_s and $m_l \to 0$?

HotQCD, PRL 123 (2019) 062002





$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L) \ ,$$

$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L) \ .$$

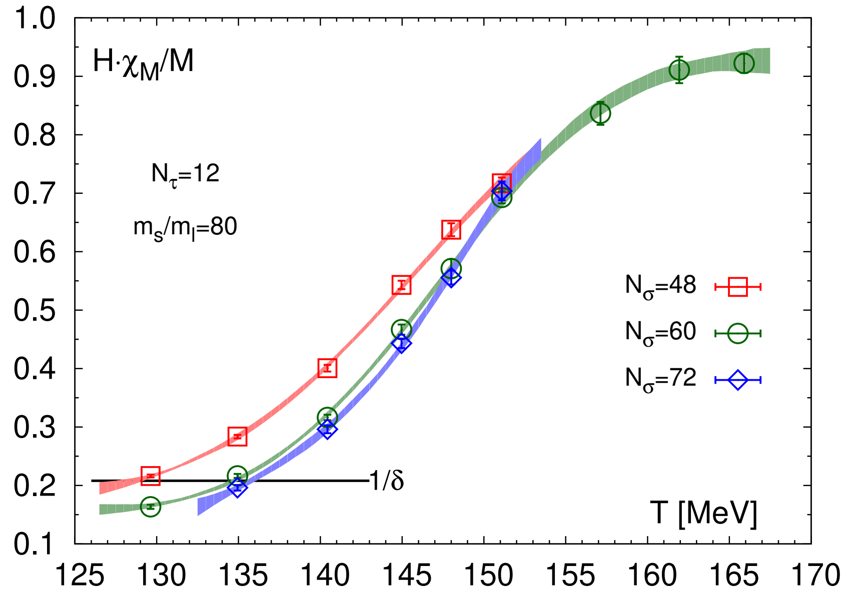
Finite volume:

$$z_L = l_0/(L h^{\nu/\beta\delta})$$

$$T_p(H, L) = T_c^0 \left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta}\right) \ + \ \text{sub leading}$$

New estimators:

$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0}\right) H^{1/\beta\delta}\right)$$

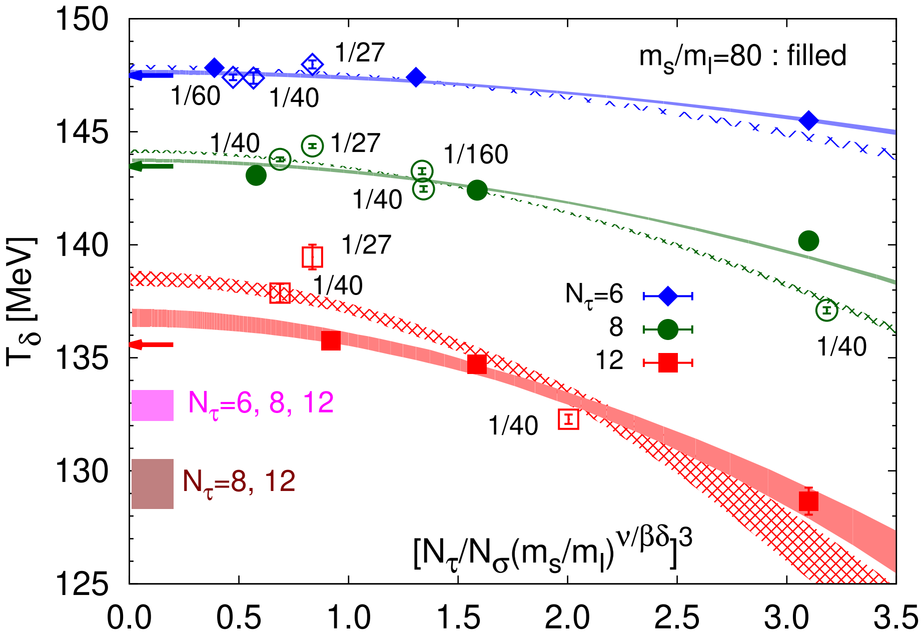


$$\frac{H\chi_M(T_\delta,H,L)}{M(T_\delta,H,L)} = \frac{1}{\delta}\,,$$

$$\chi_M(T_{60},H) = 0.6\chi_M^{max}\,.$$

$$T_X(H,L) = T_c^0\left(1+\left(\frac{z_X(z_L)}{z_0}\right)H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}\,,\quad X=\delta,\ 60$$

$$z_{60} \simeq z_\delta \simeq 0$$



Use $O(4)$ fits for m_l and volume dependence

Continuum extrapolations:

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

HotQCD, PRL 123 (2019) 062002

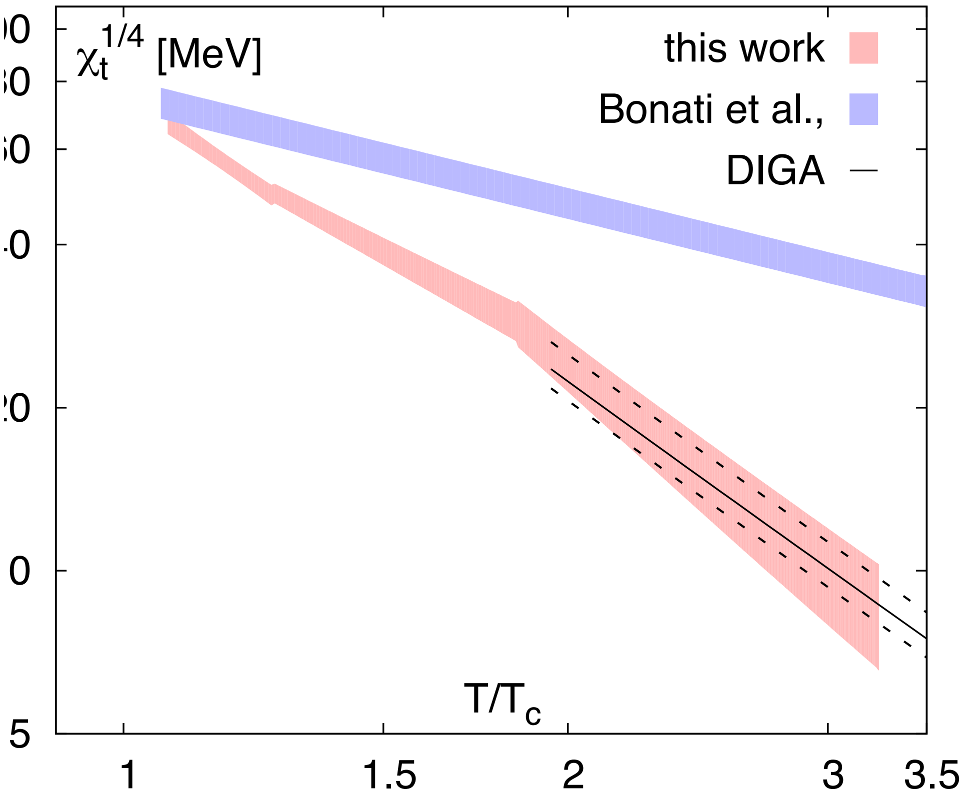
Topological Susceptibility and Instanton gas

The amount of $U_A(1)$ breaking at high T is reduced because of the reduced instanton density => dilute instanton gas approximation (DIGA),

Gross, Pisarski, Yaffe, RMP 53 (1981) 43

Topological susceptibility with HISQ action using Symanzik flow $\chi_{top} = \frac{1}{V}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$

Schadler, Sharma, PP, PLB 762 (2016) 498



$$\chi_t(T) = 2\int_0^\infty d\rho\ D(\rho)\ G(\rho\pi T),$$

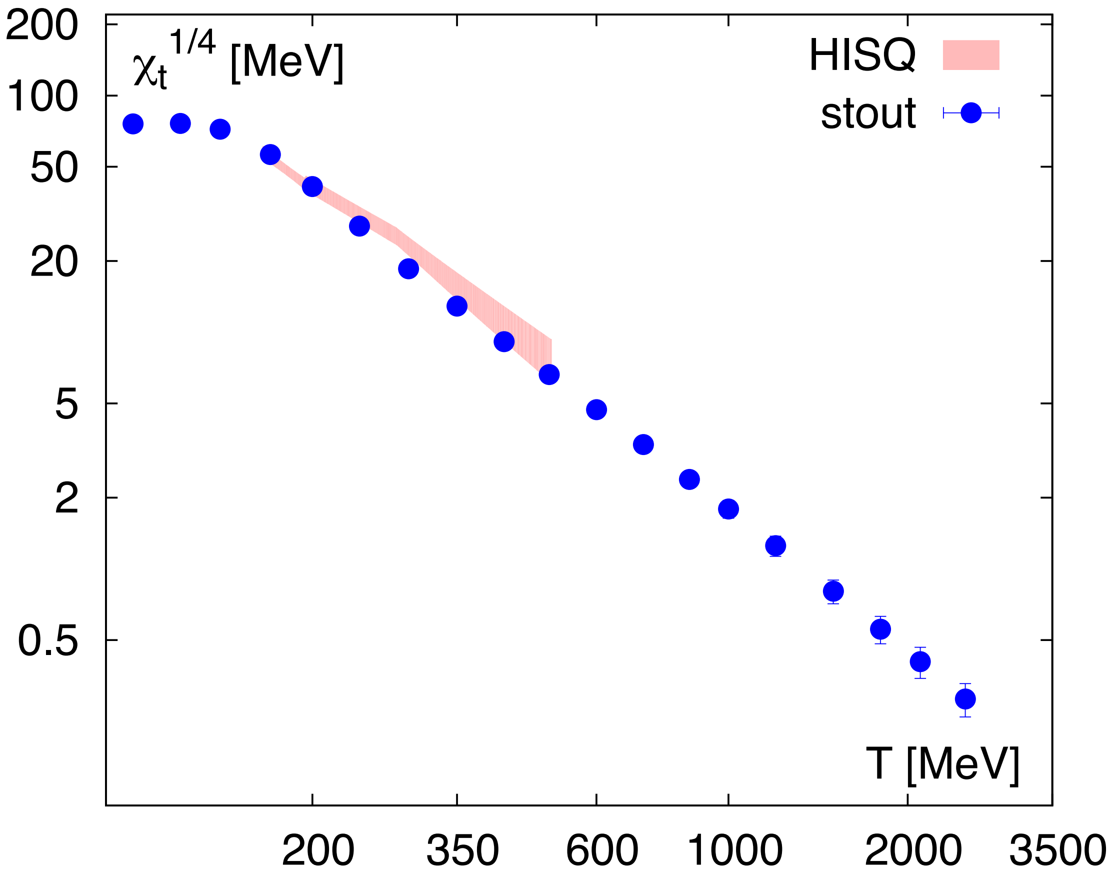
$$D(\rho) = \frac{d_{\overline{MS}}}{\rho^5}\left(\frac{2\pi}{\alpha_s(\mu)}\right)^6 e^{-\frac{2\pi}{\alpha_s(\mu)}} \times (\rho\mu)^{\beta_0+(\beta_1-12\beta_0+8N_f)\frac{\alpha_s}{4\pi}}(\rho m_i)^{N_f}.$$

Ringwald, A. and Schrempp, PLB459 (1999) 249

DIGA is compatible with the lattice results if a K factor ~1.79 is included

Similar K factor was found for SU(3) gauge theory,
Borsányi et al, PLB 752 (2016) 175

Topological Susceptibility and Instanton gas (cont'd)



Constraint on axion decay constant:

$f_a < 1.2 \cdot 10^{12}$ GeV

HISQ: PP, Sharma, Schadler, PLB 762 (2016) 498

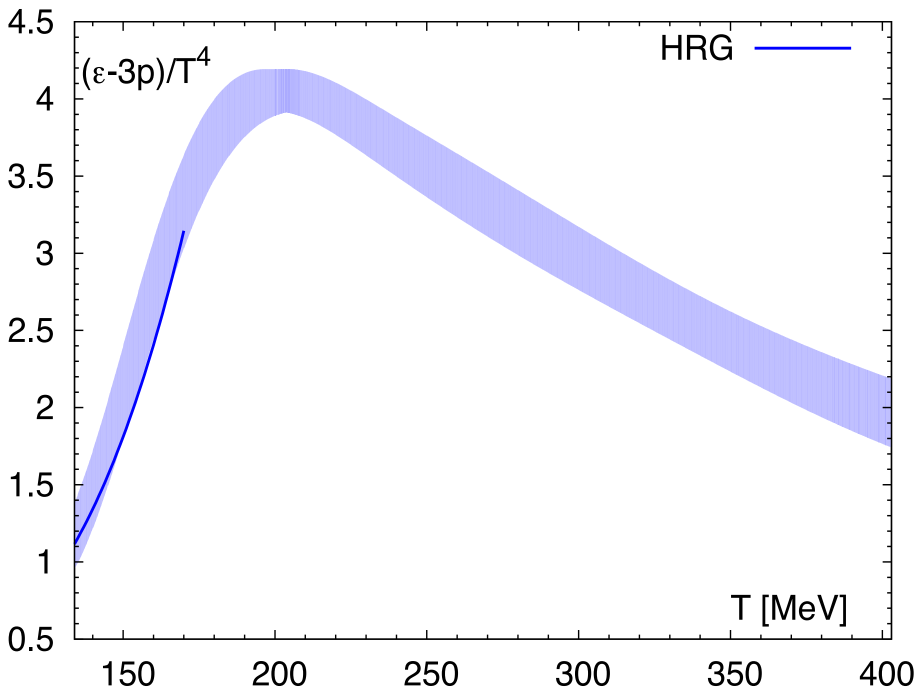
stout: Borsányi et al, Nature 539 (2016) no.7627, 69-71

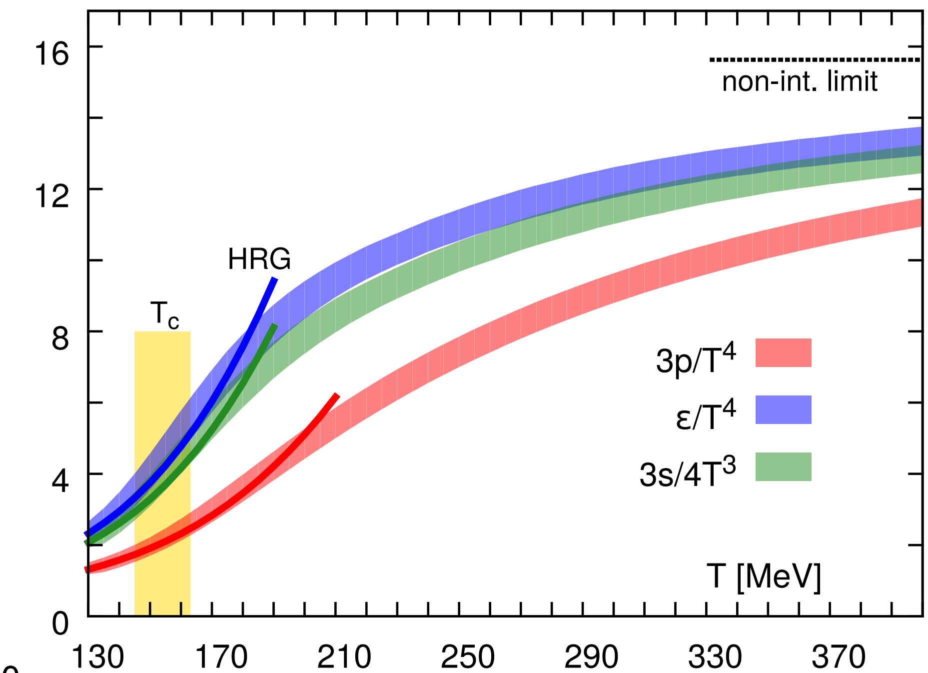
Also confirmed by recent lattice calculations: Burger et al, arXiv:1805.06001, Bonati et al, arXiv:1807.07954

QCD thermodynamics in the continuum limit

Set the lower integration limit to $T_0=130$ MeV and take $p_0=p^{HRG}(T=130$ MeV$)$ ➡ $p(T)$

Bazavov et al, PRD 90 (2014) 094503



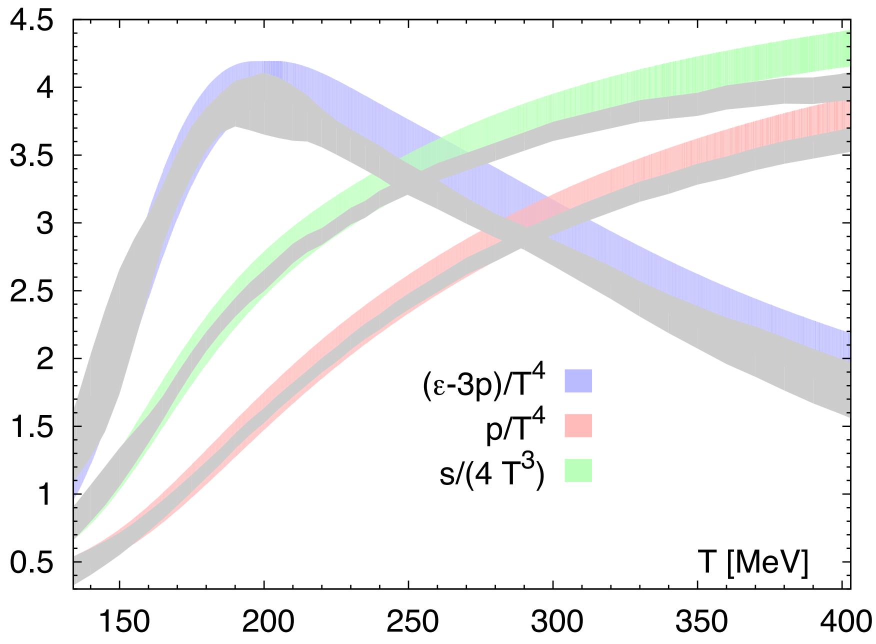


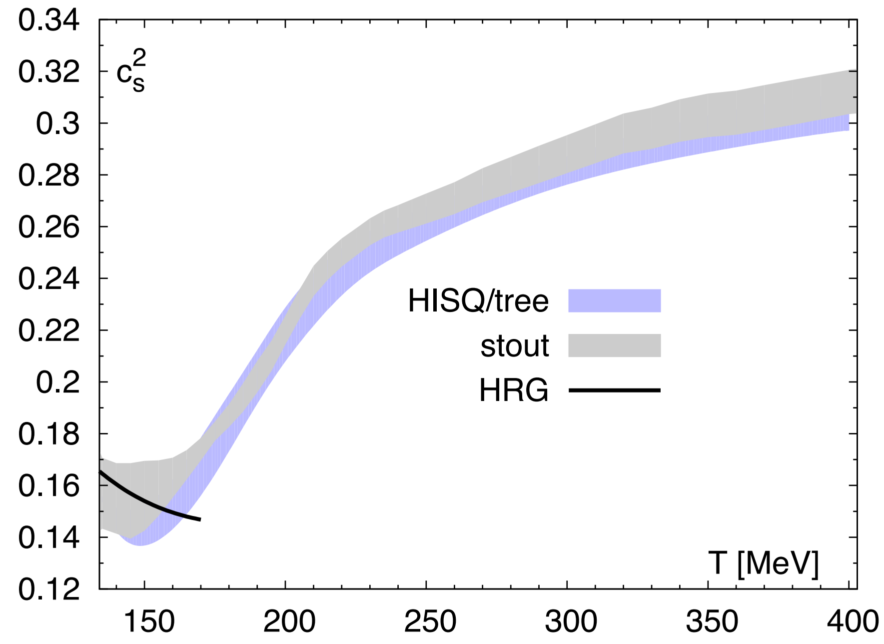
$$T_c = 156 \pm 1.5 \text{ MeV} \qquad \epsilon_{nucl} \simeq 150\text{MeV}/\text{fm}^3$$

$$\epsilon_c = 420(60) \text{ MeV}/\text{fm}^3 \qquad \epsilon_{proton} \simeq 450\text{MeV}/\text{fm}^3$$

HRG: all resonances from PDG treated as stable (zero width) particles in an ideal gas

Comparison of different continuum limit





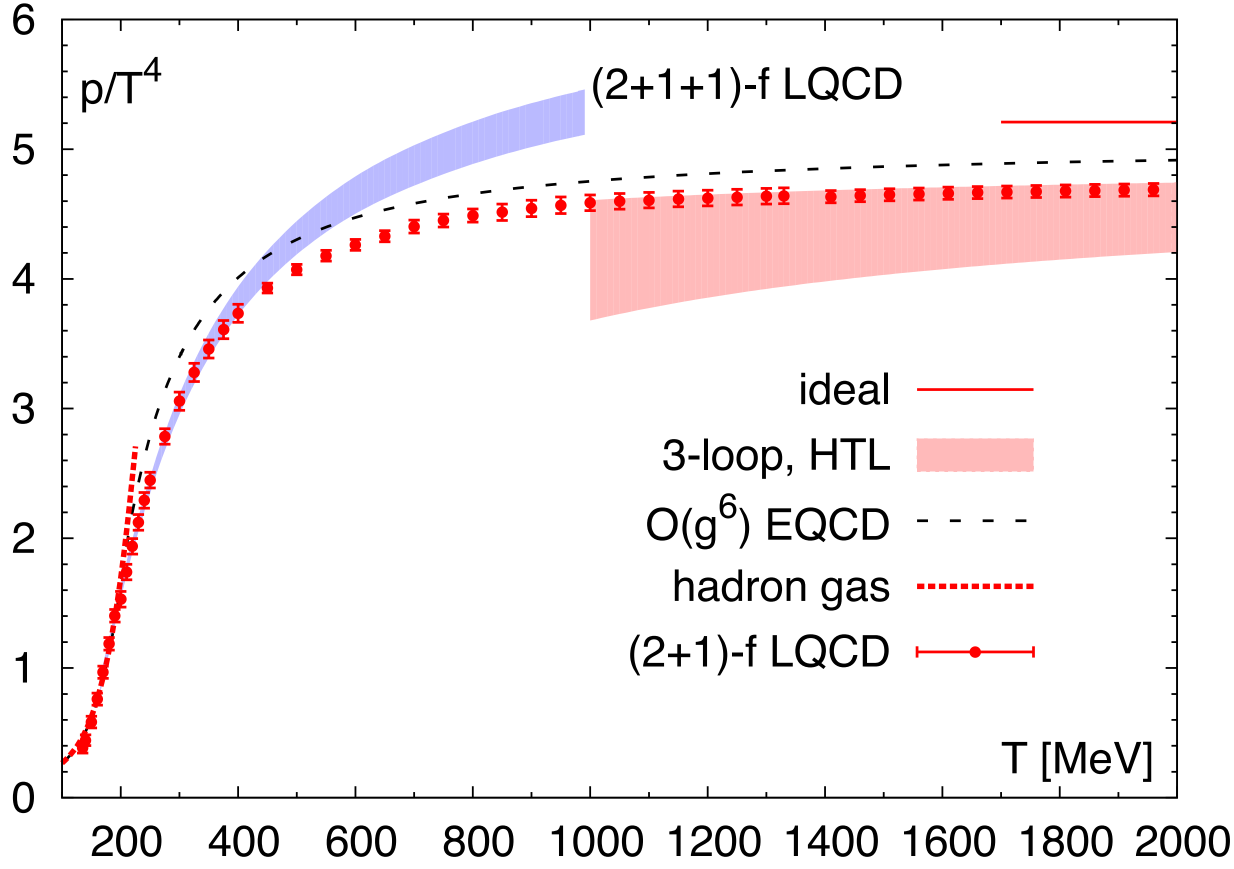
Continuum results obtained with stout and HISQ action agree reasonably well given their errors (some tension for the entropy density)

Even in the transition region the speed of sound is not much smaller than the HRG speed of sound (the EoS is never really soft)

HISQ: Bazavov et al, PRD 90 (2014) 094503
stout: Borsányi et al, PLB730 (2014) 99

Equation of state at high temperatures

2+1 flavor: Bazavov, PP, Weber, PRD 97 (2018) 014510
2+1+1 flavor (with charm quark): Borsányi et al (BW Coll.), Nature 539 (2016) 69



Lattice results are extrapolated to continuum *(a=0)*

Charm quark contribution to QCD pressure is significant for *T>400* MeV
The pressure is well described by weak coupling calculations for *T>1000* MeV

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_C)}{T^4}=\sum_{ijkl}\frac{1}{i!j!k!l!}\chi_{ijkl}^{BQSC}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k\left(\frac{\mu_C}{T}\right)^l$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4}=\sum_{ijkl}\frac{1}{i!j!k!l!}\chi_{ijkl}^{udsc}\left(\frac{\mu_u}{T}\right)^i\left(\frac{\mu_d}{T}\right)^j\left(\frac{\mu_s}{T}\right)^k\left(\frac{\mu_c}{T}\right)^l$$ quark

$$\chi_{ijkl}^{abcd}=T^{i+j+k+l}\frac{\partial^i}{\partial\mu_b^i}\frac{\partial^j}{\partial\mu_b^j}\frac{\partial^k}{\partial\mu_c^i}\frac{\partial^l}{\partial\mu_d^l}\ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d)\,|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the susceptibilities, i.e. the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X=\chi_X=\frac{1}{VT^3}\left(\langle X^2\rangle-\langle X\rangle^2\right)\qquad\chi_{11}^{XY}=\frac{1}{VT^3}\left(\langle XY\rangle-\langle X\rangle\langle Y\rangle\right)$$

⬇

information about carriers of the conserved charges (hadrons or quarks)

⬇

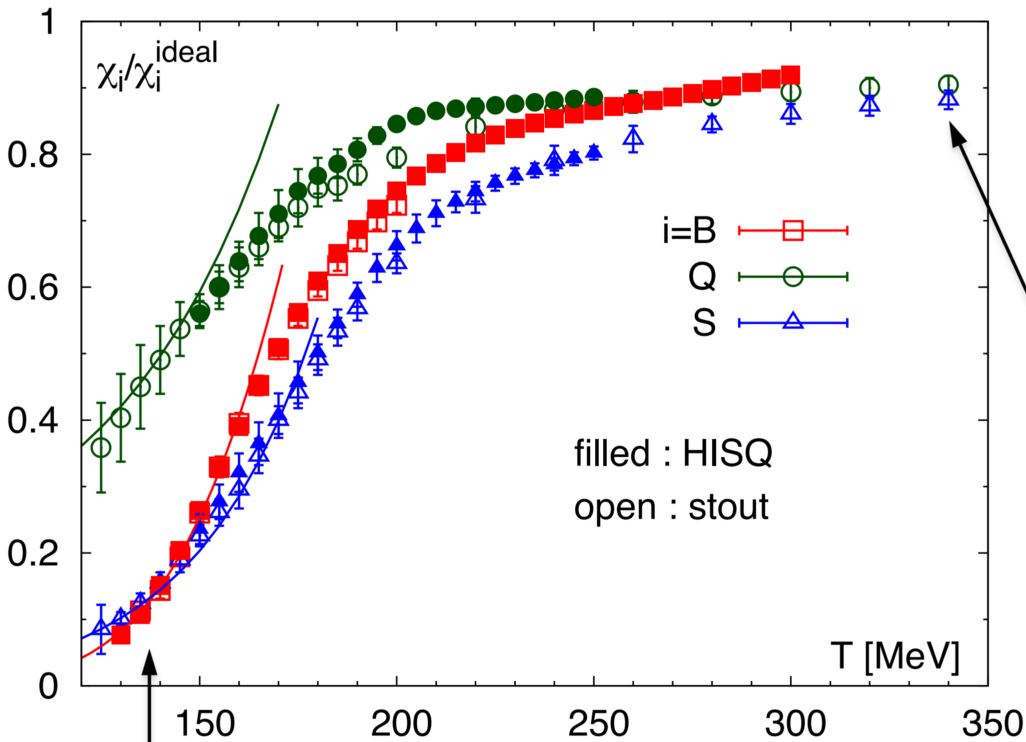
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number $$\chi_B^{\rm ideal} = \frac{1}{3}$$

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge $$\chi_Q^{\rm ideal} = \frac{2}{3}$$

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness $$\chi_S^{\rm ideal} = 1$$



HISQ: Bazavov et al, PRD86 (2012) 034509
PRD 95 (2017) 054504
stout: Borsányi et al. JHEP 1201 (2012) 138

HRG: includes missing states and repulsivemean field, work with Huovinen

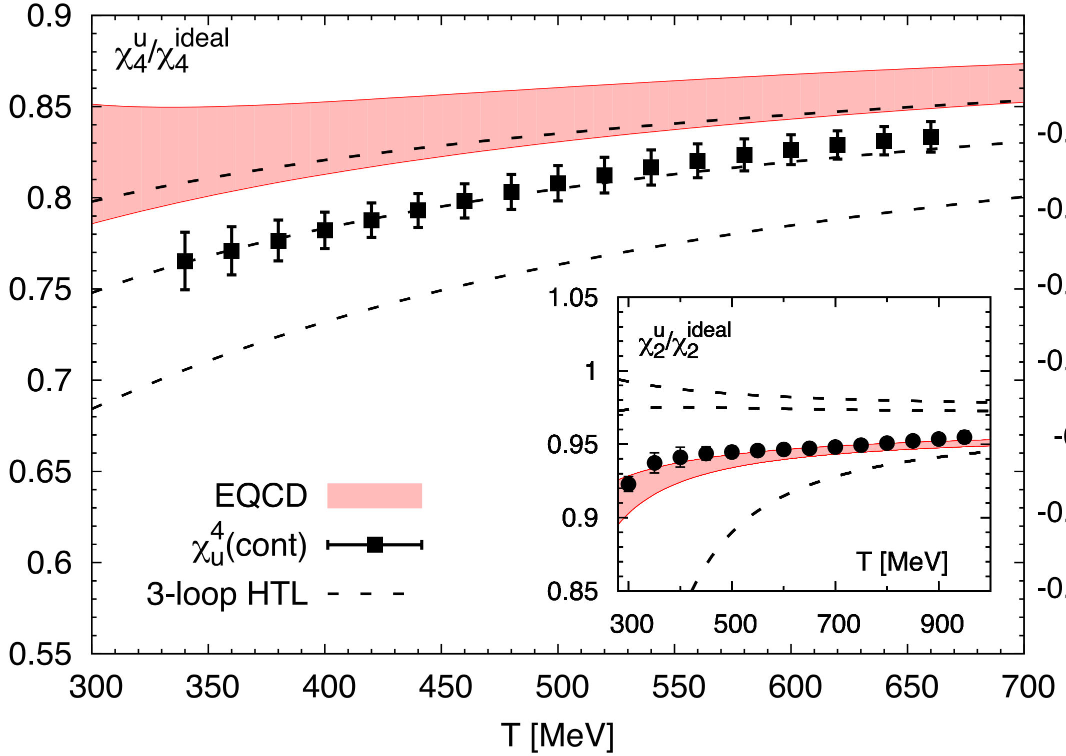
conserved charges carried by light quarks

conserved charges are carried by massive hadrons

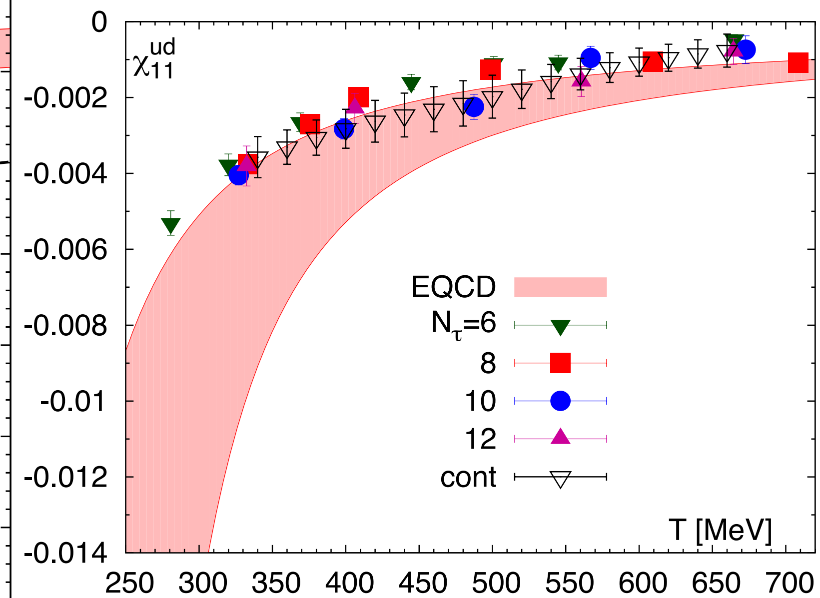
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations



- Good agreement between continuum extrapolated lattice results and the weak coupling approach
- Quark number correlations vanish at any loop order but can be calculated in EQCD and the EQCD calculations agree with the continuum extrapolated lattice results

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

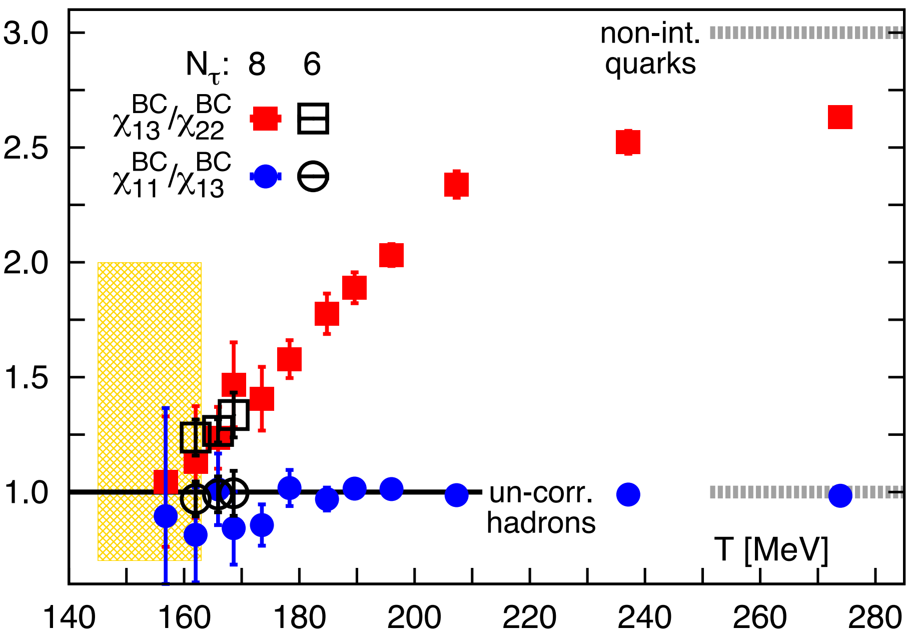
Fluctuation and correlations and deconfinement of charm

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T,\mu_X,\mu_Y,\mu_C)/T^4}{\partial\mu_X^n \partial\mu_Y^m \partial\mu_C^l}$$

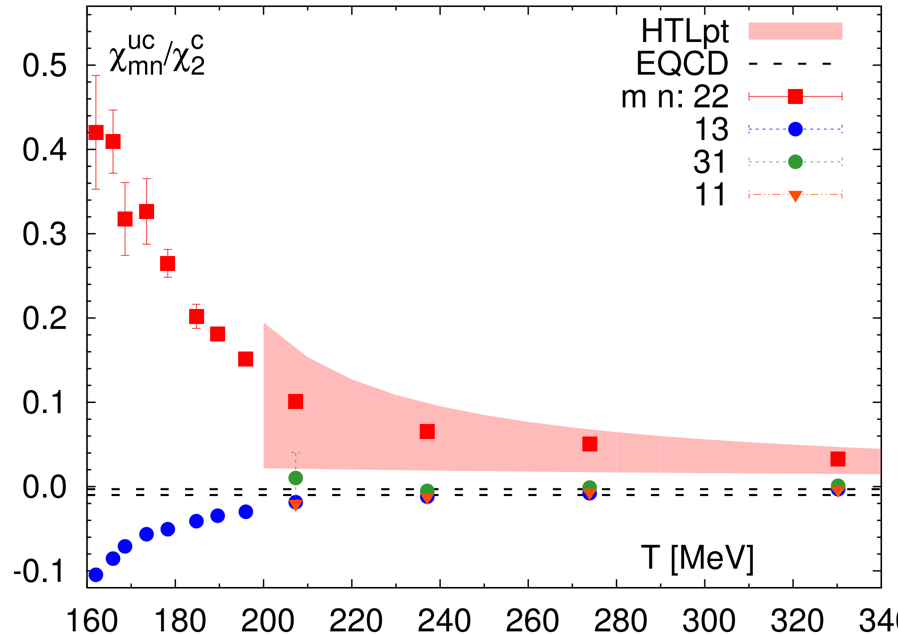
Bazavov et al, PLB 737 (2014) 210

$m_c \gg T$ ⇒ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



Hadronic description breaks down just above T_c
⇒ open charn deconfines above T_c



The charm-light quark correlations can be understood in terms of weak coupling calculations for *T>250* MeV but are much larger than the weak coupling result close to T_c

Quasi-particle model for charm degrees of freedom

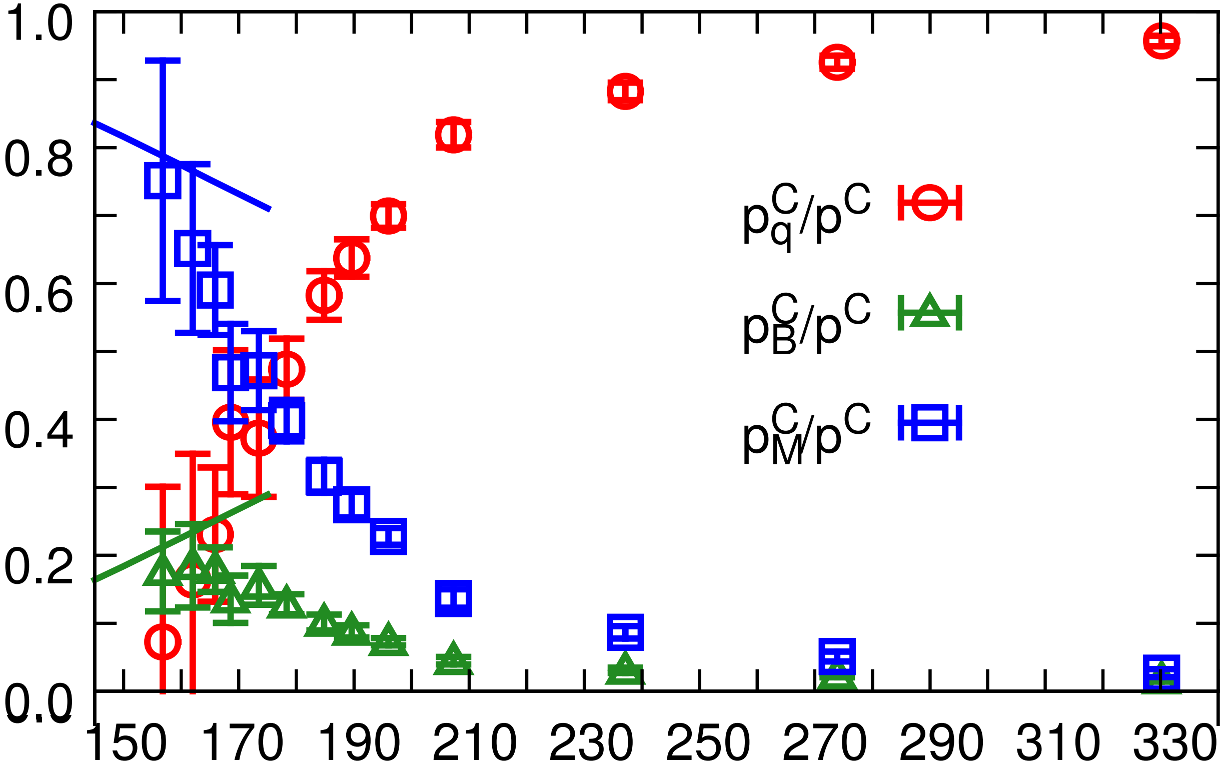
Charm dof are good quasi-particles at all T because $M_c >> T$ and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T)\cosh(\hat{\mu}_C)$$

$$\hat{\mu}_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T>200$ MeV



Partial pressures drop because hadronic cxcitations become broad at high temperatures (bound state peaks merge with the continuum)

Mukherjee, PP, Sharma, PRD93 (2016) 014502

Vice versa for quarks

Spatial Meson Correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x}, -i\tau) J(\mathbf{0}, 0)\rangle_T, \;\; G(z \to \infty, T) = A e^{-m_{scr}(T) z}$$

but related to the same spectral functions $$G(z,T) = 2\int_{-\infty}^{\infty} dp e^{ipz} \int_0^{\infty} d\omega \frac{\rho(\omega,p,T)}{\omega}$$

Low T limit :

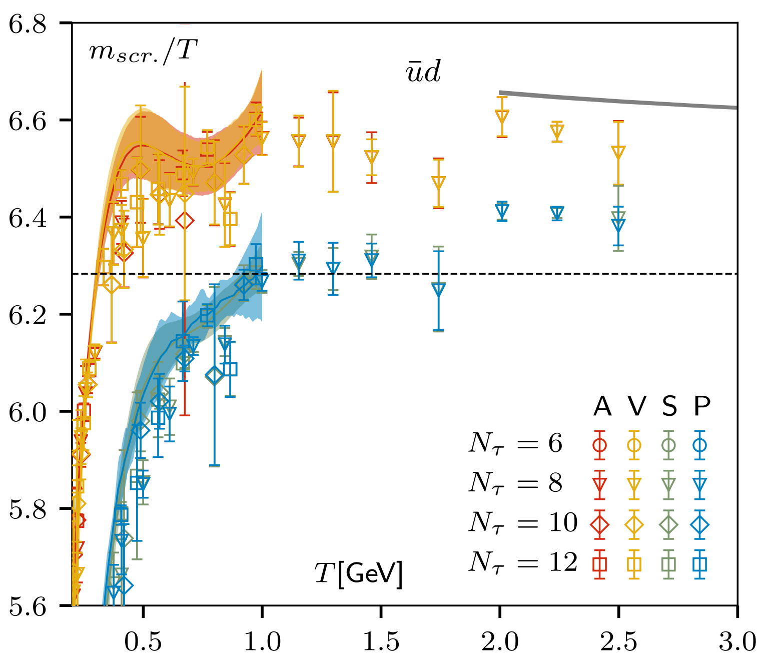
$$\sigma(\omega,p,T) \simeq A_{mes}\delta(\omega^2 - p^2 - M_{mes}^2)$$

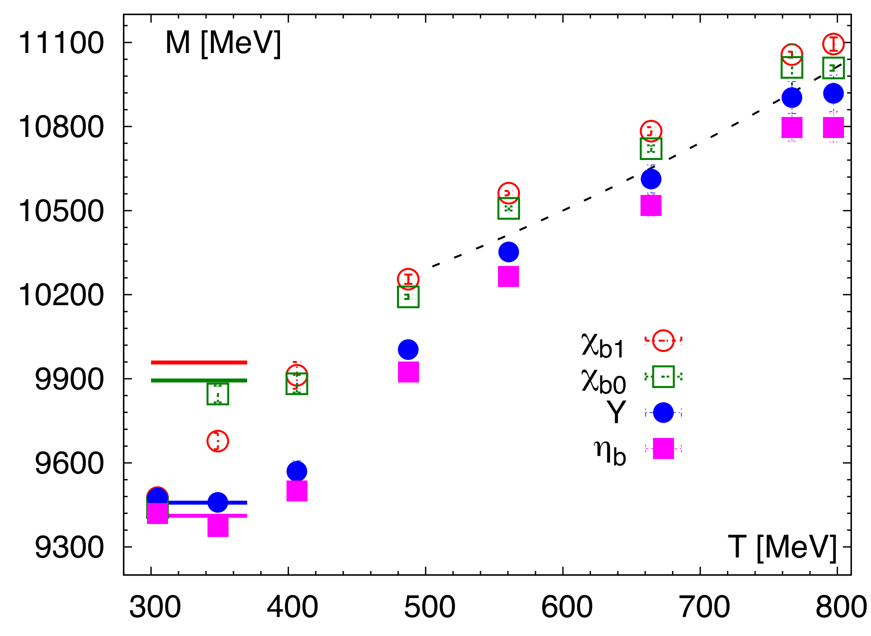
$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

High T limit :

$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

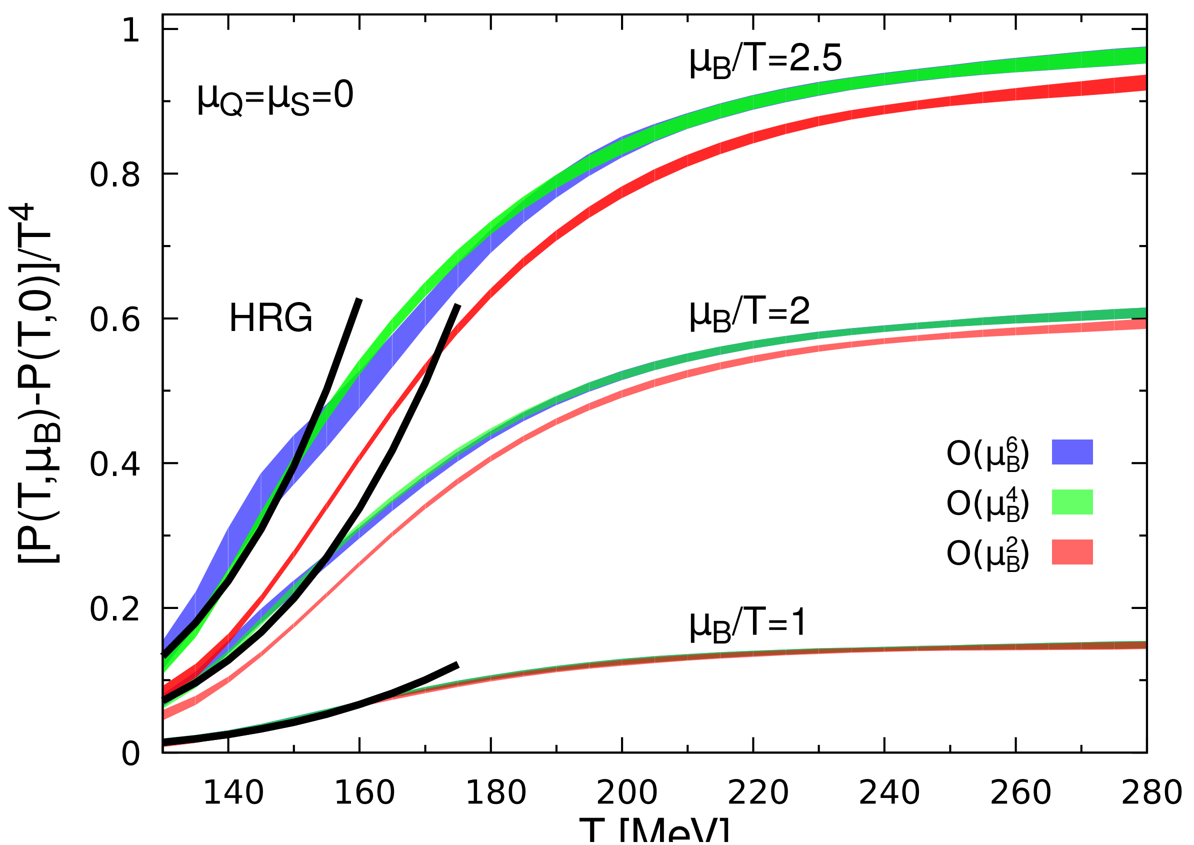
See talk by S. Sharma in parallel





Equation of state at non-zero net baryon density

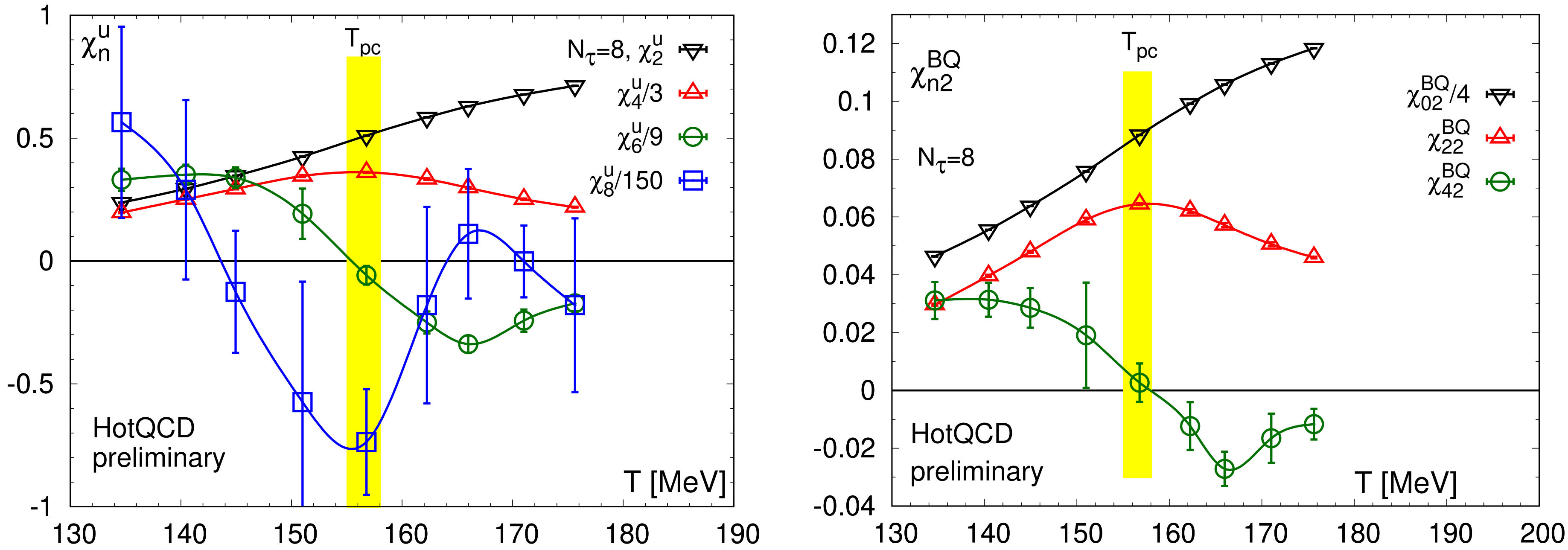
6[th] order Taylor expansion, HotQCD, PRD 95 (2017) 054504



Truncation errors of the 6[th] order Taylor expansions are small for $\mu_B/T<2.5$

Temperature dependence of Taylor expansion coefficients

Karsch, arXiv:1905.03936



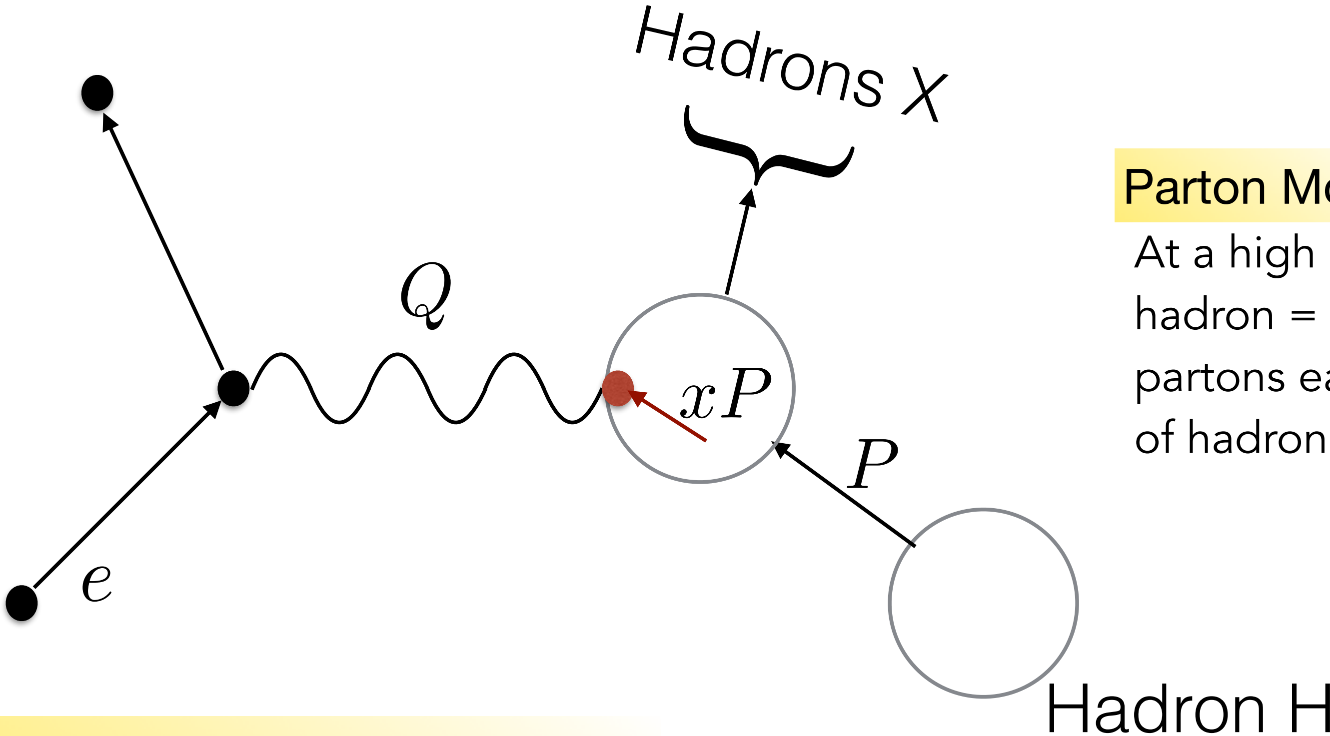
Derivative in T are very similar to derivatives in μ_B^2; expected if $O(4)$ scaling holds $t = (T - T_c^0)/T_c^0 + \kappa_B(\mu_B/T_c^0)^2$, $\partial_T \sim \partial_t \sim \partial_{\mu_B^2}$

Higher order Taylor expansion coefficients are likely to be negative for $T > 130$ MeV $\Rightarrow$ the only singularity the expansion coefficients are sensitive to is $O(4)$ transition $\Rightarrow T_c^{CEP} < T_c^0 < 132$ MeV

See also Mukherjee, Skokov, arXiv:1909.04639

Parton distribution functions

$$Q^2 = -q^2 = \mathcal{O}(\text{few GeV}) \longrightarrow \alpha_S(Q^2) = \mathcal{O}(0.1) \longrightarrow$$ Perturbative



Parton Model (Feynman '69)

At a high resolution Q^2, hadron = ensemble of massless partons each carrying fraction x of hadron momentum.

Factorization between UV and IR:

$$\sigma = \sum_i f_i(x, Q^2) \circledast \sigma \{eq_i(xP) \to eq_i(xP + q)\}$$

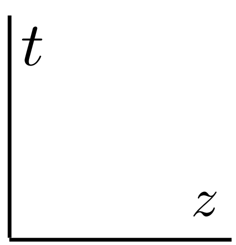
"**Total inclusive cross-section** = **probability to find a parton X cross-section to scatter from a parton**"

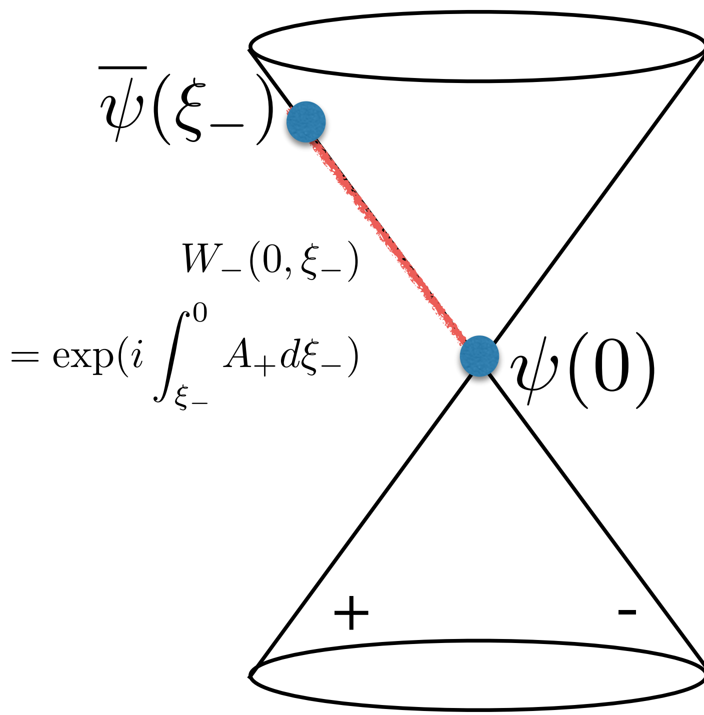
Light cone definition of PDF

Field theoretic Gauge-invariant and Lorentz invariant construction: (Soper '77)

$$f(x) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle H(P)|\overline{\psi}(\xi_-)\gamma_+ W_-(0,\xi_-)\tau\psi(0)|H(P)\rangle$$

= "Number of on-shell massless partons with energy x P^+"





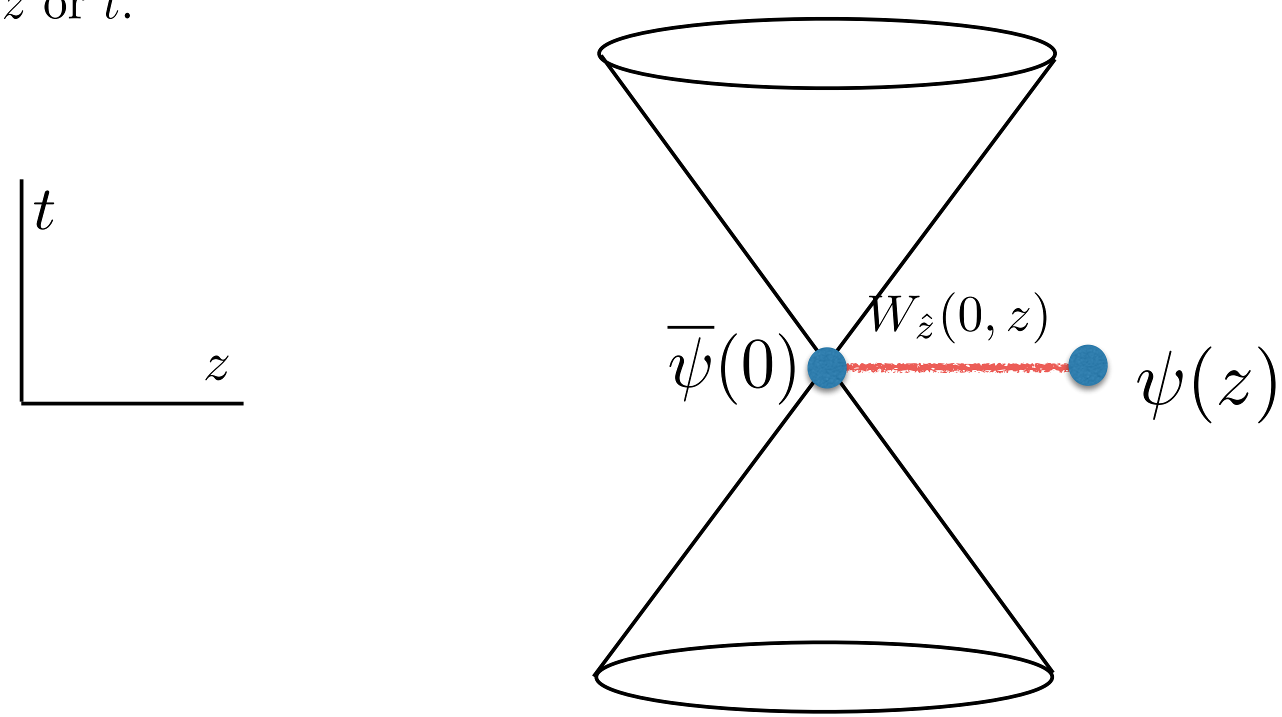
Cannot be studied on the lattice (sign problem)

Light cone PDF, quasi-PDF (qPDF) and the Euclidean lattice

Equal time correlation function that can be determined on lattice:

$$\tilde{q}(x) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle H(P_z, E)|\overline{\psi}(0)\gamma_\mu W_{\hat{z}}(0,z)\tau\psi(z)|H(P_z, E)\rangle$$

for $\mu = z \text{ or } t.$



Obtaining PDF from qPDF: the issue of limits

In 3+1d, PDF operator already is on the light-cone before regularization and renormalization.

On 4d lattice…

- one has finite lattice spacing ***a***
- At any finite ***a***, ***q(x)*** has to be renormalized at a scale P^R
- Take $a \to 0$ first, then $P_z \to \infty$

Large Momentum Effective Theory (LaMET)

1. Ensure $P_z \gg M$ and $P_z \gg \Lambda_{\rm QCD}$

2. Reliable extraction of M.E. of a fast moving hadron

3. First take continuum limit, then take $P_z \to \infty$
   In practice: Keep $(aP_z) < 1$ and renormalize

4. Matching currently to 1-loop order for quasi-PDF. Enough?

5. Extract PDF without using large quark-antiquark separations, and without modeling bias.

Perturbative matching of qPDF and PDF.

Not hopeless…

- Perturbative matching between ***q(x,P^R)*** in a regulator independent renormalization scheme at finite P_z to the infinite momentum MS-bar PDF ***f(x,μ^2)***

$$q(x; P_z, P^R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P_\perp^R}{P_z^R}, \frac{yP_z}{P_z^R}\right) f(x,\mu)$$

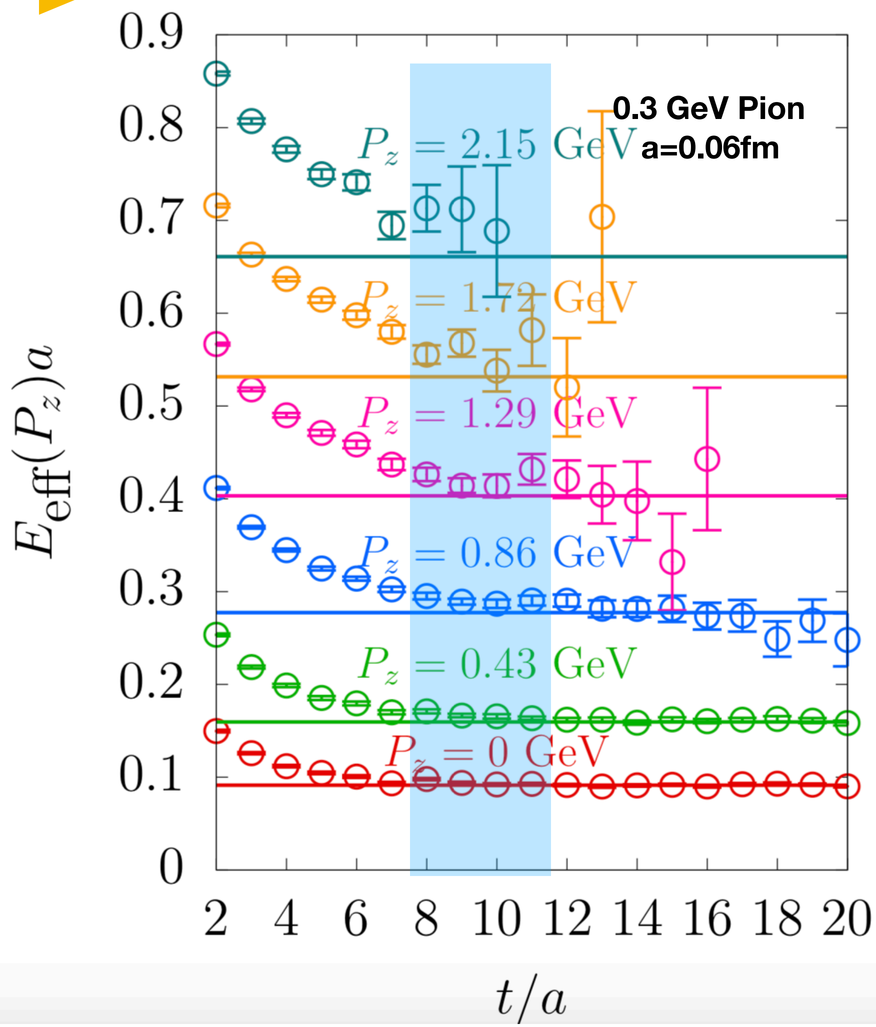
with the matching coefficient $C(\xi) = \delta(1-\xi) + \alpha_S(\mu) C^{(1)}(\xi)$

X. Ji '13, Stewart and Zhao '17

Highly boosted hadrons in lattice QCD

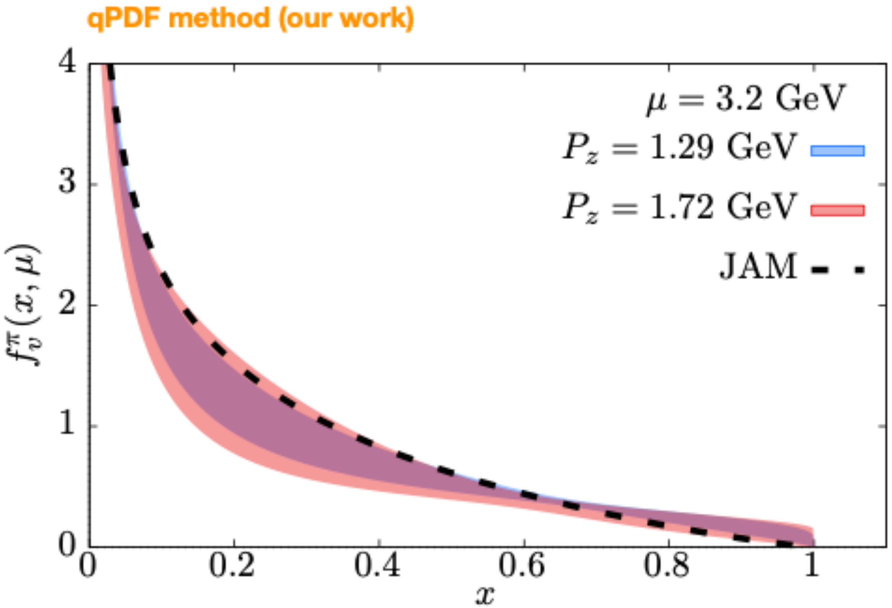
Even with optimal smearing: T_{sink} ~ 8a-12a for pion on a=0.06 fm lattice

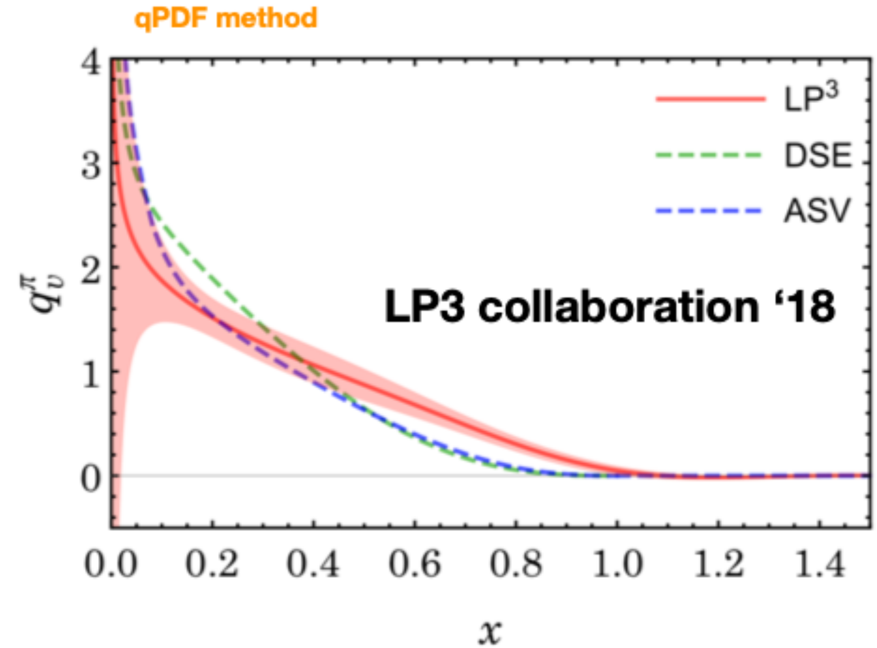
a=0.06 fm → Can push P_z ~ 3 GeV.   But, only P_z =1.29 and 1.72 GeV usable

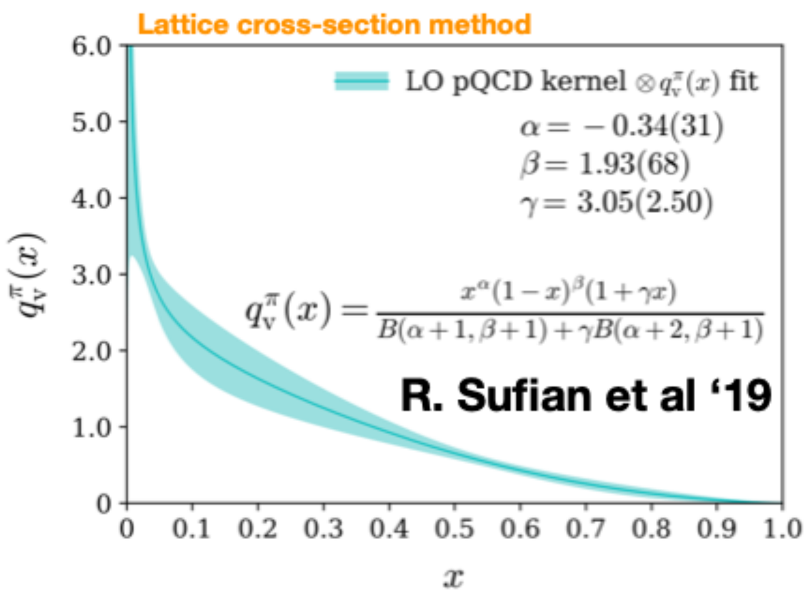


arXiv:1905.06349

Current status of pion PDF from lattice QCD







Summary

- The chiral transition temperature at the physical quark mass can be precisely determined: $T_c = 156 \pm 1.5$ MeV and can be related to chiral phase transition temperature For $m_{u,d} = 0$: $T_c = 132^{+3}_{-6}$ MeV
- Equation of state at non-zero baryon density can be obtained from the Taylor expansion and no indication of limited convergence radius for $\mu_B < 300$ MeV
- The dependence of T_c on μ_B is very small and is consistent with freeze-out curve in HI, the width of the chiral susceptibility and the peak height does not change with μ_B
- Higher order Taylor expansion coefficients are likely negative for $T > 130$ MeV $\Rightarrow T_c^{CEP} < 130$ MeV
- At high temperature QCD thermodynamics can be understood in terms of weak coupling calculations, while at low temperature it can be understood in terms of hadron gas
- LaMET may provide an unique insight into hadron structure from lattice QCD, in particular in terms of 3D imaging of hadrons
- Open challenge: real time quantities at $T > 0$ (see parallel talks)

BACKUP SLIDES

Strong CP problem and axions

A term $\theta \frac{g^2}{16\pi^2} F^a_{\mu,\nu} \tilde{F}^{\mu,\nu,a}$ can be added to QCD Largangian that break CP invariance

$$\theta \sim 1 \text{ is natural but neutron EDM suggests } \theta < 10^{-10}$$
$$\Rightarrow \text{fine tuning or strong CP problem}$$

Peccei-Quinn mechanism:

$$\theta \frac{g^2}{16\pi^2} F^a_{\mu,\nu} \tilde{F}^{\mu,\nu,a} \rightarrow (\theta + a(x)/f_A) \frac{g^2}{16\pi^2} F^a_{\mu,\nu} \tilde{F}^{\mu,\nu,a} + \frac{1}{2} (\partial_\mu a)^2$$

$$\theta + \langle a \rangle / f_A = 0$$

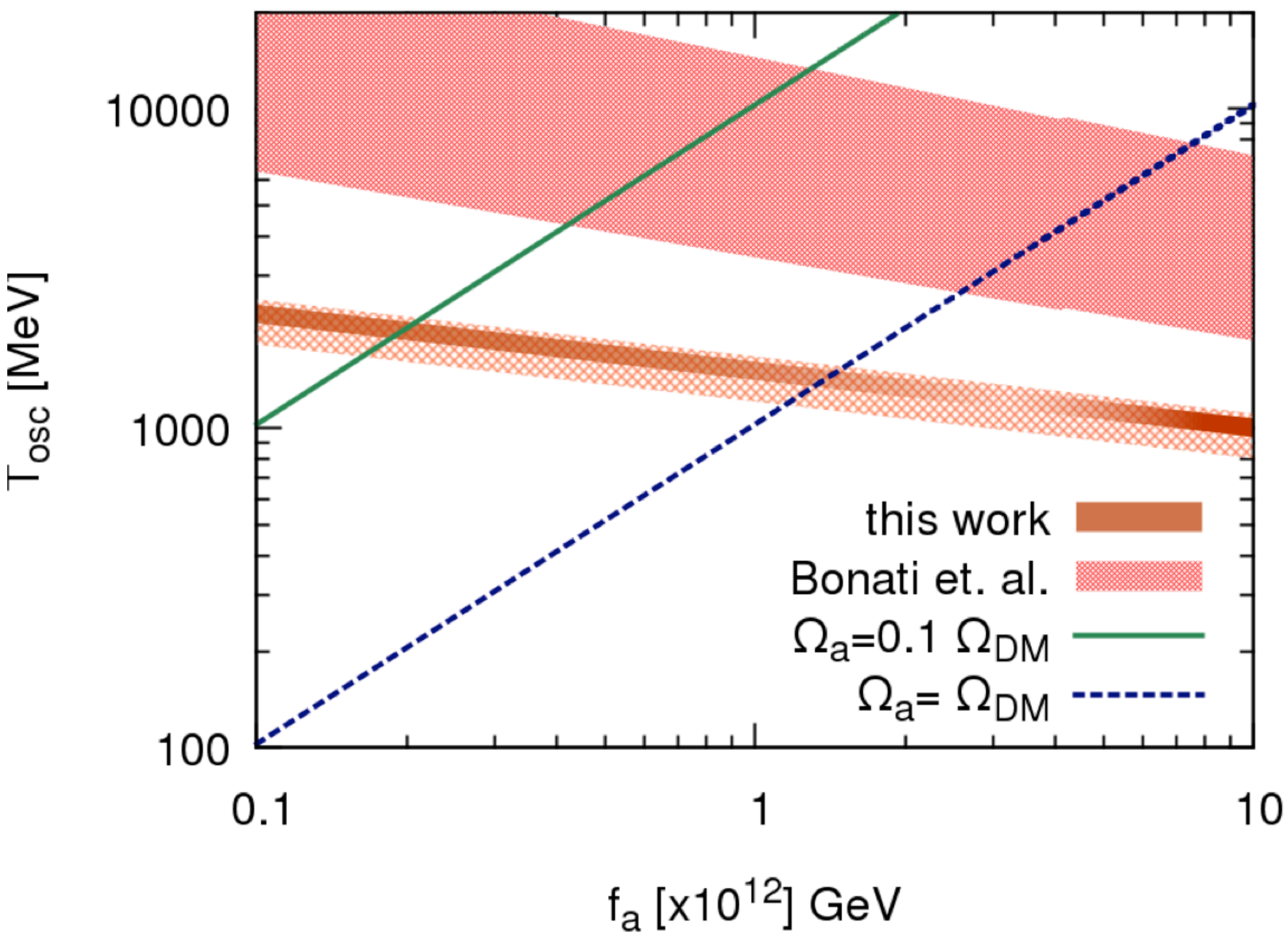
Initially axion field can assume some "random" value and as the Universe cools down it starts to feel the potential => it slowly rolls down and oscillate around the minimun at temperatures

$$3H(T_{osc}) = m_A(T_{osc}),\; m^2_A(T) = \chi_t(T)/f_A$$

Implications for axion cosmomolgy

$$3H(T_{osc}) = m_A(T_{osc}) \Rightarrow T_{osc} = T_{osc}(f_A)$$

$$\theta_0^2 = \pi^2/3$$

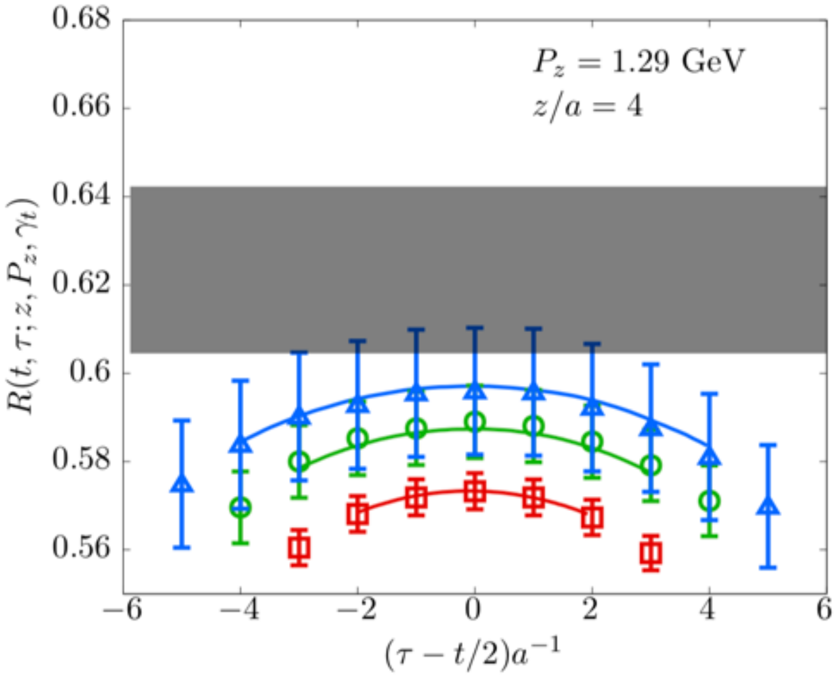


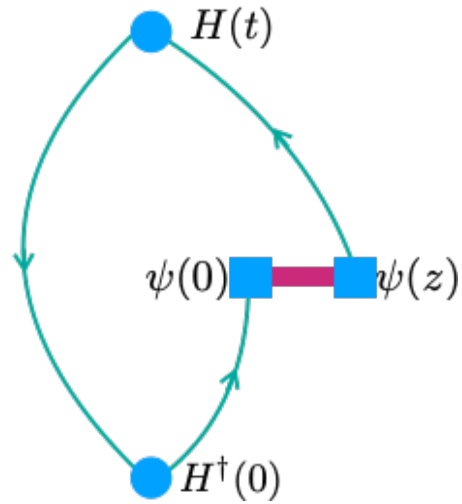
$f_A < 1.2 \times 10^{12}\mathrm{GeV}$

PP, Sharma, Schadler, PLB 762 (2016) 498

Reliable extraction of matrix element

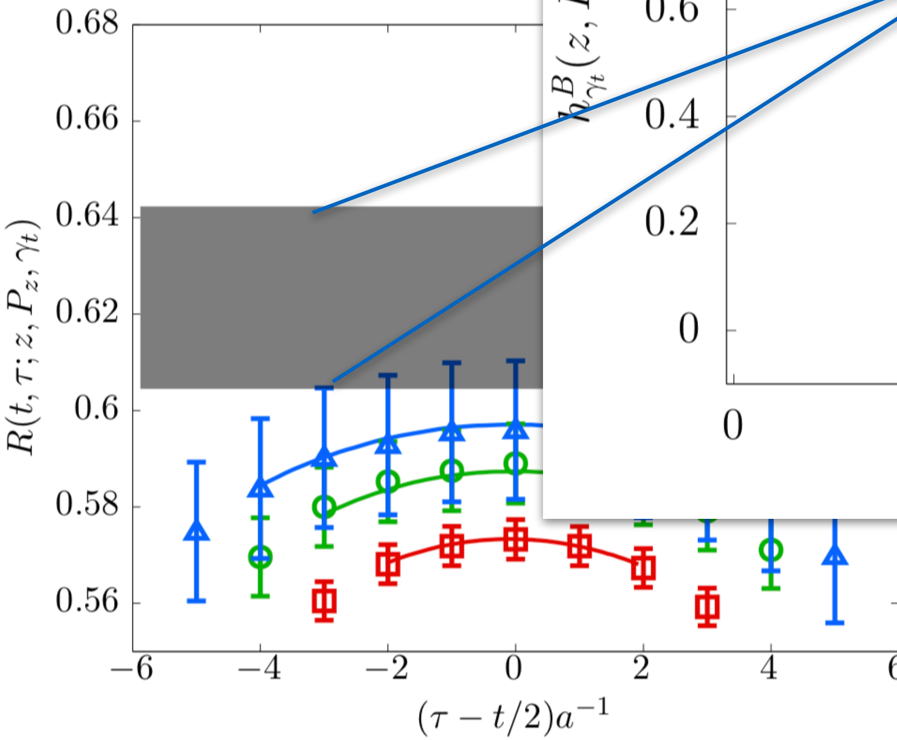
Strategies for quasi-PDF: plateau, 2- and 3-state fits to t and τ dependence, summation methods.

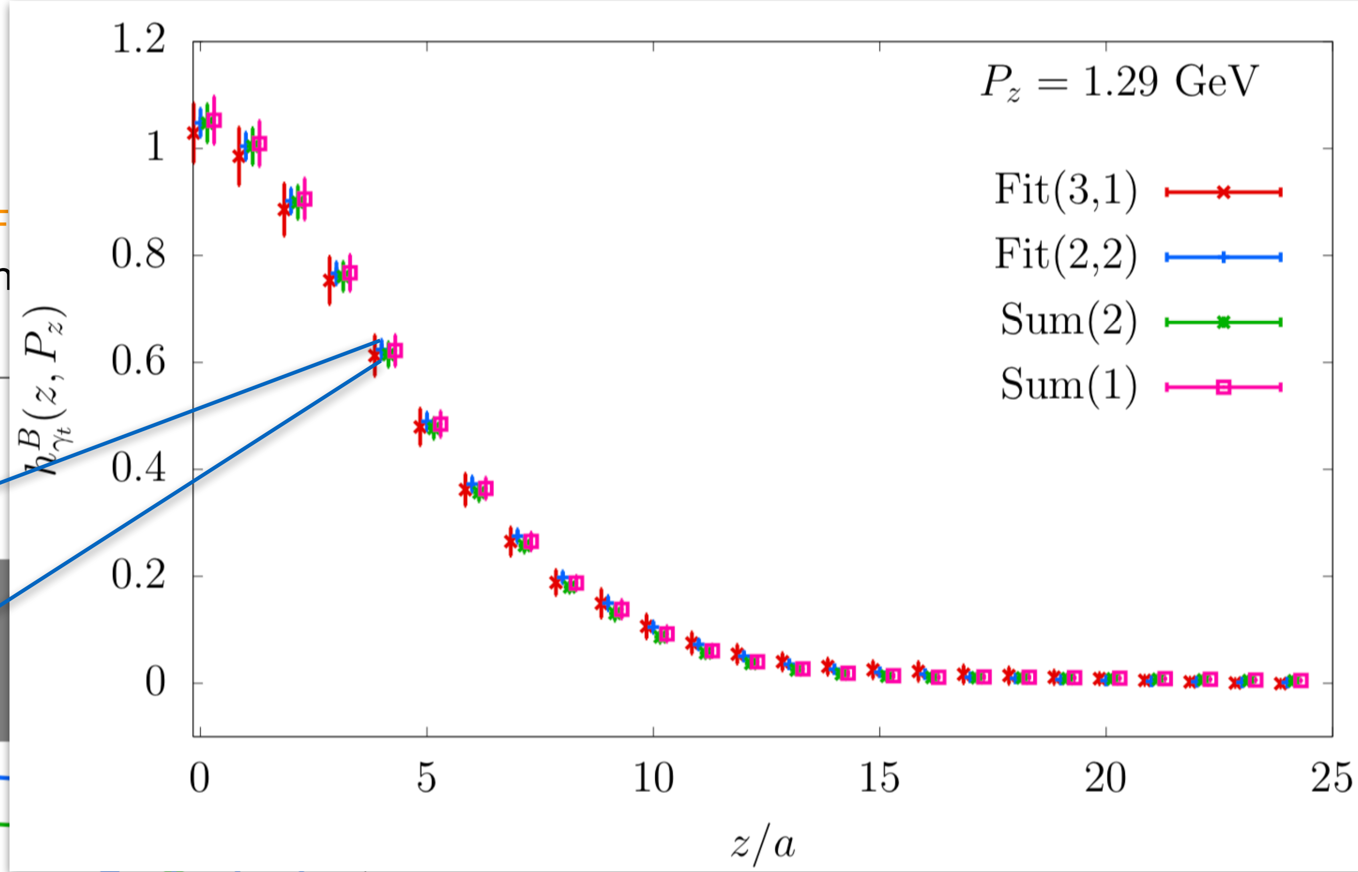




Reliable extraction of matrix element

- Strategies for quasi-PDF
dependence, summation





- Require finer lattices (a<0.05 fm) and larger P_z (>2GeV) ?
Fits indispensable. Check for fit ansatz independence.

How to renormalize the quasi-PDF ?

Renormalizability means:

$$h^R_{\gamma_t}(z; P_z, P^R) = Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{\gamma_t}(z; P_z, a)$$

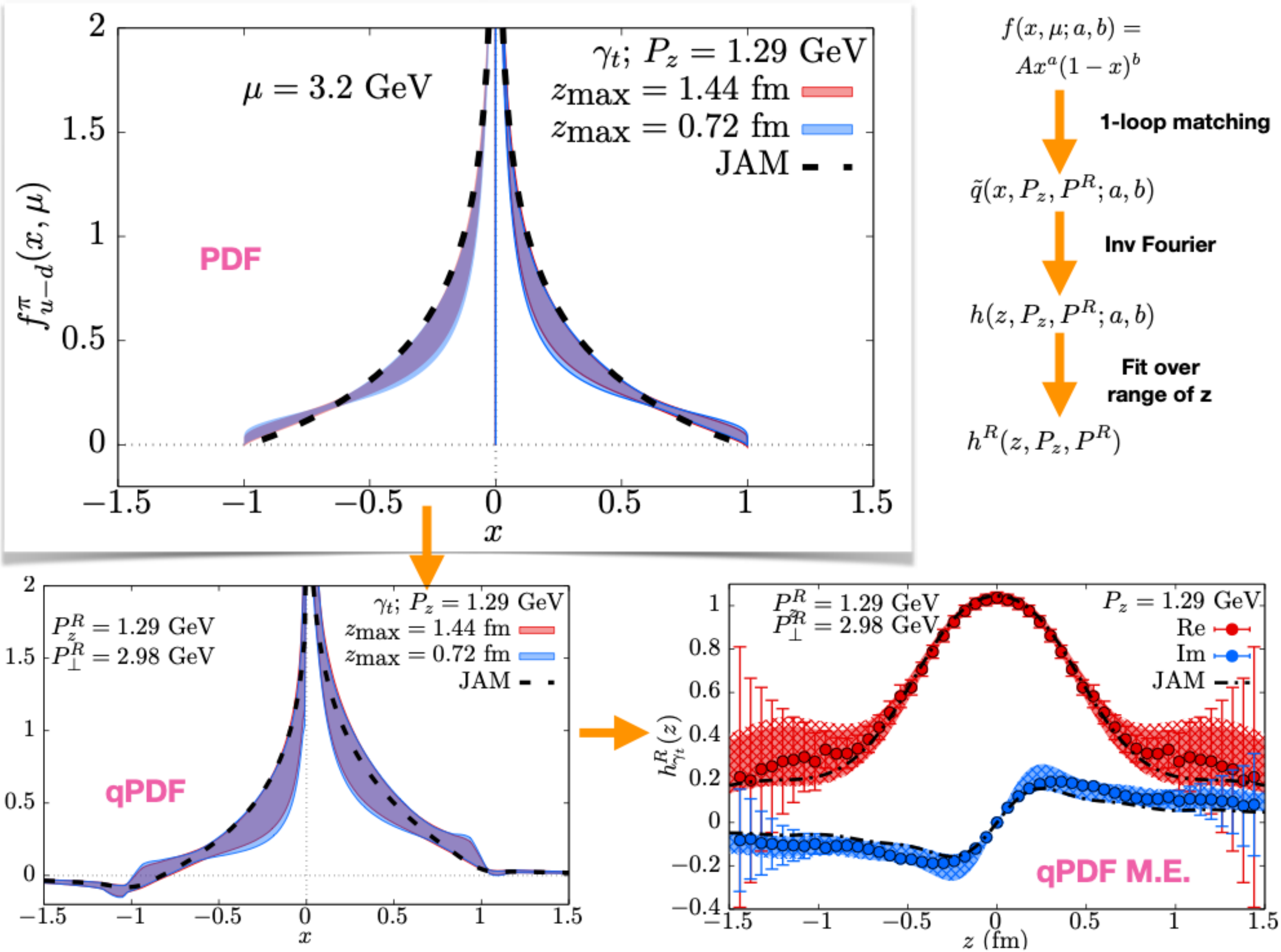
renormalized hadron qPDF

bare hadron qPDF

Non-perturbative Z-factor

⇕

LaMET perturbative framework



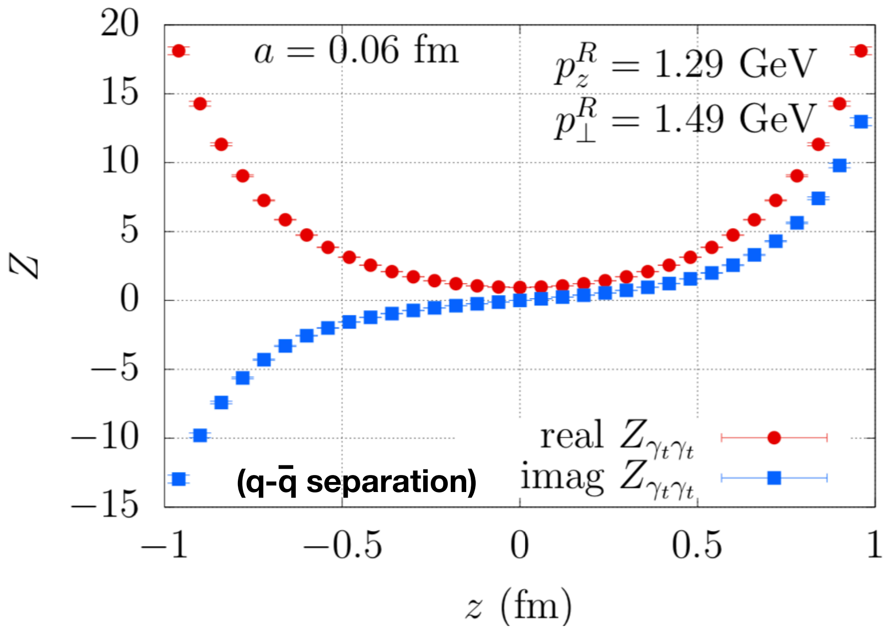
$f(x, \mu; a, b) = Ax^a(1-x)^b$
1-loop matching
$\tilde{q}(x, P_z, P^R; a, b)$
Inv Fourier
$h(z, P_z, P^R; a, b)$
Fit over range of z
$h^R(z, P_z, P^R)$
PDF
$\mu = 3.2$ GeV
γ_t; $P_z = 1.29$ GeV
$z_{\max} = 1.44$ fm
$z_{\max} = 0.72$ fm
JAM
$f^{\pi}_{u-d}(x, \mu)$
x
qPDF
$P_z^R = 1.29$ GeV
$P_\perp^R = 2.98$ GeV
γ_t; $P_z = 1.29$ GeV
$z_{\max} = 1.44$ fm
$z_{\max} = 0.72$ fm
JAM
$\tilde{q}_{u-d}(x, P_z, P^R)$
x
qPDF M.E.
$P_z^R = 1.29$ GeV
$P_\perp^R = 2.98$ GeV
$P_z = 1.29$ GeV
Re
Im
JAM
$h^R_{\gamma_t}(z)$
z (fm)

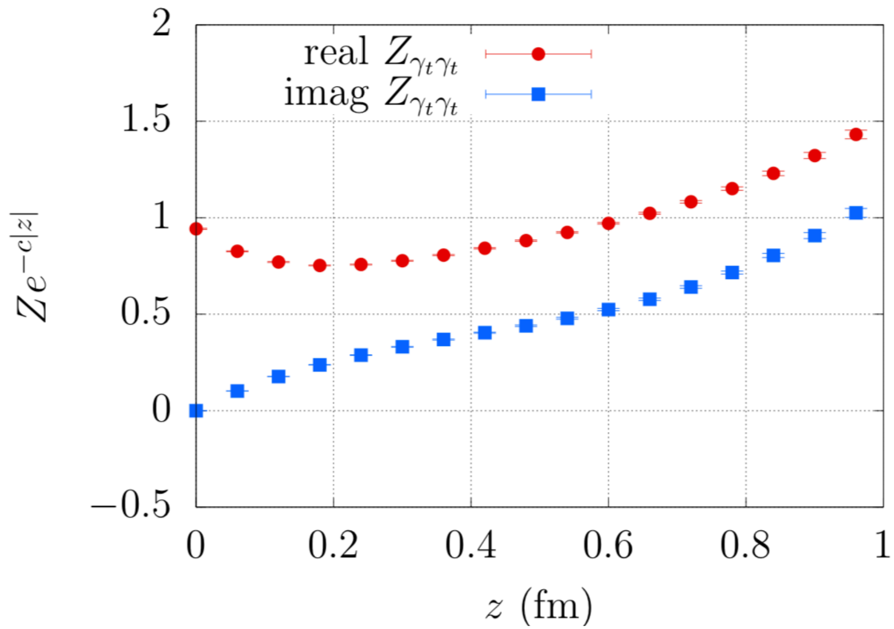
Perturbative renormalization and NPR compatible?

Renormalization in real-space → WL self-energy divergence e-$^{c|z|}$

Issue for 1-loop perturbation theory?

RI-MOM Z-factor for q-PDF → **RI-MOM Z-factor modulo WL divergence**





The residual piece modulo WL divergence is O(1)

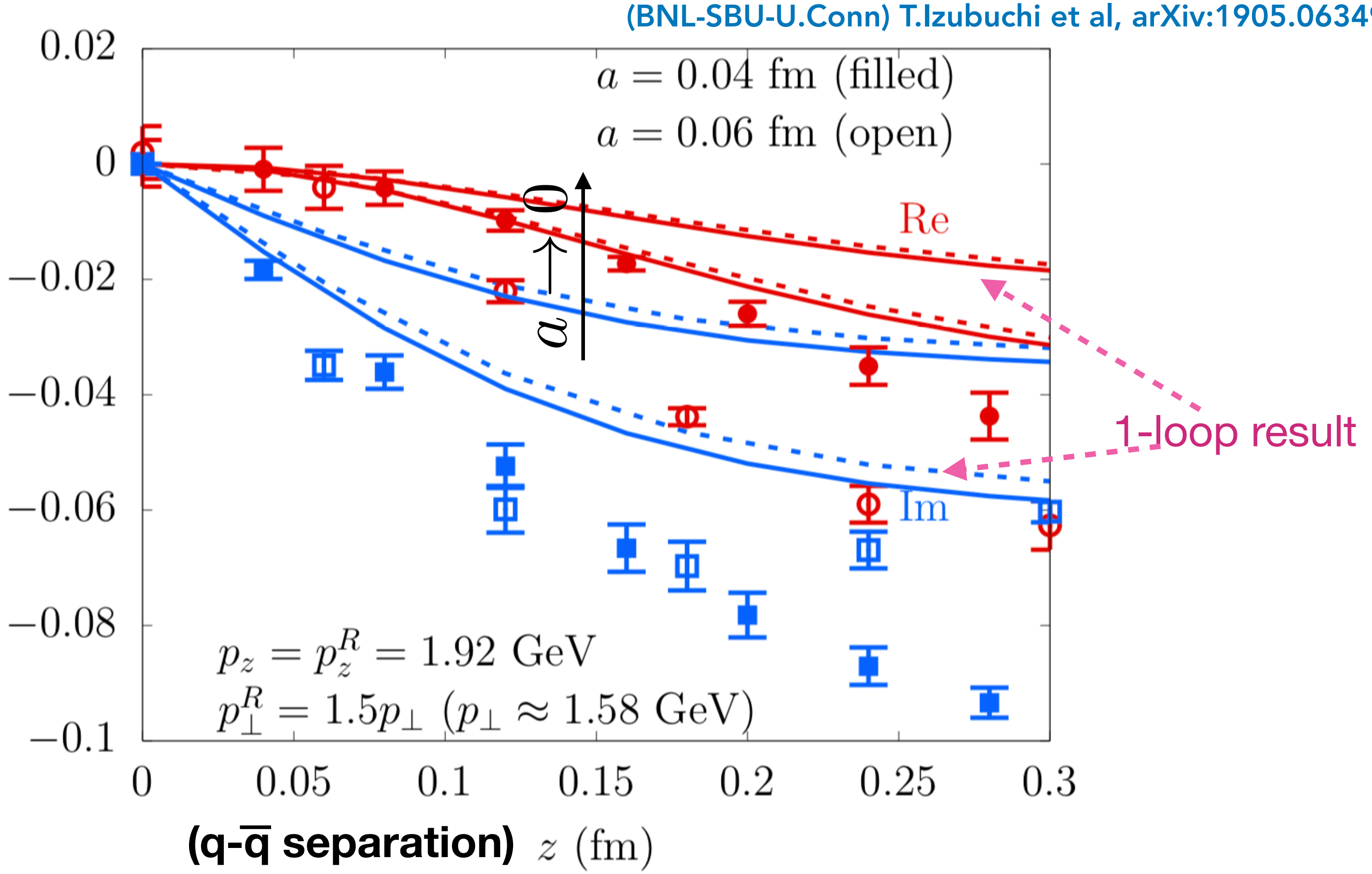
(BNL-SBU-U.Conn) T.Izubuchi et al, arXiv:1905.06349

Can this residual dependence on z be described by perturbation theory?

(BNL-SBU-U.Conn) T.Izubuchi et al, arXiv:1905.06349

"Z-factor modulo WL divergence"

$$\frac{Z(z,p^R) - Z(z,p)}{Z(z,p)}$$



- Only qualitative agreement between actual lattice data and 1-loop
- Both continuum limit as well as inclusion of 2-loop contributions could be important.

Summary and outlook

- **Theoretical framework of LaMET is now established.**
  Issues that remain are about practical implementation on lattice

Renormalization	Yes
$P_z \gg \Lambda_{\rm QCD}$	Yes
Target mass corrections	Yes
$P_z \ll 1/a$	~Ok
Excited state analysis:	$T_{\rm sink}$ <1 fm. Fits required
Continuum limit:	Not studied
Finite volume effects:	Not studied
Analysis methods:	No consensus
Matching:	1-loop. Convergence?

Things to look out for:

- Combined analysis of quasi-, pseudo- and other lattice cross-sections for PDF
- More work on GPDs and gluon PDFs — Still in very early stages
- Effects on global analysis of PDF — Sensitivity to determination of qPDF at even one value of x. (T.J.Hobbs et al, 1904.00022)
- Usage of overlap/domain wall fermion advantageous?
- Complementing experimental determination of PDF is ok. But, we also need to use lattice to *actually understand* why PDFs are the way they are!