NNLO / N^{3}LO calculations with NNLOJET

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Physics Event Generator Computing Workshop

CERN — November 27th 2018

Part 1. Antenna Subtraction @ NNLO

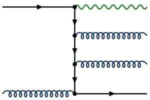
Part 2. Going beyond...

↪ q_{T} subtraction

↪ Projection-to-Born Method

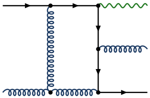
Anatomy of an NNLO calculation

$$\sigma_{\mathrm{NNLO}} = \int_{\Phi_{\mathrm{Z}+3}} \mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$$



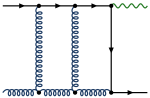
- single-unresolved
- double-unresolved

$$+ \int_{\Phi_{\mathrm{Z}+2}} \mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}$$



- single-unresolved
- $1/\epsilon^2$, $1/\epsilon$

$$+ \int_{\Phi_{\mathrm{Z}+1}} \mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$$



- $1/\epsilon^4$, $1/\epsilon^3$, $1/\epsilon^2$, $1/\epsilon$

$\sum$ finite (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

NNLO using Antenna (local subtraction)

$$
\begin{aligned}
\sigma_{\mathrm{NNLO}} = &\int_{\Phi_{\mathrm{Z}+3}} \left(\mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NNLO}}\right) \\
&+ \int_{\Phi_{\mathrm{Z}+2}} \left(\mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{NNLO}}\right) \\
&+ \int_{\Phi_{\mathrm{Z}+1}} \left(\mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{U}}_{\mathrm{NNLO}}\right)
\end{aligned}
$$

- $\mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{NNLO}}$: mimic $\mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}$ in unresolved limits
- $\mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{U}}_{\mathrm{NNLO}}$: *analytic* cancellation of poles in $\mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}$, $\mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$

$\sum$ finite -0

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

NNLOJET

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder,
T. Gehrmann, E.W.N. Glover, M. Höfer, AH, I. Majer, J. Mo, T. Morgan, J. Niehues,
J. Pires, D. Walker, J. Whitehead

Common framework for NNLO calculations using Antenna Subtraction

- parton-level event generator
- based on *antenna subtraction*
  - ↪ fully analytic
  - ↪ *all* building blocks known
  - ↪ *colour decomposition*
- *fully differential* calculation
- test & validation framework

- driver written in FORTRAN
  - ↪ core in 90/95 standard
  - ↪ `gcc 4.8+` (OpenMP)
- scripts typically in PYTHON
  - ↪ post-processing of data
  - ↪ grid submission
- dependencies
  - ↪ LHAPDF, (fastJet, Openloops)

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Available processes

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ NNLO
  $\hookrightarrow V \to \ell\bar{\ell} \quad (V = \mathrm{Z}/\gamma^*, \mathrm{W}^\pm)$
- $\mathrm{pp} \to$ jets (inc. jets, 2j) @ NNLO
- $\mathrm{ep} \to 1j$ @ N^{3}LO
- $\mathrm{ep} \to 2j$ @ NNLO
- $\mathrm{e}^+\mathrm{e}^- \to 3$ jets @ NNLO
- $\mathrm{pp} \to \mathrm{H}$ (ggH) @ N^{3}LO
- $\mathrm{pp} \to \mathrm{H} + j$ (ggH) @ NNLO
- $\mathrm{pp} \to \mathrm{H} + 2j$ (VBF) @ NNLO
  $\hookrightarrow \mathrm{H} \to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$
- $\mathrm{pp} \to V\mathrm{H}$ @ NNLO
  $\hookrightarrow \mathrm{H} \to \mathrm{b}\bar{\mathrm{b}}$
- …

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Available processes

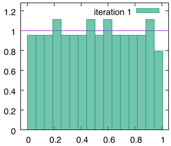
- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$
  - $\hookrightarrow V \to \ell\bar{\ell}$
- pp
- $\mathrm{ep} \to$
- $\mathrm{ep} \to$ @ NNLO
- $\mathrm{e^+e^-} \to$ @ NNLO

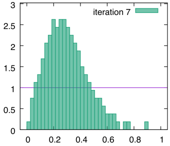
- @ N^{3}LO
- @ NNLO
- j (VBF) @ NNLO
- $\to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$
- $\mathrm{pp} \to V\mathrm{H}$ @ NNLO
  - $\hookrightarrow \mathrm{H} \to \mathrm{b\bar{b}}$
- …

NNLO subtraction set up for:

$\mathrm{pp} \to$ "colour neutral" $+\ 0, 1, 2$ jets

Three steps to NNLO — the NNLOJET workflow





1. WARMUP: VEGAS grid adaption
   - ↪ parallel execution using OpenMP
   - ↪ switch off: Histograms, scales, …
   - ↪ 6 parts: $\sigma^{(0)}$(“LO”), $\sigma^{(1)}$(“V”, “R”), $\sigma^{(2)}$(“VV”, “RV”, “RR”)
   - × dedicated runs: special bins, reweighting (p_{T}, $\hat{H}_{\mathrm{T}}$, …), …
   - ⇝ typically $\mathcal{O}$(few min) – $\mathcal{O}$(few days) on 20–40 cores

2. PRODUCTION: “poor man’s parallelisation”
   - ↪ *MANY* single-core jobs with *different random-number seeds*
   - ↪ VEGAS grid from previous step *frozen*
   - ⇝ runtime process dependent: DOMINANT part

3. POST-PROCESSING: combine the seeds
   - ↪ treatment of outliers, checks there is no stat. bias, …
   - ⇝ $\mathcal{O}$(1 min) – $\mathcal{O}$(10 h)

Drell–Yan @ NNLO

$$pp \to Z/\gamma^* \to \ell^- \ell^+$$

	pt.	cross section	CPU time
$\sigma^{(0)}$		$480.76(14)$ pb	< 1 min
$\sigma^{(1)}$	"V"	$71.81(6)$ pb	~ 1 min
	"R"	$-22.91(14)$ pb	~ 10 min
$\sigma^{(2)}$	"VV"	$-1.46(3)$ pb	~ 1 h
	"RV"	$-9.62(13)$ pb	~ 2.5 h
	"RR"	$11.04(17)$ pb	~ 19 h
σ^{NNLO}		$529.6(3)$ pb	~ 23 h

- `Intel(R) Xeon(R) W-2155 CPU @ 3.30GHz`
- fixed scale $\mu_R = \mu_F = M_Z$

* differential distributions: $\mathcal{O}(100$ CPU h$)$

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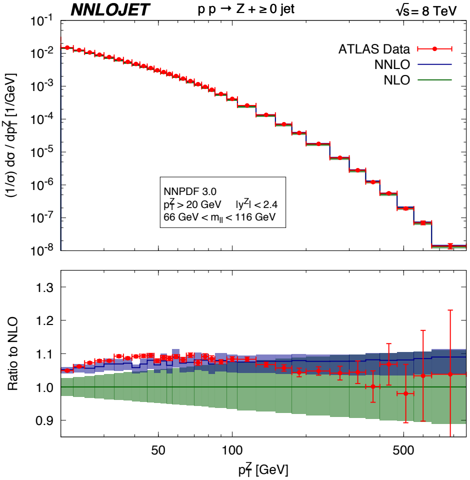
V+jet production — p_{T} spectrum

[Gehrmann–De Ridder, Gehrmann, Glover, AH, Morgan '16]



- "hard" region: $p_{\mathrm{T}}^{\mathrm{Z}} > 20\ \mathrm{GeV}$

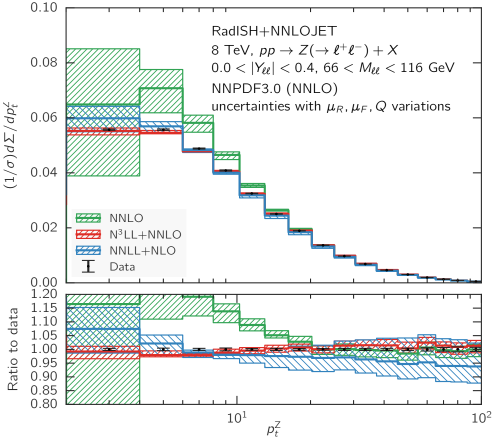
NNLO

$\mathcal{O}(100\mathrm{k}\ \text{CPU h})$

- matching to N^{3}LL requires $p_{\mathrm{T}}^{\mathrm{Z}} \gtrsim 1\ \mathrm{GeV} \quad \rightsquigarrow \quad \times 5$

V+jet production — p_{T} spectrum

[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, AH, Monni, Re, Rottoli, Torrielli ’18]



- “hard” region: $p_{\mathrm{T}}^{\mathrm{Z}} > 20$ GeV

NNLO

$\mathcal{O}(100\mathrm{k}$ CPU h$)$

- matching to N^{3}LL requires $p_{\mathrm{T}}^{\mathrm{Z}} \gtrsim 1$ GeV $\rightsquigarrow$ $\times 5$

Bottle neck & challenge — double-real emissions

$$\int_{\Phi_{Z+3}} \left(d\sigma^{RR}_{NNLO} - d\sigma^{S}_{NNLO} \right)$$

1. STABILITY:
   - $\hookrightarrow$ highest integration dimensionality
   - $\hookrightarrow$ *numerical cancellations* in unresolved limits
   - $\hookrightarrow$ much more intricate than @ NLO
   - $\rightsquigarrow$ cancellations within the counter-term

2. PERFORMANCE:
   - $\hookrightarrow$ *many* "counter events" (caching critical) $\rightsquigarrow$ $\mathcal{O}(100)\ \widetilde{\Phi}_n$

$$
\begin{array}{lllll}
\Phi_{Z+3} & \xrightarrow{X_3^0} & \widetilde{\Phi}_{Z+2} & & \\
 & \xrightarrow{X_4^0} & \widetilde{\Phi}_{Z+1} & & \\
 & \xrightarrow{X_3^0} & \widetilde{\Phi}_{Z+2} & \xrightarrow{X_3^0} & \widetilde{\widetilde{\Phi}}_{Z+1}
\end{array}
$$

   - $\hookrightarrow$ "very dynamical" scale: LHAPDF

Bottle neck & challenge — double-real emissions

1. S

2. F

$$\mathtt{B3g0Z}(1_q, 2_g, i_g, j_g, k_{\bar{q}}, Z) - \mathtt{qgB3g0ZXS}(1_q, 2_g, i_g, j_g, k_{\bar{q}}, Z)$$

```
+gf30FI(2,i,j)   * B2g0Z(1,[2],[i,j],Q,Z)
+d30FF(Q,j,i)    * B2g0Z(1,2,[j,i],[Q,j],Z)
+qd30IF(1,j,i)   * B2g0Z([1],[j,i],2,Q,Z)
+gf30FI(2,i,j)   * B2g0Z(1,[j,i],[2],Q,Z)
-qgA30II(1,2,Q)  * B2g0Z([1],j,i,[2],Z)

+D40(1,2,i,j)                           * B1g0Z([1],[2],Q,Z)
-gf30FI(2,i,j)   * qgD30II(1,[2],[i,j]) * B1g0Z([1],[[2]],Q,Z)
-qd30IF(1,j,i)   * qgD30II([1],[j,i],2) * B1g0Z([[1]],[2],Q,Z)

+D40(Q,j,i,2)                            * B1g0Z(1,[2],[i,j,Q],Z)
-d30FF(Q,j,i)    * gD30FI([Q,j],2,[j,i]) * B1g0Z(1,[2],[[j,i],[Q,j]],Z)
-gf30FI(2,i,j)   * gD30FI(Q,[2],[i,j])   * B1g0Z(1,[[2]],[[i,j],Q],Z)

-At40(1,2,j,Q)                          * B1g0Z([1],[2],i,Z)
+qA30IF(1,j,Q)   * qgA30II([1],2,[j,Q]) * B1g0Z([[1]],[2],i,Z)

-A40(1,i,2,Q)                         * B1g0Z([1],j,[2],Z)
+qgD30II(1,i,2)  * qgA30II([1],[2],Q) * B1g0Z([[1]],j,[[2]],Z)
+qgA30II(1,2,Q)  * qqA30II([1],i,[2]) * B1g0Z([[1]],j,[[2]],Z)

-A40(1,i,2,Q)                         * B1g0Z([1],[2],j,Z)
+qgD30II(1,i,2)  * qgA30II([1],[2],Q) * B1g0Z([[1]],[[2]],j,Z)

+gf30FI(2,i,j)    * qgA30II(1,[2],Q)      * B1g0Z([1],[[2]],[i,j],Z)
+d30FF(Q,i,j)     * qgA30II(1,2,[Q,i])    * B1g0Z([1],[2],[j,i],Z)
-qgD30II(1,i,2)   * qgA30II([1],[2],Q)    * B1g0Z([[1]],[[2]],j,Z)
-qA30IF(1,i,Q)    * qgA30II([1],2,[Q,i])  * B1g0Z([[1]],[2],j,Z)
+2*qd30IF(1,i,j)  * qgA30II([1],2,Q)      * B1g0Z([[1]],[2],[i,j],Z)

+2*qgA30II(1,2,Q) * qqA30II([1],i,[2]) * B1g0Z([1],[2],j,Z)
-2*qgA30II(1,2,Q) * qd30IF([2],i,j)    * B1g0Z([1],[[2]],[i,j],Z)
-2*qgA30II(1,2,Q) * qd30IF([1],i,j)    * B1g0Z([[1]],[2],[i,j],Z)

+gf30FI(2,i,j)   * qgA30II(1,[2],Q)   * B1g0Z([1],[i,j],[[2]],Z)
+qd30IF(1,j,i)   * qgA30II([1],2,Q)   * B1g0Z([[1]],[j,i],[2],Z)
-qgD30II(1,i,2)  * qgA30II([1],[2],Q) * B1g0Z([[1]],j,[[2]],Z)

+(-SIF(2,i,[i,j])
```

Bottle neck & challenge — double-real emissions

$$\int_{\Phi_{Z+3}} \left(d\sigma^{RR}_{NNLO} - d\sigma^{S}_{NNLO} \right)$$

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$$\begin{aligned}
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&\xrightarrow{X_4^0} \widetilde{\Phi}_{Z+1} \\
&\xrightarrow{X_3^0} \widetilde{\Phi}_{Z+2} \xrightarrow{X_3^0} \widetilde{\widetilde{\Phi}}_{Z+1}
\end{aligned}$$

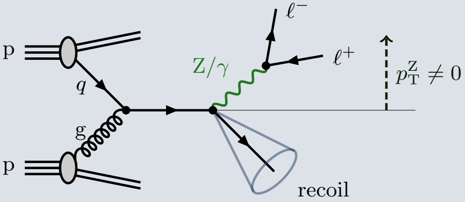
   - $\hookrightarrow$ "very dynamical" scale: LHAPDF

Part 1. Antenna Subtraction @ NNLO

Part 2. Going beyond...

- ↪ q_{T} subtraction
- ↪ Projection-to-Born Method

q_{T} subtraction



q_T subtraction @ N^{3}LO

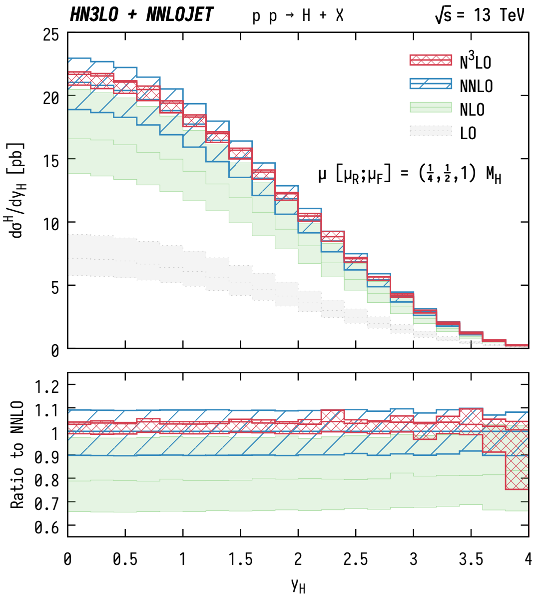
$$\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}} = \underbrace{\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}}\Big|_{q_\mathrm{T}<q_\mathrm{T}^\mathrm{cut}}}_{q_\mathrm{T}\ \text{resummation}} + \underbrace{\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}}\Big|_{q_\mathrm{T}>q_\mathrm{T}^\mathrm{cut}}}_{\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathrm{NNLO}}}$$

$$\simeq \mathcal{H}^{\mathrm{H}}_{\mathrm{N^3LO}} \otimes \mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{LO}} + \Big[\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{H,\,CT}}_{\mathrm{N^3LO}}\Big]_{q_\mathrm{T}>q_\mathrm{T}^\mathrm{cut}}$$

Competing interests

- $q_\mathrm{T}^\mathrm{cut}$ as small as possible
  suppress power-corrections that spoil above picture
- $q_\mathrm{T}^\mathrm{cut}$ as large as possible
  numerical stability

y_{H} distribution @ N³LO



[Cieri, Chen, Gehrmann, Glover, AH '18]

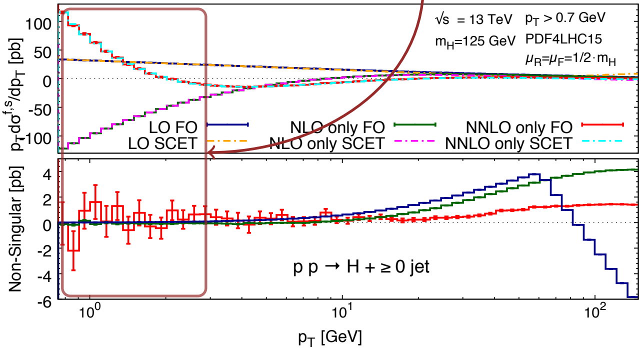
- **LO**
  $\hookrightarrow \mathcal{O}(1$ CPU min$)$
- **NLO** [Antenna]
  $\hookrightarrow \mathcal{O}(30$ CPU min$)$
- **NNLO** [Antenna]
  $\hookrightarrow \mathcal{O}(100$ CPU h$)$
- **N³LO** [q_{T} sub.]
  $\hookrightarrow \mathcal{O}(7\mathrm{M}$ CPU h$)$

q_T subtraction @ N^{3}LO

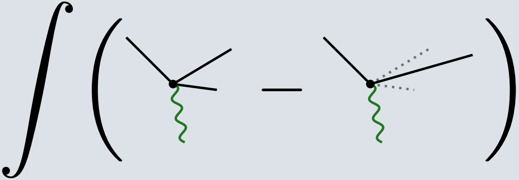
$$\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}} = \underbrace{\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}}\Big|_{q_T < q_T^{\mathrm{cut}}}}_{q_T \text{ resummation}} + \underbrace{\mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{N^3LO}}\Big|_{q_T > q_T^{\mathrm{cut}}}}_{\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathrm{NNLO}}}$$

$$\simeq \mathcal{H}^{\mathrm{H}}_{\mathrm{N^3LO}} \otimes \mathrm{d}\sigma^{\mathrm{H}}_{\mathrm{LO}} + \left[\mathrm{d}\sigma^{\mathrm{H+jet}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{H,\,CT}}_{\mathrm{N^3LO}}\right]_{q_T > q_T^{\mathrm{cut}}}$$

[Chen, Gehrmann, Glover, AH, Li, Neill, Schulze, Stewart, Zhu '18]

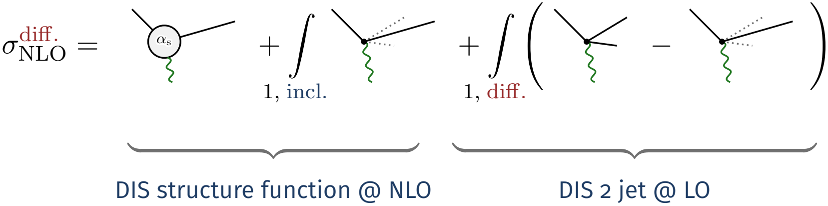


Projection-to-Born Method



Projection-to-Born

differential DIS (w/ IR treatment)

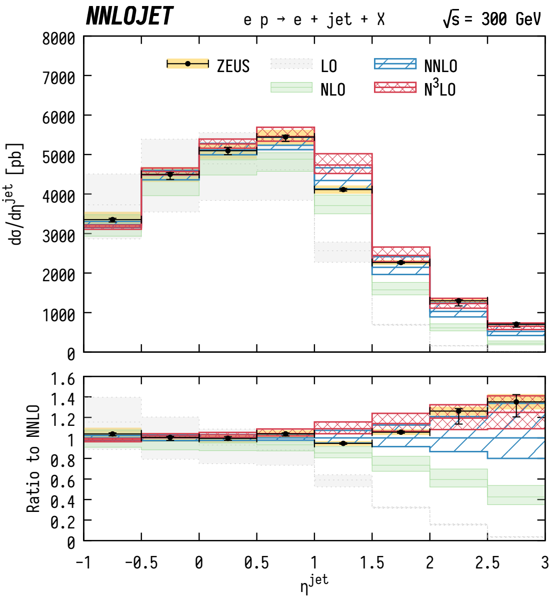


only "special" processes *but* not restricted to any order

$$\left.\begin{array}{lll} & \text{inclusive } X & @\ \mathrm{N^nLO} \\ + & X + \text{jet} & @\ \mathrm{N^{n\text{-}1}LO} \end{array}\right\} \rightsquigarrow \quad X\ @\ \mathrm{N^nLO}$$

Born kinematics: $Q^2 = -q^2 > 0, \quad x = \dfrac{-Q^2}{2P\cdot q}$

DIS jets @ N^{3}LO



- **LO**

  ↪ $\mathcal{O}(1$ CPU min$)$

- **NLO** [Antenna, P2B]

  ↪ $\mathcal{O}(1$ CPU h$)$

- **NNLO** [Antenna, P2B]

  ↪ $\mathcal{O}(1\text{k}$ CPU h$)$

- **N^{3}LO** [P2B]

  ↪ $\mathcal{O}(300\text{k}$ CPU h$)$

tl;dr

- NNLOJET
  - $\hookrightarrow$ boson-production (Drell–Yan, ggH) $\rightsquigarrow$ $\mathcal{O}(100$ CPU h$)$
  - $\hookrightarrow$ V+jet, dijet $\rightsquigarrow$ $\mathcal{O}(100\mathrm{k}$ CPU h$)$
- Antenna @ NNLO set up for: $\mathrm{pp} \to$ "colour neutral" $+ 0, 1, 2$ jets
  - $\hookrightarrow$ *new frontier:* $\mathrm{pp} \to 3j$ $\leftrightarrow$ requires improvements
  - $\hookrightarrow$ availability of 2-loop amplitudes
- differential N^{3}LO
  - $\hookrightarrow$ q_T *subtraction* [ggH] very challenging due to non-local cancellations
  - $\hookrightarrow$ *Projection-to-Born* [DIS jets] scaling is "ok" (thanks to locality?)
  - $\hookrightarrow$ *Projection-to-Born* [ggH] coming soon
- so far: main focus on *stability* $\rightsquigarrow$ performance
- next steps: *performance* & *usability*
  - $\hookrightarrow$ colour-decomposition: $\sigma = A + \frac{1}{N_c^2} B + \frac{N_f}{N_c^2} C + \ldots$ ($\Rightarrow$ multi-channel)
  - $\hookrightarrow$ `APPLfast-NNLO` interface ($\times 2$) [Britzger, Gwenlan, AH, Morgan, Sutton, Rabbertz]