Transport coefficients from in medium quarkonium dynamics

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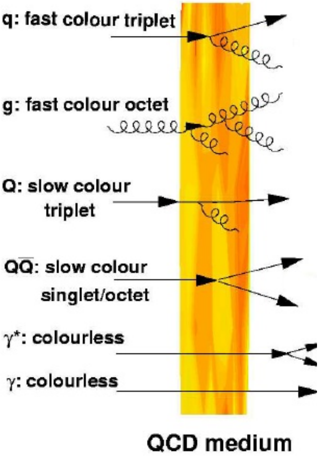
Outline

Hard probes



Picture taken from d'Enterria (2007)

Probes that are created at the beginning of the collision (typically because its creation needs a high energy) that get modified in a substantial way and that are relatively easy to detect. In this talk we focus in the ones related with heavy quarks

- Heavy quark diffusion.

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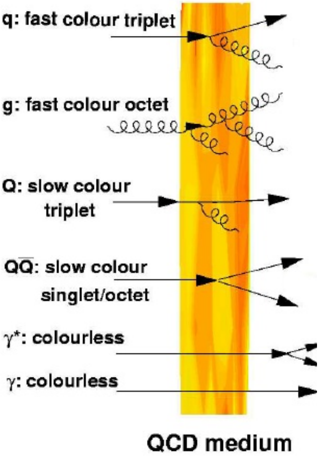

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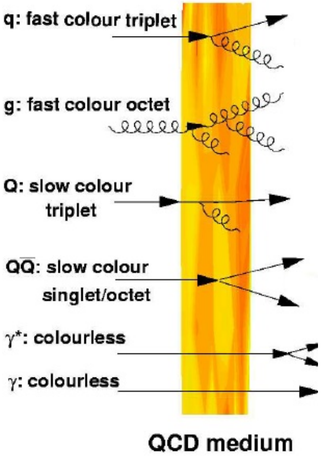

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- Heavy quark diffusion.
- Quarkonium suppression.

We will show that the heavy quark diffusion coefficient κ is also relevant for the physics of quarkonium. We will also discuss another transport coefficient important for quarkonium physics.

Heavy quark diffusion

Heavy quarks will diffuse in space in the presence of a medium. This can be quantified by the parameter D_s

$$\langle x^2(t) \rangle = 6 D_s t$$

This can be related with the transport coefficient related with diffusion in momentum space $\kappa = \frac{2T^2}{D_s}$.

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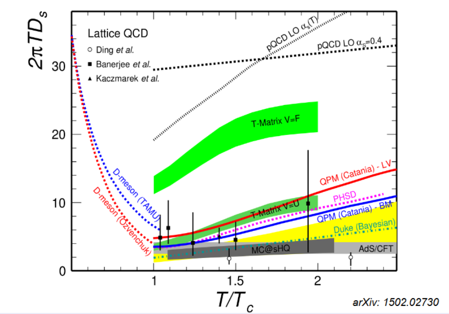

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  - κ is the heavy quark diffusion coefficient and is proportional to the decay width of quarkonium in this regime.
  - γ is proportional to the thermal mass shift of quarkonium.
- Using lattice QCD data of the decay width and thermal mass shift of quarkonium we can extract a non-perturbative determination of κ and γ.

The use of Effective Field Theories to study heavy quarks

Reminder

- The mass of a heavy quark m is much bigger than Λ_{QCD}. The production or annihilation of heavy quarks is a perturbative process.
- The temperature T of the medium is much smaller than m.
- In the case of quarkonium, other energy scales appear. The inverse of the typical radius $\frac{1}{r}$ and the binding energy E.

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Effective Field Theories

The appearance of different and very separated energy scales in a system can be a problem.

- Breaking of naive perturbation theory.
- All the relevant scales need to fit in the lattice. Large lattices, small lattice step.

This can be solved using EFTs.

Integrating out the heavy quark mass

- Integrating out the scale m can be useful both to study heavy quark diffusion and quarkonium suppression.
- This step can always be done perturbatively and is not affected by the presence of the medium. $m \gg \Lambda_{QCD}, T$.

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Remark

An EFT is not only determined by its Lagrangian. The power counting is also crucial. There are EFTs that are considered to be different but have the same Lagrangian.

- Non-relativistic QCD (NRQCD)[a]. Suitable to study quarkonium. $p \sim \frac{1}{r}$.
- Heavy quark effective theory (HQET)[b]. Suitable to study heavy-light mesons. $p \sim \Lambda_{QCD}$

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[a] Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)
[b] Eichten and Hill (1990)

NRQCD

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{d_2}{m_Q^2}F^a_{\mu\nu}D^2F^{\mu\nu a} + d_g^3\frac{1}{m_Q^2}gf_{abc}F^a_{\mu\nu}F^{\mu b}_\alpha F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger\left(iD_0 + c_2\frac{\mathbf{D}^2}{2m_Q} + c_4\frac{\mathbf{D}^4}{8m_Q^3} + c_F g\frac{\sigma\mathbf{B}}{2m_Q} + c_D g\frac{\mathbf{DE}-\mathbf{ED}}{8m_Q^2}\right.$$
$$\left. + ic_S g\frac{\sigma(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_Q^2}\right)\psi$$

$$\mathcal{L}_\chi = c.c\ of\ \mathcal{L}_\psi$$

$$\mathcal{L}_{\psi\chi} = \frac{f_1({}^1S_0)}{m_Q^2}\psi^\dagger\chi\chi^\dagger\psi + \frac{f_1({}^3S_1)}{m_Q^2}\psi^\dagger\sigma\chi\chi^\dagger\sigma\psi + \frac{f_8({}^1S_0)}{m_Q^2}\psi^\dagger T^a\chi\chi^\dagger T^a\psi$$
$$+\frac{f_8({}^3S_1)}{m_Q^2}\psi^\dagger T^a\sigma\chi\chi^\dagger T^a\sigma\psi$$

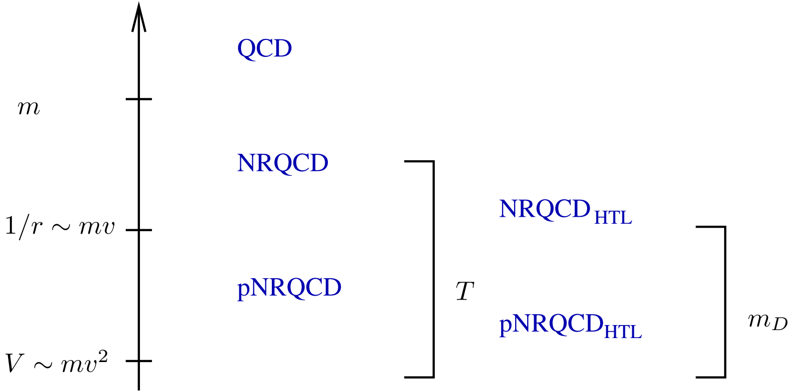
potential NRQCD Lagrangian at T=0

pNRQCD (Brambilla, Pineda, Soto and Vairo, NPB566 (2000) 275). Starting from NRQCD and integrating out the scale $\frac{1}{r}$.

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left.+O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)\, Tr(O^\dagger \mathbf{r} g \mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O)$$
$$+\frac{V_B(r)}{2} Tr(O^\dagger \mathbf{r} g \mathbf{E} O + O^\dagger O \mathbf{r} g \mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

- Degrees of freedom are singlet and octets.
- Allows to obtain manifestly gauge-invariant results. Simplifies the connection with Lattice QCD.
- If $1/r \gg T$ we can use this Lagrangian as starting point. In other cases the matching between NRQCD and pNRQCD will be modified.

EFTs to study quarkonium in a medium



Brambilla, Ghiglieri, Vairo and Petreczky (PRD78 (2008) 014017)
M. A. E and Soto (PRA78 (2008) 032520)

How to compute what we measure?

Experimentally, the most common way to detect quarkonium is thought its decay into leptons. What is the pNRQCD operator related with this observable?

$$Tr(J_{el}^{\mu}(t,\mathbf{0})J_{el,\mu}(t,\mathbf{0})\rho) \propto Tr(S^{\dagger}(t,\mathbf{0})S(t,\mathbf{0})\rho)$$

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Reinterpretation

We can understand $Tr(S^{\dagger}(t,\mathbf{x})S(t,\mathbf{x}')\rho)$ as the projection of the density matrix to the subspace in which we have a singlet. Quarkonium is an open quantum system interacting with a bath.

How to describe quarkonium in a medium?

Potential model approach

$$i\partial_t\Psi = \left(\frac{p^2}{M} + V(r,T)\right)\Psi$$

Only screening:

- Compute whether, at a given temperature, the potential is compatible with the existence of a given bound states.
- Sequential meeting picture.

Including the effect of inelastic collisions:

- Encoded in an imaginary part of the potential.
- Identify the norm of the wave-function with survival probability. Zero at large times?

How to describe quarkonium in a medium?

Transport equation approach

$$\partial_t p^{1S} = f(p^{free}) - \Gamma p^{1S} \sim \Gamma(p^{1S}_{eq} - p^{1S})$$

[2]

- At large times it arrives to an equilibrium state.
- All quantum coherence information is lost. Can not fully include screening effects in this way. Quantum mechanics is needed to solve the bound state problem.

[2] p^{1S} is the probability to find a 1S state and $f(p^{1S})$ is the probability that a 1S is generated by the decay of other particles.

How to describe quarkonium in a medium?

Open quantum system approach

$$\partial_t \rho = -i\,[H, \rho] + \mathcal{D}(\rho)$$

- Quantum version of a transport equation.
- The state is represented by a density matrix.
- Can include in the same equation screening and dissociation by collisions.

Example: The Lindblad equation

Gorini, Kossakowski and Sudarshan (1976), Lindblad (1976)

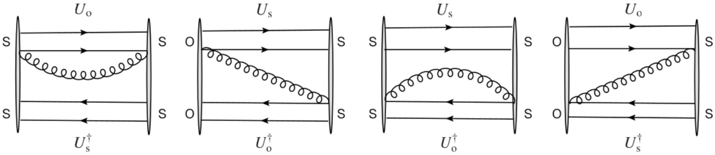
$$\partial_t \rho = -i\,[H,\rho] + \sum_i \left(C_i \rho C_i^\dagger - \frac{1}{2}\left\{ C_i^\dagger C_i, \rho \right\} \right)$$

Equation fulfilled by any evolution which:

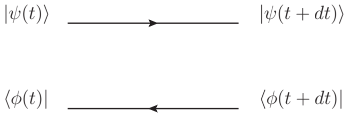
- Is Markovian (no memory).
- Conserves the trace (sum of all probabilities is equal to 1).
- Is a completely positive map (no negative probabilities).

The evolution of the density matrix

4 diagrams that connect any state at time t with a singlet at time $t + dt$.

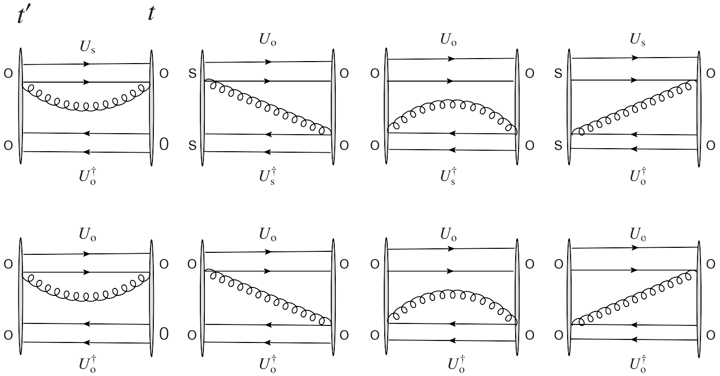


These diagrams represent the evolution of the density matrix



The evolution of the density matrix

8 diagrams that connect whatever state at time t with an octet at time $t + dt$.



The $\frac{1}{r} \gg T, m_D \gg E$ regime

Brambilla, M.A.E., Soto and Vairo (2017-2018)

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form. [3]

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa)C_k(\kappa), \rho\})$$

$$\kappa = \frac{g^2}{6\,N_c} \mathrm{Re} \int_{-\infty}^{+\infty} ds \, \langle \mathrm{T}\, E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

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κ coincides with the heavy quark diffusion coefficient.

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From κ and γ to phenomenological predictions

We can take values of κ (in this case we use lattice QCD results of Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)) and γ (we use $\gamma = 0$).

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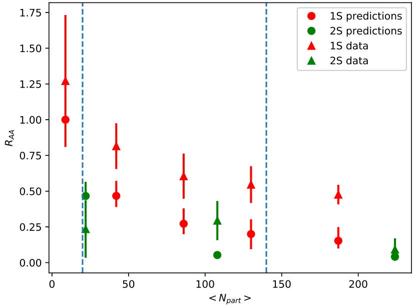

Comparison to CMS data at $\sqrt{s} = 2.76\ TeV$ (Phys.Lett. B770 (2017) 357-379), computation done in Brambilla, M.A.E., Soto and Vairo (2017-2018).

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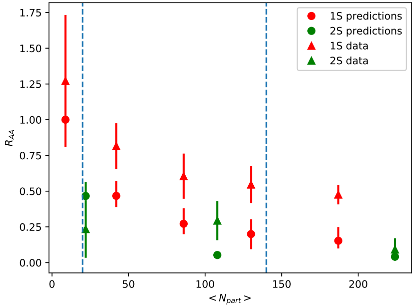

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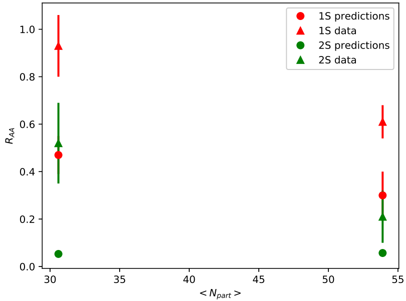

Comparison to CMS data at $\sqrt{s} = 5.02\ TeV$ (Phys. Lett. B 790, 270-293 (2019)), computation shown in Hard Probes 2018.

Determining κ and γ from quarkonium properties

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- In the case of κ, we have a determination which is based on an independent set of assumptions which can be compared with what is found studying heavy quark diffusion.
- In the case of γ, it is the first non-perturbative determination.
- The main assumption is that we are in the regime $\frac{1}{r} \gg T, m_D \gg E$ and that the bound states are Coulombic.

Determining κ and γ from quarkonium properties

Equations for κ and γ

$$\Gamma = \kappa \langle r^2 \rangle \qquad \delta M = \frac{1}{2}\gamma \langle r^2 \rangle$$

$\langle r^2 \rangle$ is computed assuming that the wave function is well described with a Coulombic potential.

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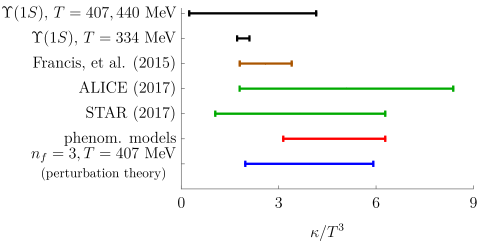
Lattice QCD data

- We use the results of Kim, Petreczky and Rothkopf (2018) for the thermal mass shift and as a lower bound for the decay width.
- We use the results of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) as upper bound for the decay width.
- Data at $T = 334\,MeV$, not used originally in our paper, is taken from Larsen, Meinel, Mukherjee and Petreczky (2019).

Determination of κ



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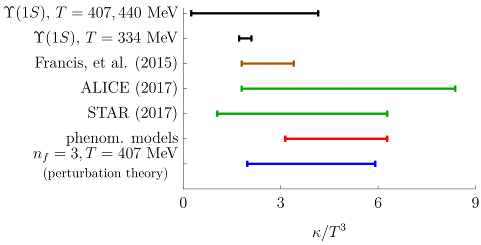

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- We took the value of Kim, Petreczky and Rothkopf (2018) at $T = 407\ MeV$ as a lower bound and the value of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) at the highest temperature available as an upper bound.

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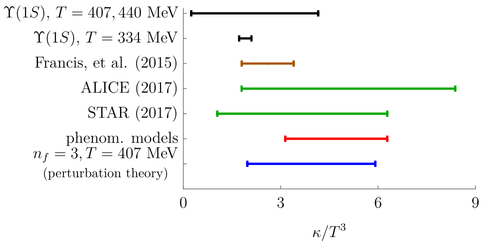

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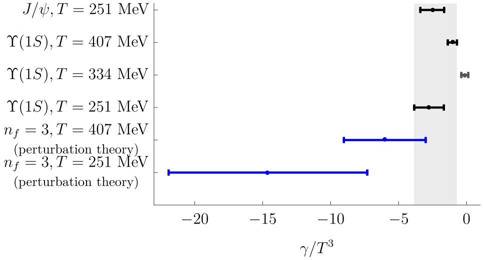
- Our result compares reasonable to other determinations.

Determination of γ



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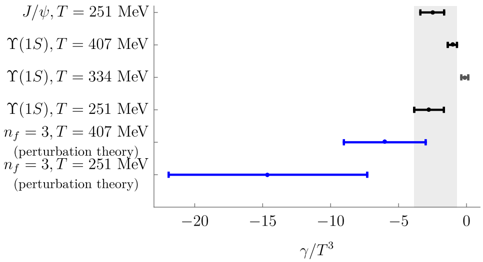

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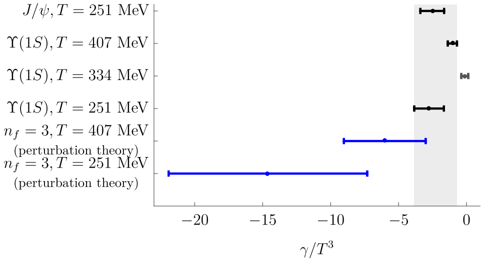

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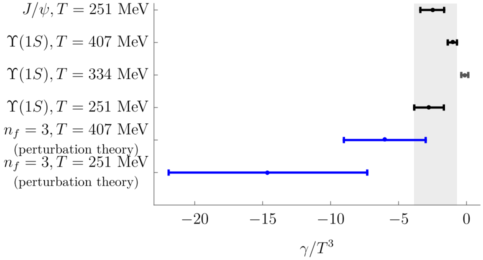

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- Results from different quarkonium state at the same temperature ($T = 251\, MeV$) are compatible with each other.
- Hint that $\frac{\gamma}{T^3}$ is not a constant.
- Lattice extracted results are much smaller than perturbative calculations.

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- We have determined κ and γ non-perturbatively using lattice QCD data.
- More on Effective Field Theories and lattice QCD on Nora Brambilla's plenary talk on Thursday.