Kinetic and Chemical Equilibration of QGP

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Kinetic and Chemical Equilibration

- Initial Collision
- Off-Thermal
- Gluon Saturation

- QCD Kinetic Solver
  - Thermalization
  - Chemical Equilibration

- Hydrodynamic
  - Thermal
  - Gluon/Quarks
Based on Effective Kinetic Theory (AMY) of QCD at LO

\[ \left( \frac{\partial}{\partial t} - \frac{p}{t} \frac{\partial}{\partial p} \right) f_s(\vec{p}, t) = -C_s^{2\leftrightarrow2}[f](\vec{p}, t) - C_s^{1\leftrightarrow2}[f](\vec{p}, t) \]

Explicitly solve Boltzmann equation for gluon and 3 light quarks/anti-quarks as an integro-differential equation including 2-2 elastic processes and 1-2 inelastic processes

- Evolution of phase space density for discretized momenta for spatially homogenous QGP
- Exact conservation of particle number (elastic) and energy (elastic + inelastic)
  - Based on weight function algorithm
  - Scale invariant evolution with massless QCD particles in current QCD Kinetic Solver
Elastic Collisions

- Elastic 2-2 Collision Integrals:  
  \[ C_{a\to2}^{b\to2}[f](\vec{p}_1) = \frac{1}{2\nu_a} \frac{1}{2E_{p_1}} \sum_{cd} \int d\Pi_{2\to2} |M_{cd}^{ab}(\vec{p}_1, \vec{p}_2|\vec{p}_3, \vec{p}_4)|^2 F_{cd}^{ab}(\vec{p}_1, \vec{p}_2|\vec{p}_3, \vec{p}_4) \]

\( gg \leftrightarrow gg \)
\( qg \leftrightarrow qg, \overline{q}g \leftrightarrow \overline{q}g \)
\( q\overline{q} \leftrightarrow gg \)
\( qq \leftrightarrow qq, \overline{q}\overline{q} \leftrightarrow \overline{q}\overline{q} \)
\( q_1q_2 \leftrightarrow q_1q_2, q_1\overline{q}_2 \leftrightarrow q_1\overline{q}_2, \overline{q}_1q_2 \leftrightarrow \overline{q}_1q_2, \overline{q}_1\overline{q}_2 \leftrightarrow \overline{q}_1\overline{q}_2 \)

Screening masses match to HTL calculations

Kurkela, Mazeliauskas, PRD99 (2019) 054018

Amy, JHEP01 (2003) 030
Inelastic Collisions

- Inelastic 1-2 Collision Integrals:

\[
C_{a \rightarrow 2}^1[f](p) = \frac{1}{2 \nu_a} \int_0^1 dz \left[ \sum_{bc} \frac{d \Gamma_{bc}^a}{dz} (p, z) \nu_a F_{bc}^a(p | z p, \bar{z} p) - \frac{1}{z^3} \frac{d \Gamma_{bc}^c}{dz} (\frac{p}{z}, \bar{z}) \nu_c F_{bc}^c(p | \frac{p}{z}, \bar{z} p) - \frac{1}{\bar{z}^3} \frac{d \Gamma_{bc}^c}{dz} (\frac{p}{\bar{z}}, z) \nu_c F_{bc}^c(p | \frac{p}{\bar{z}}, z p) \right]
\]

Effective splitting rates

Statistical Factor

Effective in-medium splitting rates accounting for LPM effect, via vertex resummation (AMY)
Kinetic and Chemical Equilibration without Long. Expansion
Gluon and Quark Chemical Equilibration

System initially in kinetic equilibrium but not in chemical equilibrium

Initially thermal gluon only, no quark
Initially thermal quark, anti-quark only, no gluon

Equilibration relaxation time

\[ \tau_R = \frac{4\pi\eta/s}{T_{eq}} \]

controls kinetic equilibration/hydrodynamization

- Chemical equilibration controlled by the same time scale as kinetic equilibration: 1.5~2.0 \( t_R \)

\[ \lambda = 4\pi\alpha_s N_c \]

\[ \eta/s = 1/(\lambda = 10) \]

\[ \eta/s = 35/(\lambda = 1) \]

Kurkela, Mazeliauskas, PRD99 (2019) 054018
KOMPOST, PRC99 (2019) 034910
System initially thermal quark/anti-quark with finite chemical potential & no gluon

 Scaling of Relaxation time as $\mu = 0$

$$\tau_R(T, \mu) = \frac{4\pi}{T_{eq}} \left( \frac{\eta(T, \mu)T}{e + p} \right) \sim \frac{4\pi}{T_{eq}} \left( \frac{\eta(T, \mu)T}{e + p} \right)_{eq} \sim \frac{4\pi}{T_{eq}} \left( \frac{\eta(T_{eq}, \mu = 0)}{s} \right)$$

1. Quark anti-quark quickly radiates gluon
2. Quark anti-quark annihilation follows
3. Chemical equilibration occurs on the same time scale for zero and finite charge density
System initially features excess of low momentum gluon (over-occupied gluon & no quark)
first consider weak coupling/ large scale separation: \(\langle p \rangle_0 = 0.2 \lambda = 0.1\)

**Self-similar evolution of gluon towards equilibrium**

Berges, Boguslavski, Schlichting, Venugopalan, PRD89(2014)114007
Abraao York, Kurkela, Lu, Moore, PED89(2014)074036

Quarks distribution follows the gluon with thermal spectra at low momentum
System initially features excess of low momentum gluon (over-occupied gluon & no quark)

- Different distinct stages:
  1. Initial memory loss
  2. Self-similar universal scaling
  3. Equilibration
- Scaling not limited to pure Yang-Mills evolution but also QCD evolution.
- Same thermalization patterns even work for moderately strongly coupled system

Berges, Boguslavski, Schlichting, Venugopalan, PRD89(2014)114007
Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036
System initially features only high-momentum quark (mini-jet) & no gluon \( <p>_0 = 3.0 \lambda = 1.0 \)

- Bottom-up thermalization
  1. Emission of (soft) quarks and gluons
  2. Radiative breakup by multiple branchings -> build up soft thermal bath
  3. Mini-jet energy loss -> heating up thermal bath

- Same pattern as for in-medium jet evolution with unified description of soft and hard sector

Kinetic and Chemical Equilibration with Long. Expansion
System initially **highly anisotropic with CGC inspired gluon dist.** & **finite charge density** $\lambda=10.0$

Onset of hydrodynamic behavior at: $t \sim 1.5 \, t_R$

Finite density thermalize at the same time scale as for zero density.

Chemical equilibration ($t \sim 2 \, t_R$) occurs on roughly the same time scale as/稍晚 than hydrodynamization ($t \sim 1.5 \, t_R$)
Conclusions And Outlook

• Generally the nonequilibrium evolution for 3-flavor QCD is similar to pure Yang-Mills
  • Over-occupied system follows a self-similar universal scaling, even for moderately strongly coupled system
  • Under-occupied system follows a bottom-up thermalization

• Chemical and kinetic equilibration for expanding QGP occurs on roughly the same time scale as for zero and finite charge density.
  • No significant changes in pressure equilibration for finite density compared to zero density
  • Detailed study of finite density in progress…

• Embrace the future: About QCD Kinetic Solver
  • Have all species of light particles in QCD, allowing finite density calculation, capable of realistic matching to hydro (KoMPoST)
  • Highly extensible machinery for including more particles (heavy quarks, …)
  • Capable of a universal description of soft and hard sectors for jet energy loss
\begin{align*}
\frac{m_D^2}{d_A} &= \frac{4g^2}{(2\pi)^3} \frac{1}{2p} \left[ \nu_G C_A f_G(p, T) + \sum_{Q=u,d,s} \nu_Q C_F \left( f_Q(p, T, \mu_Q) + f_Q(p, T, \mu_Q) \right) \right] \\
\frac{m_Q^2}{d_A m_D^2} &= \frac{g^2}{(2\pi)^3} \frac{1}{2p} \left[ 2f_G(p, T) + \left( f_Q(p, T, \mu_Q) + f_Q(p, T, \mu_Q) \right) \right] \\
T^* &= \frac{g^2}{d_A m_D^2} \int \frac{d^3p}{(2\pi)^3} \left[ \nu_G C_A f_G(p, T)(1 + f_G(p, T)) + \sum_{Q=u,d,s} \nu_Q C_F \left( f_Q(p, T, \mu_Q)(1 - f_Q(p, T, \mu_Q)) + (\bar{Q}) \right) \right] = T \\
eq \frac{\pi^2}{2} T^4 \\
eq \frac{3T^4 \text{Li}_4(-e^{-\mu/T})}{\pi^2} = \frac{7\pi^2}{240} T^4 \\
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\frac{n_G^{eq}}{(2\pi)^3} f_G(p, T) = \frac{\zeta(3)}{\pi^2} T^3 \\
n_Q^{eq}(p, T, \mu) &= \frac{T^3 \text{Li}_3(-e^{-\mu/T})}{\pi^2} = \frac{3\zeta(3)}{4\pi^2} T^3 \\
n_Q^{eq}(p, T, \mu) &= \frac{T^3 \text{Li}_3(-e^{-\mu/T})}{\pi^2} = \frac{3\zeta(3)}{4\pi^2} T^3
\end{align*}
Assuming initially thermal quark/anti-quark with finite chemical potential & no gluon
1. Quick elimination of anisotropy
2. Thermalization