Kinetic and Chemical Equilibration of QGP

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Hard Probes 2020, Austin, TX, USA
06/03/2020

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DFG Deutsche Forschungsgemeinschaft
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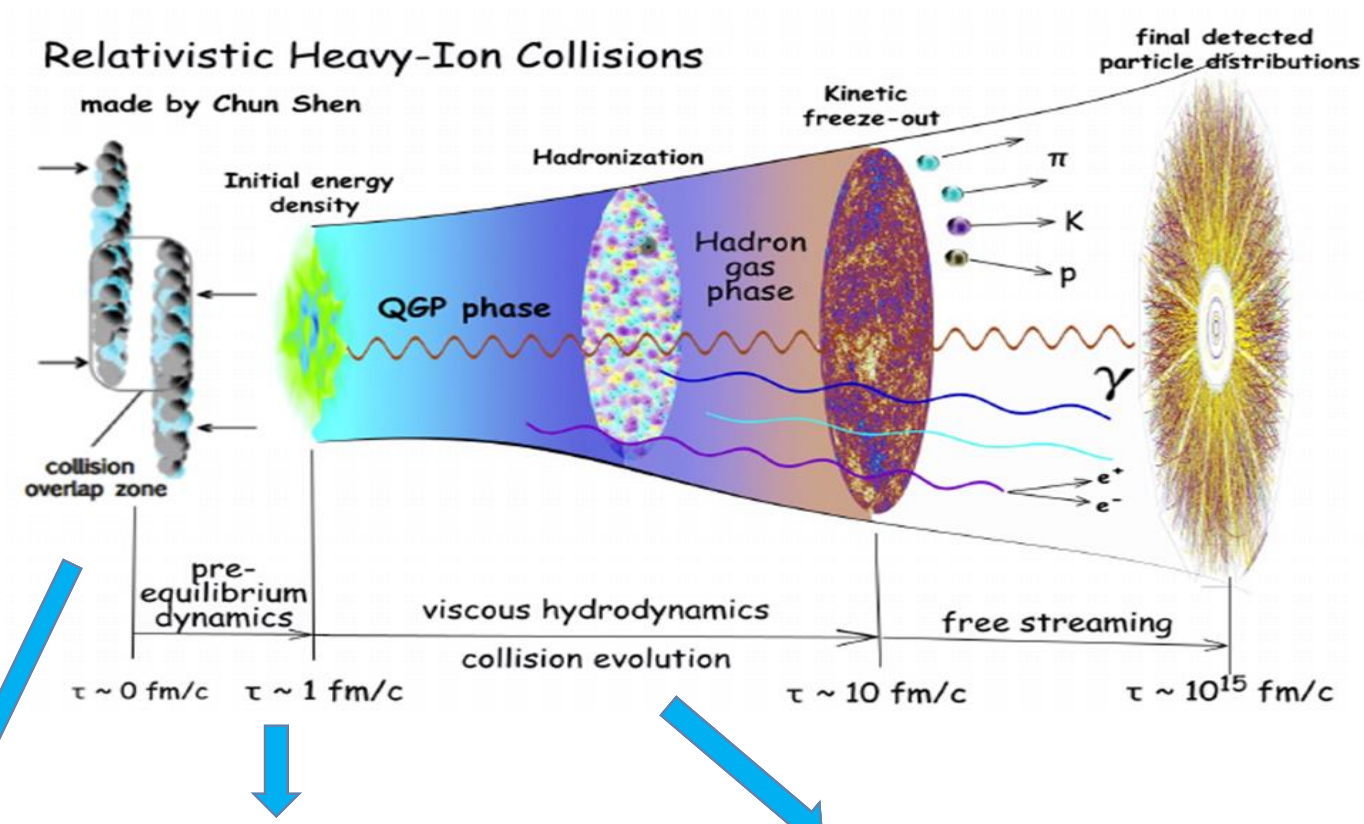
CRC-TR 211
Strong-interaction matter under extreme conditions

Kinetic and Chemical Equilibration



- Initial Collision
- Off-Thermal
- Gluon Saturation

- QCD Kinetic Solver
- Thermalization
- Chemical Equilibration

- Hydrodynamic
- Thermal
- Gluon/Quarks

QCD Kinetic Solver

Based on Effective Kinetic Theory (AMY) of QCD at LO

AMY,JHEP01 (2003) 030
AMY,JHEP0206(2002)030
Kurkela, Mazeliauskas,PRD99 (2019) 054018

$$\left(\frac{\partial}{\partial t} - \frac{p_\parallel}{t}\frac{\partial}{\partial p_\parallel}\right) f_s(\vec{p},t) = -C_s^{2\leftrightarrow 2}[f](\vec{p},t) - C_s^{1\leftrightarrow 2}[f](\vec{p},t)$$

$$s = g, u, \bar{u}, d, d, s, \bar{s}$$

Explicitly solve Boltzmann equation for gluon and 3 light quarks/anti-quarks as an integro-differential equation including 2-2 elastic processes and 1-2 inelastic processes

- Evolution of phase space density for discretized momenta for spatially homogenous QGP

- Exact conservation of particle number (elastic) and energy (elastic + inelastic)
  - Based on weight function algorithm

Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036
E Lu, PhD Thesis

- Scale invariant evolution with massless QCD particles in current QCD Kinetic Solver

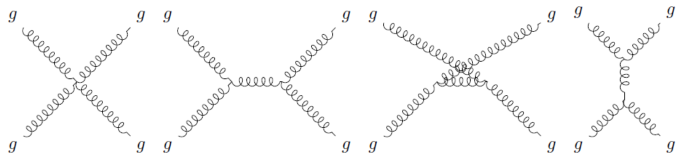
Elastic Collisions

- Elastic 2-2 Collision Integrals: pQCD amplitudes Statistical Factor

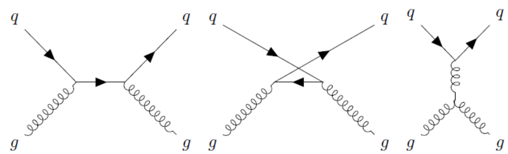
$$C_a^{2\leftrightarrow 2}[f](\vec{p}_1) = \frac{1}{2\nu_a}\frac{1}{2E_{p_1}}\sum_{cd}\int d\Pi_{2\leftrightarrow 2}\boxed{|\mathcal{M}^{ab}_{cd}(\vec{p}_1,\vec{p}_2|\vec{p}_3,\vec{p}_4)|}^2\boxed{F^{ab}_{cd}(\vec{p}_1,\vec{p}_2|\vec{p}_3,\vec{p}_4).}$$

AMY,JHEP01 (2003) 030

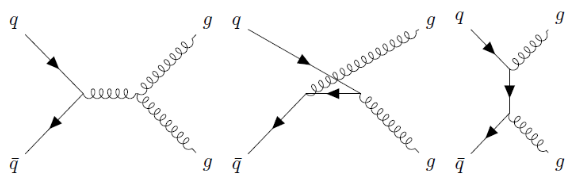
$gg \leftrightarrow gg$



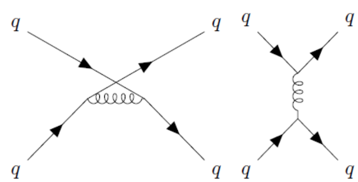
$qg \leftrightarrow qg,\ \bar{q}g \leftrightarrow \bar{q}g$



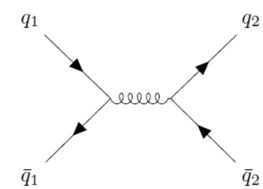
$q\bar{q} \leftrightarrow gg$



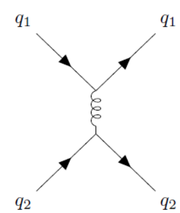
$qq \leftrightarrow qq,\ \bar{q}\bar{q} \leftrightarrow \bar{q}\bar{q}$



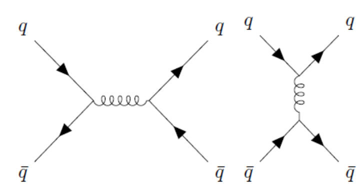
$q_1\bar{q}_1 \leftrightarrow q_2\bar{q}_2$



$q_1q_2 \leftrightarrow q_1q_2,\ q_1\bar{q}_2 \leftrightarrow q_1\bar{q}_2,$
$\bar{q}_1q_2 \leftrightarrow \bar{q}_1q_2,\ \bar{q}_1\bar{q}_2 \leftrightarrow \bar{q}_1\bar{q}_2$



$q\bar{q} \leftrightarrow q\bar{q}$



Screening masses match to HTL calculations

Kurkela, Mazeliauskas, PRD99 (2019) 054018

Inelastic Collisions

- Inelastic 1-2 Collision Integrals:

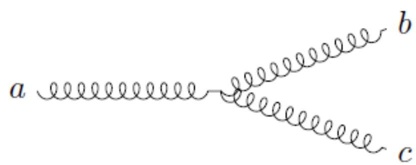
Effective splitting rates

Statistical Factor

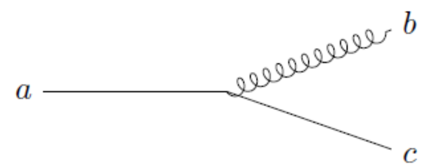
$$C_a^{1\leftrightarrow 2}[f](p) = \frac{1}{2\nu_a}\int_0^1 dz \left[\sum_{bc} \frac{d\Gamma^a_{bc}}{dz}(p,z)\nu_a F^a_{bc}(p|zp,\bar{z}p) - \frac{1}{z^3}\boxed{\frac{d\Gamma^c_{ab}}{dz}\left(\frac{p}{z},z\right)}\nu_c F^c_{ab}\left(\frac{p}{z}|p,\frac{\bar{z}}{z}p\right) - \frac{1}{\bar{z}^3}\frac{d\Gamma^c_{ab}}{dz}\left(\frac{p}{\bar{z}},\bar{z}\right)\nu_c \boxed{F^c_{ab}\left(\frac{p}{\bar{z}}|p,\frac{z}{\bar{z}}p\right)}\right]$$

AMY,JHEP0206(2002)030

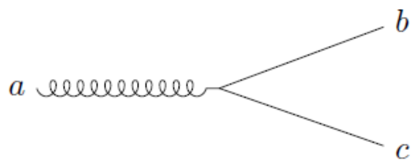
$g \leftrightarrow gg$



$q \leftrightarrow gq$



$g \leftrightarrow q\bar{q}$



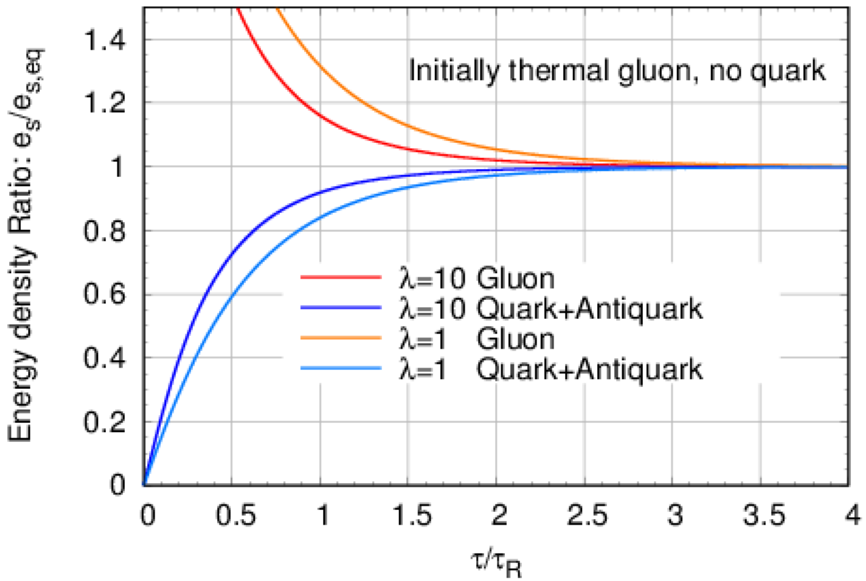
Effective in-medium splitting rates accounting for LPM effect, via vertex resummation (AMY)

Kinetic and Chemical Equilibration without Long. Expansion

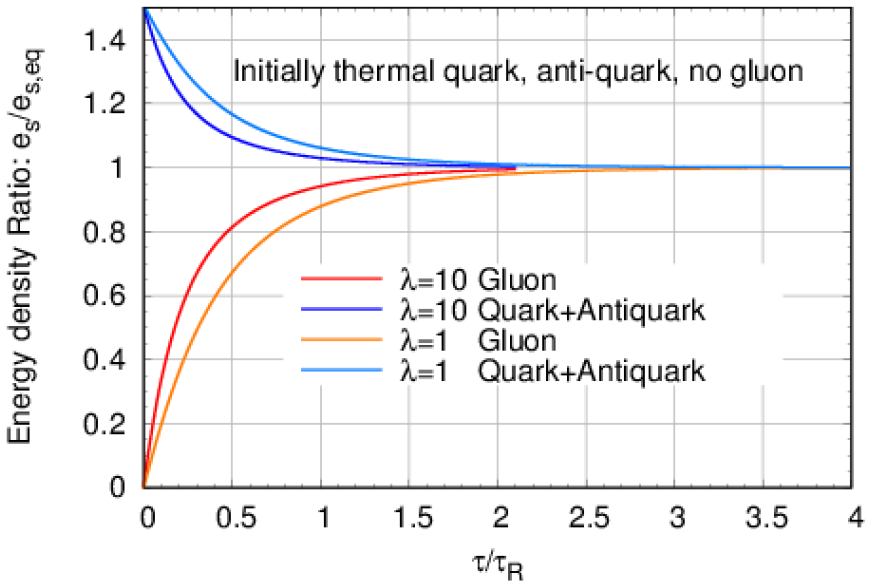
Gluon and Quark Chemical Equilibration

System initially in kinetic equilibrium but not in chemical equilibrium

Initially thermal gluon only, no quark



Initially thermal quark, anti-quark only, no gluon



Equilibration relaxation time

$$\tau_R = \frac{4\pi\eta/s}{T_{eq}}$$

controls kinetic equilibration/hydrodynamization

t'Hooft coupling

$$\lambda = 4\pi\alpha_s N_c$$

$\eta/s = 1(\lambda = 10)$

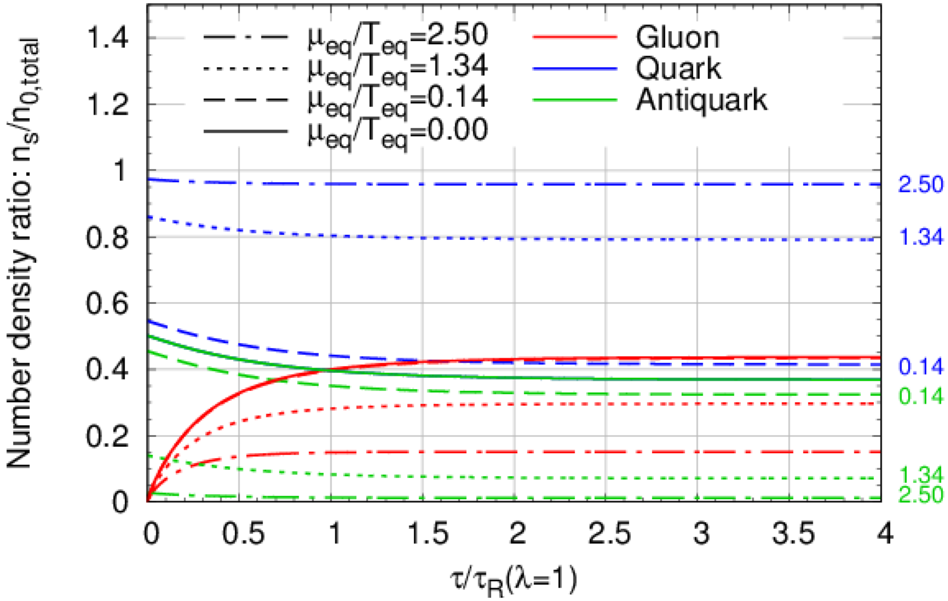
$\eta/s = 35(\lambda = 1)$

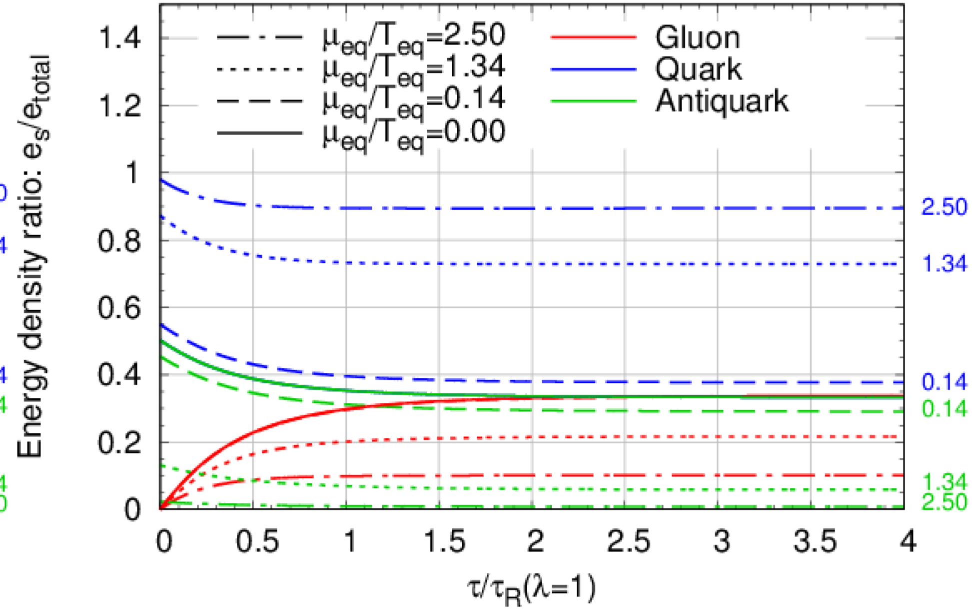
Kurkela, Mazeliauskas,PRD99 (2019) 054018
KOMPOST, PRC99 (2019) 034910

- Chemical equilibration controlled by the same time scale as kinetic equilibration: 1.5~2.0 t_R

Chemical Equilibration at Finite Density

System initially thermal quark/anti-quark with finite chemical potential & no gluon





Scaling of Relaxation time as $\mu = 0$

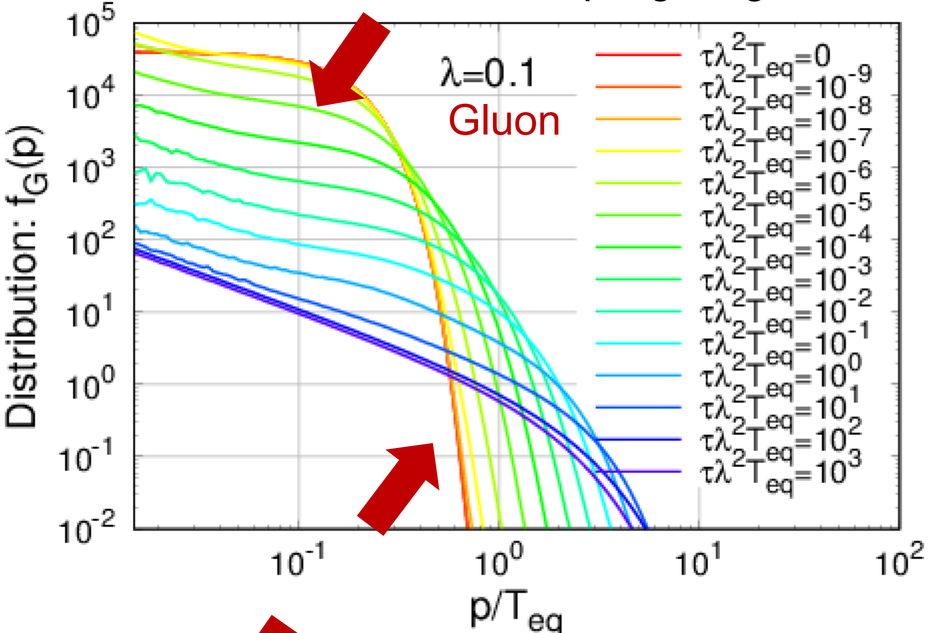
$$\tau_R(T,\mu) = \frac{4\pi}{T_{eq}}\left(\frac{\eta(T,\mu)T}{e+p}\right) \sim \frac{4\pi}{T_{eq}}\left(\frac{\eta(T,\mu)T}{e+p}\right)_{eq} \sim \frac{4\pi}{T_{eq}}\left(\frac{\eta(T_{eq},\mu=0)}{s}\right)$$

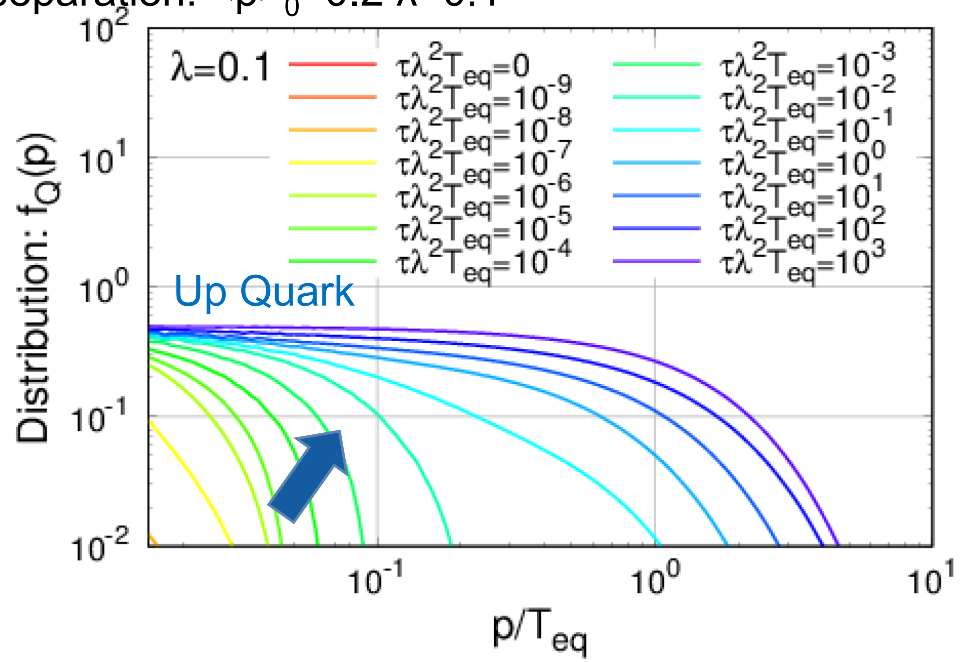
1. Quark anti-quark quickly radiates gluon
2. Quark anti-quark annihilation follows
3. Chemical equilibration occurs on the same time scale for zero and finite charge density

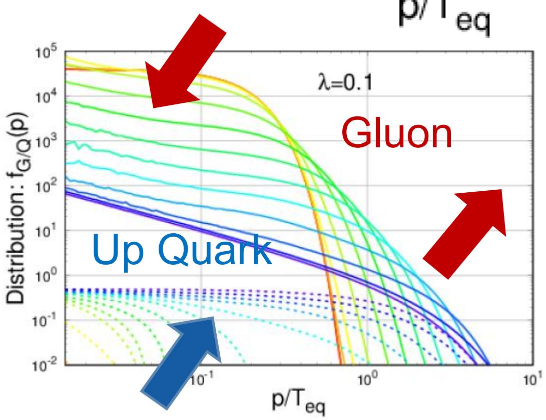
Over-Occupied System: Spectra Evolution

System initially features excess of low momentum gluon (over-occupied gluon & no quark)

first consider weak coupling/ large scale separation: $\langle p \rangle_0=0.2$ $\lambda=0.1$







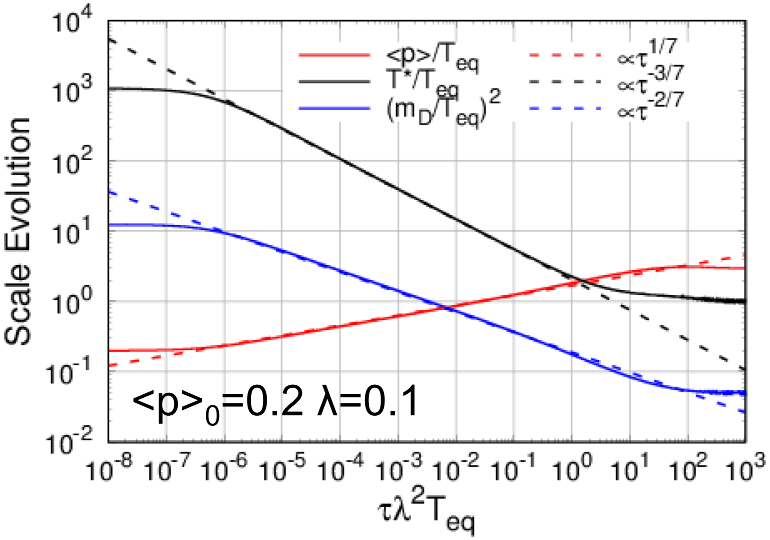
Self-similar evolution of gluon towards equilibrium

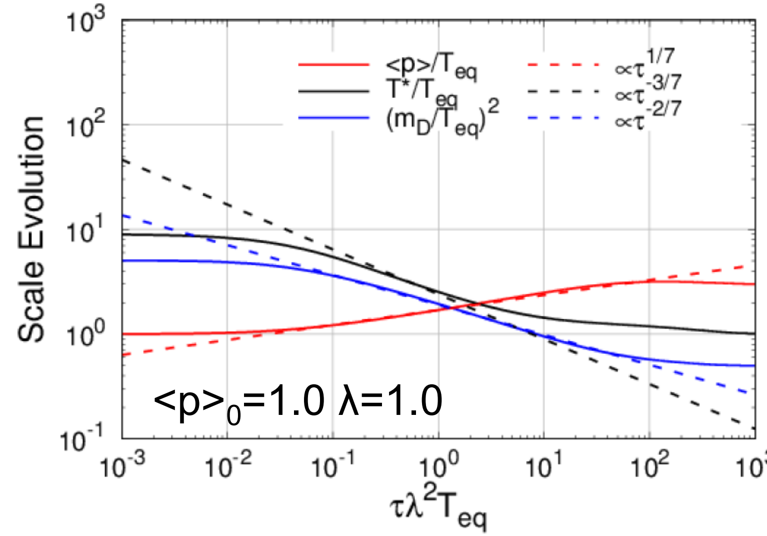
Berges, Boguslavski, Schlichting, Venugopalan, PRD89(2014)114007
Abraao York, Kurkela, Lu, Moore, PED89(2014)074036

Quarks distribution follows the gluon with thermal spectra at low momentum

Over-Occupied System: Self-Similarity

System initially features excess of low momentum gluon (over-occupied gluon & no quark)

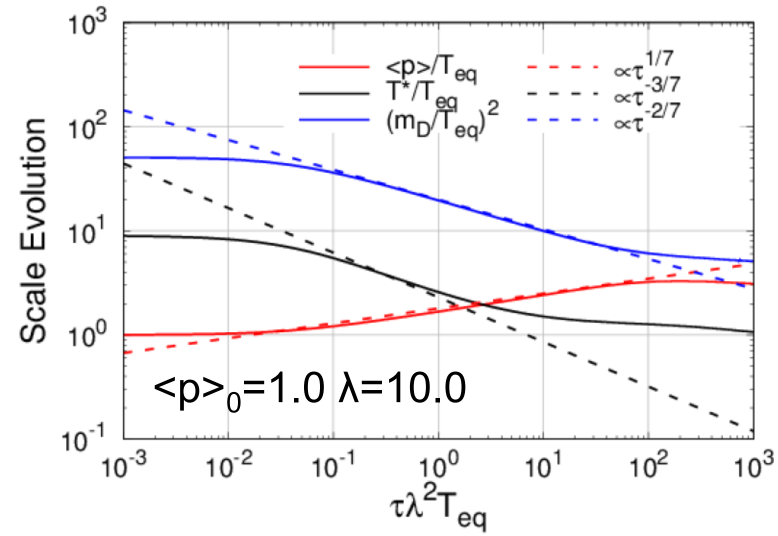




- Different distinct stages :

1. Initial memory loss
2. Self-similar universal scaling
3. Equilibration

- Scaling not limited to pure Yang-Mills evolution but also QCD evolution.
- Same thermalization patterns even work for moderately strongly coupled system

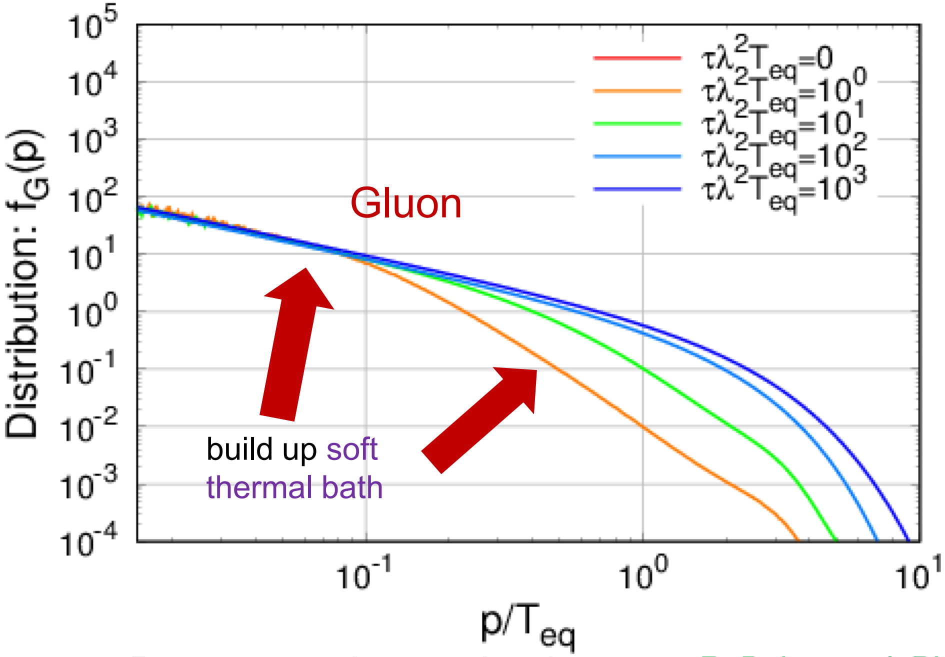


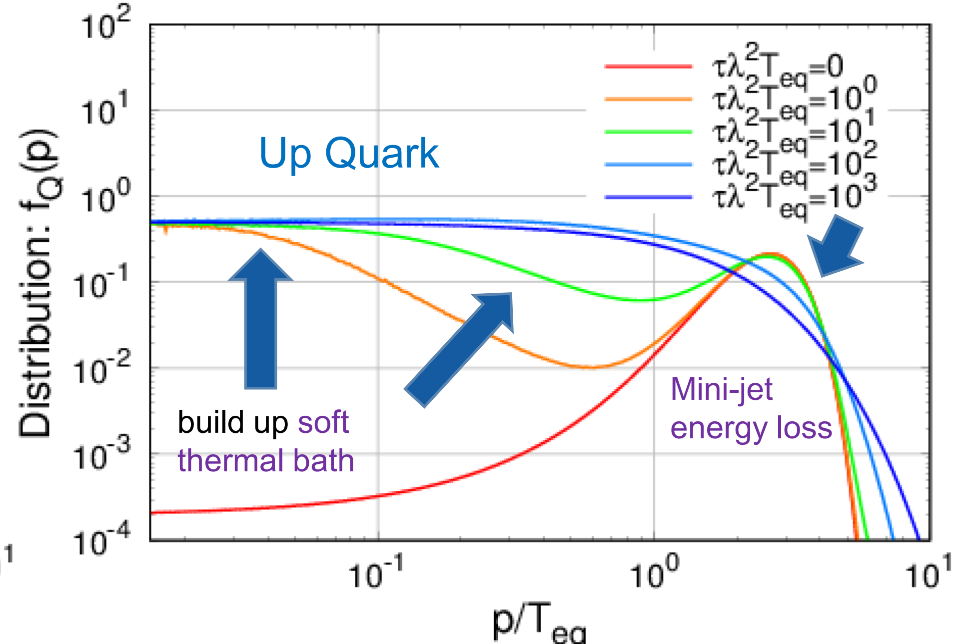
Berges, Boguslavski, Schlichting, Venugopalan, PRD89(2014)114007
Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036

Under-Occupied System: Spectra Evolution

System initially features only high-momentum quark (mini-jet) & no gluon

$\langle p \rangle_0 = 3.0$ $\lambda = 1.0$

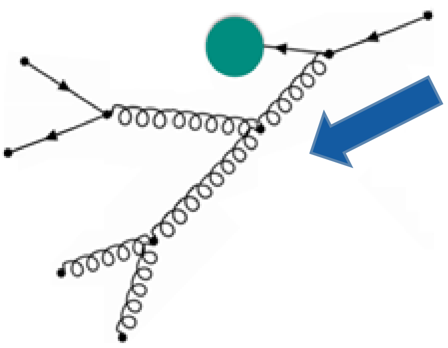




R. Baier, et al. PLB502(2001)51

- Bottom-up thermalization

1. Emission of (soft) quarks and gluons
2. Radiative breakup by multiple branchings -> build up soft thermal bath
3. Mini-jet energy loss -> heating up thermal bath



- Same pattern as for in-medium jet evolution

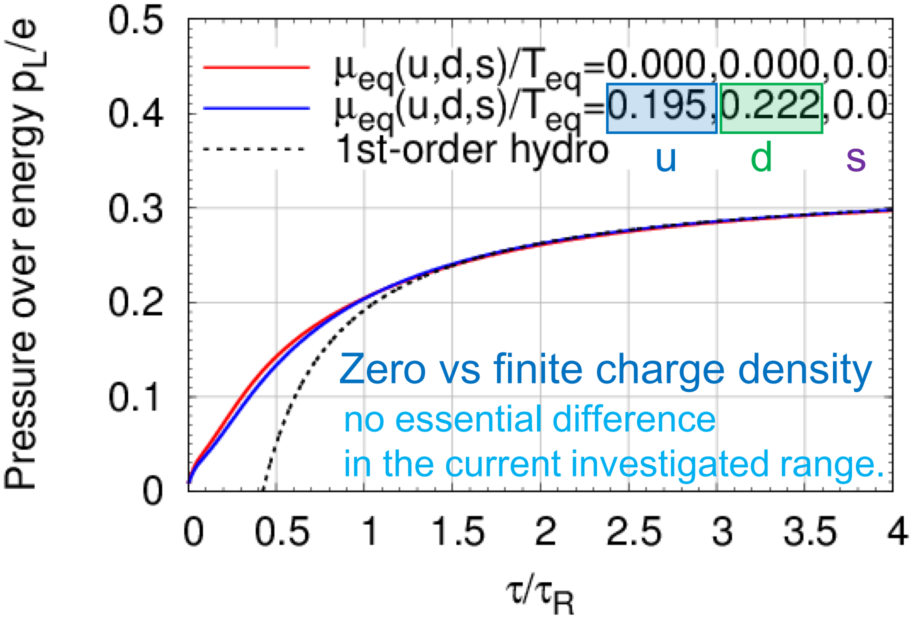
with unified description of soft and hard sector

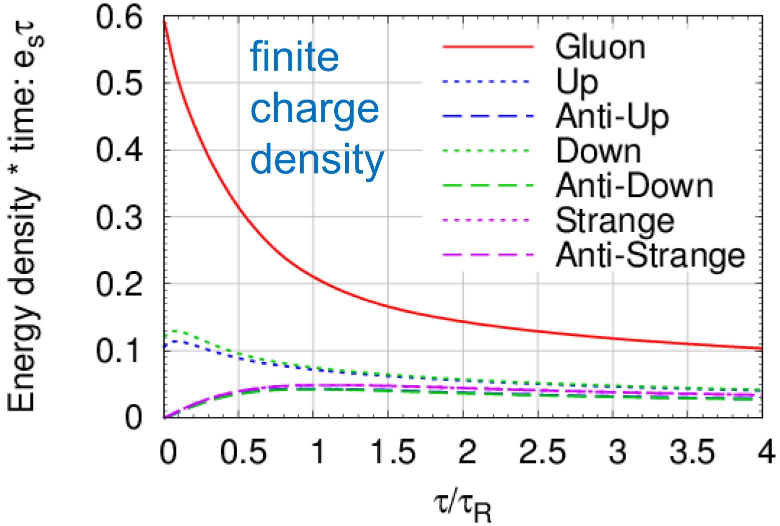
Kinetic and Chemical Equilibration with Long. Expansion

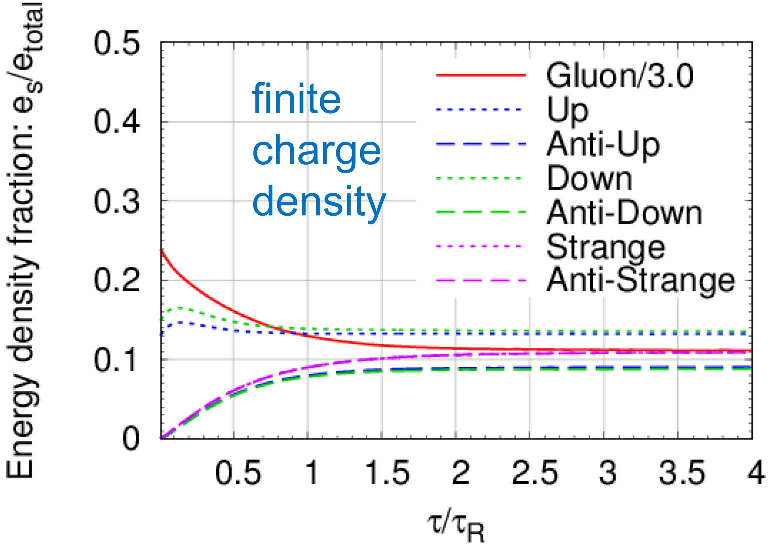
Expansion System with Finite Density

System initially highly anisotropic with CGC inspired gluon dist. & finite charge density λ=10.0

Pressure isotropization/Chemical equilibration?
(see also A. Mazeliauskas's talk)



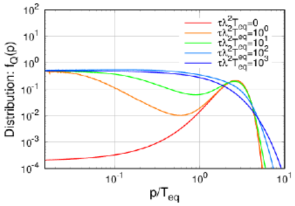




- Onset of hydrodynamic behavior at: $t \sim 1.5\ t_R$
- Finite density thermalize at the same time scale as for zero density.
- Chemical equilibration ($t \sim 2\ t_R$) occurs on roughly the same time scale as/slightly later than hydrodynamization ($t \sim 1.5\ t_R$)

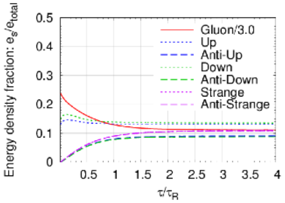
Conclusions And Outlook

- Generally the nonequilibrium evolution for 3-flavor QCD is similar to pure Yang-Mills
  - Over-occupied system follows a self-similar universal scaling, even for moderately strongly coupled system
  - Under-occupied system follows a bottom-up thermalization



- Chemical and kinetic equilibration for expanding QGP occurs on roughly the same time scale as for zero and finite charge density.
  - No significant changes in pressure equilibration for finite density compared to zero density
  - Detailed study of finite density in progress…

- Embrace the future: About QCD Kinetic Solver
  - Have all species of light particles in QCD, allowing finite density calculation, capable of realistic matching to hydro (KoMPoST)
  - Highly extensible machinery for including more particles (heavy quarks, …)
  - Capable of a universal description of soft and hard sectors for jet energy loss



Backup: Thermodynamic Quantities

$$m_D^2 = \frac{4g^2}{d_A}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left[\nu_G C_A f_G(p,T) + \sum_{Q=u,d,s}\nu_Q C_F\left(f_Q(p,T,\mu_Q)+f_{\bar{Q}}(p,T,\mu_Q)\right)\right]$$

$$m_Q^2 = g^2 C_F\int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left[2f_G(p,T) + \left(f_Q(p,T,\mu_Q)+f_{\bar{Q}}(p,T,\mu_Q)\right)\right]$$

$$T^* = \frac{g^2}{d_A m_D^2}\int \frac{d^3p}{(2\pi)^3}\left[\nu_G C_A f_G(p,T)(1+f_G(p,T)) + \sum_{Q=u,d,s}\nu_Q C_F\left(f_Q(p,T,\mu_Q)(1-f_Q(p,T,\mu_Q))+(\bar{Q})\right)\right] \overset{eq}{=} T$$

$$e_G^{\rm eq} = \int \frac{d^3p}{(2\pi)^3} p f_G^{\rm eq}(p,T) = \frac{\pi^2}{30}T^4$$

$$e_Q^{\rm eq}(p,T,\mu) = \int \frac{d^3p}{(2\pi)^3} p f_Q^{\rm eq}(p,T,\mu) = -\frac{3T^4 Li_4(-e^{\mu/T})}{\pi^2} \overset{\mu=0}{=} \frac{7\pi^2}{240}T^4$$

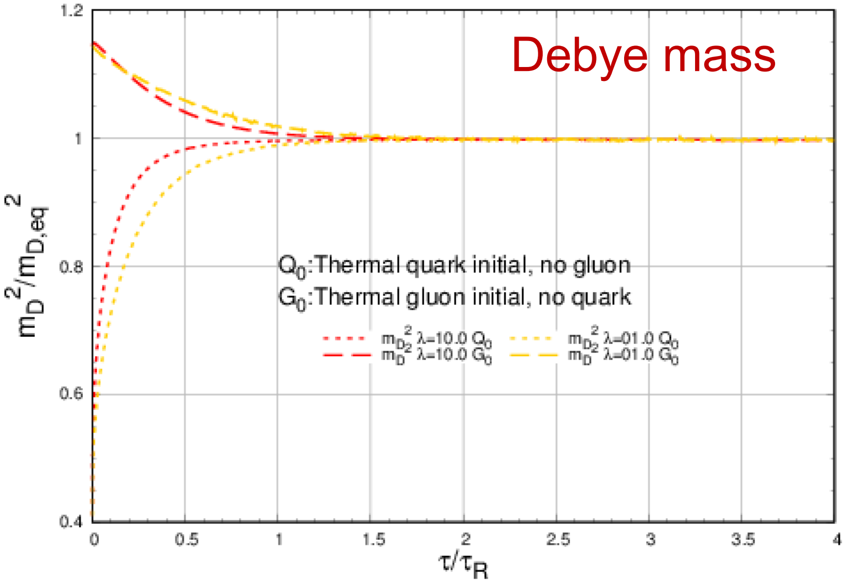
$$e_{\bar{Q}}^{\rm eq}(p,T,\mu) = \int \frac{d^3p}{(2\pi)^3} p f_{\bar{Q}}^{\rm eq}(p,T,\mu) = -\frac{3T^4 Li_4(-e^{-\mu/T})}{\pi^2} \overset{\mu=0}{=} \frac{7\pi^2}{240}T^4$$

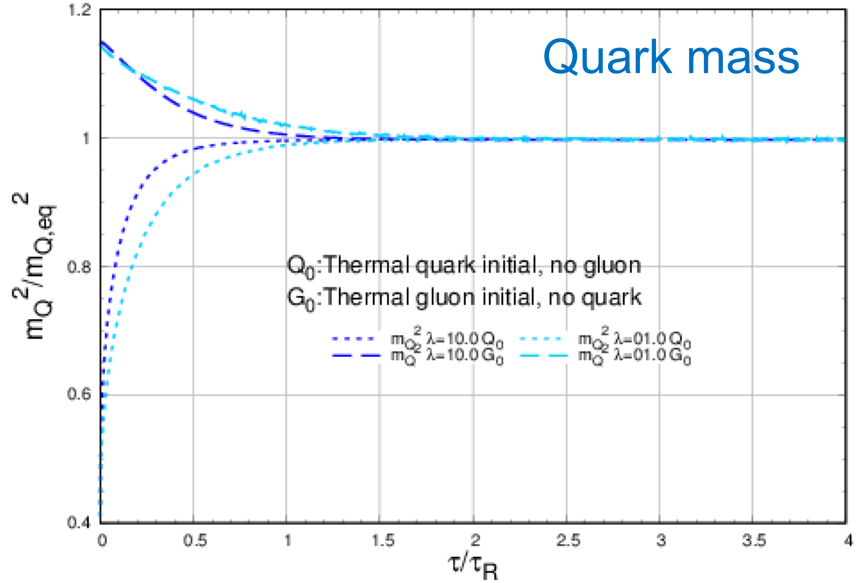
$$n_G^{\rm eq} = \int \frac{d^3p}{(2\pi)^3} f_G^{\rm eq}(p,T) = \frac{\zeta(3)}{\pi^2}T^3$$

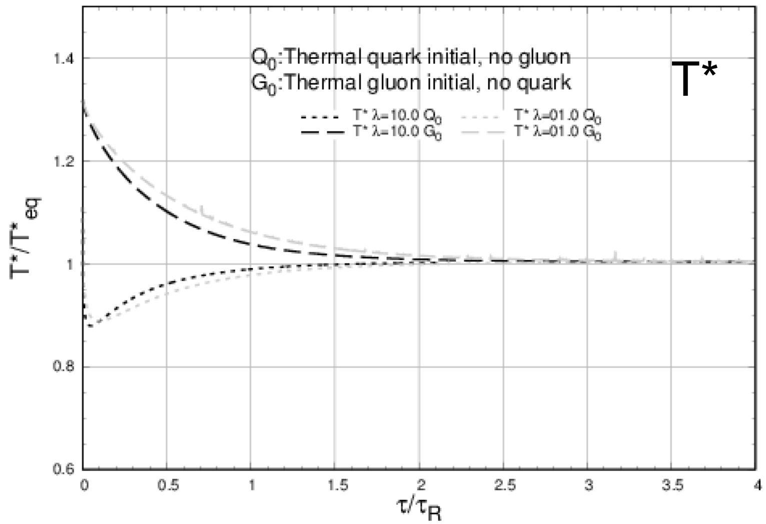
$$n_Q^{\rm eq}(p,T,\mu) = \int \frac{d^3p}{(2\pi)^3} f_Q^{\rm eq}(p,T,\mu) = -\frac{T^3 Li_3(-e^{\mu/T})}{\pi^2} \overset{\mu=0}{=} \frac{3\zeta(3)}{4\pi^2}T^3$$

$$n_{\bar{Q}}^{\rm eq}(p,T,\mu) = \int \frac{d^3p}{(2\pi)^3} f_{\bar{Q}}^{\rm eq}(p,T,\mu) = -\frac{T^3 Li_3(-e^{-\mu/T})}{\pi^2} \overset{\mu=0}{=} \frac{3\zeta(3)}{4\pi^2}T^3$$

Backup: Kinetic Equilibration Scale Evoluion

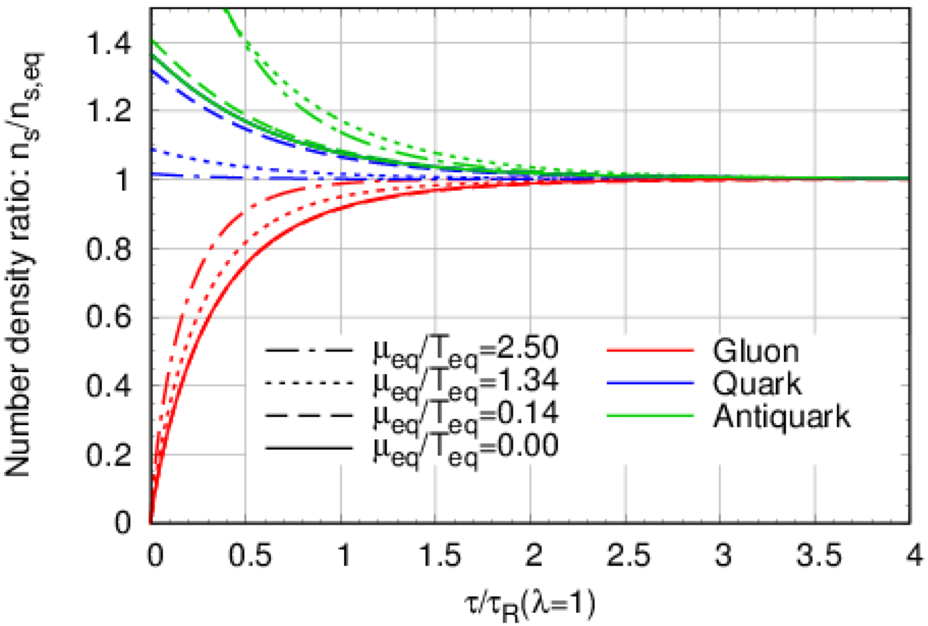


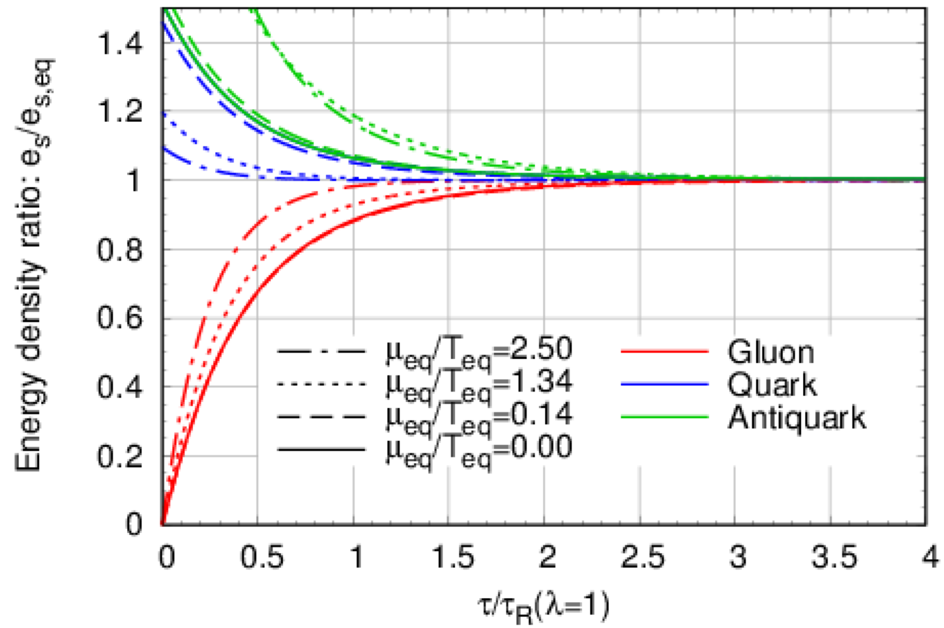




Backup: Finite Density Equilibration

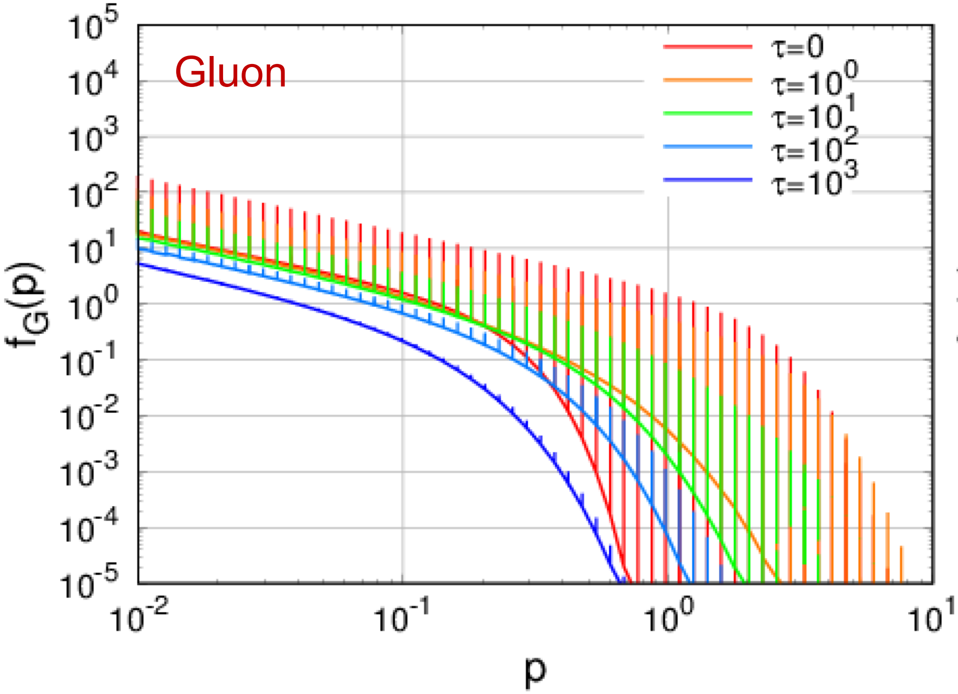
Assuming initially thermal quark/anti-quark with finite chemical potential & no gluon

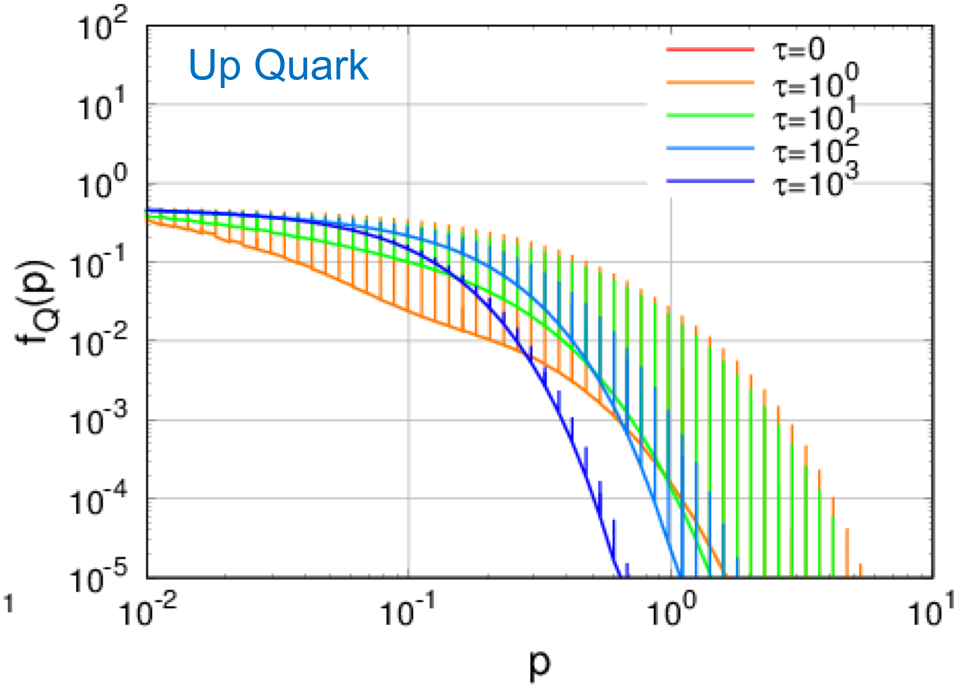




Backup: Anisotropy

System initially CGC provided gluon anisotropic distribution & no quark





1. Quick elimination of anisotropy
2. Thermalization