Ultrarelativistic quark-nucleus scattering in a light-front Hamiltonian approach

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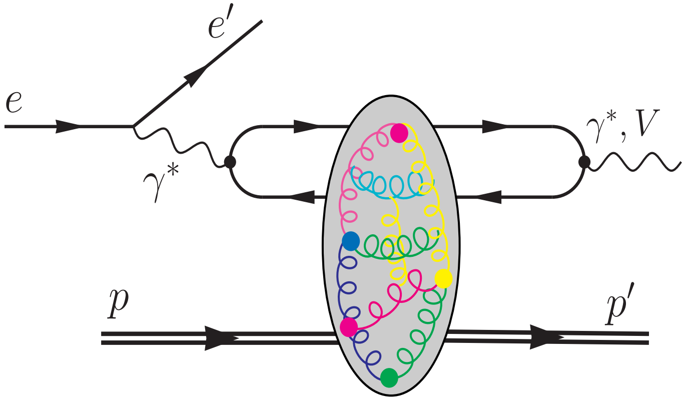
Introduction

- Studying the quark-nucleus scattering establishes a foundation for understanding more complicated processes

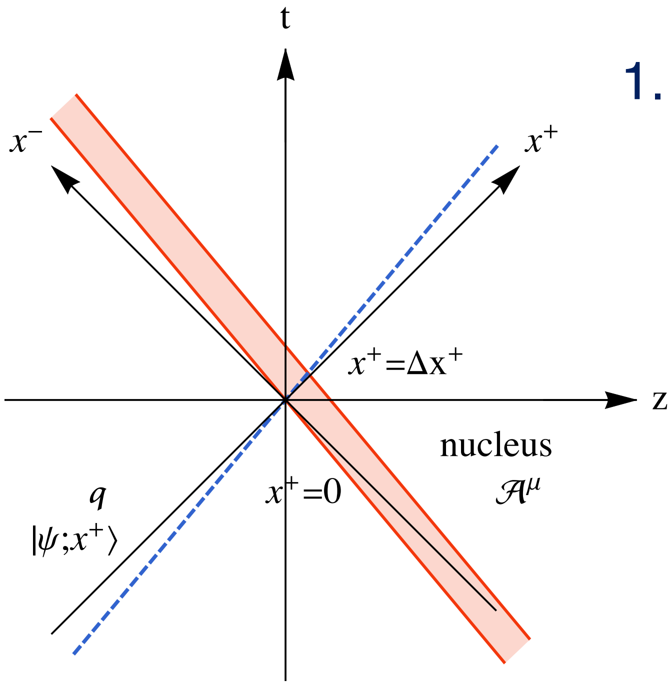


- In the deep inelastic scattering, the virtual photon fluctuates into a quark-antiquark pair, which further interacts with the heavy ion

- What are the intermediate states of the quark during the interaction?
- What sub-eikonal effects can we observe if the quark has a finite p^+?

Methodology: time-dependent Basis Light-front Quantization (tBLFQ)

We consider scattering of a high-energy quark moving in the positive z, on a high-energy nucleus moving in the negative z.



1. Derive the light-front Hamiltonian from the QCD Lagrangian with background field in the $|q\rangle$ space

$$\mathcal{L}_q = \bar{\Psi}(i\gamma^\mu \boldsymbol{D}_\mu - \boldsymbol{m})\Psi,$$

where $D_\mu = \partial_\mu + ig\mathcal{A}_\mu.$

$$P^-(x^+) = \underbrace{P^-_{QCD}}_{\text{full QCD}} + \underbrace{V(x^+)}_{\text{background field}}$$

Methodology : time-dependent Basis Light-front Quantization (tBLFQ)

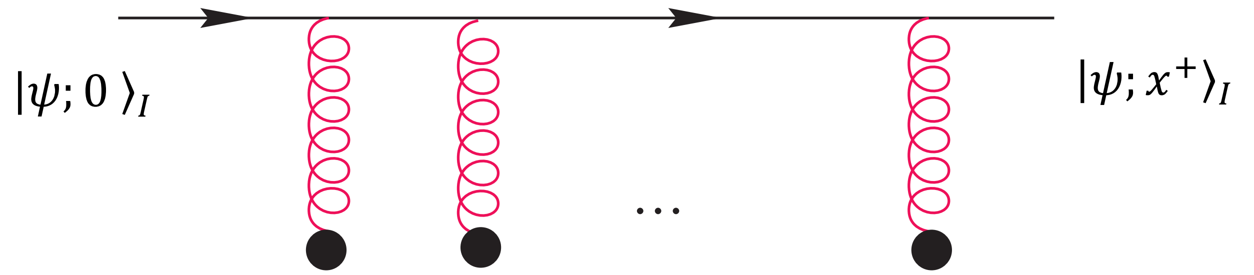
2. Solve the time-evolution equation

$$\frac{1}{2}P^-(x^+)|\psi;x^+\rangle = i\frac{\partial}{\partial x^+}|\psi;x^+\rangle$$

in the interaction picture

- states: $\left|\psi;x^+\right\rangle_I = e^{\frac{i}{2}P^-_{\text{QCD}}x^+}\left|\psi;x^+\right\rangle$
- interaction: $V_I(x^+) = e^{\frac{i}{2}P^-_{\text{QCD}}x^+}V(x^+)e^{-\frac{i}{2}P^-_{\text{QCD}}x^+}$
- evolution: $\left|\psi;x^+\right\rangle_I = \mathcal{T}_+ e^{-\frac{i}{2}\int_0^{x^+}\mathrm{d}z^+ V_I(z^+)}\left|\psi;0\right\rangle_I$

$$\rightarrow \left[1-\tfrac{i}{2}V_I(x_n^+=x^+)\delta x^+\right]\cdots\left[1-\tfrac{i}{2}V_I(x_1^+=0)\delta x^+\right]|\psi;0\rangle_I$$



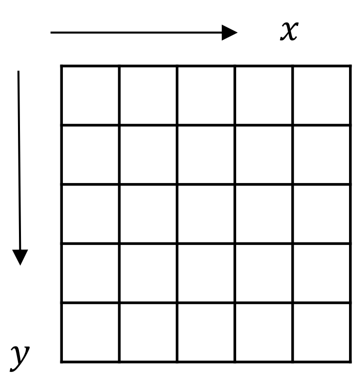
Methodology : time-dependent Basis Light-front Quantization (tBLFQ)

3. Construct basis states $|\psi; x^+\rangle_I = \sum_\beta c_\beta(x^+)|\beta\rangle$

An optimal basis should preserve the symmetries of the system and approximate the eigenfunctions of the Hamiltonian.

Here we choose the momentum states of the quark as the basis states

$$P^-_{\mathrm{QCD}}|\beta\rangle = P^-_\beta|\beta\rangle.$$



- Discretization of the transverse space:
  - Transverse grid range: [-L,L]
  - Grid index: [-N,-N+1,…,N-1]
  - Grid interval: $a = \frac{L}{N}$

4. Evaluate observables $O = \langle\psi; x^+|\hat{O}|\psi; x^+\rangle$

e.g. cross section, space distribution

Gluon field as the color glass condensate (CGC)

The background field from the nucleus, $\mathcal{A}(x^+, \vec{x}_\perp)$, is a classical gluon field described by the color glass condensate model[1]

Color charges are stochastic variables with correlations

$$< \rho_a(x^+, \vec{x}_\perp)\rho_b(y^+, \vec{y}_\perp) > = g^2\mu^2\delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)$$

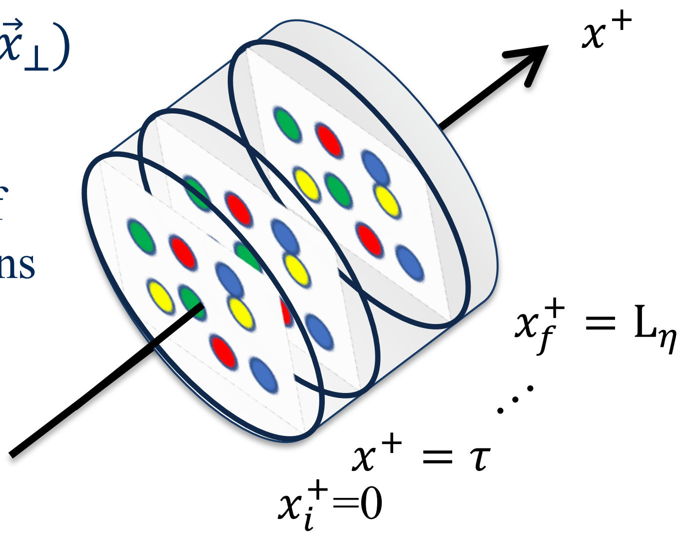
The color field is solved from

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+, \vec{x}_\perp) = \rho_a\,(x^+, \vec{x}_\perp)$$

where m_g is a chosen infrared (IR) regulator.

The color sources of different layers are independent of each other, simulating the quarks from different nucleons of the heavy ion

- The duration of the interaction: $x^+ = [0, L_\eta]$
- Number of layers: N_η
- The duration of each layer: $\tau = L_\eta/N_\eta$



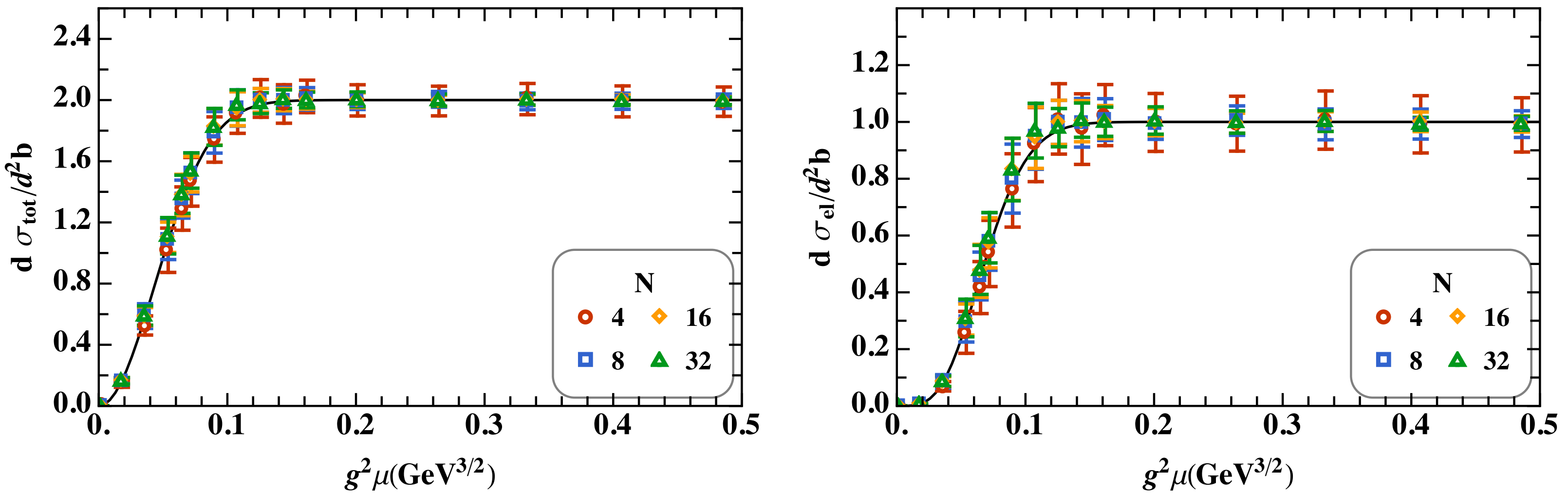
[1] L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D50, 2225 (1994).

Results: the cross sections

- ***Verify the eikonal limit:*** the total and elastic cross sections agree with the eikonal expectations (solid line)

$$\sigma_{tot} = < \int dp_f \left| {}_I\langle \psi(p_f); x_f^+ | \psi(p_i); x_i^+ \rangle_I - I \right|^2 >_{events}$$

$$\sigma_{el} = \int dp_f \left| < {}_I\langle \psi(p_f); x_f^+ | \psi(p_i); x_i^+ \rangle_I - I >_{events} \right|^2$$



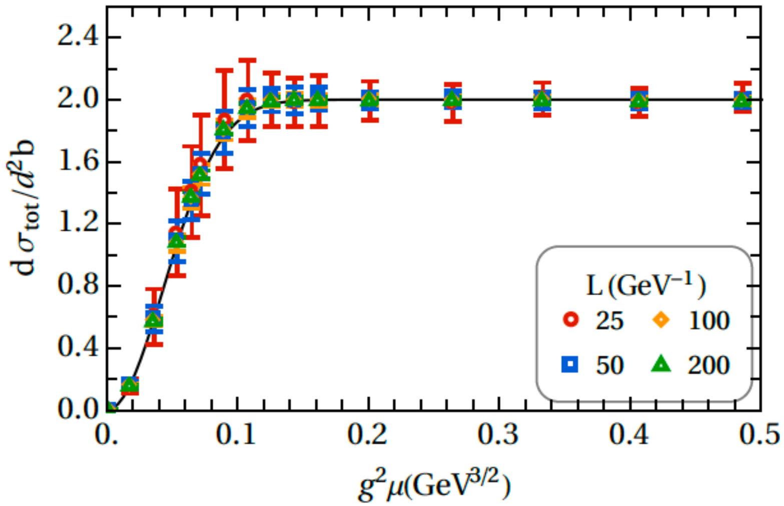
Parameter: total evolution time $L_\eta = 50\ GeV^{-1}$, $N_\eta = 4$; grid size $L = 50\ GeV^{-1}$; grid number N; $m_g = 0.1\ GeV$; $p^+ = \infty$; averaged over 100 configurations, the standard deviations are shown as uncertainties.

Results: the cross sections

Sensitivity on numerical parameters

- Grid size L

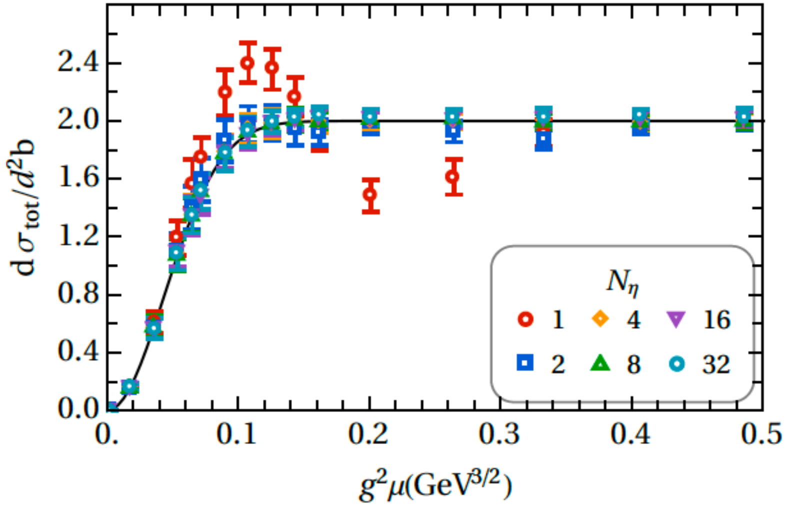
The cross section is not sensitive to the grid size L with a fixed lattice spacing



$(a = L/N = 6.25\ GeV^{-1})$

- Field layers N_η

Multiple layers of the CGC field is required to achieve the uncorrelated structure of the field along x^+



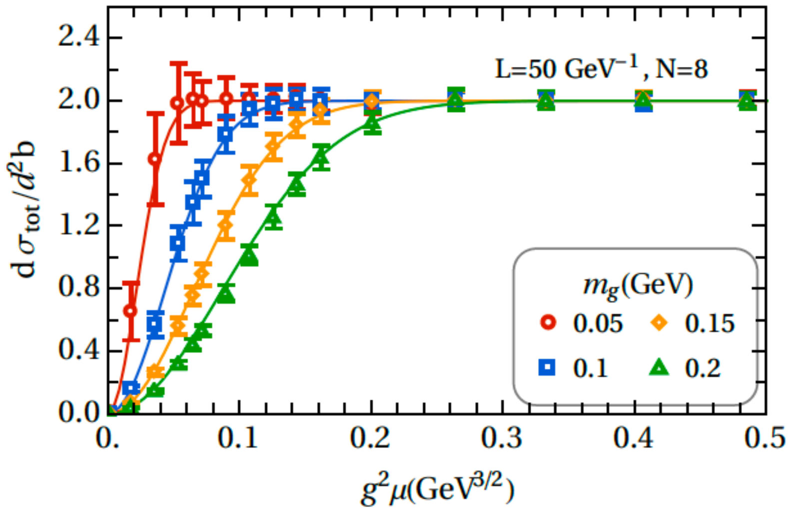
$(L = 50\ GeV^{-1},\ N = 8)$

Results: the cross sections

Sensitivity on numerical parameters

- Gluon mass/IR regulator m_g

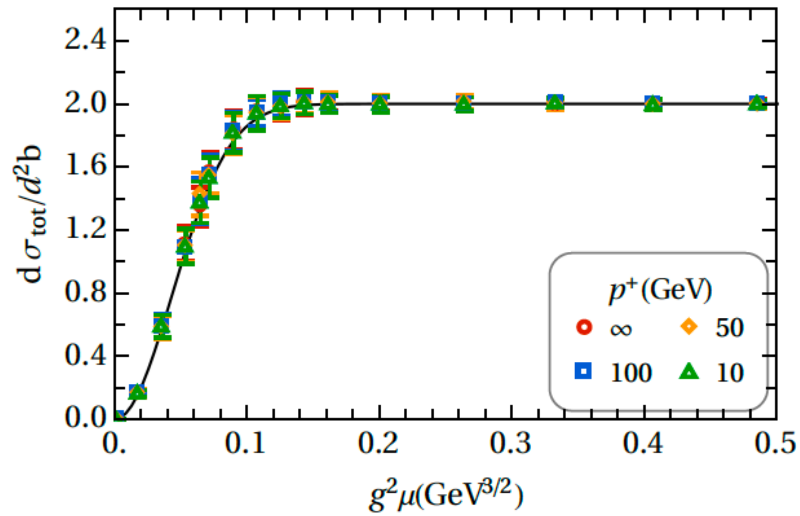
The CGC field with a smaller IR regulator shows a stronger effect on the quark



$(L = 50\ GeV^{-1}, N = 8)$

- Quark's p^+

Relaxing the eikonal condition $p^+ = \infty$ to finite values does not give noticeable effect on the cross section

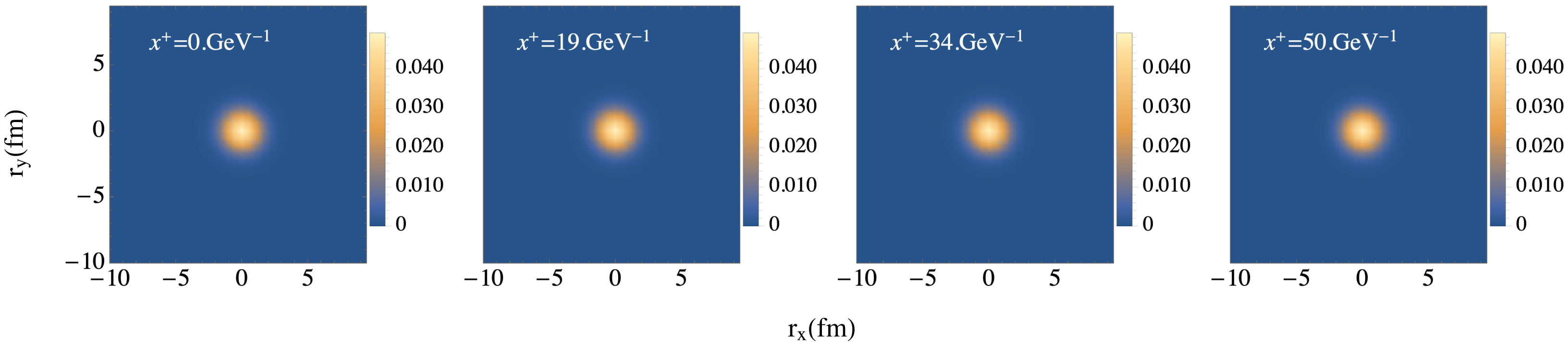


$(L = 50\ GeV^{-1}, N = 8)$

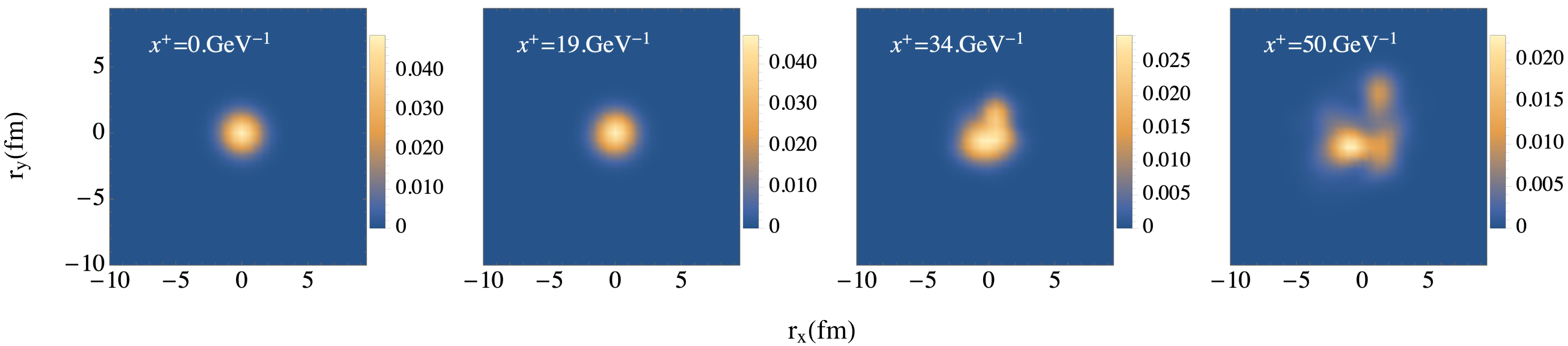
Results: evolution in the coordinate space

- ***Sub-eikonal effects*:** the transverse coordinate distribution of the quark changes over time at a finite energy scale

$p^+ = \infty$, no change in $\vec{r}_\perp$ distribution



$p^+ = 10$ GeV, $\vec{r}_\perp$ distribution changes (single event)



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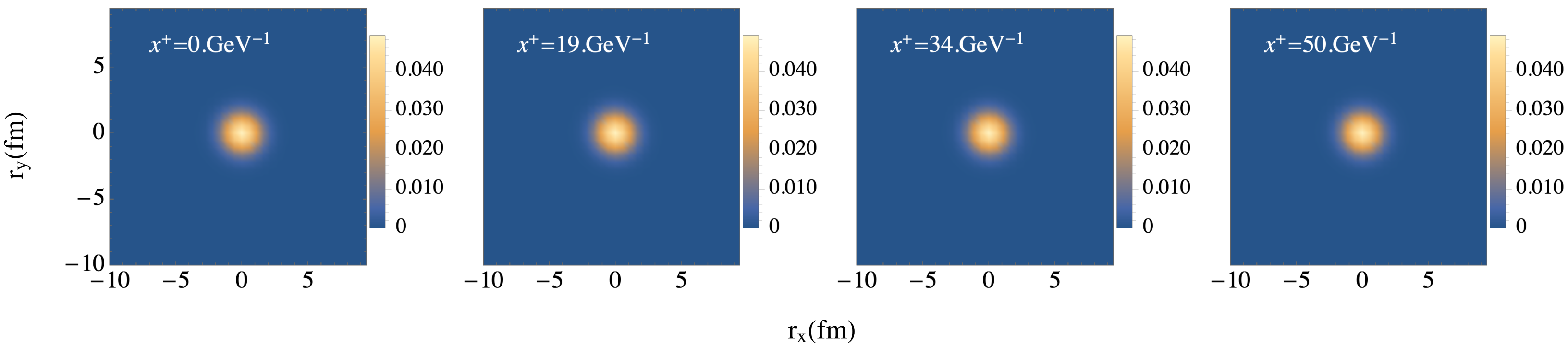
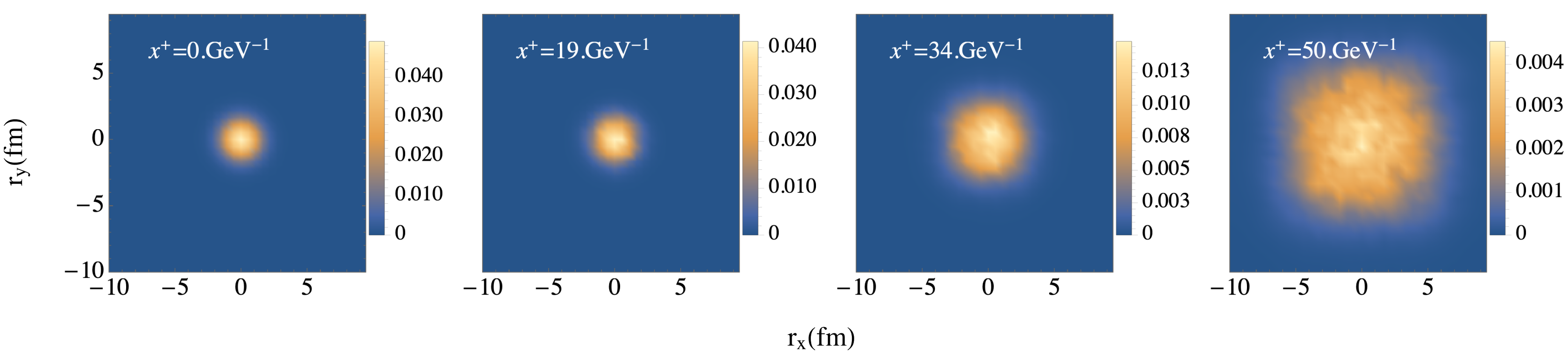


$p^+ = 10$ GeV, $\vec{r}_\perp$ distribution changes (50 events average)



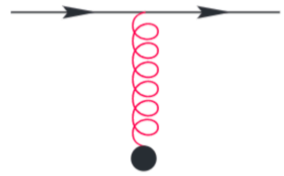
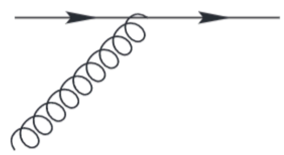
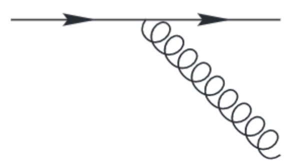
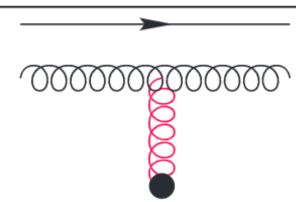
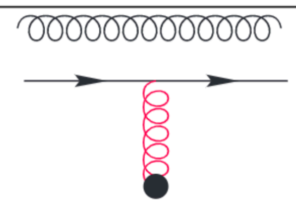
Work in progress

Quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space:

The QCD Lagrangian now contains the dynamical gluon

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

This adds new interesting aspects to our study, including gluon radiation/absorption, the change of quark's spin projection, and the effect on p^+.

Fock sector	$\vert q\rangle$	$\vert qg\rangle$
$\langle q\vert$		
$\langle qg\vert$		 

Preliminary result $|q\rangle + |qg\rangle$

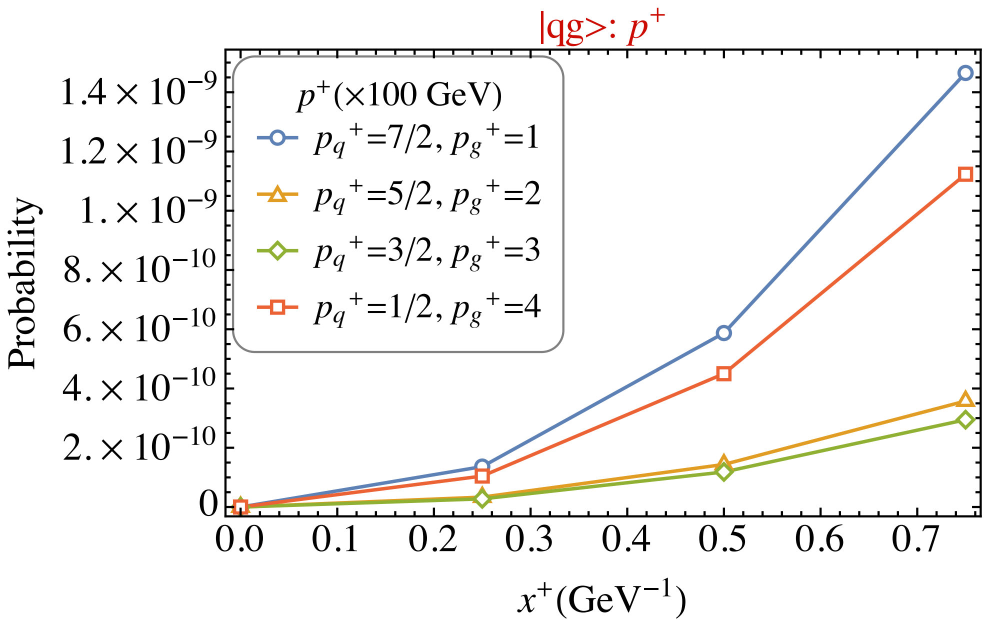
We use the simplest non-trivial grid: $L_\perp = 50\ \text{GeV}^{-1}$, $N_\perp = 2$ (4 by 4 transverse grid)

$L^- = \frac{2\pi}{100}\text{GeV}^{-1}, N^- = 4, \Delta p^+ = \frac{2\pi}{L^-} = 100\ \text{GeV}$ (4 p^+configurations for $|qg\rangle$)

Initial state: single quark with $p_\perp = 0$, $\boldsymbol{p^+ = \frac{9}{2} * 100\ \text{GeV}}, s = -\frac{1}{2}, c = 1, m_q = 0.001\ \text{GeV}$

Background field: $g^2\mu = 0.26\ \text{GeV}^{3/2}$

Evolution time: $1\ \text{GeV}^{-1}$, with 80 time step



Different longitudinal momentum configurations appear in the $|qg\rangle$ sector.

Summary

1. We study the quark-nucleus scattering with a non-perturbative Hamiltonian approach, and used the single quark sector $|q\rangle$ to carry out the calculations as an initial investigation
2. The total and differential cross sections are in good agreement with the analytical expectations under the eikonal condition
3. We use the intermediate states to analyze the time-dependent process
4. We observe the sub-eikonal effect that the transverse location of the quark changes
5. By expanding the Fock space to $|q\rangle + |qg\rangle$, we aim to study gluon emission and absorption during the collision process

M. Li, X. Zhao, P. Maris, G. Chen, Y. Li, K. Tuchin, and J. P. Vary, Phys. Rev. D101, 076016 (2020)

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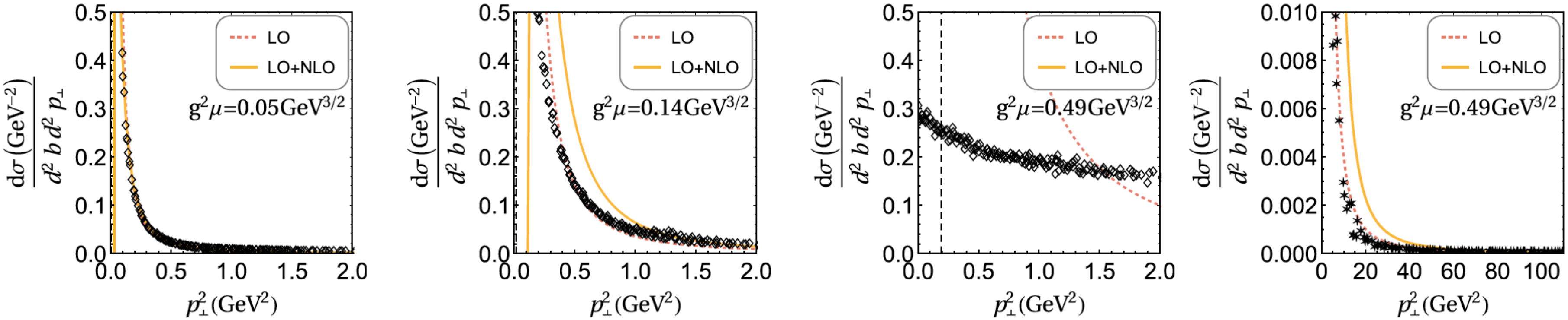
Thank you!

M. Li, X. Zhao, P. Maris, G. Chen, Y. Li, K. Tuchin, and J. P. Vary, Phys. Rev. D101, 076016 (2020)

Backup: differential cross section

The differential cross section $d\sigma/(d^2bd^2p_\perp)$ of qA could be convoluted with the quark distribution function of the proton at the factorization scale to study the pA cross section.



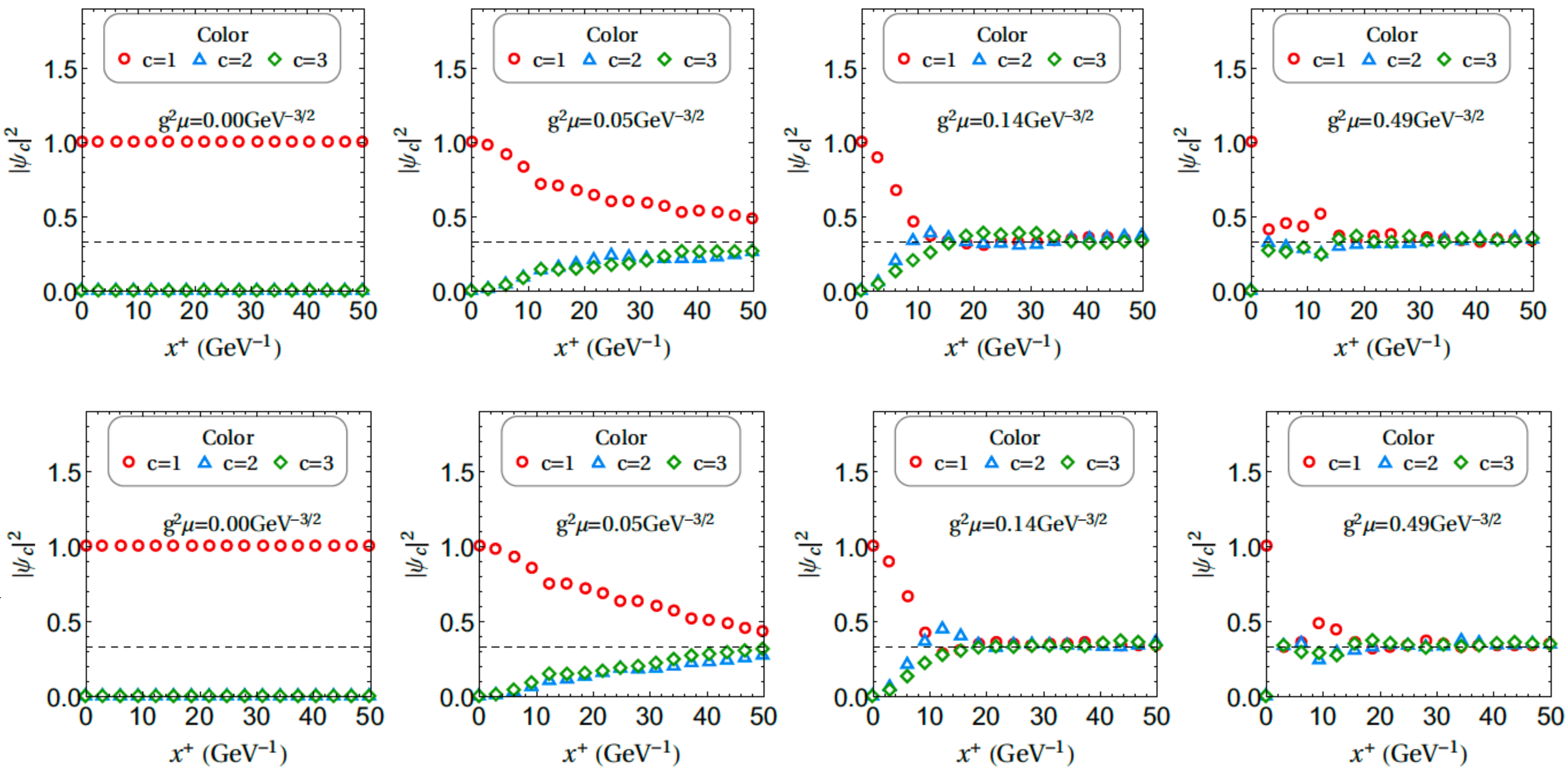
LO (NLO) is the leading (next-to leading) order expansion on $Q_s^2/p_\perp^2$, where $Q_s^2 = (g^2\mu)^2 L_\eta/(2\pi^2)$ is the dashed line. This perturbative expansion works for large $p_\perp$. The black markers are full results from tBLFQ.

Backup: evolution in the color space

In the color space, the quark evolves towards a uniformly distributed state, $|\psi_c|^2 = 1/3$, (c = 1, 2, 3)

$p^+ = \infty$

$p^+ = 10$ GeV



Initial state: single color state