

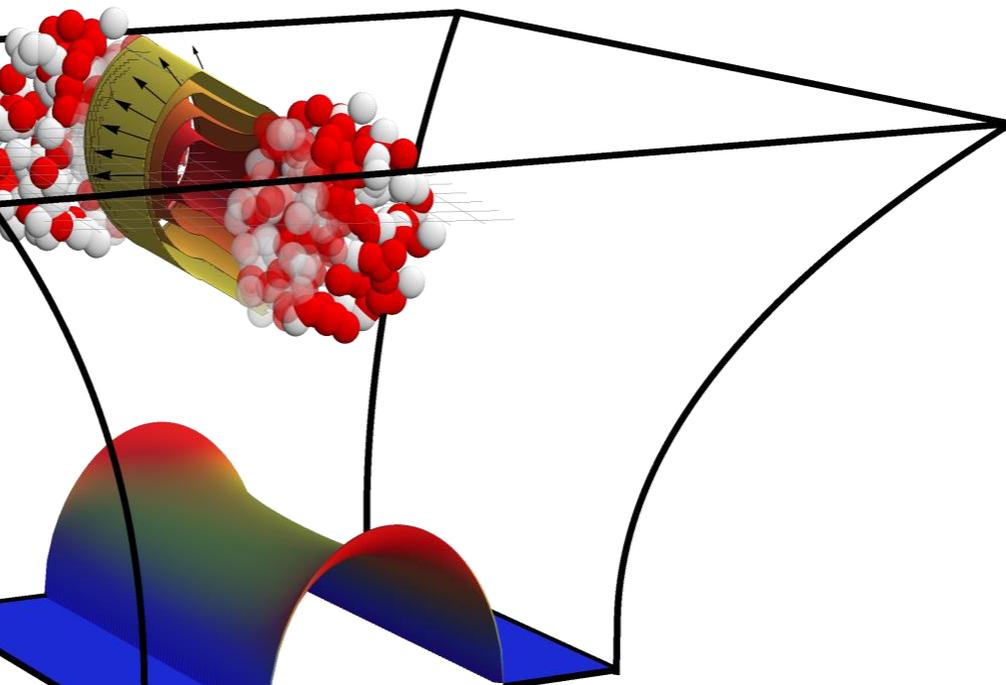


WHAT ATTRACTS TO ATTRACTORS?

FROM FAR-FROM-EQUILIBRIUM TO HYDRODYNAMICS

Based on work with Aleksi Kurkela, Urs Wiedemann and Bin Wu

Reference: 1907.08101 (PRL 2020)



Wilke van der Schee
Hard Probes 2020
Austin, Texas, 3 June 2020

HYDRO AND QGP

Exploring the limits of hydrodynamics

BIG QUESTIONS

1. How does QGP form and hydrodynamize within 1 fm/c? What are the qualitative differences, if any, between the description of hydrodynamization in a heavy ion collision obtained by assuming a weakly coupled initial stage versus a strongly coupled holographic calculation? Note that perturbative calculations typically treat $\alpha_s = 0.3$ as small while holographic calculations treat the corresponding 't Hooft coupling $\lambda \approx 11$ as large. What can we learn about the timescales and dynamics of hydrodynamization, and hence QGP formation, by analyzing the wakes that jets leave behind as they traverse a droplet of QGP?
2. What are the limits of the applicability of hydrodynamics? Can it be applied even to systems of size a fermi or less? What is the smallest droplet of QGP that behaves hydrodynamically, and how does the answer to this question change at very high temperatures where $\eta/s > 1$ and QGP is no longer a strongly coupled liquid?

THE BIG QUESTIONS

When is hydrodynamics applicable?

- And what kind of hydrodynamics? (1st, 2nd or infinite order?)
- Also: dependent on position

What is the initial fluid profile at this time?

- Presumably depending (strongly) on initial stage model
 - Strong versus weak? Constituent `quarks`?
- Often argued: memory loss due to attractor
 - But is this really so?
 - → dynamics is not necessarily on the attractor
 - → even on attractor: lot of 3+1D freedom left

HYDRODYNAMICS AND ATTRACTORS

Solve Muller-Israel-Stewart or Boltzmann equation

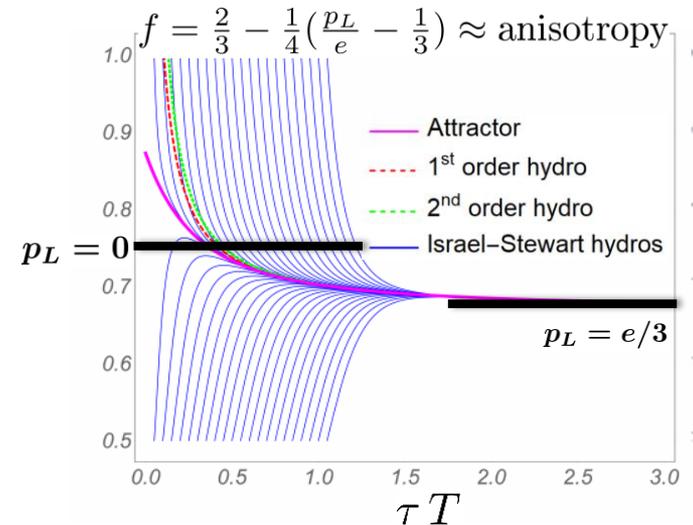
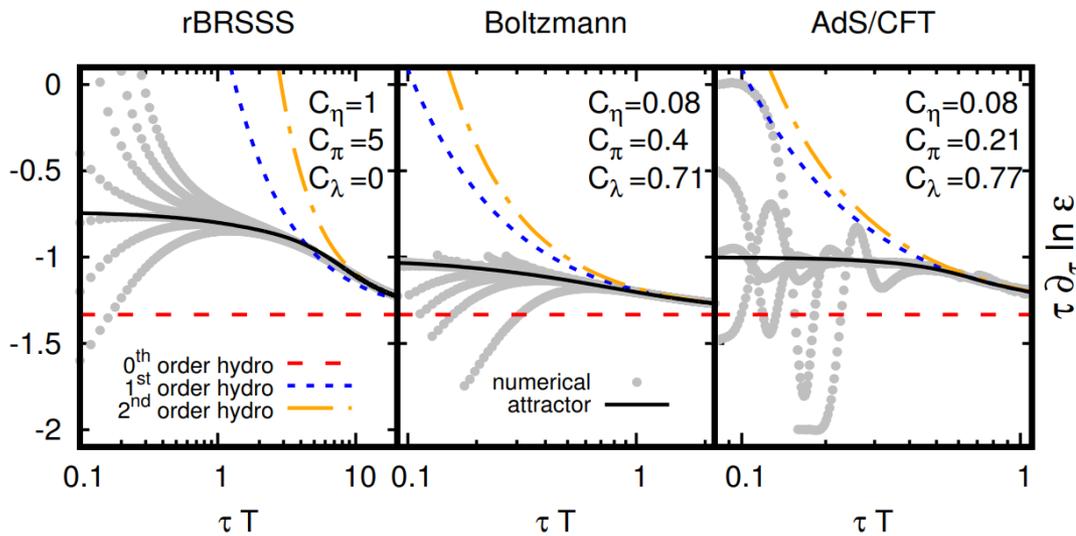
Hydro:

$$\partial_\tau \varepsilon = -\frac{1}{\tau} [\varepsilon + p_L]$$

$$\partial_\tau \phi + \frac{4}{3} \frac{\phi}{\tau} = -\frac{1}{\tau_R} \left[\phi - \frac{4}{3} \frac{\eta}{\tau} \right], \quad \phi \equiv \frac{1}{3} \varepsilon - p_L$$

RTA:

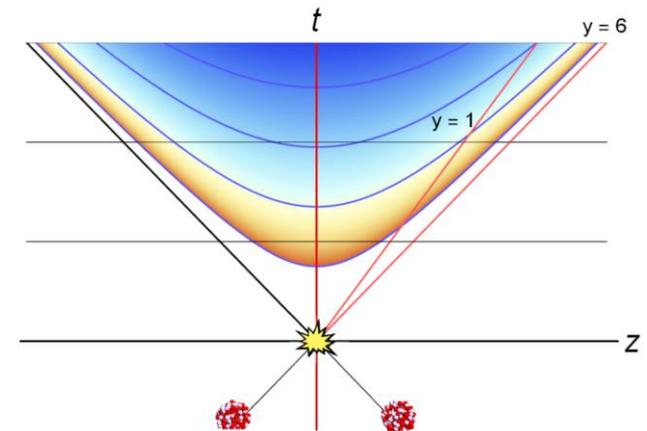
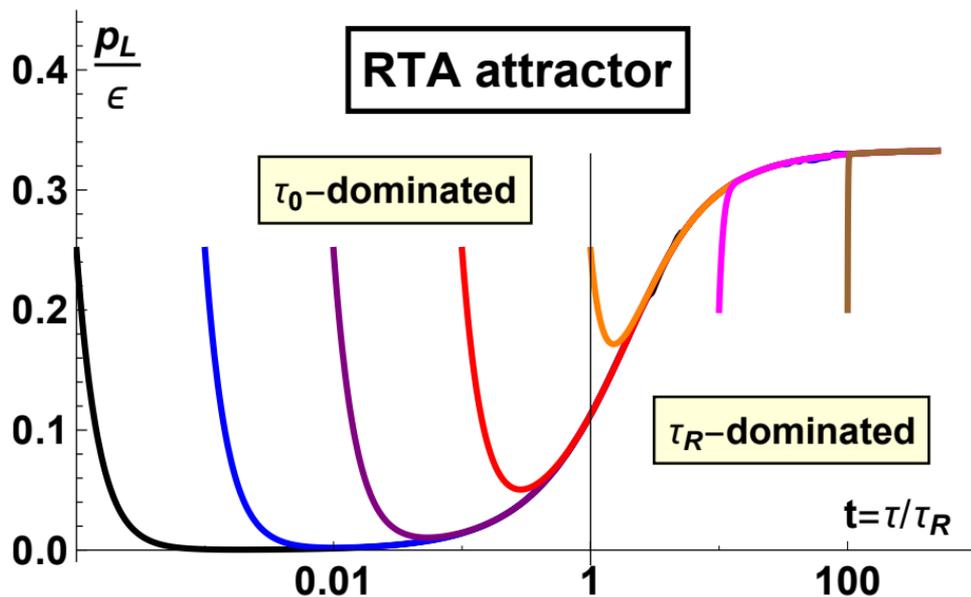
$$\partial_\tau f + \vec{v}_\perp \cdot \partial_{\vec{x}_\perp} f - \frac{p_z}{\tau} \partial_{p_z} f = -\frac{(-v_\mu u^\mu)}{\tau_R} [f - f_{eq}]$$



EARLY TIME DYNAMICS IN RTA

Depends crucially on initialization time:

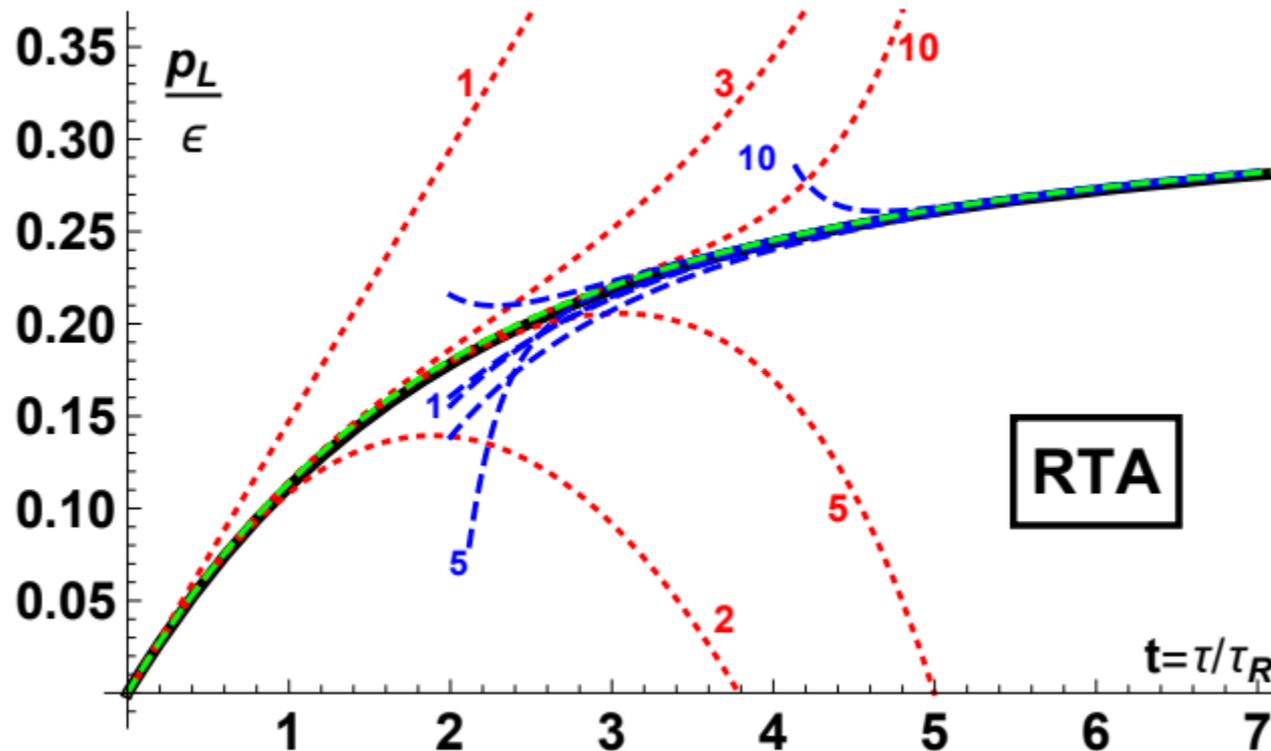
- Decays to attractor solution in a time of order τ_0
 - Expansion 'selects' particles with zero longitudinal momentum
- Curves get steeper once $\tau_0 > \tau_R$ (on log scale)



EARLY TIME EXPANSION

Expand attractor around early and late times:

- Starts at 0 (free streaming), grows (single hit), converges to isotropy
- Green-dashed: Pade approximant of early time expansion
- Converges for all times

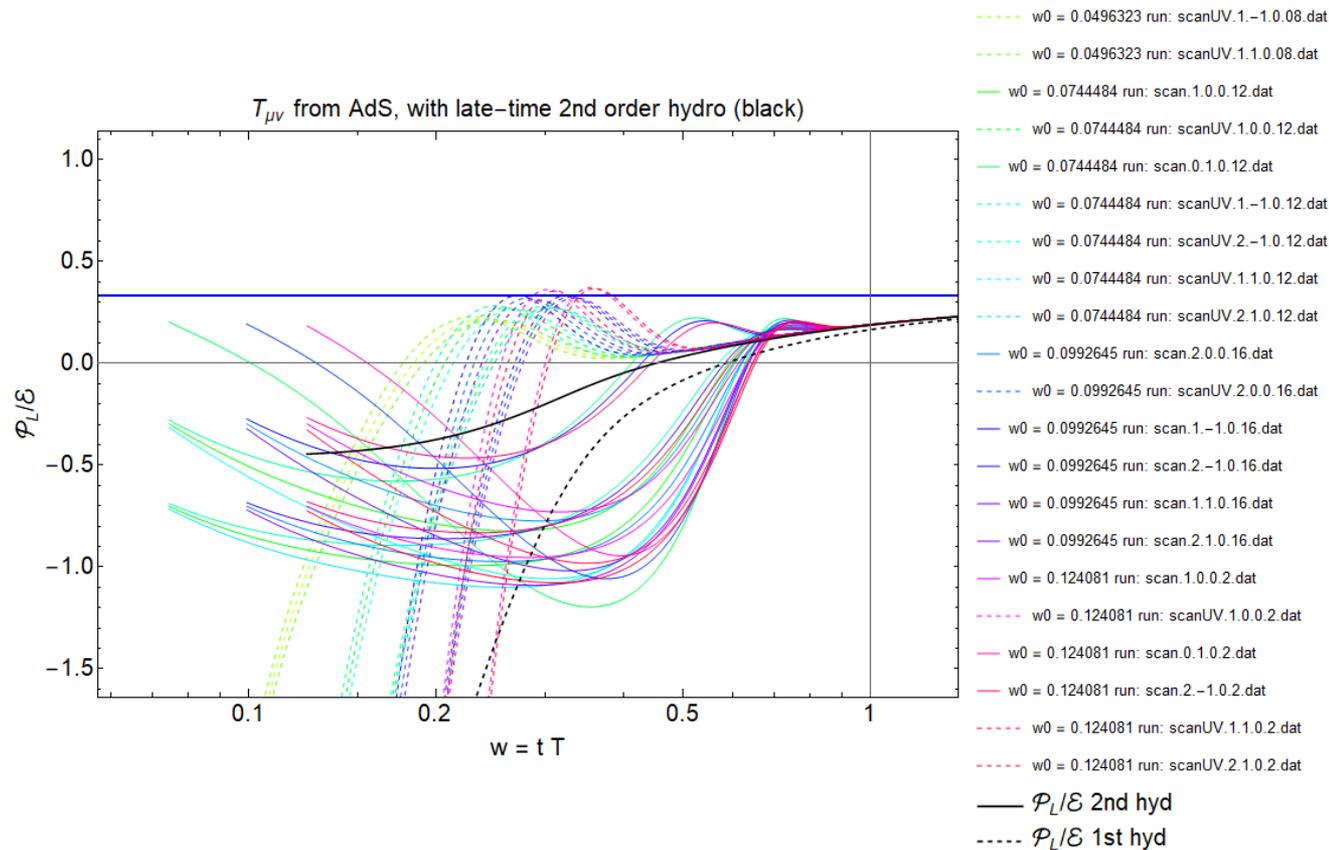


ATTRACTOR IN ADS/CFT REVISITED

Try a wide range of initial profiles

- IR profile: $B_{IR}(\rho, \tau_0) = \rho^4(a + b\rho)$
- UV profile: $B_{UV}(\rho, \tau_0) = \rho^4 e^{-40\rho}(a + b\rho)$

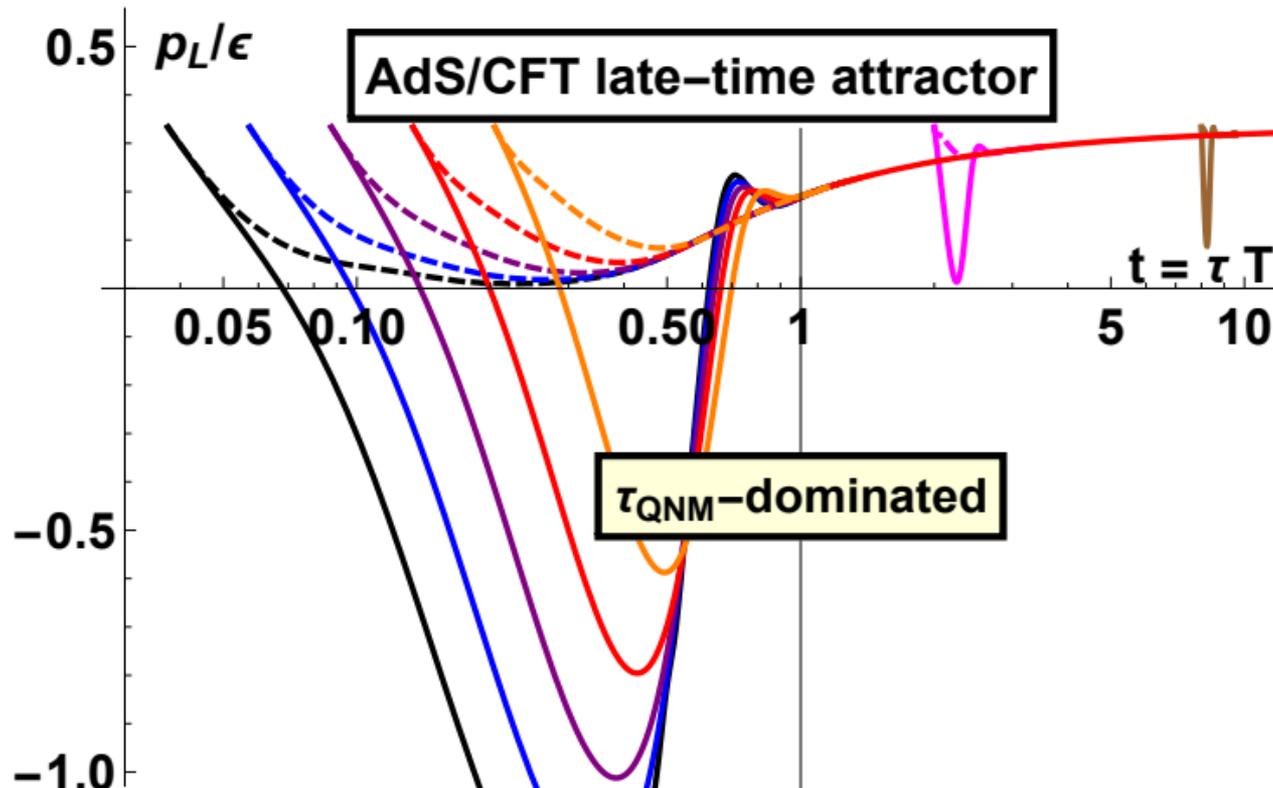
Could in principle follow from a real model, e.g. colliding sheets



ATTRACTOR IN ADS/CFT REVISITED

Start at seven different initial times

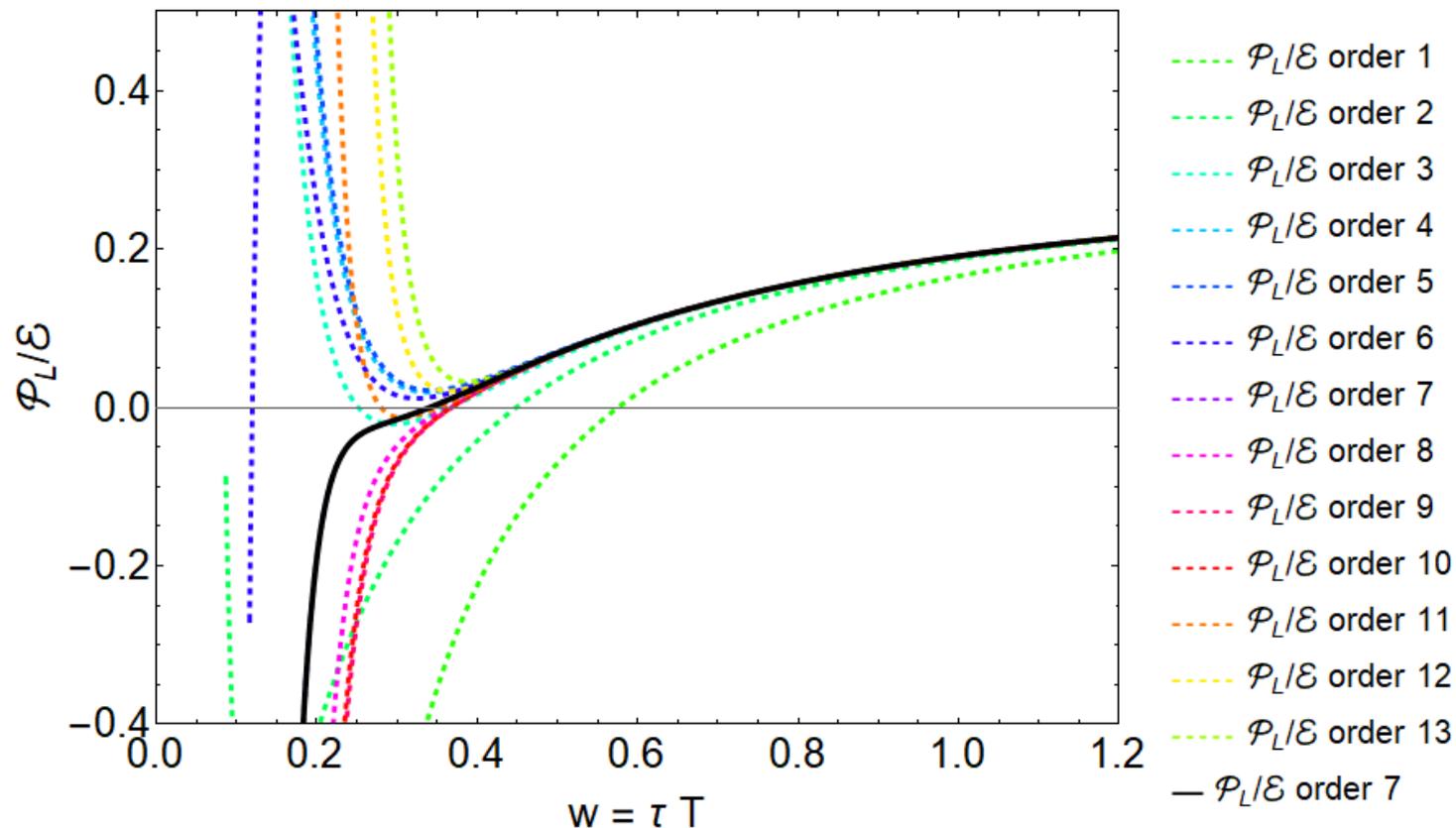
- UV + IR profiles, fixing initial isotropy + derivative
- Can distinguish a 'profile-dependent' attractor (?)



APPROACH TO HYDRODYNAMICS

Many order hydrodynamics in Bjorken strongly coupled $\mathcal{N} = 4$

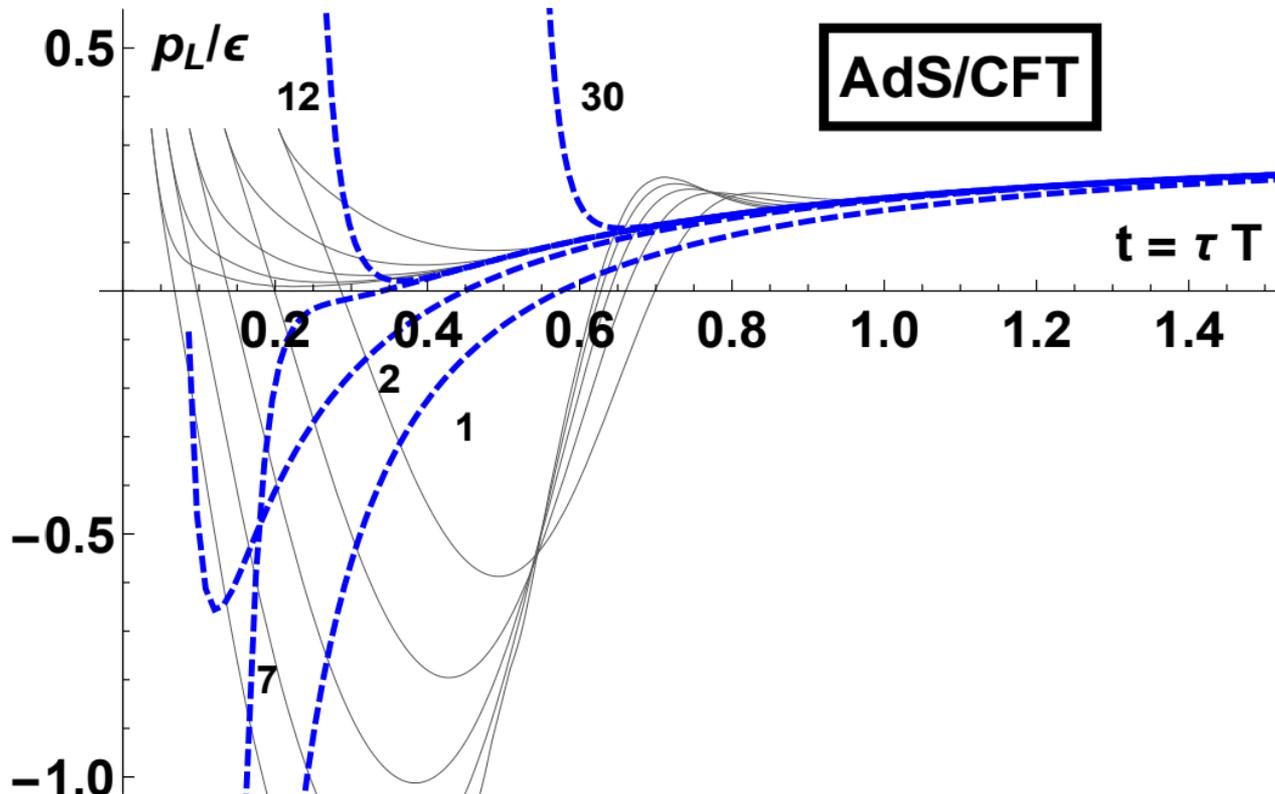
- Order 7 seems optimal (?)



APPROACH TO HYDRODYNAMICS

Same AdS/CFT evolutions as before

- Quite curious: UV agree with 7th order at $\sim t = 0.4$
- For general profiles going beyond 2nd order does not improve
- Even more curious: is there an early time attractor with free streaming? But why? (see also 1704.08699)



ATTRACTOR SUMMARY

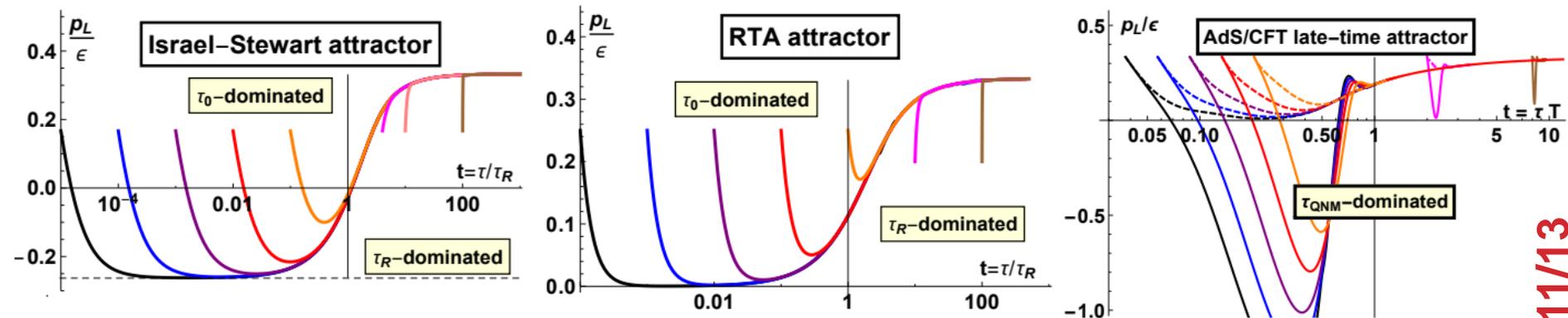
Israel-Stewart and RTA:

- Decay to attractor on time scale τ_0
- RTA: expansion dominated: free streaming ($\sim p_L = 0$)

Strong coupling

- Decay to attractor on time scale $1/T$
- Initial dynamics determined by initial condition (IC)
 - 'UV' profile converges faster

Attractor itself dominated by interaction/transport (τ_R)



WHY DOES ADS/CFT DECAY 'SLOWLY'?

UV and IR profiles contain higher-point correlation functions

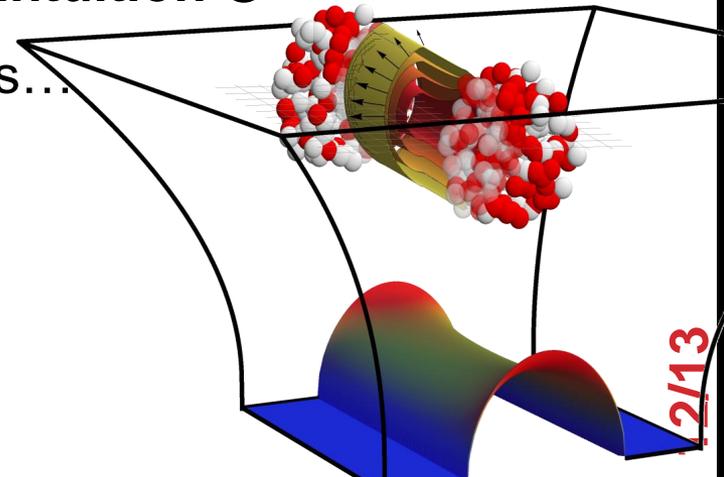
- Impossible to put correlations on lengths $\gg 1/T$ (because of horizon)
- IR: strong correlation on scale $1/T$
- UV: constructed to have only small-scale correlations

Causality: system cannot relax before a time $1/T$ has passed (IR)

- RTA can relax much faster

Somewhat against weak/strong coupling intuition ☺

- But perhaps it's all about initial conditions...



THE BIG QUESTIONS

When is hydrodynamics applicable?

- RTA: within $2/\tau_0$, strong coupling: within $1/T$
- And what kind of hydrodynamics? (1st, 2nd or infinite order?)
- RTA: attractor (τ_R), strong coupling: 2nd order, or 7th order in rare cases
- Also: dependent on position

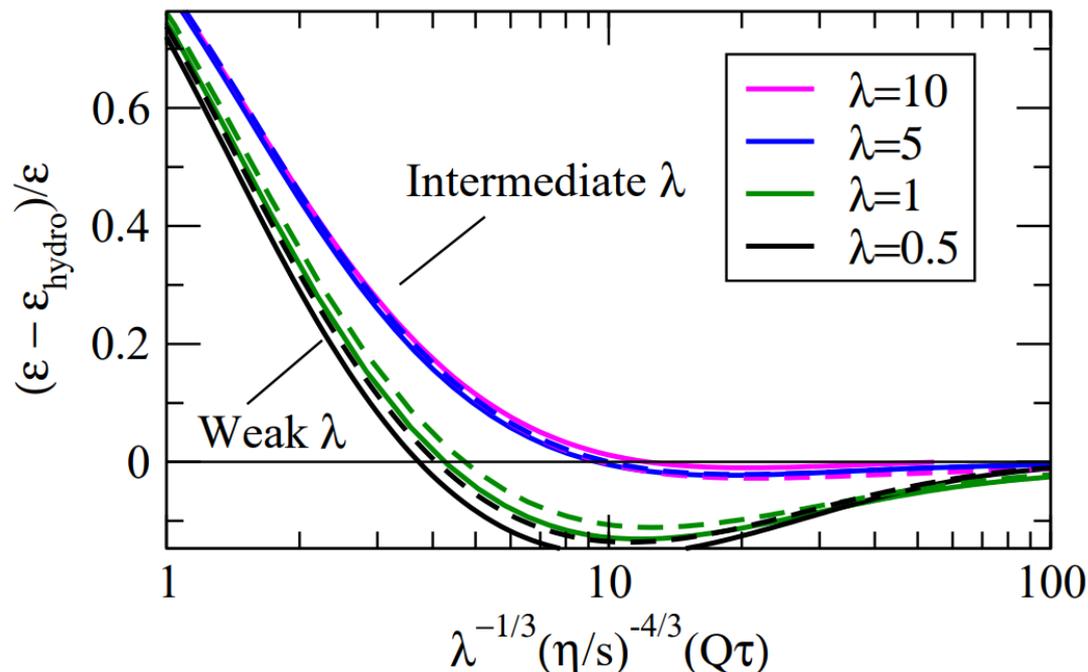
What is the initial fluid profile at this time?

- Presumably depending (strongly) on initial stage model
 - Strong coupling: evolution depending on higher-point correlations: may need refined knowledge initial stage (this work: arbitrary IC)
- Often argued: memory loss due to attractor
 - In the end everything goes to hydro (at least in 1+1D CFT)
 - RTA: IC at early enough τ_0 is truly lost (but note assumption of RTA)
 - Strong coupling: IC will influence final distribution of energy
 - *Conservation of misery remains true...*

WEAK COUPLING BEYOND RTA

Relaxation time approximation gives limited dynamics

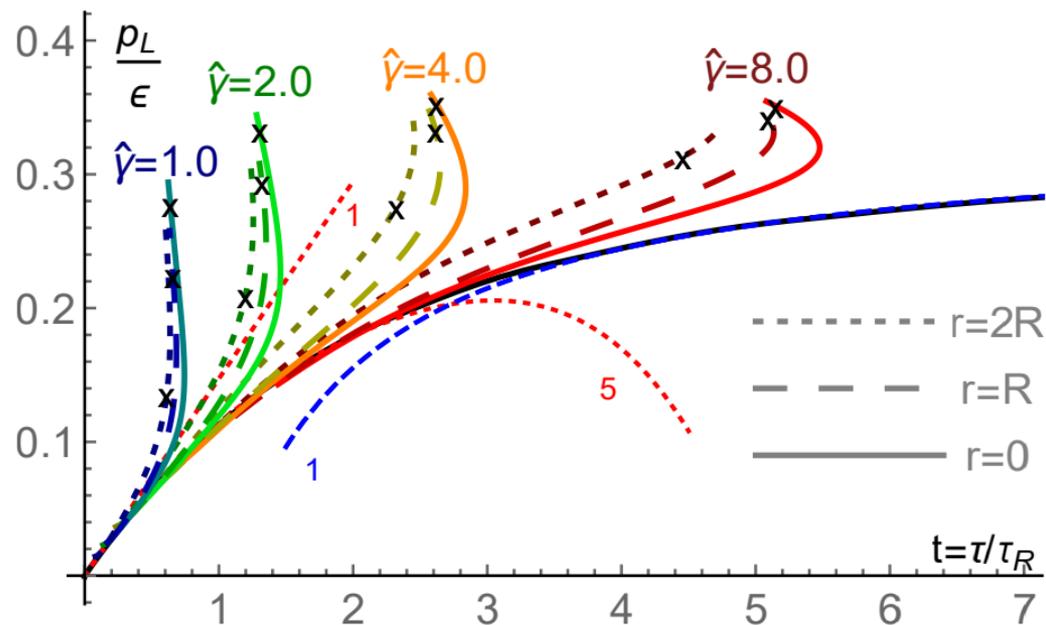
- All moments decay at same time: quick decay to attractor
- Different in full-fledged kinetic theory (YM):
(also showing bottom-up thermalization)



3+1 ADVERTISEMENT

Follow anisotropy across system sizes position in plane

- Relevant parameter: opacity: $\hat{\gamma} = \gamma R^{3/4} (\epsilon_0 \tau_0)^{1/4}$
- Depending on coupling strength γ
- Crosses indicate $t = 2R$, after which system decouples



ADS/CFT ON A LINEAR SCALE

Some dynamics is clearer on a linear scale:

