Dipole model at Next-to-Leading Order meets HERA data

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In collaboration with
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arXiv:2006.xxxxx

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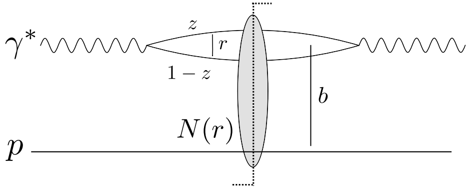
DIS in the Dipole Picture at leading order



Leading order $\gamma^* - p$ scattering

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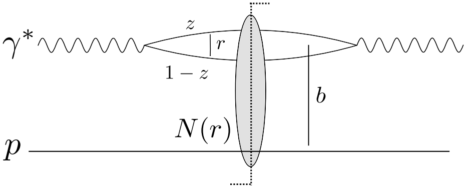

Leading order $\gamma^* - p$ scattering

In Dipole Picture at Leading Order $\gamma^* p$ cross section using *optical theorem*:

$$\sigma^{\mathrm{LO}}_{L,T}(x_{Bj}, Q^2) \sim 4 N_{\mathrm{c}} \alpha_{em} \sum_f e_f^2 \int_0^1 \mathrm{d} z_1 \int_{\mathbf{x}_0, \mathbf{x}_1} \left| \Psi_{\gamma^*_{L,T} \to q\bar{q}} \right|^2 N(\mathbf{x}_{01}),$$

$$1 - N(\mathbf{x}_{01}) \equiv S_{01} := 1/N_{\mathrm{c}} \left\langle \mathrm{Tr}\, U(\mathbf{x}_0) U^\dagger(\mathbf{x}_1) \right\rangle_x$$

$$U = \text{Wilson line}$$

Target evolution: BK equation

Target evolution is described approximatively[1] by the Balitsky-Kovchegov (BK) equation:

$$\partial_y \langle S_{01} \rangle_y = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2 \mathbf{x}_2 \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{02}^2 \mathbf{x}_{21}^2} \left[\langle S_{02} \rangle_y \langle S_{21} \rangle_y - \langle S_{01} \rangle_y \right].$$

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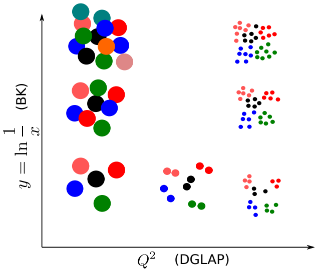

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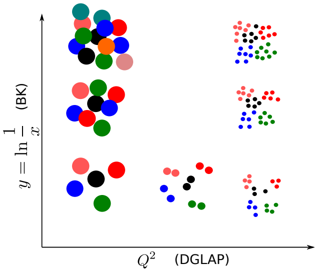

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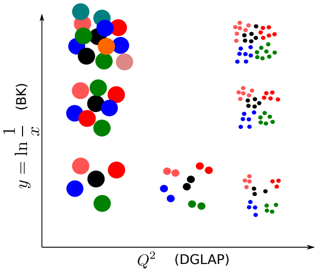

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- In a nutshell
  - Describe inclusive HERA data well
  - Simultaneous description of HERA heavy quark data not as good

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Beyond LO: Evolution in projectile rapidity Y

- Projectile rapidity $Y \sim \ln W^2$

[2] E. Iancu et al., Phys. Lett. B 744 (2015) 293
[3] G. Beuf, Phys. Rev. D 89 (2014) no. 7 074039

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Collinear resummation of large transverse logs leads to "ResumBK" [2]

$$\partial_Y S(\mathbf{x}_{01}, Y) = \int \mathrm{d}^2\mathbf{x}_2 K_{\mathrm{DLA}} K_{\mathrm{STL}} K_{\mathrm{BK}} [S(\mathbf{x}_{02}) S(\mathbf{x}_{21}) - S(\mathbf{x}_{01})].$$

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Another technique leads to a kinematic constraint (KCBK) and non-local equation [3]

$$\partial_Y S(\mathbf{x}_{01}, Y) = \int \mathrm{d}^2\mathbf{z} K_{\mathrm{BK}} \theta \left(Y - \Delta_{012} - Y_{0,\mathrm{if}}\right) \times [S(\mathbf{x}_{02}, Y - \Delta_{012}) S(\mathbf{x}_{21}, Y - \Delta_{012}) - S(\mathbf{x}_{01}, Y)]$$

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Beyond LO: Evolution in target rapidity η

Recent study[4] argues that evolution should be expressed in $\eta \sim \ln \frac{1}{x_{Bj}}$:

$$\partial_\eta \bar{S}(\mathbf{x}_{01}, \eta) = \int \mathrm{d}^2\mathbf{x}_2 K_{\mathrm{BK}} \theta(\eta - \eta_0 - \delta)[\bar{S}(\mathbf{x}_{02}, \eta - \delta_{02})\bar{S}(\mathbf{x}_{21}, \eta - \delta_{21}) - \bar{S}(\mathbf{x}_{01}, \eta)]$$

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- Evolution in η, DIS impact factors in Y: need shift $\eta = Y - \rho$
  - $\rho \equiv \ln \frac{1}{\min\{1, \mathbf{x}_{ij}{}^2 Q_0^2\}}$
- LO DIS fits done to HERA data with good results[5].

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NLO DIS cross section in the Dipole Picture

Next-to-Leading Order $\gamma^* p$ cross section can be partitioned as

$$\sigma^{\text{NLO}}_{L,T} = \sigma^{\text{IC}}_{L,T} + \sigma^{qg}_{L,T} + \sigma^{\text{dip}}_{L,T},$$

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$$\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{IC}} + \sigma_{L,T}^{qg} + \sigma_{L,T}^{\mathrm{dip}},$$

where the NLO contributions are[6] [7]:

$$\sigma_{L,T}^{qg} = 8 N_{\mathrm{c}} \alpha_{em} \frac{\alpha_s C_{\mathrm{F}}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{z_{2,\min}}^{1-z_1} \frac{\mathrm{d}z_2}{z_2} \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathcal{K}_{L,T}^{\mathrm{NLO}} \left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2\right),$$

$$\sigma_{L,T}^{\mathrm{dip}} = 4 N_{\mathrm{c}} \alpha_{em} \frac{\alpha_s C_{\mathrm{F}}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} \mathcal{K}_{L,T}^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1) \left[\frac{1}{2} \ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right],$$

z_2 = gluon momentum fraction. $z_{2,\min} = e^{Y_{0,\mathrm{if}}} x_{Bj} \frac{Q_0^2}{Q^2}$

N.B. Evolution range is controlled by $z_{2,\min}$ at NLO.

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Initial condition and fit schemes

- Resolve the transient effect[8] ($\sigma^{qg} \to 0$, $\sigma^{\text{dip}} \neq 0$) at $z_{2,\text{min}} \approx 1$ by setting $Y_{0,\text{if}} = 0$
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- Running coupling prescriptions
  - *smallest dipole*: Balitsky prescription in LOBK and smallest dipole elsewhere
    - Shortest lenght scale $\sim$ largest momentum scale dominates
  - *parent dipole*

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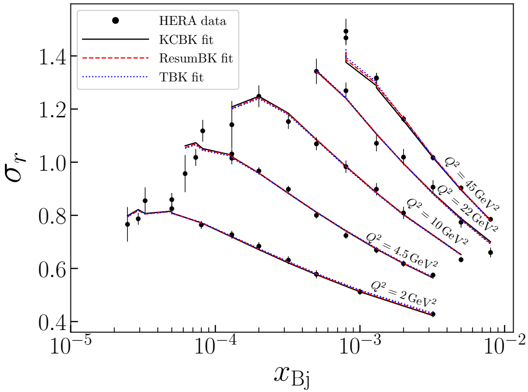
Fits to HERA data



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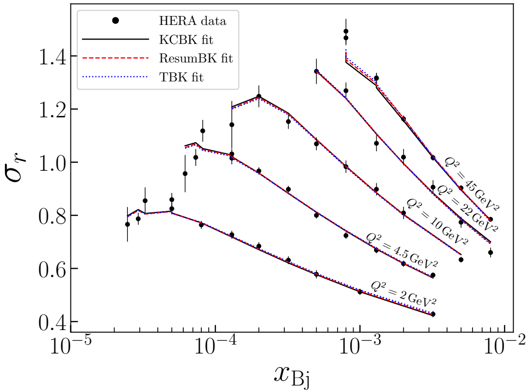

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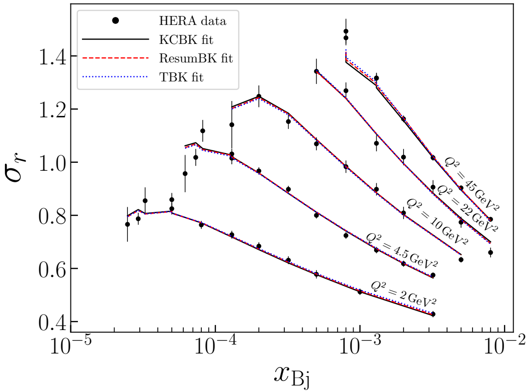

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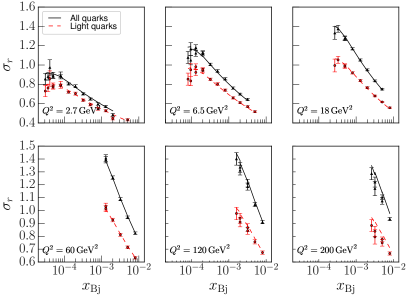
- Even combined HERA data cannot differentiate between BK equations and running coupling scheme choices
- Balitsky + smallest dipole prescription used overall slightly worse in χ^2/N

Subtracting heavy quarks from HERA data



The solid and dashed lines show the calculated cross sections from the IPsat fit that are used to generate the pseudodata.

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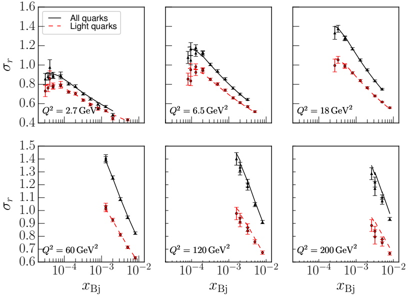

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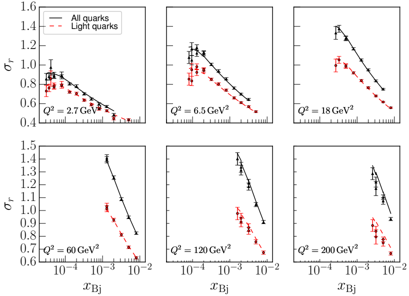

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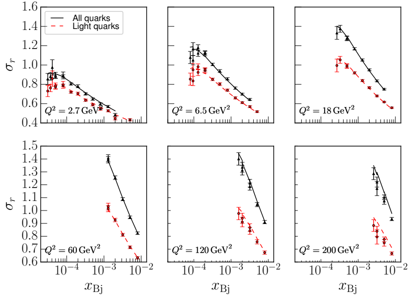

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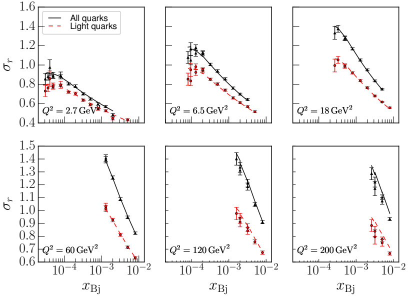

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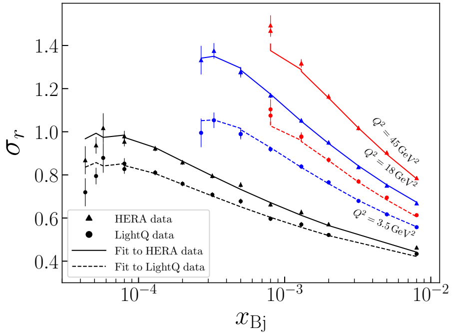
[a] H. Mäntysaari and P. Zurita, Phys.Rev.D **98** 036002 (2018)

Fits to light quark data



NLO CGC can fit light quark data as well.

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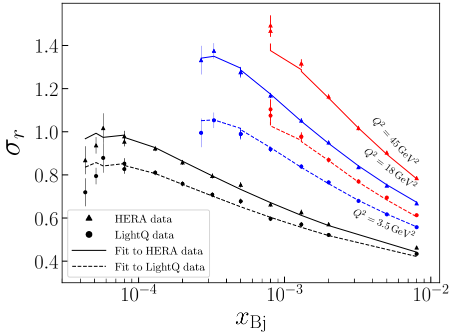

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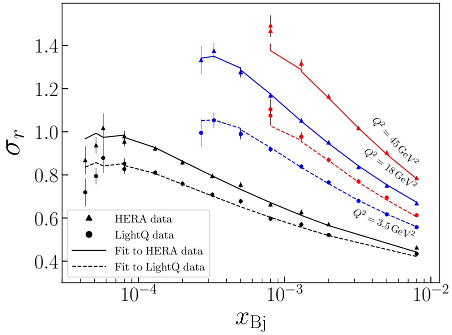

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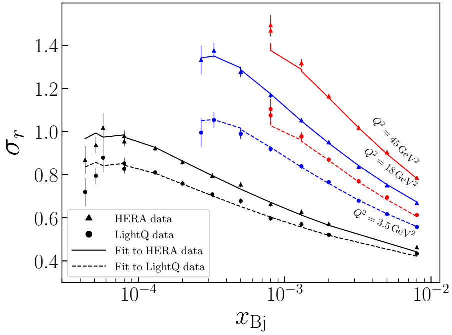

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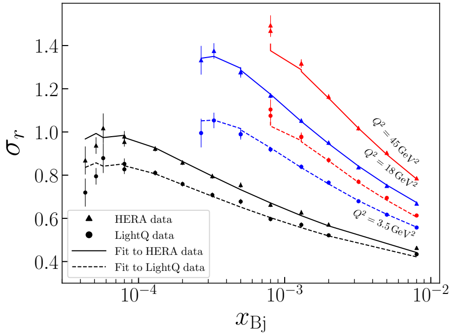

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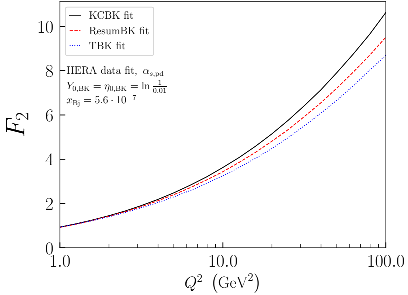
Data	α_s	χ^2/N	$Q^2_{s,0}$	C^2	γ	$\sigma_0/2$ [mb]
HERA	parent	1.85	0.0833	3.49	0.98	9.74
light-q	parent	1.58	0.0753	37.7	1.25	18.41

BK predictions at LHeC kinematics



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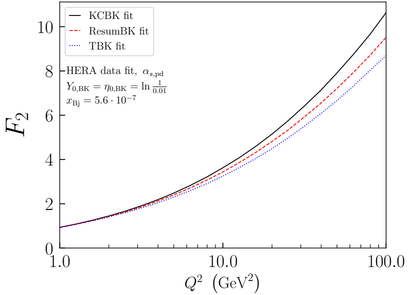

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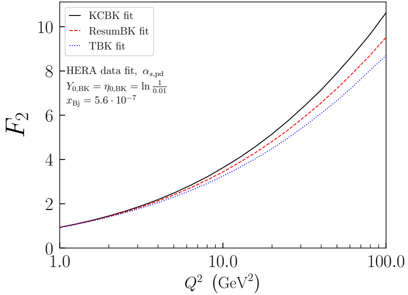

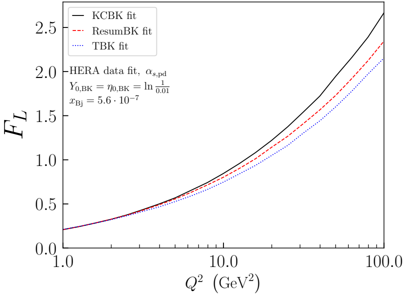

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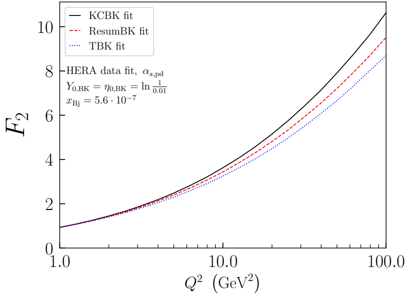

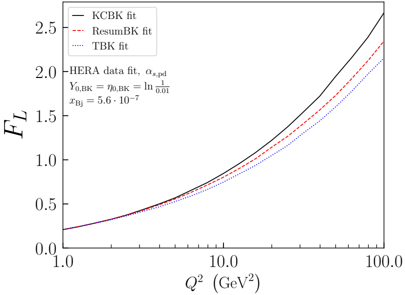

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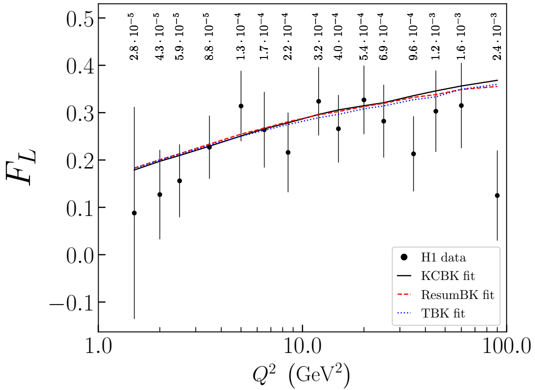
- Differences moderate even at LHeC kinematics

- Effect between Y and η evolution slightly enhanced
- F_L is sensitive to smaller dipoles

Fit comparison to H1 F_L data



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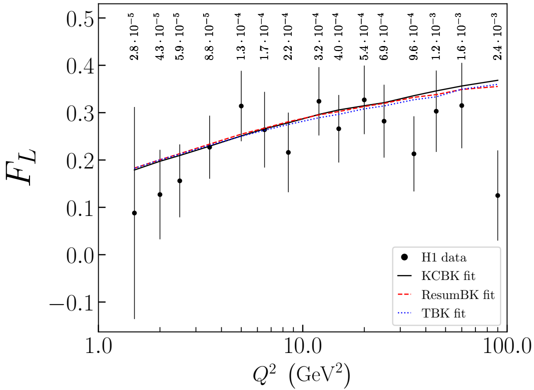

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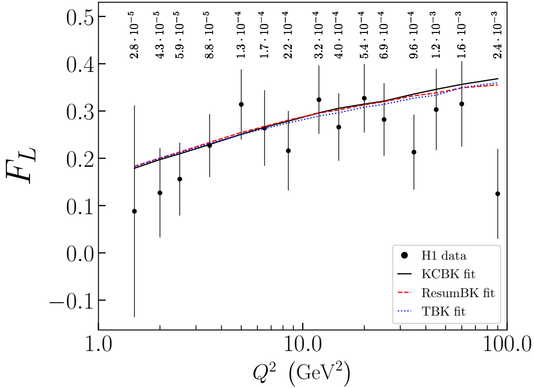

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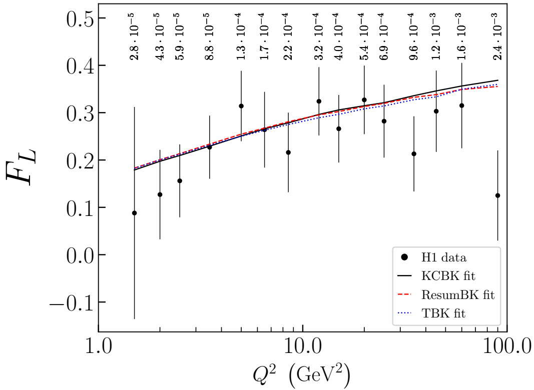

- F_L computed with HERA $\sigma_{\rm red}$ fits compared to H1 F_L data
- Fits describe F_L nicely
- KCBK, ResumBK and TBK equivalent
- Would start to see differences between evolutions at smaller x_{Bj}, moderately high Q^2

Conclusions

- NLO DIS cross section and small-x evolution: first NLO fits to HERA data
  - KCBK, ResumBK, and TBK all describe the combined HERA data well

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- Important test for CGC at NLO
- Would be preferrable to fit precise $F_{2,c}$
  - Will include massive quarks when NLO impact factors become available
- Precise F_2 and F_L data over a wide kinematical range in x and Q^2 can help to constrain the evolution

Thank you!

Backup slides

Fit results: KCBK

Data	α_s	$Y_{0,\mathrm{BK}}$	χ^2/N	$Q^2_{s,0}$	C^2	γ	$\sigma_0/2$ [mb]
HERA	parent	$\ln\frac{1}{0.01}$	1.85	0.0833	3.49	0.98	9.74
light-q	parent	$\ln\frac{1}{0.01}$	1.58	0.0753	37.7	1.25	18.41
HERA	parent	0	1.24	0.0680	79.9	1.21	18.39
light-q	parent	0	1.18	0.0664	1340	1.47	27.12
HERA	smallest	$\ln\frac{1}{0.01}$	1.89	0.0905	0.846	1.21	8.68
light-q	smallest	$\ln\frac{1}{0.01}$	2.63	0.0720	1.91	1.55	12.44
HERA	smallest	0	1.49	0.1114	0.846	1.94	8.53
light-q	smallest	0	1.69	0.1040	2.87	7.70	12.09

Fits to HERA and light quark data with Kinematically Constrained BK.

Fits: ResumBK

Data	α_s	$Y_{0,\mathrm{BK}}$	χ^2/N	$Q^2_{s,0}$	C^2	γ	$\sigma_0/2$ [mb]
HERA	parent	$\ln\frac{1}{0.01}$	2.24	0.0964	1.21	0.98	7.66
light-q	parent	$\ln\frac{1}{0.01}$	1.62	0.0755	11.7	1.24	16.53
HERA	parent	0	1.12	0.0721	89.5	1.37	19.68
light-q	parent	0	1.18	0.0794	1480	1.92	26.69
HERA	smallest	$\ln\frac{1}{0.01}$	2.37	0.0950	0.313	1.24	7.85
light-q	smallest	$\ln\frac{1}{0.01}$	2.21	0.0796	0.684	1.81	11.34
HERA	smallest	0	2.35	0.0530	0.486	1.56	10.10
light-q	smallest	0	3.19	0.0566	1.27	9.35	14.27

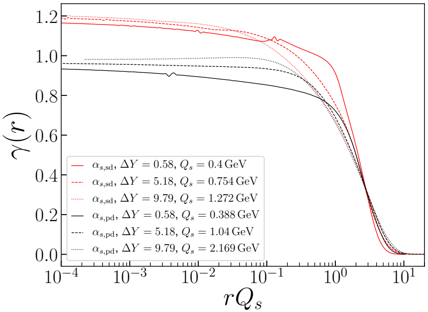
Fits to HERA and light quark data with Collinearly Resummed BK.

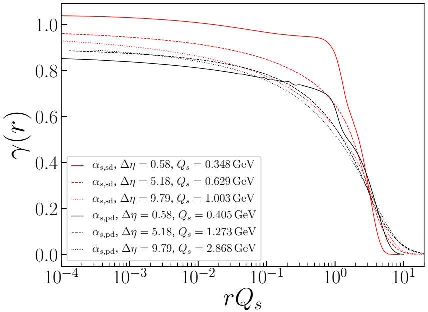
Fits: TBK

Data	α_s	$\eta_{0,\mathrm{BK}}$	χ^2/N	$Q^2_{s,0}$	C^2	γ	$\sigma_0/2$ [mb]
HERA	parent	$\ln \frac{1}{0.01}$	2.76	0.0917	0.641	0.90	6.19
light-q	parent	$\ln \frac{1}{0.01}$	1.61	0.0729	14.4	1.19	16.45
HERA	parent	0	1.03	0.0820	209	1.44	19.78
light-q	parent	0	1.26	0.0731	8050	1.86	29.84
HERA	smallest	$\ln \frac{1}{0.01}$	2.48	0.0678	1.23	1.13	10.43
light-q	smallest	$\ln \frac{1}{0.01}$	1.90	0.0537	3.55	1.59	16.85
HERA	smallest	0	2.77	0.0645	3.67	6.37	14.14
light-q	smallest	0	1.82	0.0690	822	8.35	29.26

Fits to HERA and light quark data with Target momentum fraction BK.

Evolution of anomalous dimension $\gamma(r) = \frac{\mathrm{d}\ln N(r)}{\mathrm{d}\ln r^2}$





- In Y, at $r \sim 1/Q_s$, parent dipole increases, smallest dipole decreases γ
- At asymptotically small dipoles γ fixed
- Evolved γ meet on a curve that fits the data

- In η, evolution at $r \sim 1/Q_s$ decreasing with either coupling
- Evolves towards asymptotic $\gamma \sim 0.6$ at large η