Finite N_c Corrections in NLO BK

using the Gaussian Truncation

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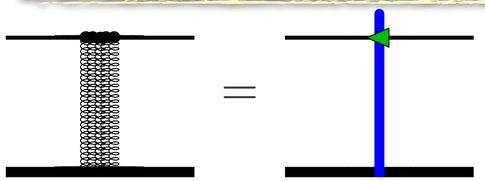
Introduction

- Context: Colour Glass Condensate
- Wilson lines are basic building blocks of calculations

Marquet, Weigert [Nucl.Phys. A843 (2010) 68-97]

Blaizot, Gelis, Venugopalan [Nucl.Phys. A743 (2004) 57-91]

$$U^\dagger_{\boldsymbol{x}} := P\exp\left\{ig\int_{x^+} \alpha^a_{\boldsymbol{x}}(x^+)t^a\right\} =$$



- They appear within expectation values (average over all possible target colour field configurations) e.g. DIS

$$\sigma_{\mathrm{DIS}}(Y,Q^2) = 2\int_{\boldsymbol{r}}\int_0^1 d\alpha \left|\Psi(\alpha,r^2,Q^2)\right|^2 \int_{\boldsymbol{b}} \left\langle 1 - S^{(2)}_{\boldsymbol{x},\boldsymbol{y}}\right\rangle_Y$$

$$S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} := \frac{1}{N_{\mathrm{c}}}\mathrm{tr}\left\{U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}}\right\}$$

Evolution Equations

Jalilian-Marian, Kovner, McLerran, Weigert [Phys.Rev. D55 (1997) 5414-5428],
Iancu, Leonidov, McLerran [Nucl.Phys. A692 (2001) 583-645]

- JIMWLK equation $\Longleftrightarrow$ Balitsky Hierarchy: evolution of correlators in rapidity
- Balitsky Hierarchy is infinite set of equations; $\mathcal{O}(n)$ equation needs input from $\mathcal{O}(n+1)$
- First equation is Balitsky equation for $S^{(2)}_{\boldsymbol{x},\boldsymbol{y}}$

$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle = \frac{\alpha_s}{\pi^2} \int_{\boldsymbol{z}} \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \left(\left\langle \frac{1}{N_c} U^{ab}_{\boldsymbol{z}} \mathrm{tr} \left\{ t^a U_{\boldsymbol{x}} t^b U^\dagger_{\boldsymbol{y}} \right\} \right\rangle - C_f \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle \right)$$

- Use Fierz identity for 3-point correlator and take large-N_c limit: $\langle \mathrm{tr}\,\{\}\,\mathrm{tr}\,\{\}\rangle \approx \langle \mathrm{tr}\,\{\}\rangle\langle \mathrm{tr}\,\{\}\rangle$

  $\rightarrow$ LO BK equation

$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle = \frac{\alpha_s}{\pi^2} \frac{N_c}{2} \int_{\boldsymbol{z}} \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \left(\left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle \right)$$

Balitsky [Phys. Rev. D60 (1999) 014020],
Kovchegov [Phys. Rev. D60 (1999) 034008],
Kovchegov [Phys. Rev. D61 (2000)]

NLO BK Equation

Balitsky, Chirilli [Nucl. Phys. B822:45-87 (2009)]

- Proceed as above, but keep terms $\mathcal{O}(\alpha_s^2)$

$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle = \frac{\alpha_s N_c}{2\pi^2} \int_{\boldsymbol{z}} K_1^{\mathsf{BC}} \langle D_1 \rangle + \frac{\alpha_s^2 {N_c}^2}{16\pi^4} \int_{\boldsymbol{z},\boldsymbol{z}'} \left(K_{2,1} \langle D_{2,1} \rangle + K_{2,2} \langle D_{2,2} \rangle \right) + \mathcal{O}(n_f)$$

- Correlator pieces:

$$\langle D_1 \rangle = \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle \xrightarrow{\text{Large } N_c} \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle$$

$$\langle D_{2,1} \rangle = \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle - \frac{1}{{N_c}^2} \left\langle S^{(6)}_{\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}',\boldsymbol{y},\boldsymbol{z},\boldsymbol{z}'} \right\rangle - (z' \to z) \xrightarrow{\text{Large } N_c} \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle$$

$$\langle D_{2,2} \rangle = \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle - (z' \to z) \xrightarrow{\text{Large } N_c} \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} \right\rangle \left\langle S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle$$

- Kernels $K_1^{\mathsf{BC}}, K_{2,1}, K_{2,2}$ are simple functions of lengths ($\boldsymbol{z}$ and $\boldsymbol{z}'$ are gluon coordinates)

$$r^2 = (\boldsymbol{x}-\boldsymbol{y})^2 \qquad X^2 = (\boldsymbol{x}-\boldsymbol{z})^2 \qquad Y^2 = (\boldsymbol{y}-\boldsymbol{z})^2 \qquad Z^2 = (\boldsymbol{z}-\boldsymbol{z}')^2 \qquad X'^2 = (\boldsymbol{x}-\boldsymbol{z}')^2 \qquad Y'^2 = (\boldsymbol{y}-\boldsymbol{z}')^2$$

Gaussian Truncation

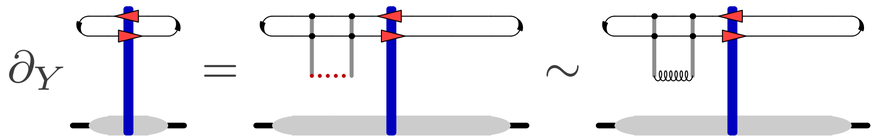
- Need to calculate 6-point correlators in $\langle D_{2,1}\rangle$ and $\langle D_{2,2}\rangle$
- Truncate infinite hierarchy of evolution equations by some parametrisation

  $\rightarrow$ Gaussian approximation

$$\left\langle \hat{\mathcal{O}} \right\rangle_Y := \exp\left\{ -\frac{1}{2}\int^Y dY' \int_{\boldsymbol{uv}} G_{Y',\boldsymbol{uv}} L^a_{\boldsymbol{u}} L^a_{\boldsymbol{v}} \right\} \hat{\mathcal{O}}$$

- E.g. Dipole parametrisation

$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle_Y = -\frac{1}{2}\left\langle \int_{\boldsymbol{uv}} G_{Y,\boldsymbol{uv}} L^a_{\boldsymbol{u}} L^a_{\boldsymbol{v}} S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle$$



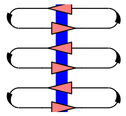
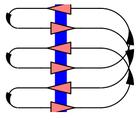
$$\Rightarrow \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle_Y = e^{-C_{\mathrm{F}}\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}}} \qquad \text{where} \qquad \mathcal{G}_{\boldsymbol{x},\boldsymbol{y}} := \int^Y \mathrm{d}Y' \left(G_{\boldsymbol{x},\boldsymbol{y}}(Y') - \frac{1}{2}\left[G_{\boldsymbol{x},\boldsymbol{x}}(Y') + G_{\boldsymbol{y},\boldsymbol{y}}(Y') \right] \right)$$

- Legitimacy of truncation provided by group theory constraints (automatically satisfied in coincidence limits)

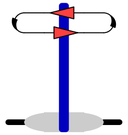
Iancu, Leonidov, McLerran [Nucl.Phys. A692 (2001) 583-645]
Fujii [Nucl.Phys. A709 (2002) 236-250]
Dumitru, Dusling, Gelis, Jalilian-Marian, Lappi, Venugopalan [Phys.Lett. B697 (2011) 21-25]
Dusling, Mace, Venugopalan [Phys.Rev. D97 (2018) no.1, 016014]

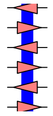
Analytical Calculation

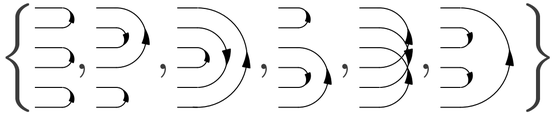
- NLO BK requires 6-point correlators

$$\left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle =$$  and $$\left\langle S^{(6)}_{\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}',\boldsymbol{y},\boldsymbol{z},\boldsymbol{z}'} \right\rangle =$$ 

with 2 repeated coordinates (4 unique coordinates remain)

- Now 6×6 matrix of correlators, not just 

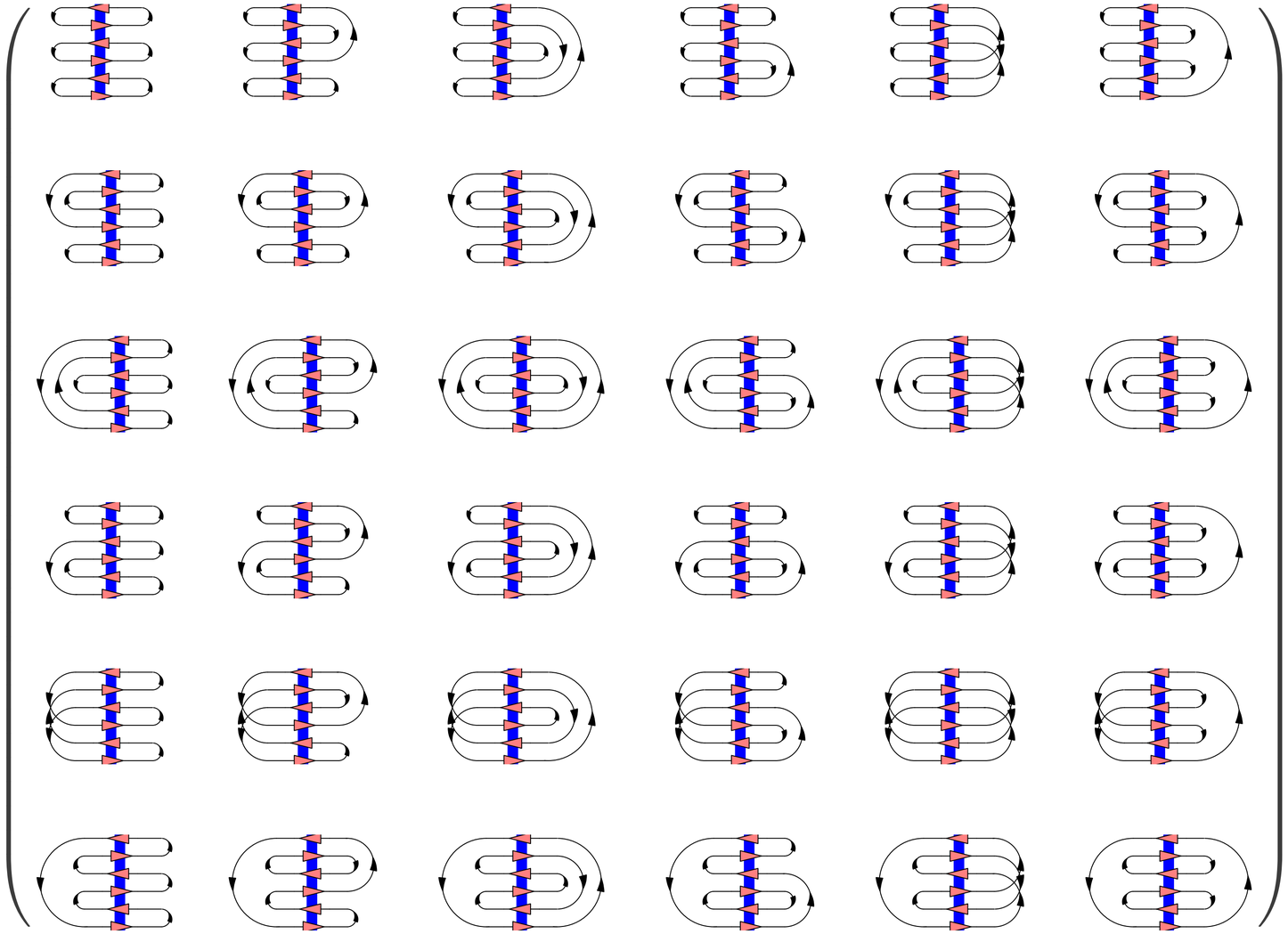
- **STEP 1**: choose orthonormal basis for operator 

$\rightarrow$ naive choice is simplest orthonormal set 

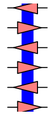
Matrix of Correlators

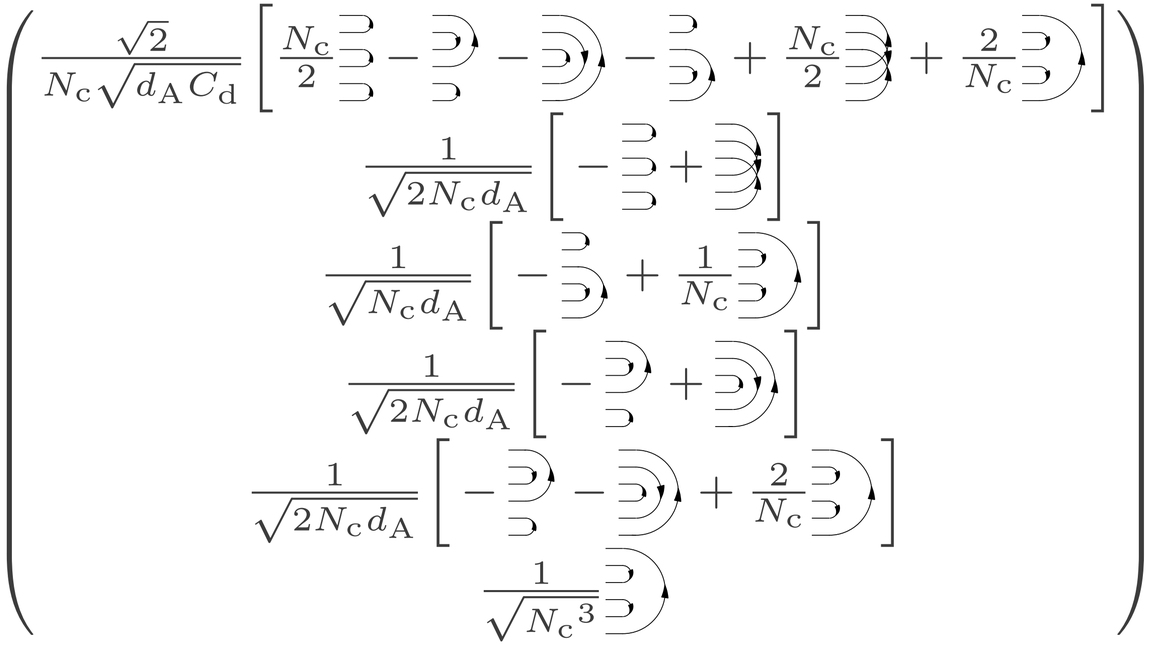
- **STEP 2**: Construct matrix of correlators

$\mathcal{A}(Y) :=$



Better Basis Choice

- **STEP 3**: construct transition matrix by taking sum of all possible diagrams with 1 gluon added
- **STEP 4**: exponentiate 6×6 transition matrix to get parametric equation for each of 36 correlators in $\mathcal{A}(Y)$
- Better idea! Exploit coordinates that appear twice in $\left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle$ and $\left\langle S^{(6)}_{\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}',\boldsymbol{y},\boldsymbol{z},\boldsymbol{z}'} \right\rangle$
- Back to **STEP 1**: choose better basis for 



Matrix Differential Equation

- Need to solve symmetric 6×6 matrix equation $\partial_Y \mathcal{A}(Y) = -\mathcal{M}(Y)\mathcal{A}(Y)$
- Clever basis choice $\implies$ transition matrix $\mathcal{M}$ block diagonalises

$$\lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \mathcal{M}(Y) = \begin{pmatrix} \mathcal{M}_1^{(3\times3)}(Y) & 0 & 0 \\ 0 & \mathcal{M}_2^{(2\times2)}(Y) & 0 \\ 0 & 0 & \mathcal{M}_3^{(1\times1)}(Y) \end{pmatrix}$$

- 1×1 transition matrix $\mathcal{M}_3(Y) = C_\mathrm{F}\mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_2}$ gives well-known Gaussian parametrisation for dipole

$$\left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle = e^{-C_\mathrm{F}\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}}}$$

- Solution can be inverted to express other, more complicated correlators in terms of dipole
- 2×2 transition matrix $\mathcal{M}_2(Y)$ gives known 4-point parametrisation

$$\left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle = \frac{1}{{N_\mathrm{c}}^2} e^{-C_\mathrm{F}\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}}} + \frac{2C_\mathrm{F}}{N_\mathrm{c}} e^{-C_\mathrm{F}\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}}} e^{-\frac{N_\mathrm{c}}{2}(\mathcal{G}_{\boldsymbol{x},\boldsymbol{z}}+\mathcal{G}_{\boldsymbol{y},\boldsymbol{z}}-\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}})}$$

3×3 Equation

- Need to solve $\partial_Y \mathcal{A}_1(Y) = -\mathcal{M}_1(Y)\mathcal{A}_1(Y)$
- Transition matrix is

$$\mathcal{M}_1(Y) = \begin{pmatrix} \frac{N_\mathrm{c}}{4}\Gamma_1 & \frac{\sqrt{N_\mathrm{c} C_\mathrm{d}}}{4}\Gamma_2 & 0 \\ \cdots & \frac{N_\mathrm{c}}{4}\Gamma_1 & -\frac{1}{\sqrt{2}}\Gamma_2 \\ \cdots & \cdots & \Gamma_0 \end{pmatrix}$$

where

$$\Gamma_0 = N_\mathrm{c}\mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{y}_3} + C_\mathrm{F}\mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_2}$$

$$\Gamma_1 = \mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{x}_3} + \mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{y}_2} + 2\mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{y}_3} - \frac{2}{{N_\mathrm{c}}^2}\mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_2} + \mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_3} + \mathcal{G}_{\boldsymbol{y}_2,\boldsymbol{y}_3}$$

$$\Gamma_2 = \mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{x}_3} - \mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{y}_2} - \mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_3} + \mathcal{G}_{\boldsymbol{y}_2,\boldsymbol{y}_3}$$

Cubic Polynomial

- Matrix exponentiation procedure reduces to finding roots of polynomial

$$p(z) = 4{N_\mathrm{c}}^2\Gamma_0\Gamma_1^2 - 4C_\mathrm{d}N_\mathrm{c}\Gamma_0\Gamma_2^2 - 8N_\mathrm{c}\Gamma_1\Gamma_2^2 + 8N_\mathrm{c}\Gamma_0\Gamma_1 z + {N_\mathrm{c}}^2\Gamma_1^2 z - 8\Gamma_2^2 z - C_\mathrm{d}N_\mathrm{c}\Gamma_2^2 z + 4\Gamma_0 z^2 + 2N_\mathrm{c}\Gamma_1 z^2 + z^3$$

- Roots are

$$z_1 = \frac{1}{3}\left\{ -2(2\Gamma_0 + N_\mathrm{c}\Gamma_1) + \frac{1}{Z}\left[Y + 3({N_\mathrm{c}}^2 + 4)\Gamma_2^2\right] + Z \right\}$$

$$z_2 = \frac{1}{12}\left\{ -8(2\Gamma_0 + N_\mathrm{c}\Gamma_1) - \frac{1}{Z}2i(\sqrt{3} - i)\left[Y + 3({N_\mathrm{c}}^2 + 4)\Gamma_2^2\right] + 2i(\sqrt{3} + i)Z \right\}$$

$$z_3 = \frac{1}{12}\left\{ -8(2\Gamma_0 + N_\mathrm{c}\Gamma_1) + \frac{1}{Z}2i(\sqrt{3} + i)\left[Y + 3({N_\mathrm{c}}^2 + 4)\Gamma_2^2\right] - 2i(\sqrt{3} - i)Z \right\}$$

where

$$X = 4(-4\Gamma_0 + N_\mathrm{c}\Gamma_1)^2\left[Y - 9({N_\mathrm{c}}^2 - 8)\Gamma_2^2\right]^2 - 4\left[Y + 3({N_\mathrm{c}}^2 + 4)\Gamma_2^2\right]^3$$

$$Y = (N_\mathrm{c}\Gamma_1 - 4\Gamma_0)^2$$

$$Z = \left[\frac{\sqrt{X}}{2} - (4\Gamma_0 - N_\mathrm{c}\Gamma_1)\left(Y - 9({N_\mathrm{c}}^2 - 8)\Gamma_2^2\right)\right]^{1/3}$$

3×3 Matrix Exponentiation

- Exponentiate transition matrix

$$\mathcal{A}_1(Y) = e^{-\mathcal{M}_1(Y)} = \begin{pmatrix} \sum_{i=1}^{3} \frac{m_{11}}{d}(z_i) & -\sqrt{C_\mathrm{d} N_\mathrm{c}}\Gamma_2 \sum_{i=1}^{3} \frac{m_{12}}{d}(z_i) & -2\sqrt{2C_\mathrm{d} N_\mathrm{c}}\Gamma_2^2 \sum_{i=1}^{3} \frac{m_{13}}{d}(z_i) \\ \cdots & \sum_{i=1}^{3} \frac{m_{22}}{d}(z_i) & 2\sqrt{2}\Gamma_2 \sum_{i=1}^{3} \frac{m_{23}}{d}(z_i) \\ \cdots & \cdots & \sum_{i=1}^{3} \frac{m_{33}}{d}(z_i) \end{pmatrix}$$

- Functions of roots are

$$m_{11}(z) = e^{z/4}\left(4N_\mathrm{c}\Gamma_0\Gamma_1 - 8\Gamma_2^2 + (4\Gamma_0 + N_\mathrm{c}\Gamma_1)z + z^2\right) \qquad m_{22}(z) = e^{z/4}\left(4N_\mathrm{c}\Gamma_0\Gamma_1 + (4\Gamma_0 + N_\mathrm{c}\Gamma_1)z + z^2\right)$$

$$m_{12}(z) = e^{z/4}\left(4\Gamma_0 + z\right) \qquad m_{23}(z) = e^{z/4}\left(N_\mathrm{c}\Gamma_1 + z\right)$$

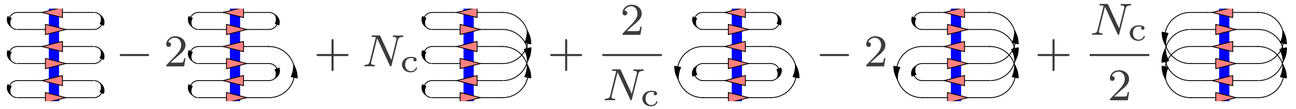
$$m_{13}(z) = e^{z/4} \qquad m_{33}(z) = e^{z/4}\left({N_\mathrm{c}}^2\Gamma_1^2 - C_\mathrm{d}N_\mathrm{c}\Gamma_2^2 + 2N_\mathrm{c}\Gamma_1 z + z^2\right)$$

and

$$d(z) = 8N_\mathrm{c}\Gamma_0\Gamma_1 + {N_\mathrm{c}}^2\Gamma_1^2 - 8\Gamma_2^2 - C_\mathrm{d}N_\mathrm{c}\Gamma_2^2 + 8\Gamma_0 z + 4N_\mathrm{c}\Gamma_1 z + 3z^2$$

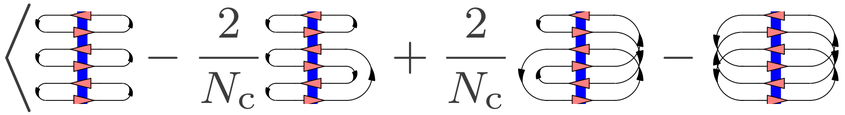
3×3 Correlator Matrix

$$\mathcal{A}_1^{(11)}(Y) = \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \frac{1}{N_\mathrm{c} d_\mathrm{A} C_\mathrm{d}} \Bigg\langle \frac{N_\mathrm{c}}{2}$$

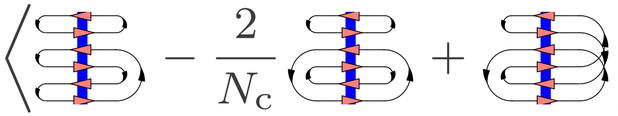


$$- \frac{2}{N_\mathrm{c} C_\mathrm{d}} e^{\frac{1}{2N_\mathrm{c}} \mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_2}} \left(e^{-\frac{N_\mathrm{c}}{2}\left(\mathcal{G}_{\boldsymbol{y}_3,\boldsymbol{x}_3}+\mathcal{G}_{\boldsymbol{y}_3,\boldsymbol{y}_2}\right)} + e^{-\frac{N_\mathrm{c}}{2}\left(\mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{x}_3}+\mathcal{G}_{\boldsymbol{x}_2,\boldsymbol{y}_2}\right)} \right)$$

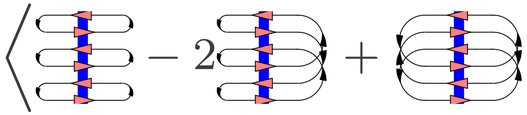
$$\mathcal{A}_1^{(12)}(Y) = \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} -\frac{1}{2 d_\mathrm{A} \sqrt{C_\mathrm{d} N_\mathrm{c}}} \Bigg\langle$$



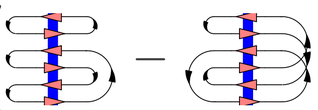
$$\mathcal{A}_1^{(13)}(Y) = \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} -\frac{1}{d_\mathrm{A} \sqrt{2 C_\mathrm{d} N_\mathrm{c}}} \Bigg\langle$$



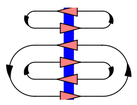
$$\mathcal{A}_1^{(22)}(Y) = \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \frac{1}{2 N_\mathrm{c} d_\mathrm{A}} \Bigg\langle$$



$$\mathcal{A}_1^{(23)}(Y) = \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \frac{1}{\sqrt{2} N_\mathrm{c} d_\mathrm{A}} \Bigg\langle$$



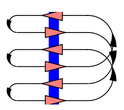
$$\mathcal{A}_1^{(33)}(Y) = \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \frac{1}{N_\mathrm{c} d_\mathrm{A}} \Bigg\langle$$



$$\Bigg\rangle - \frac{1}{d_\mathrm{A}} e^{-C_\mathrm{F} \mathcal{G}_{\boldsymbol{x}_3,\boldsymbol{y}_2}}$$

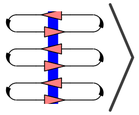
6-point Correlators for NLO BK

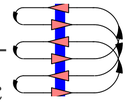
- **Final analytical solution**: two required 6-point correlators are

$$\lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \left\langle \frac{1}{N_\mathrm{c}} \right.$$  $$\left. \right\rangle = \left\langle \frac{1}{N_\mathrm{c}} \mathrm{tr}\left\{ U_{\boldsymbol{z}}^\dagger U_{\boldsymbol{z}'} U_{\boldsymbol{x}}^\dagger U_{\boldsymbol{z}} U_{\boldsymbol{z}'}^\dagger U_{\boldsymbol{y}} \right\} \right\rangle$$

$$= \frac{1}{{N_\mathrm{c}}^2} e^{-C_\mathrm{F}\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}}} + \frac{d_\mathrm{A}}{{N_\mathrm{c}}^2} e^{\frac{1}{2N_\mathrm{c}}\mathcal{G}_{\boldsymbol{x},\boldsymbol{y}}} \left(e^{-\frac{N_\mathrm{c}}{2}(\mathcal{G}_{\boldsymbol{z},\boldsymbol{x}}+\mathcal{G}_{\boldsymbol{z},\boldsymbol{y}})} + e^{-\frac{N_\mathrm{c}}{2}(\mathcal{G}_{\boldsymbol{z}',\boldsymbol{x}}+\mathcal{G}_{\boldsymbol{z}',\boldsymbol{y}})} \right)$$

$$+ C_\mathrm{F} C_\mathrm{d} \mathcal{A}_1^{(11)} - N_\mathrm{c} C_\mathrm{F} \mathcal{A}_1^{(22)} - C_\mathrm{F}\sqrt{8{N_\mathrm{c}}^3 C_\mathrm{d}}\,\mathcal{A}_1^{(13)} + 2N_\mathrm{c} C_\mathrm{F} \mathcal{A}_1^{(33)}$$

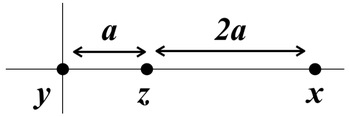
$$\lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \left\langle \frac{1}{{N_\mathrm{c}}^3} \right.$$  $$\left. \right\rangle = \left\langle \frac{1}{N_\mathrm{c}} \mathrm{tr}\left\{ U_{\boldsymbol{z}}^\dagger U_{\boldsymbol{z}'} \right\} \frac{1}{N_\mathrm{c}} \mathrm{tr}\left\{ U_{\boldsymbol{z}'}^\dagger U_{\boldsymbol{y}} \right\} \frac{1}{N_\mathrm{c}} \mathrm{tr}\left\{ U_{\boldsymbol{x}}^\dagger U_{\boldsymbol{z}} \right\} \right\rangle$$

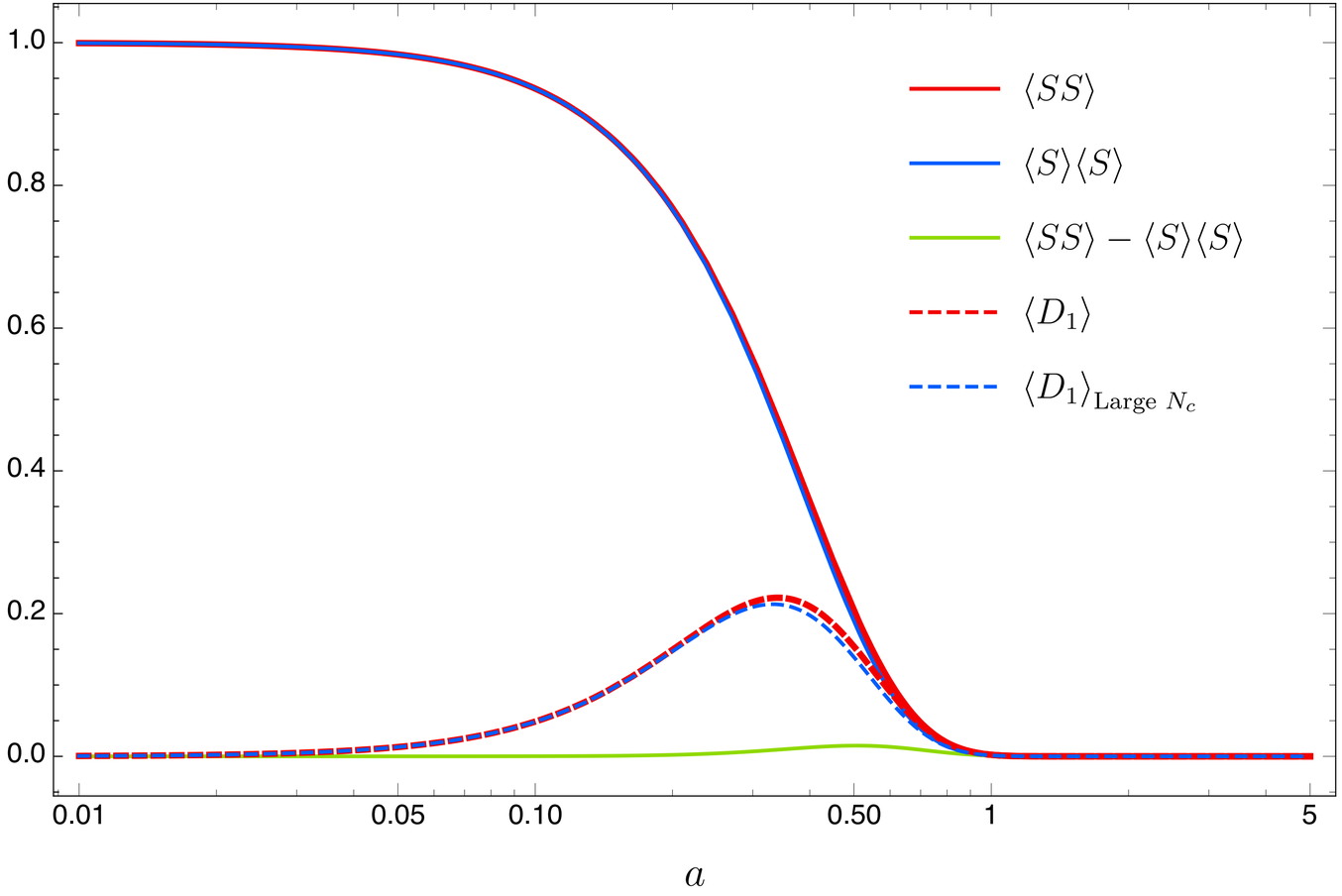
$$= \frac{1}{{N_\mathrm{c}}^2} \lim_{\substack{\boldsymbol{u}\to\boldsymbol{z}\\ \boldsymbol{v}\to\boldsymbol{z}'}} \left\langle \frac{1}{N_\mathrm{c}} \right.$$  $$\left. \right\rangle - \frac{2C_\mathrm{F}\sqrt{C_\mathrm{d}}}{\sqrt{{N_\mathrm{c}}^3}} \mathcal{A}_1^{(12)} + \frac{2C_\mathrm{F}}{N_\mathrm{c}} \mathcal{A}_1^{(22)} + \sqrt{8} N_\mathrm{c} C_\mathrm{F} \frac{1}{{N_\mathrm{c}}^3} \mathcal{A}_1^{(23)}$$

- Add these results to BK code to solve numerically

Numerical Results: LO-like Correlators

Correlators for **LO-like** BK integrand for one particular typical configuration of coordinates



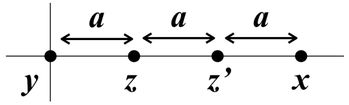


Recall BK equation:

$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle \sim \int K_1^{\text{BC}} \langle D_1 \rangle + \text{NLO-like}$$

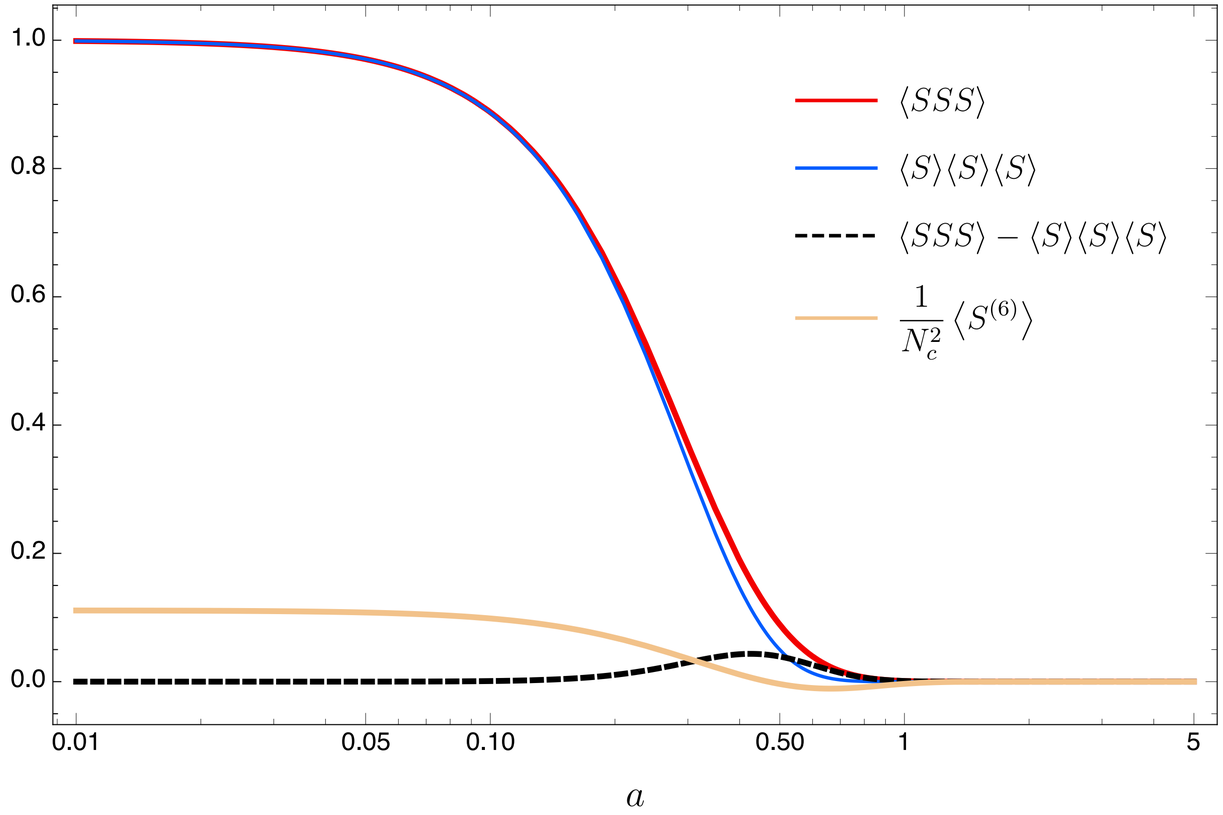
$$\langle D_1 \rangle = \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{y}} \right\rangle - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle$$

NLO-like Correlators



Correlators for **NLO-like** BK integrand for one particular typical configuration of coordinates

$\rightarrow$ very small correction from including finite-N_c piece



Recall BK equation:

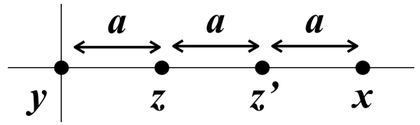
$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle \sim \int K_1^{\mathrm{BC}} \langle D_1 \rangle + \int K_{2,1} \langle D_{2,1} \rangle + \int K_{2,2} \langle D_{2,2} \rangle$$

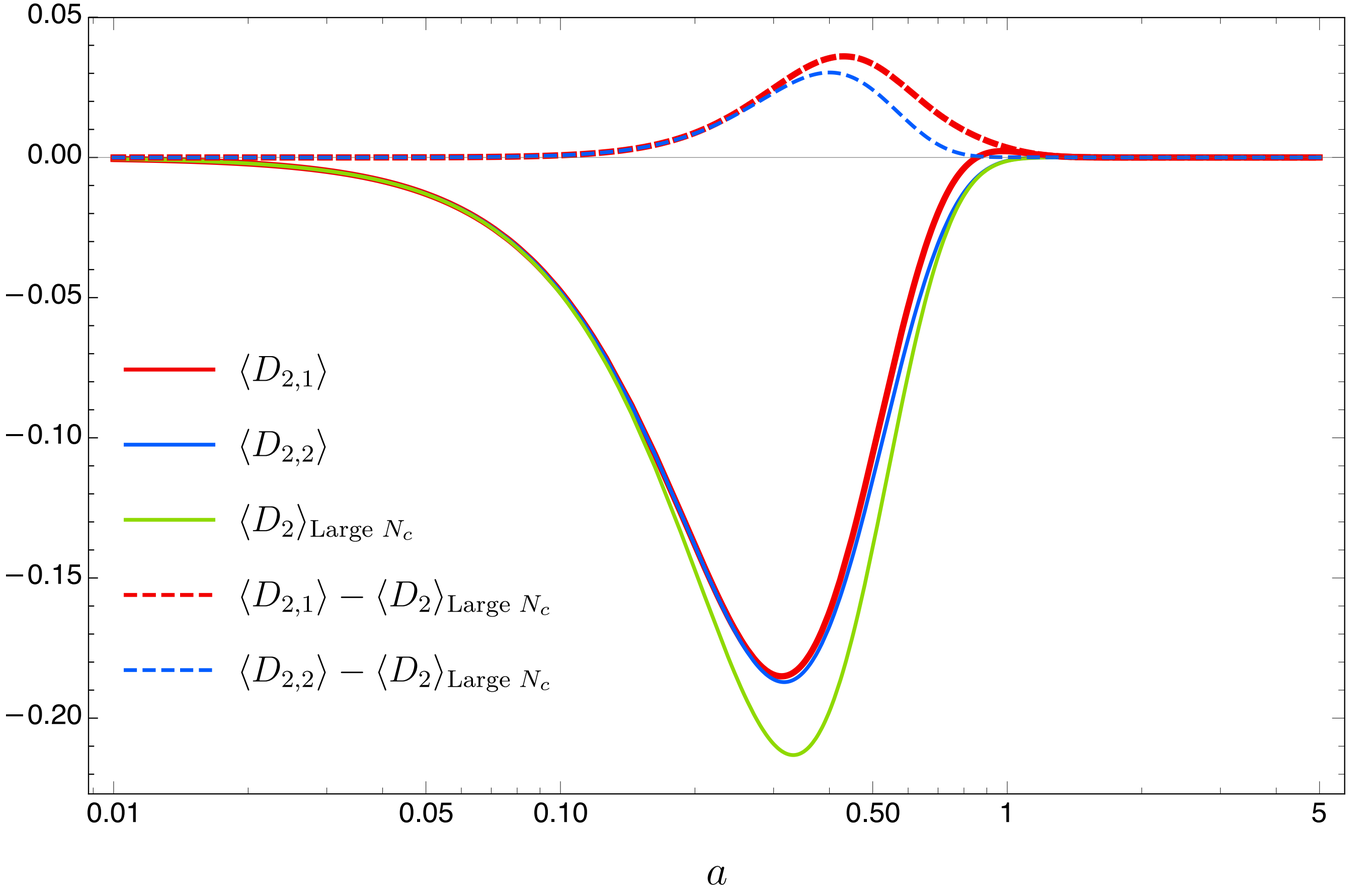
$$\langle D_{2,1} \rangle = \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle - \frac{1}{{N_c}^2} \left\langle S^{(6)}_{\boldsymbol{x},\boldsymbol{z},\boldsymbol{z}',\boldsymbol{y},\boldsymbol{z},\boldsymbol{z}'} \right\rangle - (z' \rightarrow z)$$

$$\langle D_{2,2} \rangle = \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{z}} S^{(2)}_{\boldsymbol{z},\boldsymbol{z}'} S^{(2)}_{\boldsymbol{z}',\boldsymbol{y}} \right\rangle - (z' \rightarrow z)$$

NLO Integrand

One particular typical configuration of coordinates

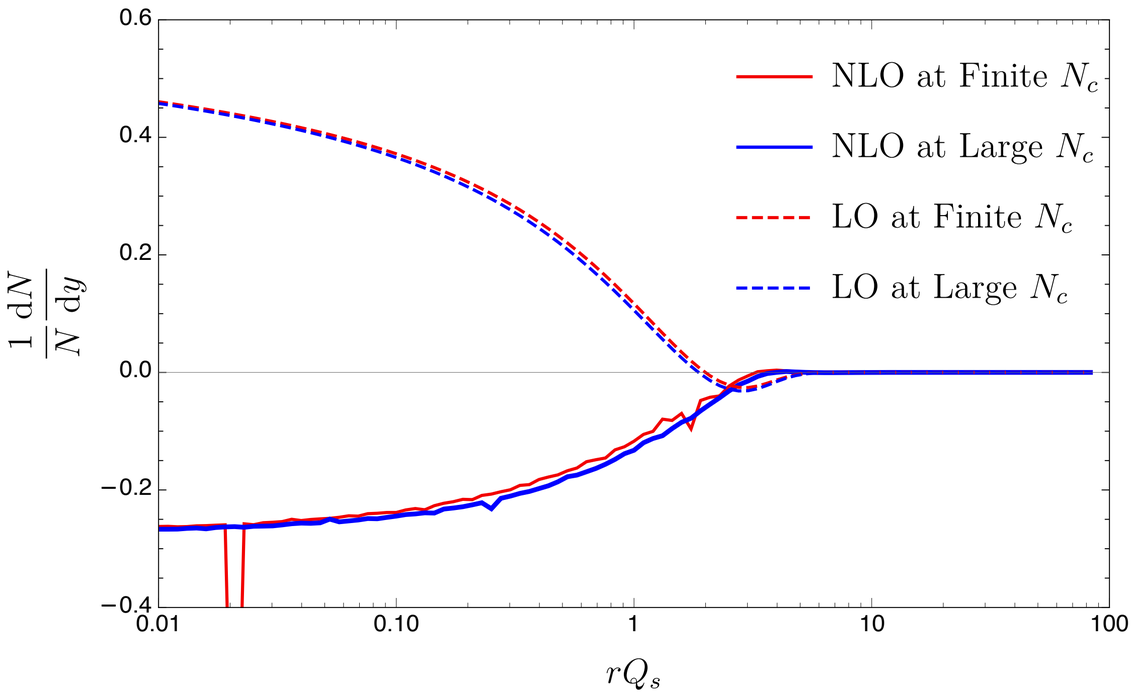




Recall BK equation:

$$\partial_Y \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle \sim \int K_1^{\text{BC}} \langle D_1 \rangle + \int K_{2,1} \langle D_{2,1} \rangle + \int K_{2,2} \langle D_{2,2} \rangle$$

Finite- vs Large-N_c dN/dy

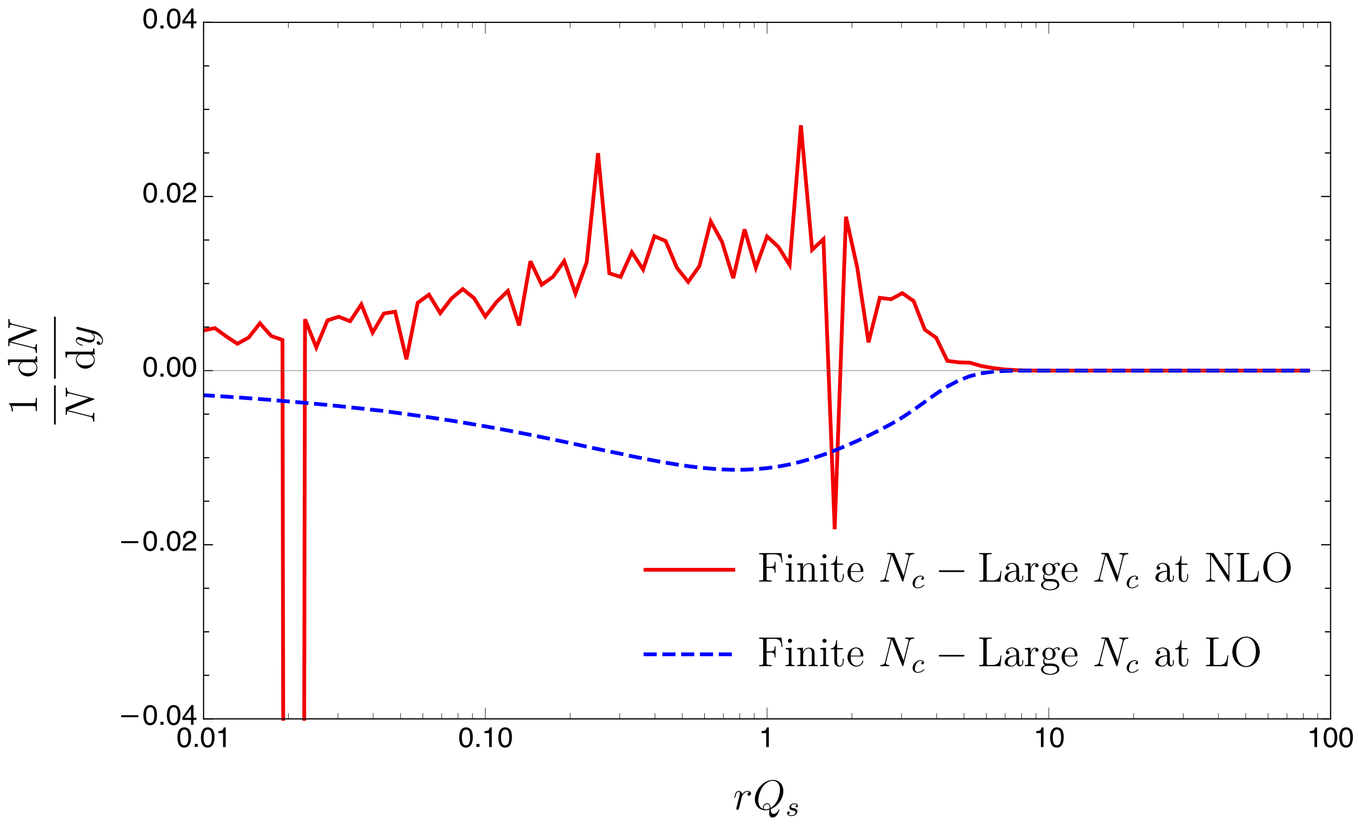


LO means only $\sim \int K_1^{\mathrm{BC}} \langle D_1 \rangle$

NLO means only

$\int K_{2,1} \langle D_{2,1} \rangle + \int K_{2,2} \langle D_{2,2} \rangle$

Finite- vs Large-N_c Difference in dN/dy



$$N_{\boldsymbol{x},\boldsymbol{y}} = 1 - \left\langle S^{(2)}_{\boldsymbol{x},\boldsymbol{y}} \right\rangle$$

LO means only $\sim \int K_1^{\mathrm{BC}} \langle D_1 \rangle$

NLO means only

$\int K_{2,1} \langle D_{2,1} \rangle + \int K_{2,2} \langle D_{2,2} \rangle$

Summary

- Studied high energy evolution of Wilson line correlators within CGC framework
- Infinite hierarchy of evolution equations can be truncated using Gaussian approximation to parametrise correlators
- NLO BK requires 6-point correlators
  $\rightarrow$ better basis choice leads to simplified calculation: 6×6 matrix equation block diagonalises in particular coincidence limits
- We have purely analytical parametrisations for correlators
- Used parametric equations for numerical studies of NLO BK and found very small difference between large-N_c and finite-N_c results
- Naive expectation before calculation: finite-N_c corrections at NLO are $\frac{1}{{N_\mathrm{c}}^2} \sim \mathcal{O}(10\%)$
  $\rightarrow$ but numerics show much smaller correction, $\sim \mathcal{O}(1\%)$ (similar to LO-like case)