

# In-medium transverse momentum broadening effects on di-jet observables

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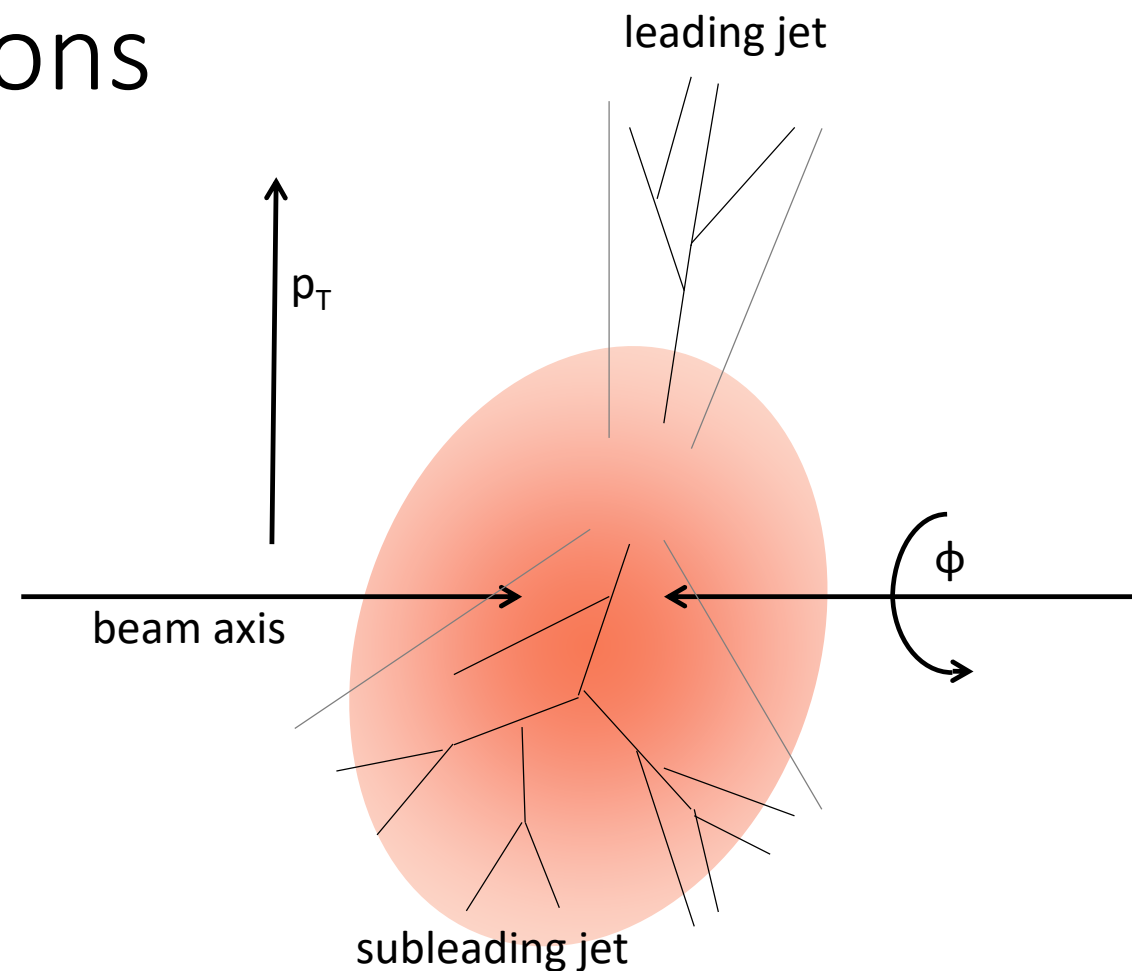
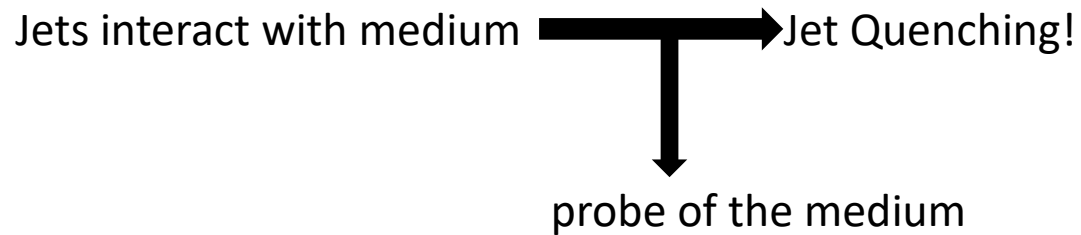
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based on: [\[arXiv:1911.05463\]](https://arxiv.org/abs/1911.05463)

# Jets in Heavy Ion collisions

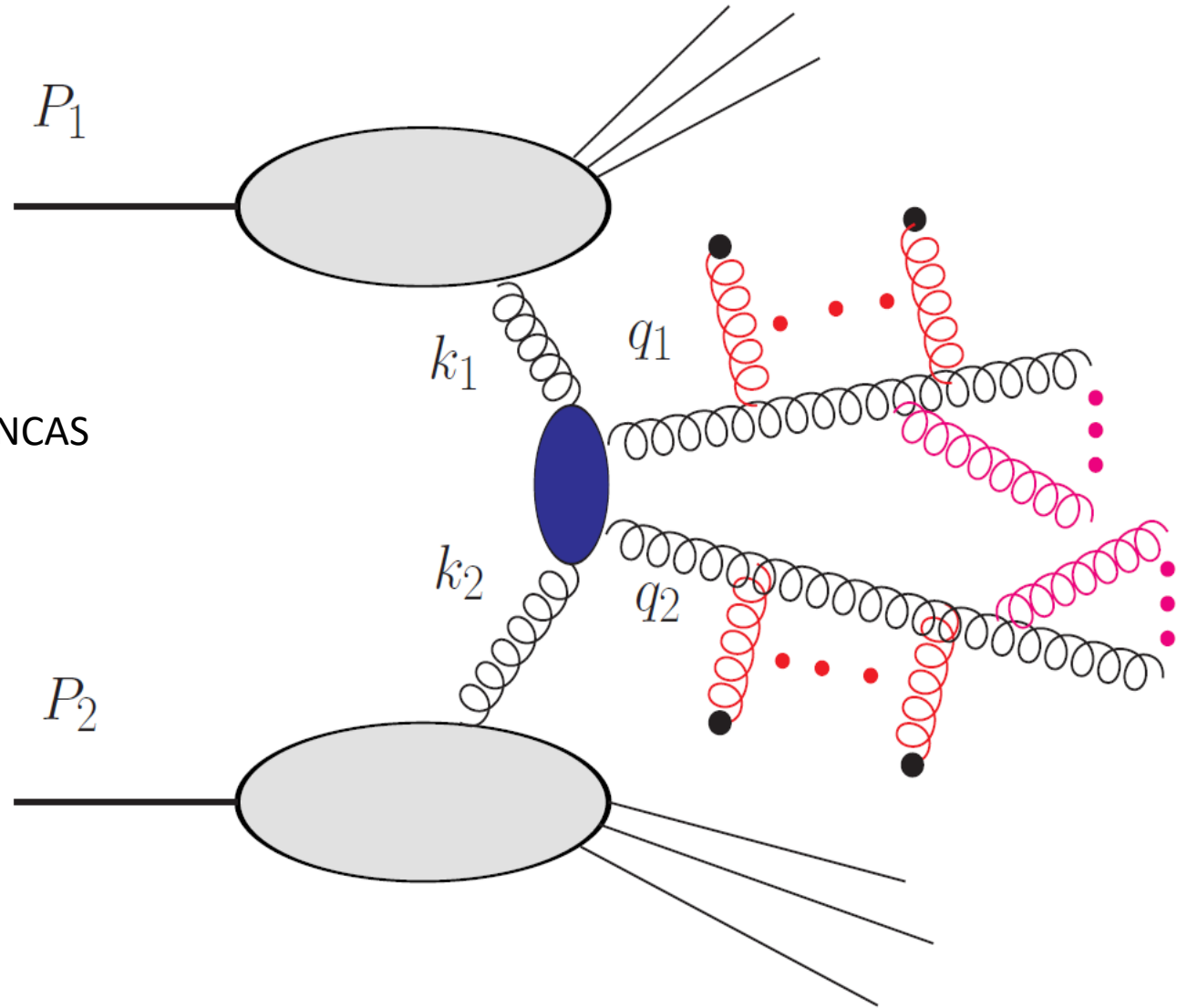


# Jet Production(1/2)

Cross section =

$(u)PDF1 * (u)PDF2$  } Here via KATIE  
\*hard cross section

\*fragmentation of jet1 } Here via MINCAS  
\*fragmentation of jet2



# Jet Production (2/2)

$k_T$  factorization:

$$\frac{d\sigma_{pp}}{dy_1 dy_2 d^2q_{1T} d^2q_{2T}} = \int \frac{d^2k_{1T}}{\pi} \frac{d^2k_{2T}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}_{g^* g^* \rightarrow gg}^{\text{off-shell}}|^2} \\ \times \delta^{(2)}(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_{1T} - \vec{q}_{2T}) \mathcal{F}_g(x_1, k_{1T}^2, \mu_F^2) \mathcal{F}_g(x_2, k_{2T}^2, \mu_F^2)$$

$\mathcal{F}_g(x, k_T^2, \mu_F^2)$  ...unintegrated parton densities

→ full phase space access at LO  
particularly relevant at low x

# Coherent Emission

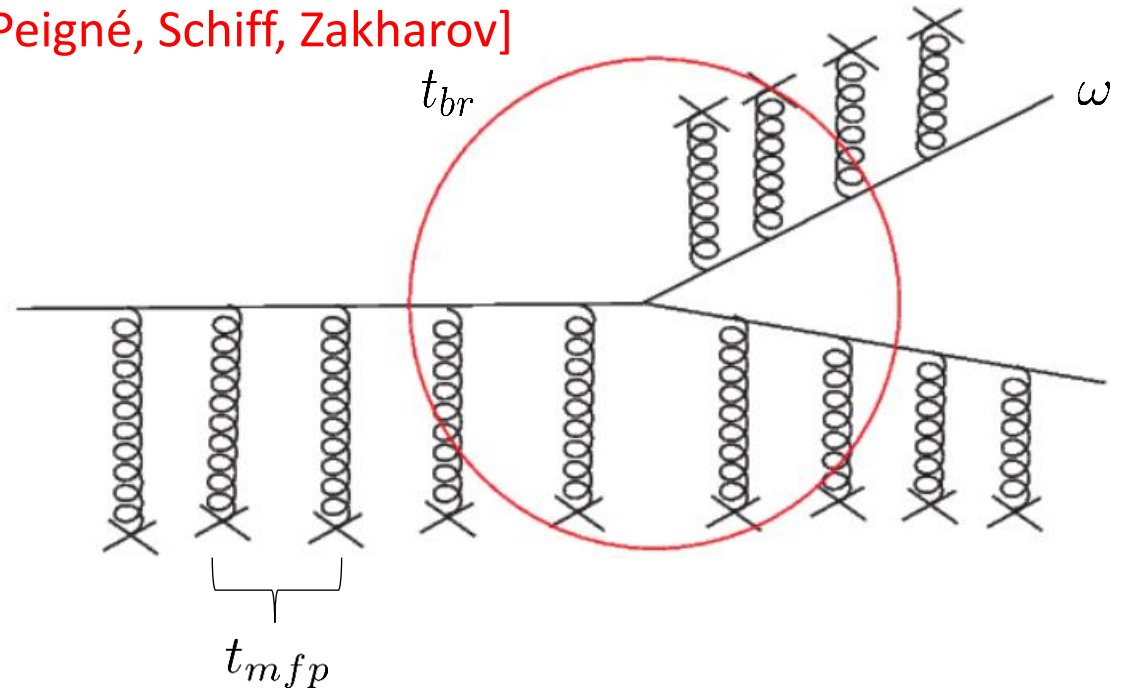
...à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

$t_{br} \sim t_{mfp}$ : one scattering + radiation  
...Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$ : coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$



Look at range:  $\omega_{BH} < \omega < \omega_c$

need effective splitting kernel

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

# BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

Generalizes BDMPS-Z approach

Includes transverse momentum broadening

For gluon-jets:

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

Induced Radiation:

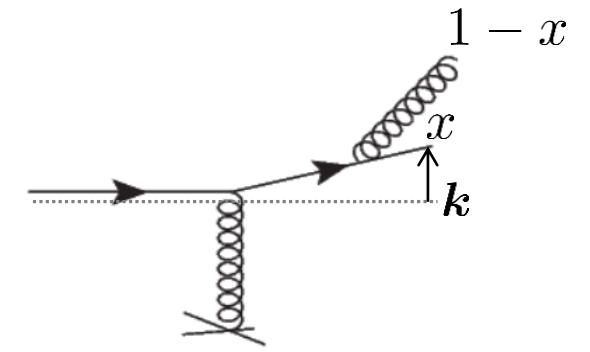
$$\mathcal{K}(z) = \frac{(1 - z + z^2)^{\frac{5}{2}}}{[z(1 - z)]^{\frac{3}{2}}}$$

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{\omega}} \propto \frac{1}{t_{br}}$$

Integration over  $\mathbf{k}$

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

$k_T$  broadening in dijets



Momentum distribution:

$$p \rightarrow xp$$

Momentum transfer:

$$p \rightarrow p + \mathbf{k}$$

Scattering:

$$C(\mathbf{q}) = w(\mathbf{q}) - \delta(\mathbf{q}) \int d^2 \mathbf{q}' w(\mathbf{q}')$$

$$\text{we use: } w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^2 (\mathbf{q}^2 + m_D^2)}$$

# BDIM Equation as Integral Equation

[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

$$\tau = \frac{t}{t^*}$$

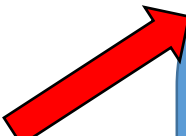
$$\begin{aligned} D(x, \mathbf{k}, \tau) = & e^{-\Psi(x)(\tau - \tau_0)} D(x, \mathbf{k}, \tau_0) \\ & + \int_{\tau_0}^{\tau} d\tau' \int_0^1 dz \int_0^1 dy \int d^2 \mathbf{k}' \int d^2 \mathbf{q} \mathcal{G}(z, \mathbf{q}) \\ & \times \delta(x - zy) \delta(\mathbf{k} - \mathbf{q} - z\mathbf{k}') e^{-\Psi(x)(\tau - \tau')} D(y, \mathbf{k}', \tau') \end{aligned}$$

$$\begin{aligned} \Phi(x) &= \frac{1}{\sqrt{x}} \int_0^{1-\varepsilon} dz z \mathcal{K}(z), \\ W &= t^* \int_{|\mathbf{q}| > q_{\min}} d^2 \mathbf{q} \frac{w(\mathbf{q})}{(2\pi)^2}, \\ \Psi(x) &= \Phi(x) + W, \end{aligned}$$

$$\mathcal{G}(z, \mathbf{q}) = \sqrt{\frac{z}{x}} z \mathcal{K}(z) \theta(1 - \varepsilon - z) \delta(\mathbf{q}) + t^* \frac{w(\mathbf{q})}{(2\pi)^2} \theta(|\mathbf{q}| - q_{\min}) \delta(1 - z)$$

# Monte-Carlo algorithm

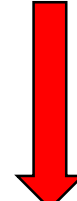
Set/Select  
 $x_0$  and  $k_0$   
at  $\tau_0$



- Select  $\tau_{i+1}$ :  
Probability density:  $\Psi(x_i)e^{-\Psi(x_i)(\tau_{i+1}-\tau_i)}$
- Splitting with probability  $p = \Phi(x_i)/\Psi(x_i)$   
otherwise: scattering.
- If splitting, select  $z = \frac{x_{i+1}}{x_i}$   
probability density:  $\frac{z\mathcal{K}(z)}{\int_0^{1-\epsilon} dz' z' \mathcal{K}(z')}$
- If scattering, select  $\mathbf{q}_{i+1}$   
probability density  $\frac{t^*}{(2\pi)^2} \frac{w(\mathbf{q}_{i+1})}{W}$

Repeat for  
next step in  
 $\tau$  and  $x$  or  $k$

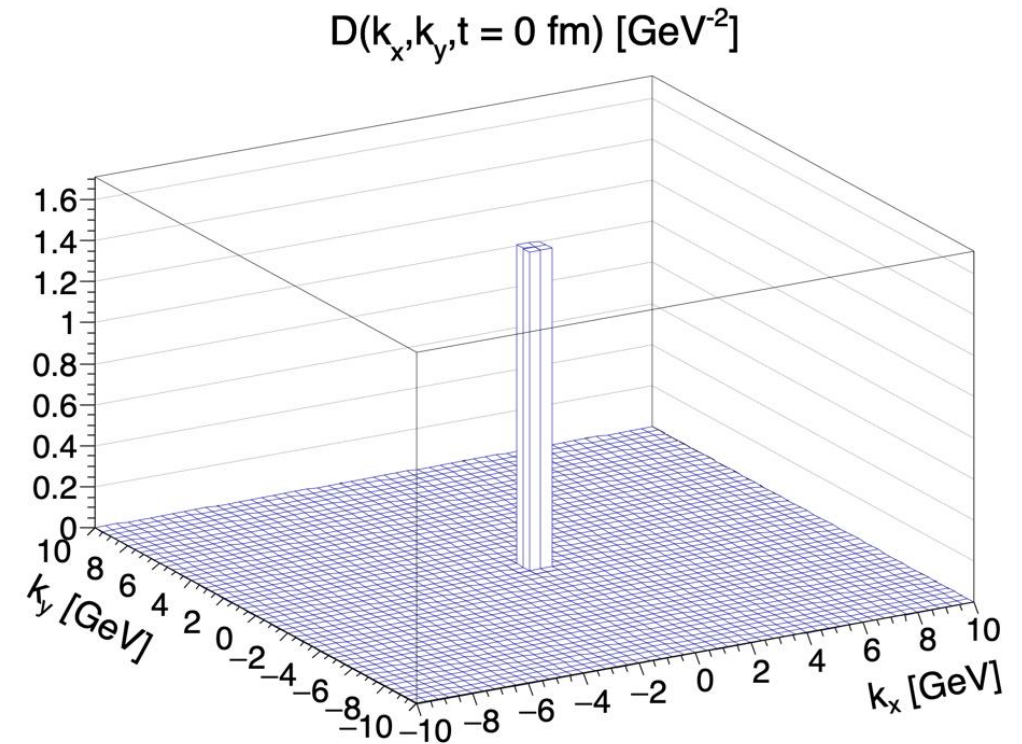
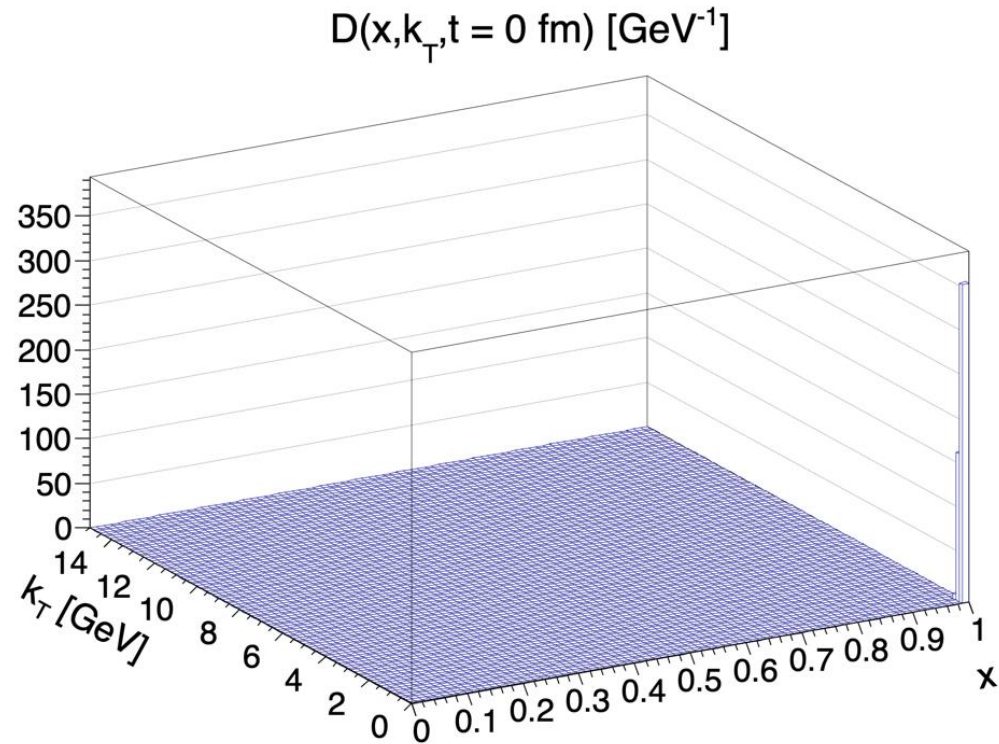
Stop once  
 $\tau > \tau_L$



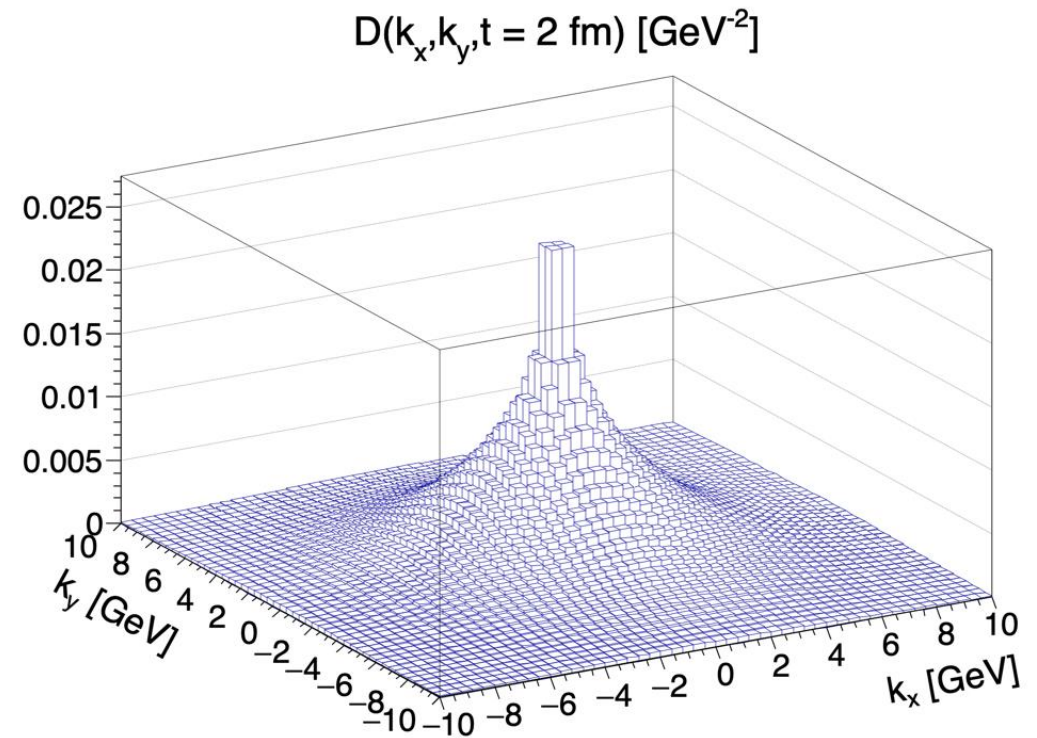
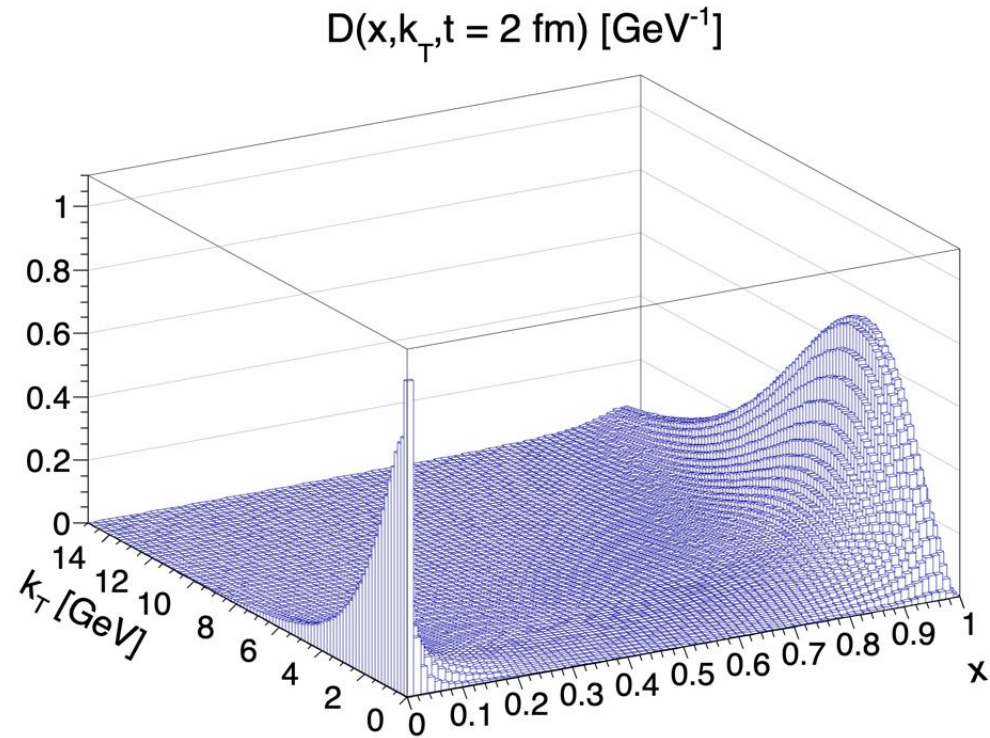
M  
I  
N  
C  
A  
S



# Evolution of $D(x, \mathbf{k}, t)$ (1/2)



# Evolution of $D(x, \mathbf{k}, t)$ (2/2)



# k Distribution

always same distribution for  
changes  $p \rightarrow p + q$   
 $\rightarrow$  central limit theorem

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t) + \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right]$$

Splitting à la  $p \rightarrow zp$   
 $\rightarrow$  perturbations of  
different sizes  
 $\rightarrow$  non Gaussian behavior

Virtual emissions

For example:  
 $p \rightarrow z_1 p \rightarrow z_1 p + \mathbf{q}_1$   
 $\rightarrow z_1 p + \mathbf{q}_1 + \mathbf{q}_2$   
 $\rightarrow z_2(z_1 + \mathbf{q}_1 + \mathbf{q}_2) \rightarrow \dots$

$k_T$  broadening in dijets

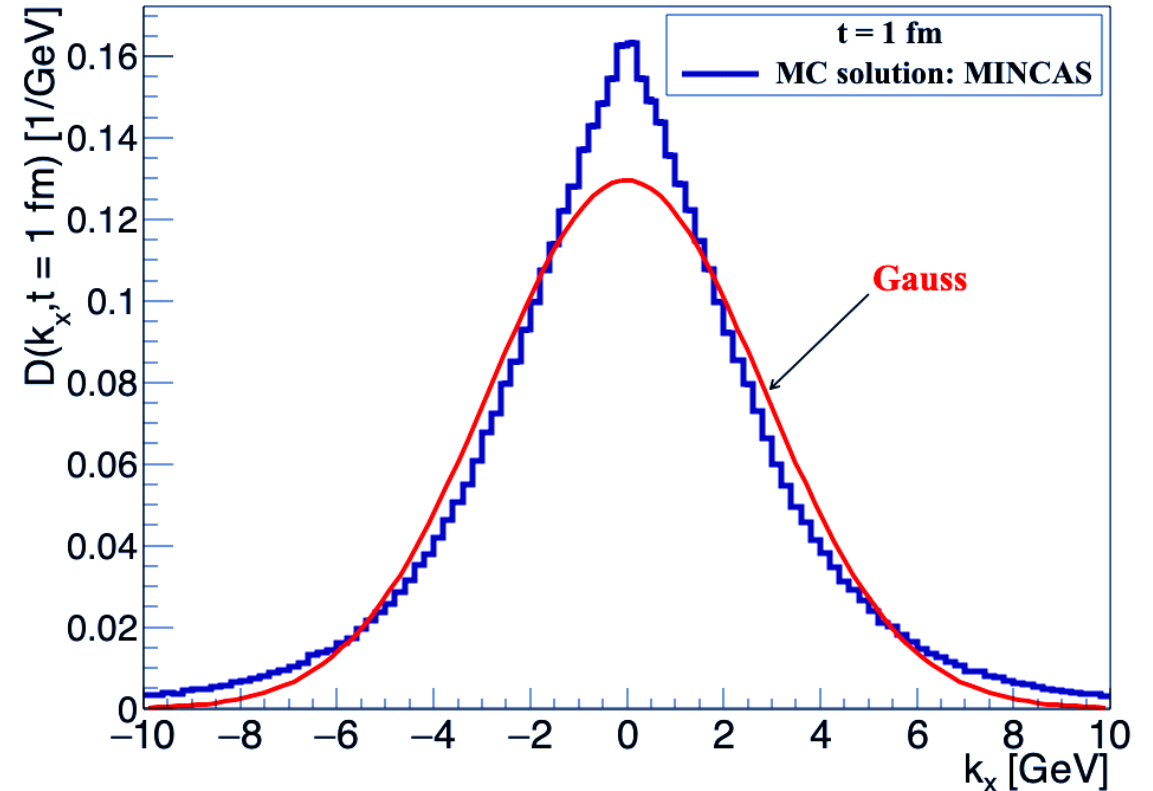


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

# Program: KATIE+MINCAS

- Use KATIE for hard initial collisions:
  - (u)PDFs for colliding nucleons
  - Hard collision cross-section (Monte-Carlo simulation)
  - Resulting particles  $\rightarrow$  initial particles of jets

[van Hameren: *Comput.Phys.Commun.* 224 (2018) 371-380]

- Jets: by MINCAS
  - Monte-Carlo simulation of BDIM equation
  - Time-evolution of jets in medium

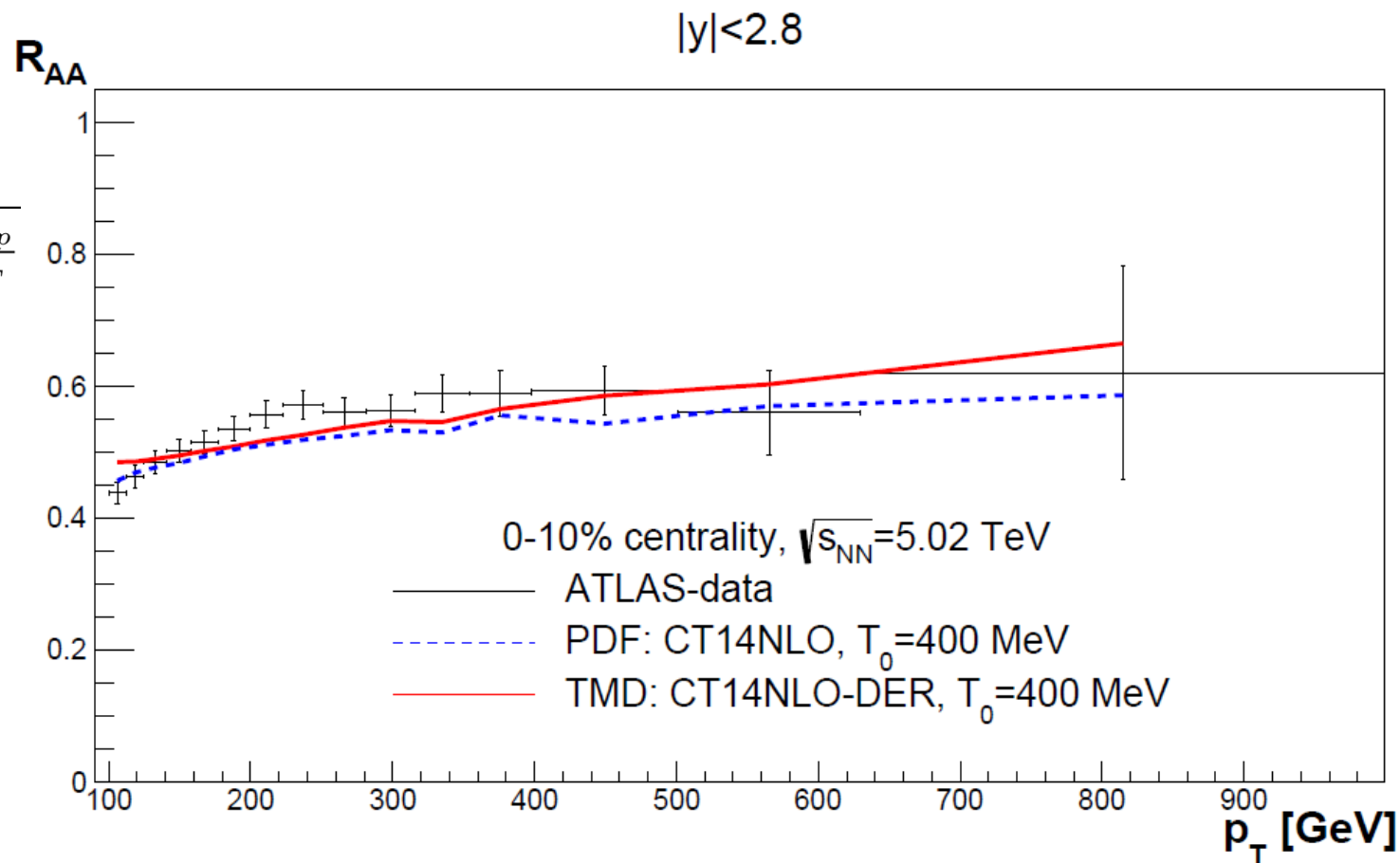
[Kutak, Płaczek, Straka: *Eur.Phys.J.* C79 (2019) no.4, 317]

Other codes implementing  
BDMPS-Z:

MARTINI, JEWEL, QPYTHIA, ...

# R<sub>AA</sub>

$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle T_{AA} \rangle \frac{d\sigma_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{AA}}{dp_T}}{\frac{d\sigma_{pp}}{dp_T}}$$



# Gaussian $k_T$ broadening

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

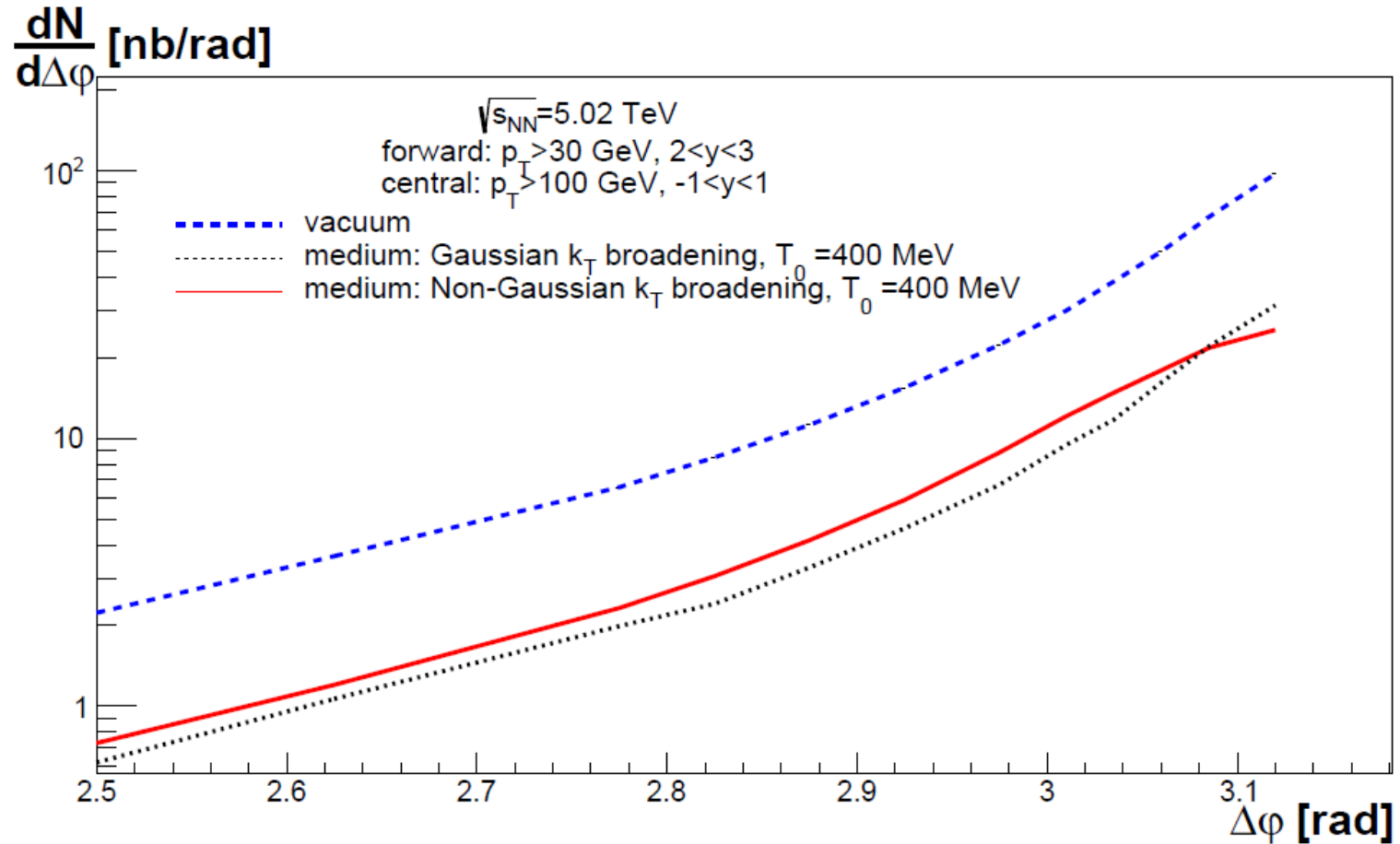
Integrate over  $d^2 \mathbf{k}$

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

For comparison with full equation: add  $k_T$  selected from Gaussian!

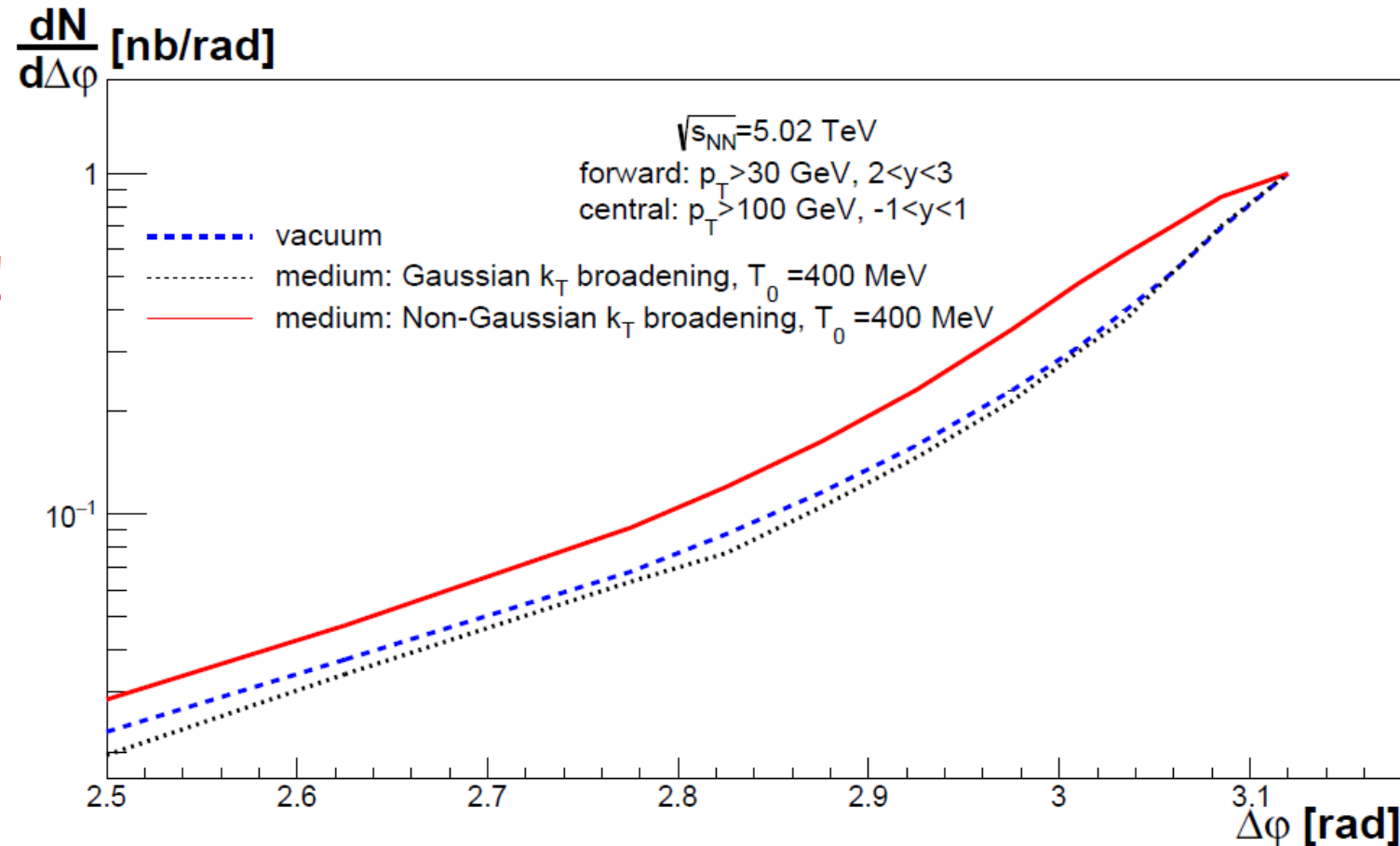
width:  $\sigma^2 \sim \hat{q}L$

# Azimuthal Decorrelations



# Azimuthal Decorrelations

**Normalized  
to maximum!**





# Summary

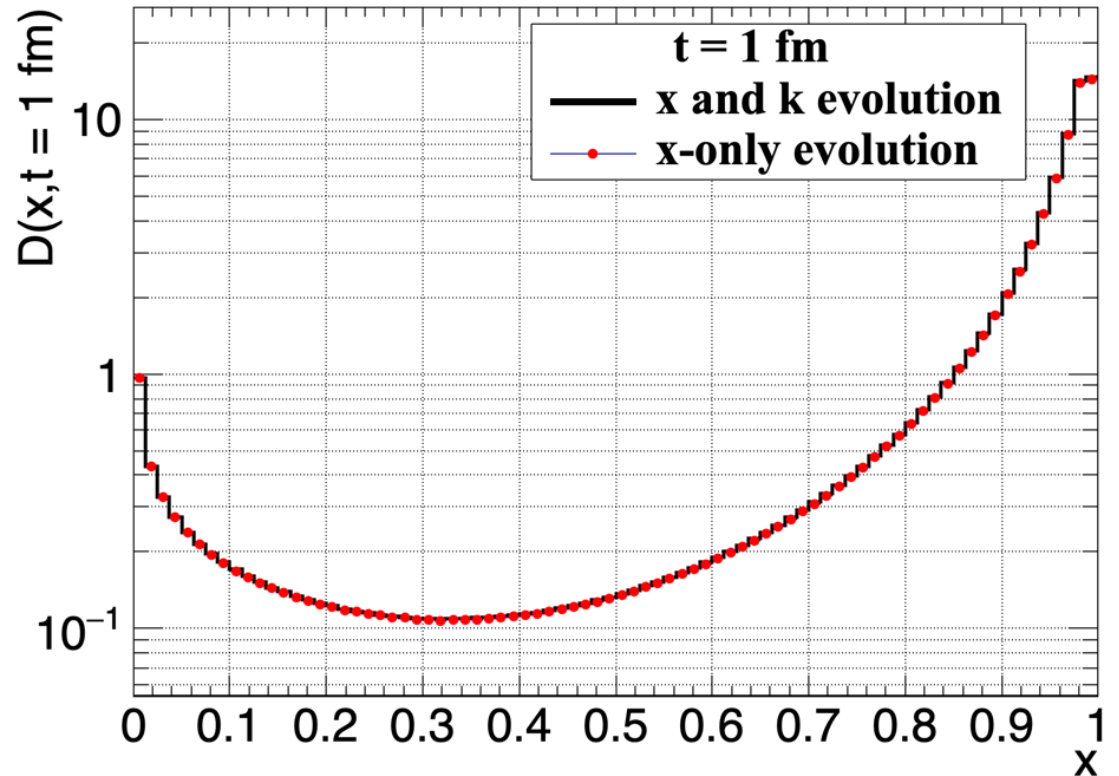
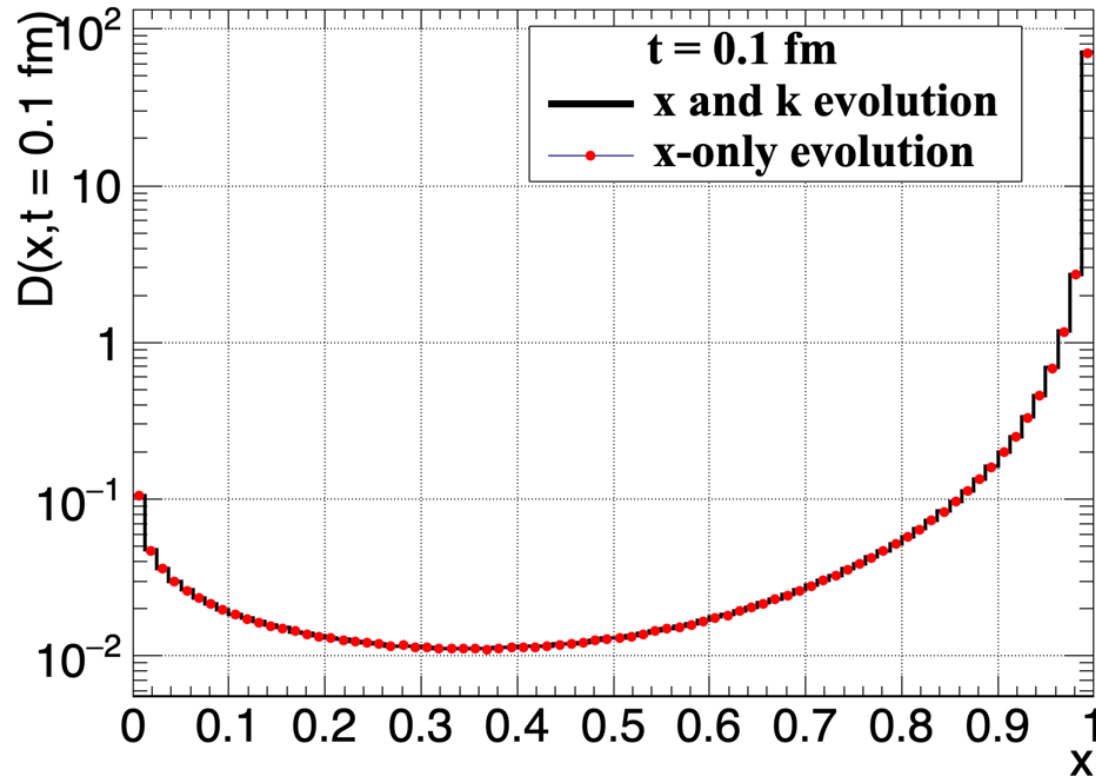
- MINCAS: jet evolution based on coherent emission and scattering
- Combination with KATIE: allows for calculation of jet-observables
- Results differ from pure Gaussian broadening...
- ...e.g.: in angular correlations of di-jets,
- But  $p_T$  distributions seem to be invariant (so far)

# Outlook

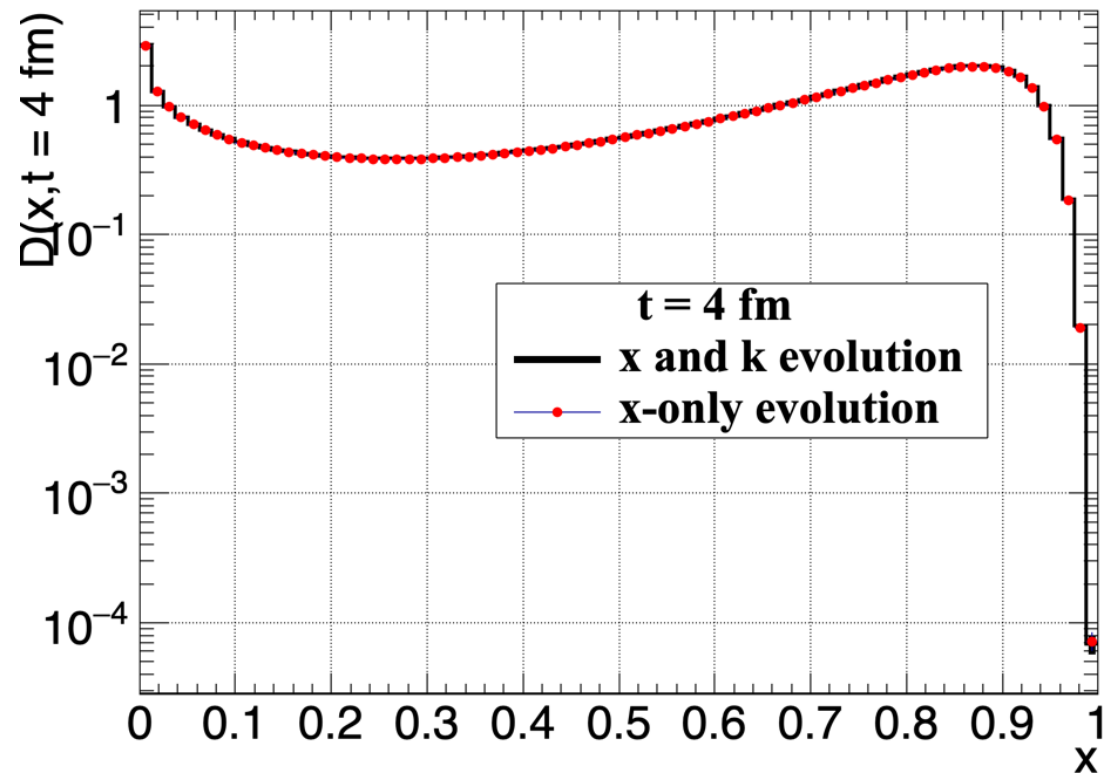
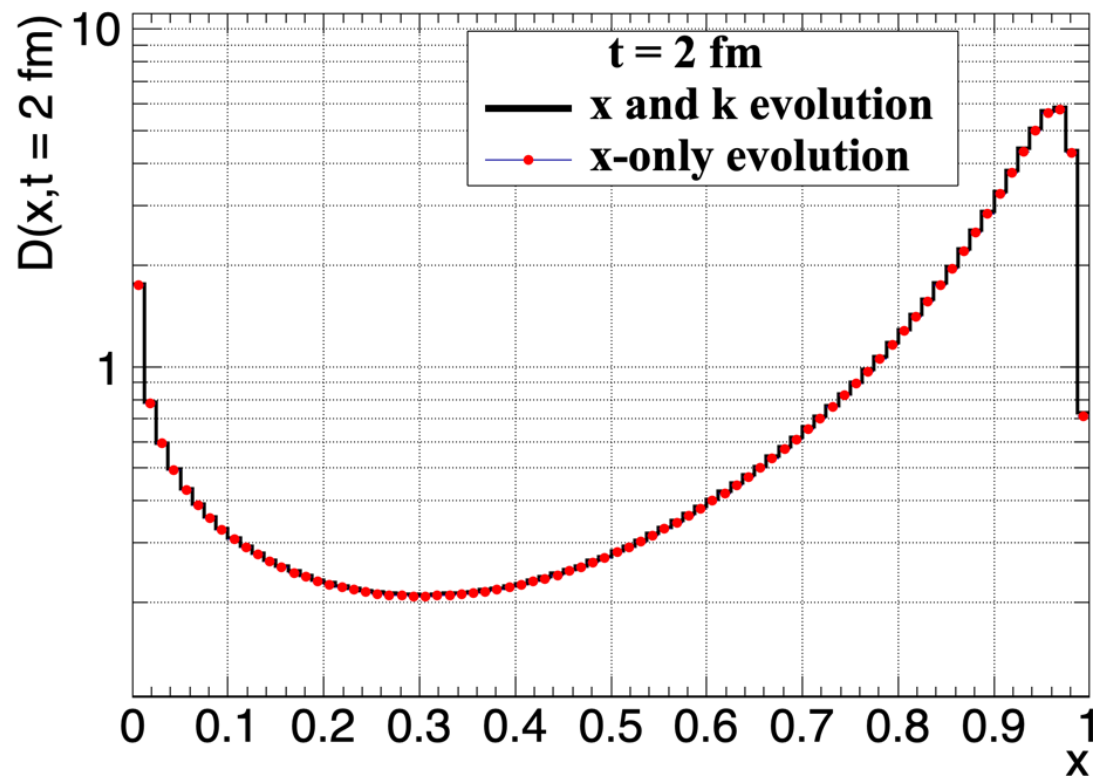
- to account for quarks
- to study more forward processes

# Back-up slides

# Turbulent behavior (1/2)



# Turbulent behavior (2/2)



# Jet Production

Factorization for AA collisions:

$$\frac{d\sigma_{AA}}{d\Omega_p} = \int d\Omega_q \int d^2\mathbf{l} \int_0^1 \frac{d\tilde{x}}{\tilde{x}} \delta(p^+ - \tilde{x}q^+) \delta^{(2)}(\mathbf{p} - \mathbf{l} - \mathbf{q}) D(\tilde{x}, \mathbf{l}, \tau(q^+)) \frac{d\sigma_{pp}}{d\Omega_q}$$

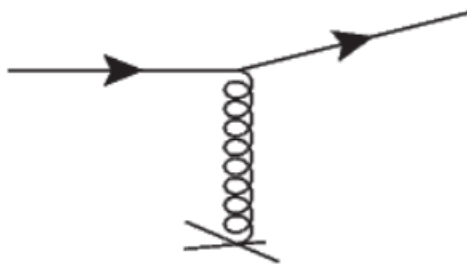
$$d\Omega_q = dq^+ d^2\mathbf{q} \qquad \tau(q^+) = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{q^+}} L$$



$$\begin{aligned} \frac{d^2\sigma_{AA}}{d\Omega_{p_1} d\Omega_{p_2}} &= \int d\Omega_{q_1} \int d\Omega_{q_2} \int d^2\mathbf{l}_1 \int d^2\mathbf{l}_2 \int_0^1 \frac{d\tilde{x}_1}{\tilde{x}_1} \delta(p_1^+ - \tilde{x}_1 q_1^+) \int_0^1 \frac{d\tilde{x}_2}{\tilde{x}_2} \delta(p_2^+ - \tilde{x}_2 q_2^+) \\ &\quad \delta^{(2)}(\mathbf{p}_1 - \mathbf{l}_1 - \mathbf{q}_1) \delta^{(2)}(\mathbf{p}_2 - \mathbf{l}_2 - \mathbf{q}_2) D(\tilde{x}_1, \mathbf{l}_1, \tau(q_1^+)) D(\tilde{x}_2, \mathbf{l}_2, \tau(q_2^+)) \frac{d^2\sigma_{pp}}{d\Omega_{q_1} d\Omega_{q_2}} \end{aligned}$$

# Processes in Jets

scattering...



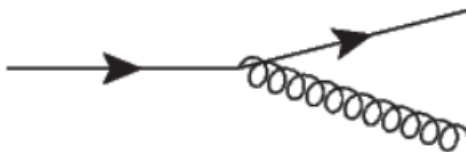
Transverse momentum transfer!

$$p \rightarrow p + k_T$$

Scattering Kernel:  $C(k_T)$

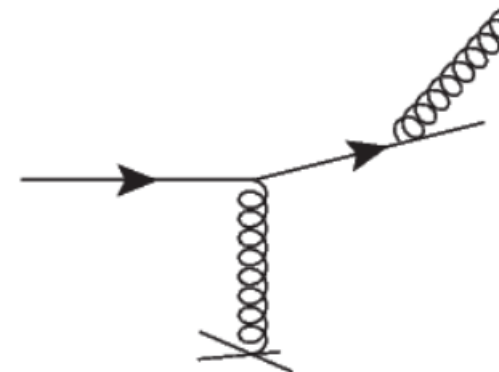
Average transfer:  $\hat{q}$

...splitting...



Bremsstrahlung as in vacuum.

...induced radiation



Momentum distribution:

$$p \rightarrow zp$$

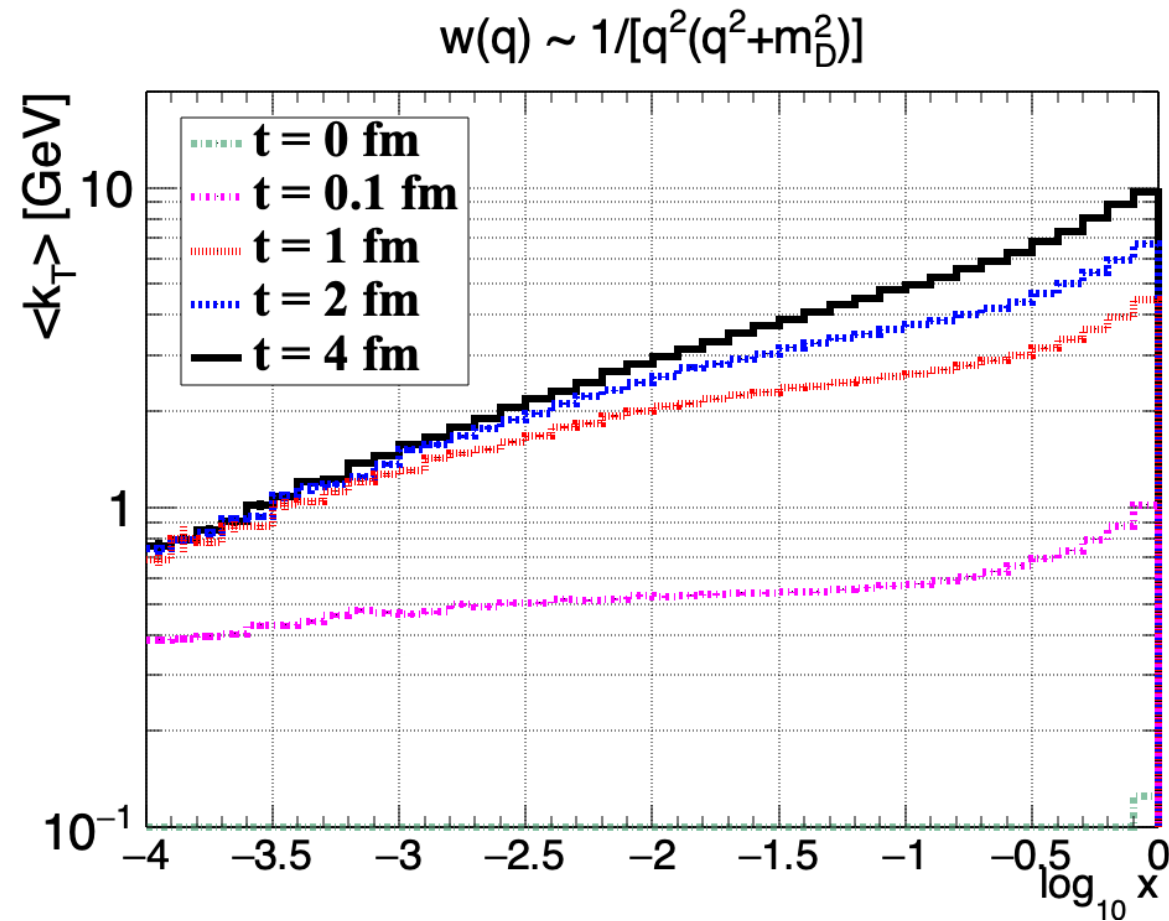
+Momentum transfer:

$$p \rightarrow zp + k_T$$

Kernel:  $\mathcal{K}(z, k_T)$

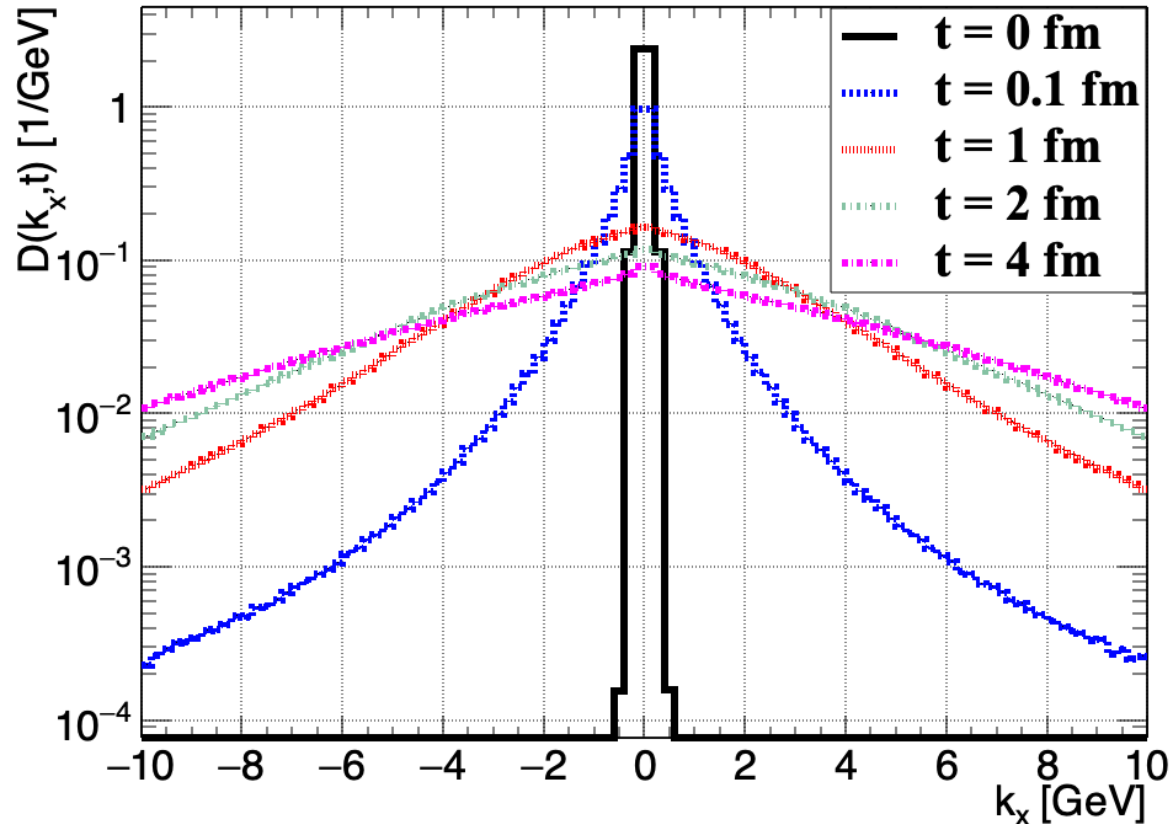
Our results: combination of scattering and induced radiation processes!

# $k_T$ Distribution

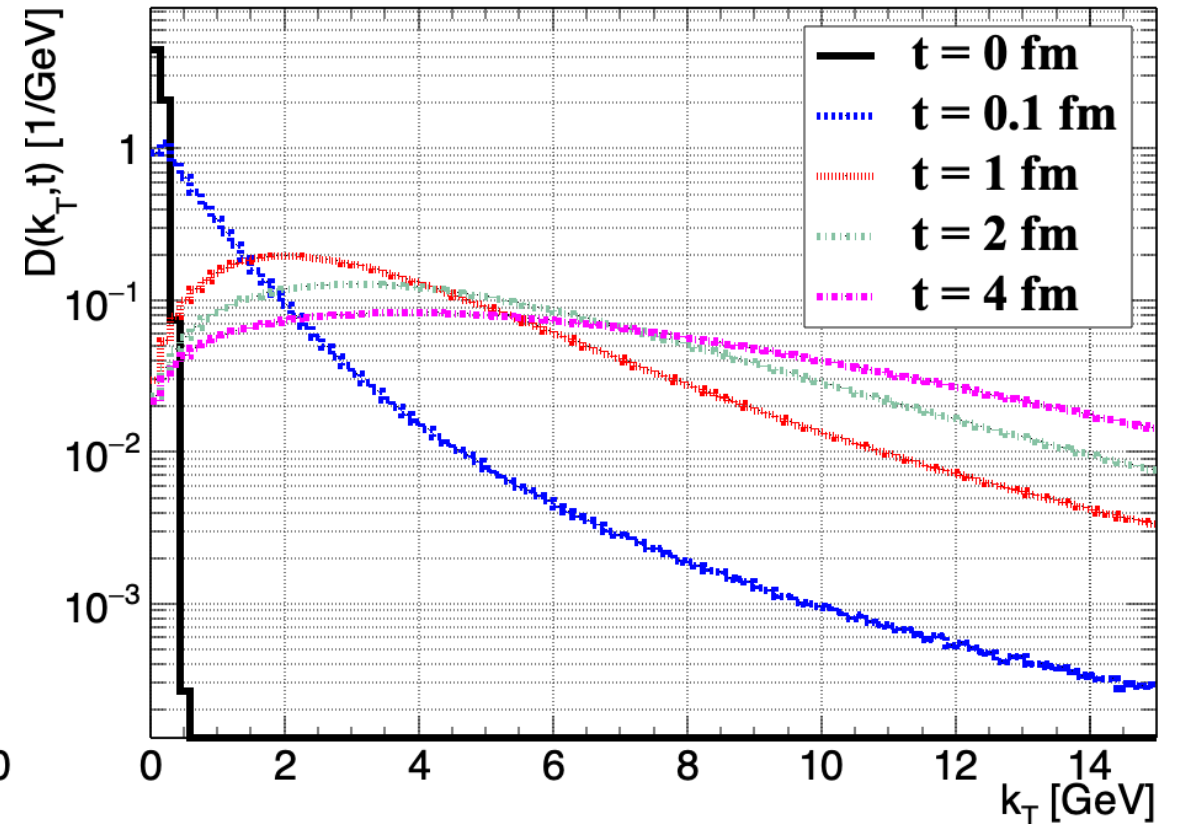


# $k_T$ Distribution

$$f(z) = 1 - z + z^2, \quad x > 10^{-4}$$



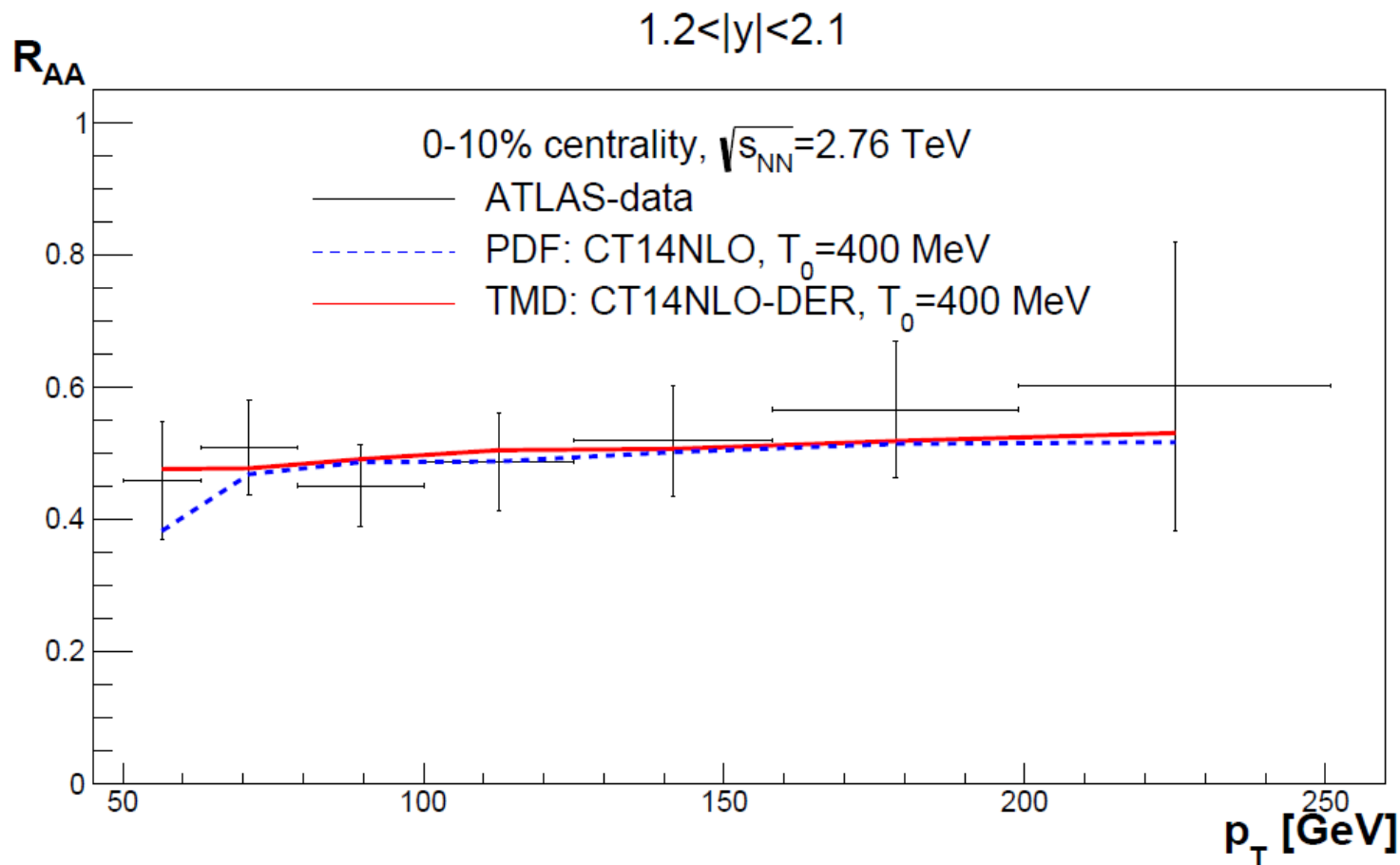
$$f(z) = 1 - z + z^2, \quad x > 10^{-4}$$





# $R_{AA}$

$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle T_{AA} \rangle \frac{d\sigma_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{AA}}{dp_T}}{\frac{d\sigma_{pp}}{dp_T}}$$



# Asymmetry $A_j$

$$A_j = \frac{p_{Tc} - p_{Tf}}{p_{Tc} + p_{Tf}}$$

