Jet quenching and scaling properties of medium-evolved gluon cascade in expanding media

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in collaboration with

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Outline of the talk

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- Medium-induced gluon radiation
- BDMPS-Z Formalism
- Single gluon emission Spectra
- Scaling Properties
- Splitting rates
- Static; exponential; Bjorken Medium
- Singular Splitting kernels
- Full Splitting kernels
- Medium Evolved Gluon Distribution
- Scaling Properties

Based on the recent work: Medium-induced cascade in expanding media; S. P. Adhya, C. A. Salgado, M. Spousta, K. Tywoniuk; arXiv: 1911.12193.
Outline of the talk

Quenching parameter for media

- In the multiple soft scattering approximation (BDMPS) formalism, we start from harmonic oscillator (HO) with a time dependent imaginary frequency \( \Omega(t) \)

\[
\frac{d^2c(t)}{dt^2} + \Omega^2(t)c(t) = 0
\]

\( c(t) \) = information about medium and its expansion

\[
\Omega^2(t) = -i \frac{\hat{q}(t)}{2z(1 - z)p} \quad \kappa(z) = \sqrt{[1 - z(1 - z)]/[z(1 - z)]}
\]

- We only consider gluon splitting.

Average quenching parameter for static and Bjorken media

\[
< \hat{q} > = \frac{2}{L^2} \int_{t_0}^{L + t_0} dt(t - t_0)\hat{q}(t)
\]

Average quenching parameter for exponential medium

\[
< \hat{q} >^{exp} = \frac{2}{L^2} \int_{0}^{\infty} dt\hat{q}(t)
\]

Maximum available gluon energy in medium \( \Rightarrow \)

\[
< \omega_c > = \frac{1}{2} < \hat{q} > L^2
\]

The Bjorken profile modelled as

\[
\hat{q}(t) = \begin{cases} 
0 & \text{for } t < t_0, \\
\hat{q}_0(t_0/t)^\alpha & \text{for } t_0 < t < L + t_0, \\
0 & \text{for } L + t_0 < t.
\end{cases}
\]
The single gluon emission spectra are given as:

\[ \frac{dI}{dz}^{static,soft} \simeq \frac{\alpha_s}{\pi} P(z) \sqrt{\frac{\omega_c}{2\omega}} \]

\[ \frac{dI}{dz}^{static} = \frac{\alpha_s}{\pi} P(z) \text{Re} \ln[\cos(\Omega_0 L)] \]

\[ \frac{dI}{dz}^{expo} = \frac{\alpha_s}{\pi} P(z) \text{Re} \ln J_0(2\Omega_0 L) \]

\[ \frac{dI}{dz}^{BJ} = \frac{\alpha_s}{\pi} P(z) \text{Re} \left[ \left( \frac{t_0}{L+t_0} \right)^{1/2} \frac{J_1(z_0)Y_0(z_L) - Y_1(z_0)J_0(z_L)}{J_1(z_L)Y_0(z_L) - Y_1(z_L)J_0(z_L)} \right] \]

\[ P_{gg} = 2C_A \frac{(1-z)(1-z)}{z(1-z)} \]

\[ \tau \equiv \sqrt{\frac{\hat{q}_0 L}{p}} \]

\[ z_0 \equiv (1-i)\kappa(z)\tau_0 \]

\[ z_L \equiv (1-i)\kappa(z)\sqrt{\tau_0(\tau + \tau_0)} , \]

Effective parameter

\[ \frac{dI}{dz}^{static,sing} \simeq \frac{dI}{dz}^{expo,sing} \simeq \frac{dI}{dz}^{BJ,sing} \]

The singular spectra can be re-scaled

\[ \omega_{eff} = \begin{cases} \frac{1}{2}\hat{q}_0 L^2 & \text{static medium} \\ 2\hat{q}_0 L^2 & \text{exponentially expansion} \\ 2\hat{q}_0 t_0 L & \text{Bjorken expansion} \end{cases} \]

\[ \hat{q}_{eff}^{expo} = 4\hat{q}_0 \]

\[ \hat{q}_{eff}^{BJ} = 4\hat{q}_0 t_0 / L \]
• The BDMPS soft ($w < w_c$) has a constant splitting rate independent of the time of evolution of the plasma.
• The rates for all the profiles except the BDMPS soft are similar at very low evolution time or length of the medium.
• In the Bjorken, the presence of pre-factor $\tau_0/(\tau_0 + \tau)$ leads to the dumping of the splitting rate for $\tau > \tau_0$. 

\[
\tau_{\text{eff}} = \begin{cases} 
\tau & \text{static medium} \\
2\tau & \text{exponential medium} \\
2\sqrt{\tau_0\tau} & \text{Bjorken medium}
\end{cases}
\]

\[\mathcal{K}(z, \tau) \equiv \frac{dI}{dz\,d\tau}\]
• At high values of $z$ the rates for different profiles differ significantly.

• At the low-$z$ values all profiles have universal slope due to $P(z)\kappa(z)$ factor present in splitting rates of all the profiles which diverges for $z \to 0$ as $z^{-3/2}$.

• We recover a universal behavior of parton evolution for expanding media in the soft gluon regime with $\tau_{\text{eff}}$. 
Medium evolved gluon spectra

- The kinematic evolution equation (GAIN + LOSS terms) in terms of gluon spectra:

\[
\frac{\partial D(x, \tau)}{\partial \tau} = \int dz K(z, \tau | p) \left[ \sqrt{z} \mathcal{D} \left( \frac{x}{z}, \tau \right) - \sqrt{z} \mathcal{D}(x, \tau) \right]
\]

- The numerical value of the BDMPS soft spectra agrees with the analytical result (check of the numerical routine).

- At low \( z \), we see a \( 1/(\sqrt{x}) \) behaviour of all the profiles \( \gg \) recovered from the similar gluon splitting at low \( z \).

\[
D(x, \tau) = \frac{\tau}{\sqrt{x} (1 - x)^{3/2}} e^{-\pi \frac{\tau^2}{1 - x}}
\]
• Scaled gluon spectra (singular and full kernels)

Gluon distribution:

A. **Singular** rates $\Rightarrow$ Nice scaling in $\tau_{\text{eff}}$.

B. **Full** rates $\Rightarrow$ No scaling in $\tau_{\text{eff}}$

$\tau_{\text{eff}} = \begin{cases} 
\tau & \text{static medium} \\
2\tau & \text{exponential medium} \\
2\sqrt{\tau_0 \tau} & \text{Bjorken medium}
\end{cases}$
Moments of the distribution and $Q_{AA}$

- The yield for the inclusive jet suppression can be obtained as a convolution of the $D(x, \tau)$ distribution (gluon spectra) with the initial parton spectra,
  \[
  \frac{d\sigma_{AA}}{dp_T} = \int dp_T' \int_0^1 \frac{dx}{x} \delta(p_T - xp_T')D\left(x, \tau \equiv \sqrt{\hat{q}/p_T'}\right) \frac{d\sigma_0}{dp_T'}
  \]

- The jet suppression factor:
  - We include only one parton species (gluons); thus $Q_{AA}$ is a proxy for $R_{AA}$
  - Approximated by power law
  \[
  d\sigma_0/dp_T \propto p_T^{-N}
  \]
  We assume that we have a steeply falling hard spectrum, with $n = 5.6$

- Analytical insight: We start with the "singular" kernels and its scaling to arrive at,
  \[
  Q(p_T) = \int_0^1 dx x^{N-1} D(x, \sqrt{x\tau})
  \]

- Due to the additional, explicit dependence on the ratio $(t_0/L)$, we conclude that in the case of the Bjorken expansion there is no universal way of rescaling the parameters to arrive at the results of the static medium.

\[
\frac{Q_{AA}^{exp}}{Q_{AA}^{static}} \simeq \exp \left[-2\tilde{\alpha} \sqrt{\pi \hat{q}_0 L^2 (n-1)/p_T}\right]
\]

\[
\frac{Q_{AA}^{Bjork}}{Q_{AA}^{static}} \simeq \exp \left[-2\tilde{\alpha} \sqrt{\pi \hat{q}_0 L^2 (n-1)/p_T} \left(2\sqrt{\frac{t_0}{L}} - 1\right)\right]
\]
• The numerical value of the BDMPS static (soft) agrees with the analytical result.

• The uncertainty band is obtained by varying $\alpha_s$ by 10%.

• A large difference can be seen for different media due to varying rate of expansions.
• Replace $\omega_c$ values for exponential case and Bjorken case by $\omega_{\text{eff}}$ for singular kernels.

• Recover nice scaling features as in singular rates and spectra.

The Bjorken profile depends on additional choice of $(\tau_0/\tau)$: No universal scaling

$\omega_{\text{opt}}$ by $\chi^2$ fit.

Good, but not perfect scaling is achieved by minimization.

Scaling for expo medium $\sim$ average scaling.
Summary and discussions

• We study the impact of the expansion of de-confined medium on single-gluon emission spectrum, its re-summation and the jet suppression factor \((QAA)\) within the BDMPS-Z formalism.

• The distribution of medium-induced gluons is calculated using an evolution equation with splitting kernels derived from the gluon emission spectra. A universal behaviour of splitting kernels seen for soft gluon emissions for effective evolution time \(\tau_{\text{eff}}\).

• For realistic spectra valid beyond the soft-gluon emission limit, these scaling features are partially replaced by a scaling expected from considering an averaged jet quenching parameter along the trajectory of propagation.

• Appropriate choice of the quenching parameter can sort out differences between different medium expansions.

• Sizable differences among the values of the quenching parameter for different types of medium and kinematical ranges point to the importance of precise modelling of the jet quenching phenomenon.

Thank you
Revealing scaling feature of the spectrum

Scaling properties of the full spectra:
• The insets provide analytical estimates from singular spectra only.

\[ \frac{dI_{\text{exp, BJ}}(\langle \hat{q} \rangle)}{d\omega} \bigg/ \frac{dI_{\text{static}}(\langle \hat{q} \rangle)}{d\omega} = \sqrt{2} \]
for \( \omega \ll \langle \hat{q} \rangle \sim L^2 \)

\[ \frac{dI_{\text{exp, BJ}}(\langle \hat{q} \rangle)}{d\omega} \bigg/ \frac{dI_{\text{static}}(\langle \hat{q} \rangle)}{d\omega} = 3/4 \]
for \( \omega \gg \langle \hat{q} \rangle \sim L^2 \)

\[ \frac{dI_{\text{exp, BJ}}(\hat{q}_{\text{eff}})}{d\omega} \bigg/ \frac{dI_{\text{static}}(\hat{q}_{\text{eff}})}{d\omega} = 3 \]
for \( \omega \ll \hat{q}_{\text{eff}} L^2 \)

\[ \frac{dI_{\text{exp, BJ}}(\hat{q}_{\text{eff}})}{d\omega} \bigg/ \frac{dI_{\text{static}}(\hat{q}_{\text{eff}})}{d\omega} = 3/16 \]
for \( \omega \gg \hat{q}_{\text{eff}} L^2 \)

• Nice scaling for hard sector in left panel.
• Nice scaling for soft sector in right panel.