Simple implementation of color coherence for the resumation of soft BDMPS-Z gluons

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Motivation

hep-ph/9607355, hep-ph/9608322, hep-ph/9704255, hep-ph/9804212
1311.5823, 1205.5739

$$k_f^2 \sim \sqrt{\omega \hat{q}}$$

$$\omega_c \sim \hat{q} L^2$$

Gluon production in the BDMPS-Z framework is dominated by the soft modes $\omega \frac{\mathrm{dI}}{\mathrm{d}\omega} \sim \sqrt{\frac{\omega_c}{\omega}}$

$\boxed{1}$ As a consequence the QM formation time $t_f \sim \sqrt{\frac{\omega}{\hat{q}}}$ for these gluons is very short when compared with the medium length L.

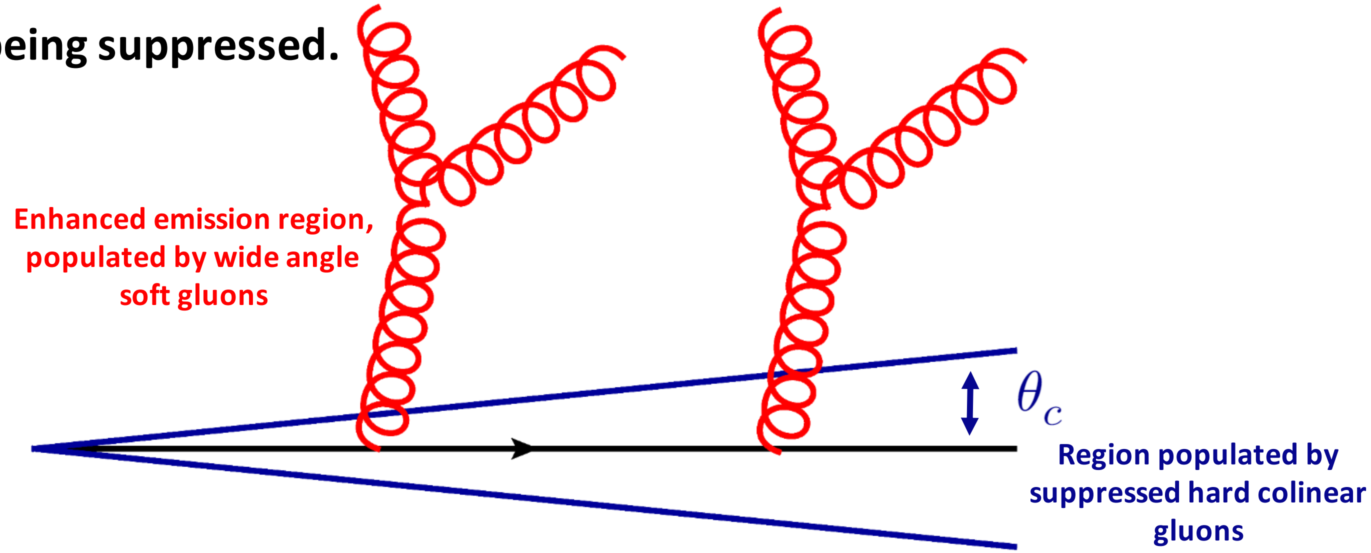
$\boxed{2}$ The typical angle of emission $\theta_f^2 \sim \sqrt{\frac{\hat{q}}{\omega^3}}$ for such gluons is parametically much larger than the critical emission angle $\theta_c^2 \sim \frac{k_f^2}{\omega_c^2} \sim \frac{1}{\hat{q} L^3}$, i.e. the typical emission angle for gluons with a formation time of the order of L.

$\boxed{3}$ In the regime $L \gg t_f$ the P.S. for emissions is propotional to L, i.e. gluons can be emitted at any point in the medium.

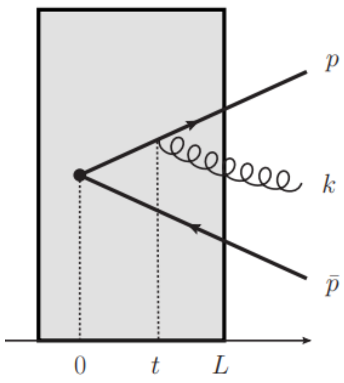
$\boxed{4}$ Interferences between emitters scale with t_f, being suppressed.

To sum up, the dominant gluons are

- Wide angled and soft $\boxed{1}$ $\boxed{2}$
- Produced decoherently, instantaneously and at any point in the medium $\boxed{3}$ $\boxed{4}$



Motivation



Later on, this picture was improved by studying the QCD antenna in the medium.

This generates an additional scale that controls decoherence

$$\theta_c < \theta < \theta_f$$

In either case, as long as $\theta \gg \theta_c$

In this regime color coherence controls interferences → The P.S. for emission will go as ← $\theta_c < \theta_f < \theta$

$$L - \max(t_f, t_d) \sim L$$

$$1 - e^{-\left(\frac{\theta}{\theta_c}\right)^2} \sim \text{“}\Theta(\theta > \theta_c)\text{”}$$

Color coherence is quickly lost and QM coherence controls interferences

(previous slide)

Focusing on color coherence effects, the “step function” picture is only true parametically and could not hold for all emissions.

Can we combine multiple soft emissions with color coherence effects?

Overview

Goal: Introduce color coherence effects into soft medium induced gluon production.

Today

How:

- Compute the spectrum for the emission of a BDMPS-Z gluon followed by a soft gluon splitting.
- Find an effective way of recasting such a spectrum in terms of modified BMDPS-Z emission kernel.
- Introduce this modification into a full shower.

- Compare to fully (color and QM) decoherent shower.

The rate equations

$$\begin{cases} \mathbf{k}' \gg k_f \to \text{Exp. suppresion} \\ \mathbf{k}' \ll k_f \to \text{LPM regime} \end{cases}$$

The dominant piece of the BDMPS-Z spectrum reads

$$\omega \frac{\mathrm{dI}}{d\omega d^2\mathbf{k}} \sim \frac{\bar{\alpha}}{\omega} \int_{t_f}^{L} dt \int_{\mathbf{k}'} \mathcal{P}(\mathbf{k}-\mathbf{k}', L-t) \sin\left(\frac{\mathbf{k}'^2}{2k_f^2}\right) e^{-\frac{\mathbf{k}'^2}{2k_f^2}}$$

Late time emissions

Classical broadening after splitting $\otimes$ Emission kernel $\mathbf{k}' \sim k_f$ $k_f^2 \sim \sqrt{\omega \hat{q}}$

can be understood as a two step process: medium induced gluon emission followed by momentum smearing.

Lifting the soft gluon approximation, this still holds in the small formation time picture. Two major approximations:

- Formation time can be ignored compared to any other time scale (in particular the medium length);
- The colour structure of the outoging states, is at leading order, that of 2 independent partons.

$\mathbf{s} \sim \sqrt{\hat{q} t_f}$

$$\omega \frac{dI}{d\Omega_k d\Omega_q} \sim \mathcal{P}_2(\mathbf{k},\mathbf{q},z;L,t_0) = 2g^2 z(1-z) \int_{t_0}^{L} \mathrm{d}t \int_{\mathbf{s}} \mathcal{K}(\mathbf{s},z,p_0^+) \int_{l} \mathcal{P}_1(\mathbf{k}-\mathbf{l}z-\cancel{\mathbf{s}};L,t)\mathcal{P}_1(\mathbf{q}-(1-z)\mathbf{l}+\cancel{\mathbf{s}};L,t)\mathcal{P}_1(\mathbf{l}-\mathbf{p}_0;t,t_0)$$

$$\mathcal{K}(\mathbf{l},p_0^+,\cancel{t}) = \frac{2}{p_0^+}\frac{P(z)}{z(1-z)} \sin\left(\frac{\mathbf{l}^2}{2k_f^2}\right) e^{-\frac{\mathbf{l}^2}{2k_f^2}}$$

$P(z)$ Vacuum splitting function

$$\begin{cases} \int_t \mathcal{K} \sim L & \text{Leading emission term} \\ \int_{\mathbf{l}} \mathcal{K} = \frac{P(z)}{2\pi}\sqrt{\frac{\hat{q}}{\omega}} \end{cases}$$

$$\mathcal{P}_1(\mathbf{k},t,s) = \frac{4\pi}{\hat{q}(t-s)} \exp\left(-\frac{\mathbf{k}^2}{\hat{q}(t-s)}\right)$$

in the harmonic approximation

The rate equations

Due to the factorization of the full emission process one can resum all emissions (as in DGLAP).

The generating functional is build up from 2 blocks: single particle broadening and in-medium branching.

$$\mathcal{Z}_{p_0}(t,t_0\,|\,u)=\sum_{n=1}^{\infty}\frac{1}{n!}\int_{\Omega_i}P_n(\vec{k}_1,\ldots,\vec{k}_n;t,t_0)u(\vec{k}_1)\ldots u(\vec{k}_n)$$

Block 1: Broadening

Diffusion equation in momentum space

$$\mathcal{C}(\mathbf{r})=-\frac{\hat{q}}{4}\mathbf{r}^2$$

$$\partial_t\mathcal{P}_1(\mathbf{k};t,t_0)=\int_{\mathbf{l}}\mathcal{C}(\mathbf{l})\mathcal{P}_1(\mathbf{k}-\mathbf{l};t,t_0)$$

Block 2: Branching

Truncate external legs

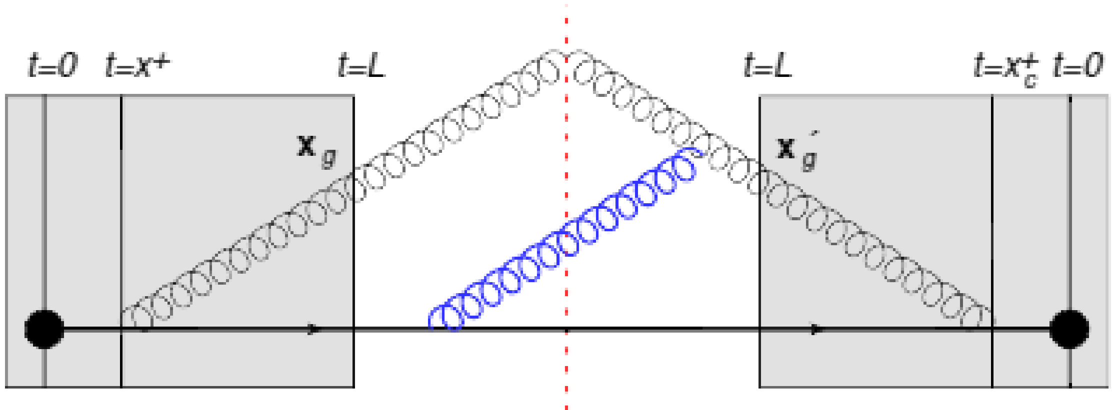
$$\widetilde{\mathcal{P}}_2(\mathbf{k},\mathbf{q},z;L,t_0)=2g^2z(1-z)\int_{t_0}^{L}dt\;\mathcal{K}(z,p_0^+)$$
$$\times(2\pi)^4\delta^{(2)}(\mathbf{k}-z\mathbf{p}_0)\delta^{(2)}(\mathbf{q}-(1-z)\mathbf{p}_0)$$

One can write down a simple time evolution equation for the gluon inclusive distribution: $D(x,\mathbf{k},t)\equiv k^+\left(\frac{\delta\mathcal{Z}_{p_0}(t,t_0\,|\,u)}{\delta u(\vec{k})}\right)_{u=1}$

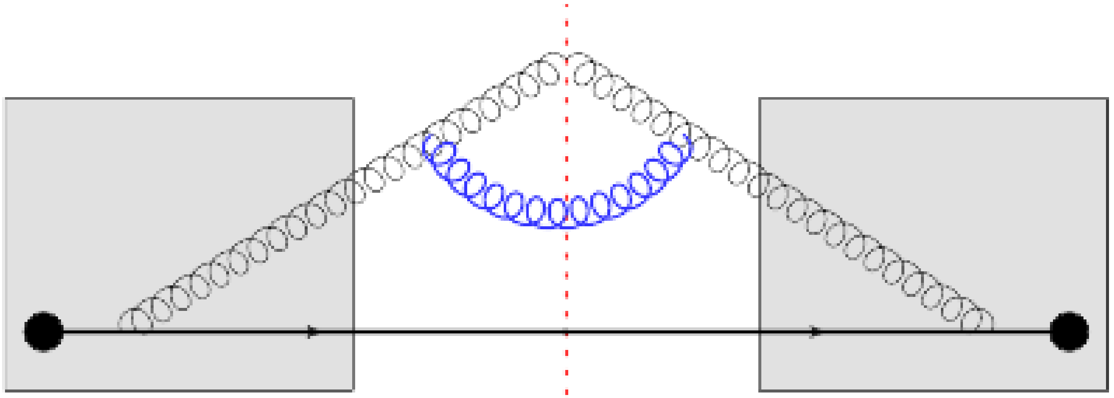
$$\partial_t D(x,\mathbf{k},t)=\int_{\mathbf{l}}C(\mathbf{l},t)D(x,\mathbf{k}-\mathbf{l},t)+\alpha_s\int_z\left[\frac{2}{z^2}\mathcal{K}\left(z,\frac{x}{z}p_0^+\right)D\left(\frac{x}{z},\frac{\mathbf{k}}{z}\right)\Theta(z-x)-\mathcal{K}\left(z,xp_0^+\right)D(x,\mathbf{k},t)\right]$$

Interferences in the soft limit

We start by computing the spectrum of emission of a medium-induced gluon, followed by a soft vacuum gluon.



Interference diagram



Direct diagram

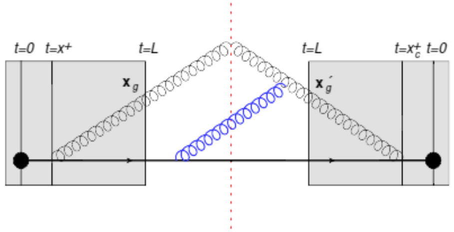
The role of the second gluon is to measure the color coherence of the system:

Small emission angles

The system keeps coherent and the new emission plays no role.

Large emission angles

The Phase Space (P.S.) for emission is vanishinghly small since both emitters decohere.



Interferences in the soft limit

$$\left(\omega\frac{dI}{d\omega d^2\mathbf{k}}\right)^{\mathrm{g}} = \frac{\alpha_s C_A}{(2\pi^2)\mathbf{k}^2}$$

We obtain

$$\omega\omega'\frac{dI}{d^2\mathbf{k}d^2\mathbf{k}'d\omega d\omega'} = \left(\omega'\frac{dI}{d\omega' d^2\mathbf{k}'}\right)^{\mathrm{g}}\frac{2C_F\alpha_s}{(2\pi^2)\omega^2 N_c(N_c^2-1)^2}\mathrm{Re}\Bigg[\int_{\mathbf{x}_g\mathbf{x}'_g x^+ x_c^+ \mathbf{z}} e^{i\mathbf{k}(\mathbf{x}'_g-\mathbf{x}_g)}\partial_{\mathbf{x}}|_0\cdot\partial_{\mathbf{x}'}|_0\,\mathrm{Tr}\langle G(\mathbf{x},\mathbf{z}\,|\,k^+)W(\mathbf{0})\rangle_{x_c^+,x^+}\,\boxed{\times f^{jia}f^{cbd}\langle W^{ac}(\mathbf{0})G^{jb}(\mathbf{z},\mathbf{x}_g\,|\,\omega)G^{\dagger id}(\mathbf{x}'_g,\mathbf{x}'\,|\,\omega)\rangle_{L,x_c^+}}\Bigg]$$

while the BDMPS-Z result reads.

$$\left(\omega\frac{dI}{d^2\mathbf{k}d\omega}\right)^{\mathrm{In-In}} = \frac{2C_F\alpha_s}{(2\pi^2)\omega^2(N_c^2-1)^2}\mathrm{Re}\Bigg[\int_{\mathbf{x}_g\mathbf{x}'_g x^+ x_c^+ \mathbf{z}} e^{i\mathbf{k}(\mathbf{x}'_g-\mathbf{x}_g)}\partial_{\mathbf{x}}|_0\cdot\partial_{\mathbf{x}'}|_0\;\mathrm{Tr}\langle G(\mathbf{x},\mathbf{z}\,|\,k^+)W(\mathbf{0})\rangle_{x_c^+,x^+}\,\boxed{\mathrm{Tr}\langle G(\mathbf{z},\mathbf{x}_g\,|\,\omega)G^{\dagger}(\mathbf{x}',\mathbf{x}'_g\,|\,\omega)\rangle_{L,x_c^+}}\Bigg]$$

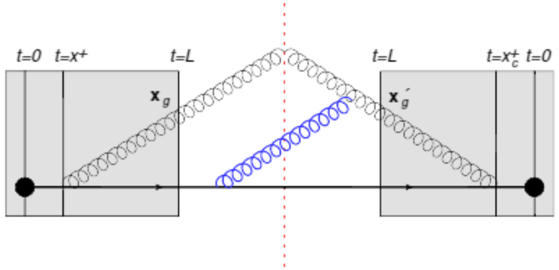
$$G(\mathbf{x},L;\mathbf{y},t_0) = \int_{\mathbf{y}}^{\mathbf{x}}\mathcal{D}\mathbf{r}(\xi)\exp\left(\frac{i\omega}{2}\int_{\xi}\dot{\mathbf{r}}^2(\xi)\right)\times W(\mathbf{r}(\xi))_{L,t_0} \qquad W(\mathbf{x})_{L,t_0} = \mathcal{P}\exp\left(ig\int_t A_-(t,\mathbf{x})\right)$$

In the harmonic approximation:

$$\boxed{\int_{\mathbf{z}}^{\mathbf{x}_g}\mathcal{D}\mathbf{r}_1\int_{\mathbf{x}'}^{\mathbf{x}'_g}\mathcal{D}\mathbf{r}_2\exp\left(\int_t\frac{i\omega}{2}(\dot{\mathbf{r}}_1^2-\dot{\mathbf{r}}_2^2)-\frac{\hat{q}}{8}\int_t(\mathbf{r}_2^2+\mathbf{r}_1^2+(\mathbf{r}_2-\mathbf{r}_1)^2)\right) = \int_{\mathbf{z}}^{\mathbf{x}_g}\mathcal{D}\mathbf{r}_1\int_{\mathbf{x}'}^{\mathbf{x}'_g}\mathcal{D}\mathbf{r}_2\exp\left(\int_t\frac{i\omega}{2}(\dot{\mathbf{r}}_1^2-\dot{\mathbf{r}}_2^2)-\frac{\hat{q}}{4}\int_t(\mathbf{r}_1\cdot\mathbf{r}_2+(\mathbf{r}_2-\mathbf{r}_1)^2)\right)}$$

Semiclassical approximation

$$\int_{\mathbf{z}}^{\mathbf{x}_g} \mathcal{D}\mathbf{r}_1 \int_{\mathbf{x}'}^{\mathbf{x}'_g} \mathcal{D}\mathbf{r}_2 \exp\left(\int_t \frac{i\omega}{2}(\dot{\mathbf{r}}_1^2 - \dot{\mathbf{r}}_2^2) - \frac{\hat{q}}{4}\int_t (\mathbf{r}_1 \cdot \mathbf{r}_2 + (\mathbf{r}_2 - \mathbf{r}_1)^2)\right)$$



The above object can be computed in the harmonic approximation exactly. After some manipulations, it gives rise to two decoupled HO path integrals:

$$\int_{\gamma(\mathbf{z}-\beta\mathbf{x}')}^{\gamma(\mathbf{x}_g-\beta\mathbf{x}'_g)} \mathcal{D}\mathbf{r}_1(t) \int_{\gamma(\mathbf{x}'-\beta\mathbf{z})}^{\gamma(\mathbf{x}'_g-\beta\mathbf{x}_g)} \mathcal{D}\mathbf{r}_2(t) \exp\left(\int_{x_c^+}^{L} dt \frac{i\omega}{2}\dot{\mathbf{r}}_1^2 - \frac{\hat{q}\sqrt{3}}{8}\mathbf{r}_1^2\right) \exp\left(\int_{x_c^+}^{L} dt - \frac{i\omega}{2}\dot{\mathbf{r}}_2^2 - \frac{\hat{q}\sqrt{3}}{8}\mathbf{r}_2^2\right) \equiv \mathcal{J}(\mathbf{b}_g, \mathbf{b})_{L,x_c^+} \mathcal{J}^\dagger(\mathbf{c}_g, \mathbf{c})_{L,x_c^+}$$

$$\mathbf{b}_g = \gamma(\mathbf{x}_g - \beta\mathbf{x}'_g) \quad , \quad \mathbf{b} = \gamma(\mathbf{z} - \beta\mathbf{x}') \quad , \quad \mathbf{c}_g = \gamma(\mathbf{x}'_g - \beta\mathbf{x}_g) \quad , \quad \mathbf{c} = \gamma(\mathbf{x}' - \beta\mathbf{z}) \qquad \beta = 2 - \sqrt{3} \quad \gamma = \frac{1}{i\sqrt{6 + 4\sqrt{3}}}$$

This result is not useful for our purposes. Therefore, we try to gauge the effects of colour coherence so that we can rewrite the above result as:

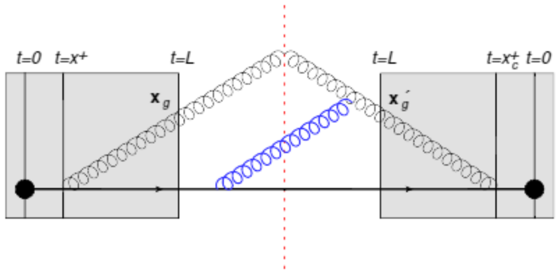
$$\boxed{\mathcal{P}_1(\mathbf{k}; t)\boxed{C(\mathbf{l}, t)}}$$

Since C is not a function of final momentum, it can be recast as a correction to the branching kernel

Factor encapsulating color coherence

Semiclassical approximation

$$\int_{\mathbf{z}}^{\mathbf{x}_g} \mathcal{D}\mathbf{r}_1 \int_{\mathbf{x}'}^{\mathbf{x}'_g} \mathcal{D}\mathbf{r}_2 \exp\left(\int_t \frac{i\omega}{2}(\dot{\mathbf{r}}_1^2 - \dot{\mathbf{r}}_2^2) - \frac{\hat{q}}{4}\int_t \big(\underbrace{\mathbf{r}_1\cdot\mathbf{r}_2}_{\text{Coherence piece}} + \underbrace{(\mathbf{r}_2-\mathbf{r}_1)^2}_{\text{Dipole piece}}\big)\right)$$



Our goal: extract the color coherence factor from the above result as a correction to the single particle broadening.

What we do: write the gluon propagator in the semiclassical approximation, i.e. $G(\mathbf{x}) \sim G_0(\mathbf{x}_{\mathrm{cl.}})W(\mathbf{x}_{\mathrm{cl.}})$

$$\mathbf{r}_1(t) = \mathbf{x} + \mathbf{z} + t\theta \sim \mathbf{x} + (t+t_f)\theta$$

$$\mathbf{r}_2(t) = \mathbf{x}' + t\theta$$

$$(\mathbf{r}_1\cdot\mathbf{r}_2)(\Delta t) \sim (\Delta t\theta)\cdot(\Delta t\theta) = (\Delta t)^2\theta^2$$

$$1 - \Delta_{med} \equiv \exp\left(-\frac{\hat{q}}{12}\theta^2(\Delta t)^3\right)$$

$$\Delta t = L - x_c^+$$

Implementation in the rate equations

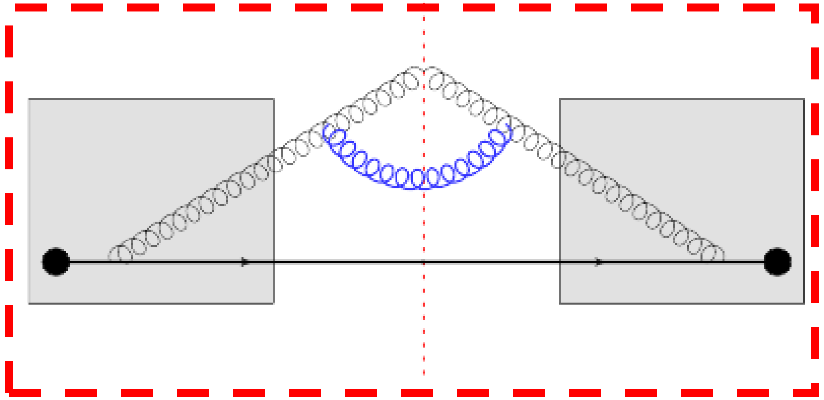
Given the previous result, we can modify the rate equations. To do this we update the branching building block using

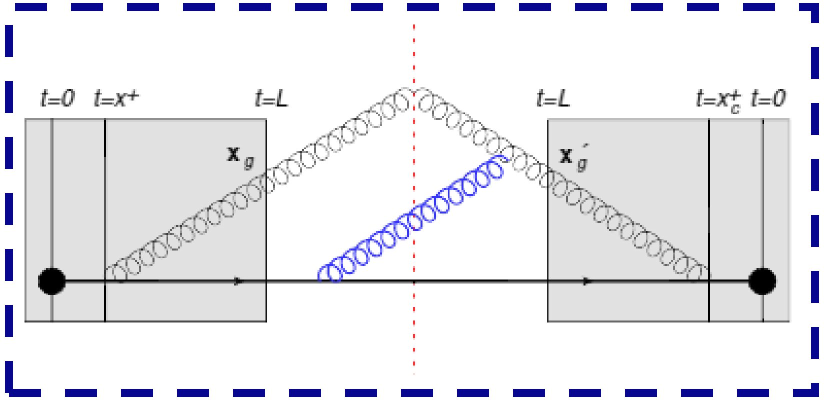
$$\mathcal{P}_1(\mathbf{k}-z\mathbf{l};L,t)\exp\left(-\frac{\hat{q}}{12}\theta^2(L-t)^3\right)$$

We still have to specify what is the emission angle

Before doing that, we must however also include the direct diagrams so that the actual correction is

$$\Delta_{med} = \boxed{1} - \boxed{\exp\left(-\frac{\hat{q}}{12}\theta^2(\Delta t)^3\right)}$$





Implementation in the rate equations

Given the previous result, we obtain a new term in the evolution for the branching building block

$$\partial_L \widetilde{\mathcal{P}_2}(\mathbf{k},\mathbf{p},z;L,t_0) = 2g^2 z(1-z)\int_{\mathbf{l}} \mathcal{K}(\mathbf{l},z,p_0^+)\delta(\mathbf{k}-\mathbf{l}-z\mathbf{p}_0)\delta(\mathbf{p}+\mathbf{l}-(1-z)\mathbf{p}_0)\,(1-1)$$
$$+\int_{t_0}^{L}\int_{\mathbf{l}} \mathcal{K}(\mathbf{l},z,p_0^+)\partial_L\Delta_{med}(z,\mathbf{l},p_0^+,L-t)\delta(\mathbf{k}-\mathbf{l}-z\mathbf{p}_0)\delta(\mathbf{p}+\mathbf{l}-(1-z)\mathbf{p}_0)$$

Here we have defined the emission angle: $\theta = \dfrac{\mathbf{l}}{z(1-z)p_0^+}$

We amputate the broadening probabilities for external legs, but this does not affect the coherence factor

After performing the last time integration we obtain a simple correction to the initial branching kernel

$$\partial_t \mathcal{Z}_{p_0}(t,t_0\mid u) = \int_{\mathbf{l}} C(\mathbf{l},t_0)u(p_0^+,\mathbf{p}_0+\mathbf{l}) + \alpha_s\int_z\int_{\mathbf{l}}\mathcal{K}(\mathbf{l},z,p_0^+)\Delta_{med}(z,\mathbf{l},t)\left[u(zp_0^+,\mathbf{p}_0\cancel{-\mathbf{l}})u((1-z)p_0^+,(1-z)\mathbf{p}_0\cancel{+\mathbf{l}}) - u(p_0^+,\mathbf{p}_0\cancel{+\mathbf{l}})\right]$$

To write this in a similar form to the fully decoherent case, we can still ignore momentum transfer in the broadening terms.

$$\mathcal{K}\to\mathcal{K}' = \int_{\mathbf{l}}(\mathcal{K}-\delta\mathcal{K})(\mathbf{l}) = \int_{\mathbf{l}}\mathcal{K}(\mathbf{l})\Delta_{med}(\mathbf{l})$$

Implementation in the rate equations

$$t_d \sim \frac{1}{(\hat{q}\theta^2)^{1/3}}$$

Given the previous result, we obtain a new term in the evolution for the branching building block

$$\mathcal{K} \to \mathcal{K}' = \int_{\mathbf{l}} (\mathcal{K} - \delta\mathcal{K})(\mathbf{l}) = \int_{\mathbf{l}} \mathcal{K}(\mathbf{l})\Delta_{med}(\mathbf{l})$$

$$\mathcal{K}'(\mathbf{l}, z, p_0^+, t) = \frac{2}{p_0^+}\frac{P(z)}{z(1-z)} \sin\left(\frac{\mathbf{l}^2}{2k_f^2}\right) e^{-\frac{\mathbf{l}^2}{2k_f^2}} (1 - e^{-\frac{\hat{q}}{12}\theta^2 t^3})$$

$\int_t \mathcal{K}' \sim L - t_d + \mathcal{O}\left(L\frac{t_d^3}{L^3}\right)$ **In accordance with the initial discussion, now with formation time replaced by decoherence time.**

Correct physical picture

Time scaling

Large angle emissions: $\mathcal{K}' \to \mathcal{K}$	L
Reduced P.S. due to interferences	$L - t_d$
Small unresolved branching: strong radiation suppression	$L \sim t_d \quad \int_t \mathcal{K}' \sim L\frac{t_d^3}{L^3}$

Shortcomings of our work

- Our derivation only provides an "effective" way of inserting color coherence effects into the rate equations. As such, it should be seen as simple procedure to gauge the effects of color coherence into multiple medium-induced gluon branching.

- Formally our result only applies to the leading logarithmic approximation for gluon emission and as such its extension to the full kinematical domain in the energy fraction z is abusive.

- The way colour coherence is introduced is not unique. This ambiguity stems from the fact that we provide an effective implementation of this effect.

- Pure QM intereferences are neglected. They of course play an important role, and a full treatment should consider this scale.

  - Moving away from this strict limit breaks the classical Markovian picture which is crucial to construct the rate equations.

Shortcomings of our work

➡ Pure QM intereferences are neglected. They of course play an important role, and a full treatment should consider this scale.

Effectively we are assuming that color decoherence times are always larger the QM decoherence scales

$$L - \max(\cancel{t_f}, t_d)$$

For example, for the color structure of the outgoing gluons, one has:

$$\mathcal{P}_1(\mathbf{k} - z\mathbf{l})\mathcal{P}_1(\mathbf{k} - (1-z)\mathbf{l})\left(1 + \mathcal{O}\left(\frac{t_f}{L}\right)\right)$$

The term we neglect is also multiplied by a coherence factor $\exp\left(-\frac{\hat{q}}{4}\int_{\Delta t} \theta^2 (\Delta t)^2\right)$

which can become dominant compared to the terms we include.

Conclusions

- We computed a simple correction to the emission kernel present in the rate equations which implements color coherence .

- This correction obeys the expected scalings for the emission P.S. .

- We ignore formation time effects, whose contributions could be of the order or larger than the effects we study.

Thank you!