How many observables are necessary to constrain light parton transport properties?

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This work is supported by NSF grant ACI-1550225 and DOE grant DE-FG02-05ER41367.

Motivation

Goal:

- Explore the **non-perturbative effects** of the soft interactions of hard partons propagating through a QCD medium
- Data-driven constraints on transport coefficients of soft interactions

Question:

- To what extent the coefficients can be constrained from measurements?
- How many observables do we need?
- What is the effect of reducing experimental uncertainties?

This work:

- **Proof of principle calculation**

Proof of principle calculation

Utilize **closure test** to demonstrate the purpose of this calculation:

- calculate observables using **known** parton transport coefficient ($\hat{q}_{\mathrm{soft},0}$)
- apply Bayesian analysis on the calculated observables
- transport coefficients are expected to be constrained at the **known** value $\hat{q}_{\mathrm{soft},0}$
- determine which observable can best constrain the transport coefficients

Closure test gives us knowledge of "real" soft transport coefficients value.

Hard-soft factorized model of parton energy loss

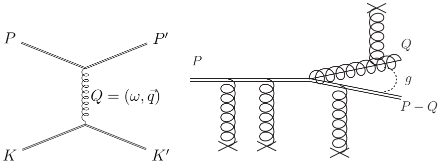
J. Ghiglieri, G. Moore, D. Teaney, JHEP03 (2016) 095

Weakly-coupled effective kinetic approach

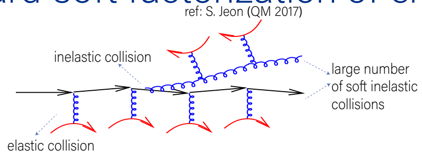
- Perturbative parton-medium interaction
- Dynamics of quasiparticles are described by transport equations
- Energy gain and loss are naturally included

Leading-order realizations (e.g. MARTINI):

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a(\mathbf{p}, \mathbf{x}, t) = -\mathcal{C}_a^{2\leftrightarrow 2}[f] - \mathcal{C}_a^{1\leftrightarrow 2}[f]$$



Hard-soft factorization of energy loss



Interactions with the medium:

- Large number of soft interactions
- Rare hard scatterings

Parton energy loss factorized as **hard interactions + diffusion process**

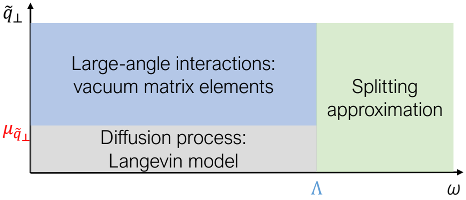
Benefits of factorized transport model

- Systematically factorized soft and hard parton-plasma interactions
- Efficient and flexible stochastic description of soft interactions
  1. Dynamical properties are encoded in a few parameters
  2. **Diffusion process** does not rely on the quasiparticle assumption
  3. Parametric **diffusion process** enables Baysian analysis

Treatments to Different Processes

q: momentum exchange, ω: energy exchange, $\tilde{q}_\perp \equiv \sqrt{q^2 - \omega^2}$

Elastic interactions:



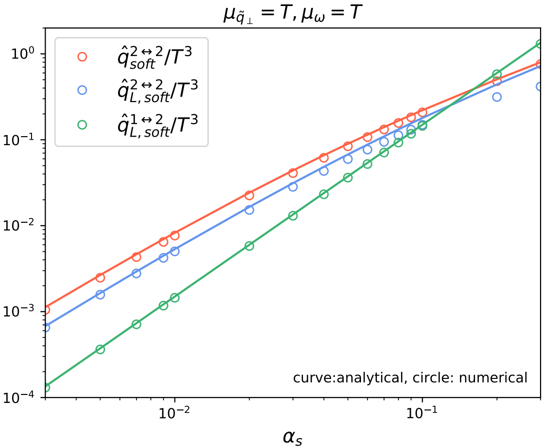
Inelastic interactions:

$$\mathcal{C}^{2\leftrightarrow 2} + \mathcal{C}^{1\leftrightarrow 2} = \mathcal{C}^{\text{large-angle}}(\mu_{\tilde{q}_\perp}, \Lambda) + \mathcal{C}^{\text{split}}(\Lambda) + \mathcal{C}^{\text{large-}\omega}(\mu_\omega) + \mathcal{C}_a^{\text{diff}}(\mu_{\tilde{q}_\perp}, \mu_\omega)$$

$$\mathcal{C}^{\text{diff}}[f] = -\frac{\partial}{\partial p^i}\left[\eta_{D,\text{soft}}(p) p^i f(p)\right] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j \hat{q}_{L,\text{soft}}(p) + \frac{1}{2}\left(\delta^{ij} - \hat{p}^i\hat{p}^j\right)\hat{q}_{\text{soft}}(p)\right) f(p)\right]$$

$\mathcal{C}_a^{\text{diff}}$ is parametric, and suitable for data-driven constraints.

Numerical vs analytical diffusion coefficients - coupling



Analytical diffusion coefficients:

$$\hat{q}^{2\to 2}_{\mathrm{soft}} = \frac{g^2 C_R T m_D^2}{4\pi} \ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{m_D}\right)^2\right]$$

$$\hat{q}^{2\to 2}_{L,\mathrm{soft}} = \frac{g^2 C_R T M_\infty^2}{4\pi} \ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{m_\infty}\right)^2\right]$$

$$\hat{q}^{1\to 2}_{L,\mathrm{soft}} = \frac{(2 - \ln 2) g^4 C_R C_A T^2 \mu_\omega}{4\pi^3}$$

Numerical diffusion coefficients:

$$\hat{q}_{\mathrm{soft}}(p) = \int_0^{\mu_{\tilde{q}_\perp}} d^2 q_\perp q_\perp^2 \frac{d\Gamma(\mathbf{p}, \mathbf{p}+\mathbf{q}, \mu_\omega)}{d^2 q_\perp}$$

$$\hat{q}_{L,\mathrm{soft}}(p) = \int_0^{\mu_\omega} dq^z (q^z)^2 \frac{d\Gamma(\mathbf{p}, \mathbf{p}+\mathbf{q}, \mu_{\tilde{q}_\perp})}{dq^z}$$

Proof of principle calculation: constraining transport coefficients

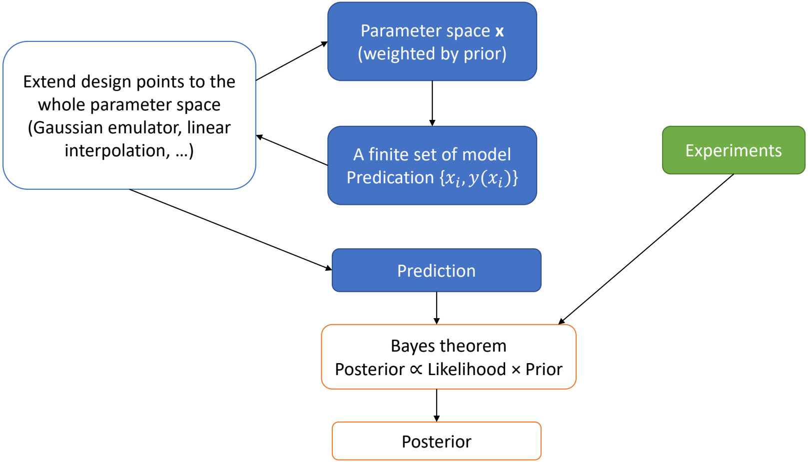
Use **closure test** instead of real data, since this gives us knowledge of "real" $\hat{q}_{\mathrm{soft}}$.

- Use pre-set $\hat{q}_{\mathrm{soft}}$ to construct "data"
- Apply Bayesian analysis on the observables to constrain $\hat{q}$
- Compare the constrained $\hat{q}_{\mathrm{soft}}$ with the known true $\hat{q}_{\mathrm{soft}}$
- Study how adding observables and reducing uncertainties improve the constraints

We perform the calculation in **JETSCAPE framework v3**[1].

[1] J. H. Putschke, et al., arXiv:1902.05934 (2019). https://github.com/JETSCAPE/JETSCAPE

Introduction to Bayesian analysis



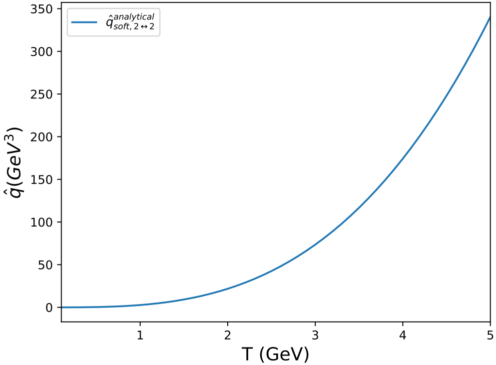
ref: modified based on W. Ke (JETSCAPE winter school 2019)

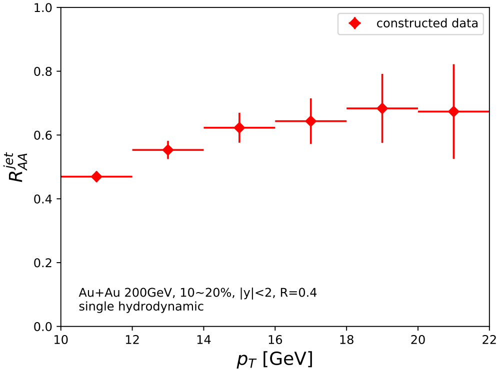
Construct "data"

- calculate observables using known soft transport coefficients:

$$\mu_\omega = \mu_{\tilde{q}_\perp} = 2T,\ \hat{q}_{\rm soft} = \hat{q}^{\rm analytical}_{\rm soft}$$

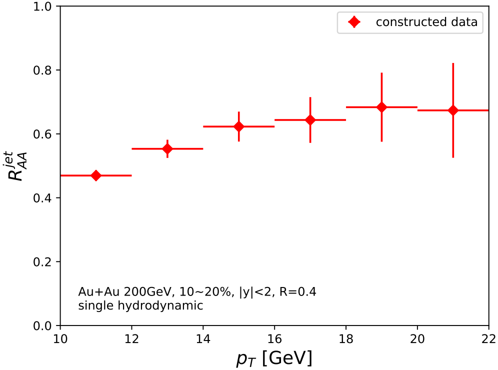
$$\text{analytical } \hat{q}^{\rm analytical}_{{\rm soft},2\to 2} = \frac{g^2 C_R T m_D^2}{4\pi} \ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{m_D}\right)^2\right]$$

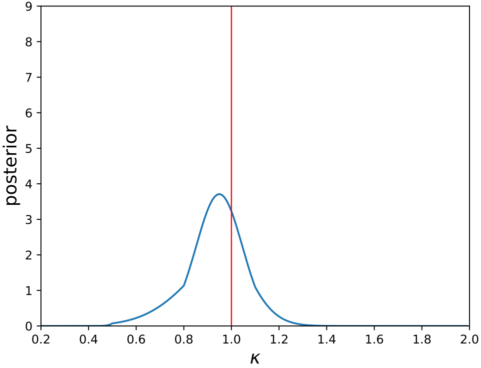




How much is the coefficient constrained?

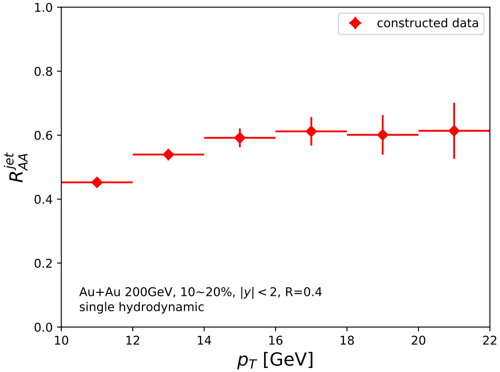
Constrain $\hat{q}_{\mathrm{soft}} = \kappa \hat{q}_{\mathrm{soft}}^{\mathrm{analytical}}$, $\kappa = 1$ is expected.

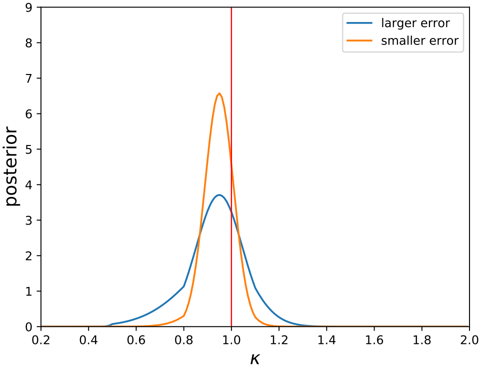




How does the data uncertainty affect the constrain?

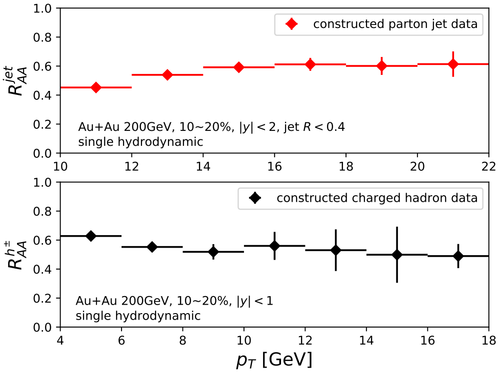
We construct the data using the same parameters, but with smaller uncertainties.

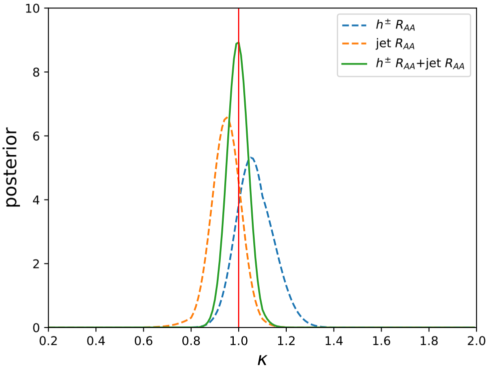




How does adding an observable affect the constrain?

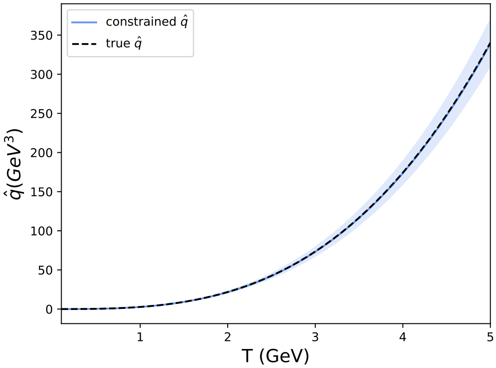
We constrain the parameters on the observables parton jet R_{AA} (R=0.4) and charged hadron R_{AA} at the same time.

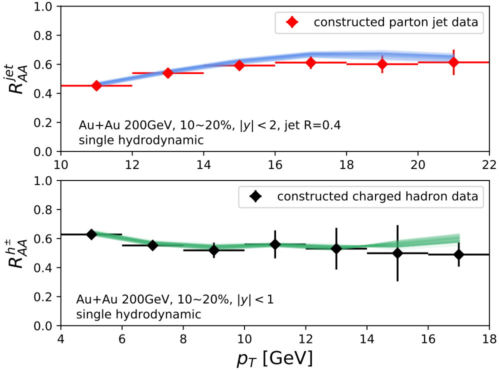




Posterior prediction

We sample the parameter using the posterior probability distribution to calculate the observable.





Conclusion & Outlook

- Energy loss model is reformulated as **hard collisions+diffusion**.
- Systematic and flexible factorization in the weakly-coupled limit.
- Bayesian analysis on soft transport coefficients is applied.
- Constrain quality of different observables is studied.

Outlook

- Study with more observables
- Use more flexible parametric format of soft transport coefficients
- Constrain other parameters at the same time (e.g. hard-soft cutoff)
- Study more centralities with realistic hydrodynamic events
- Make predictions with real data-driven constraints