PHOTON RADIATION IN HOT NUCLEAR MATTER BY MEANS OF CHIRAL ANOMALIES

Kirill Tuchin

based upon Phys.Lett.B786 (2018), Phys.Rev.Lett.121 (2018) with X-G Huang and Phys.Rev.C 99 (2019)

IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

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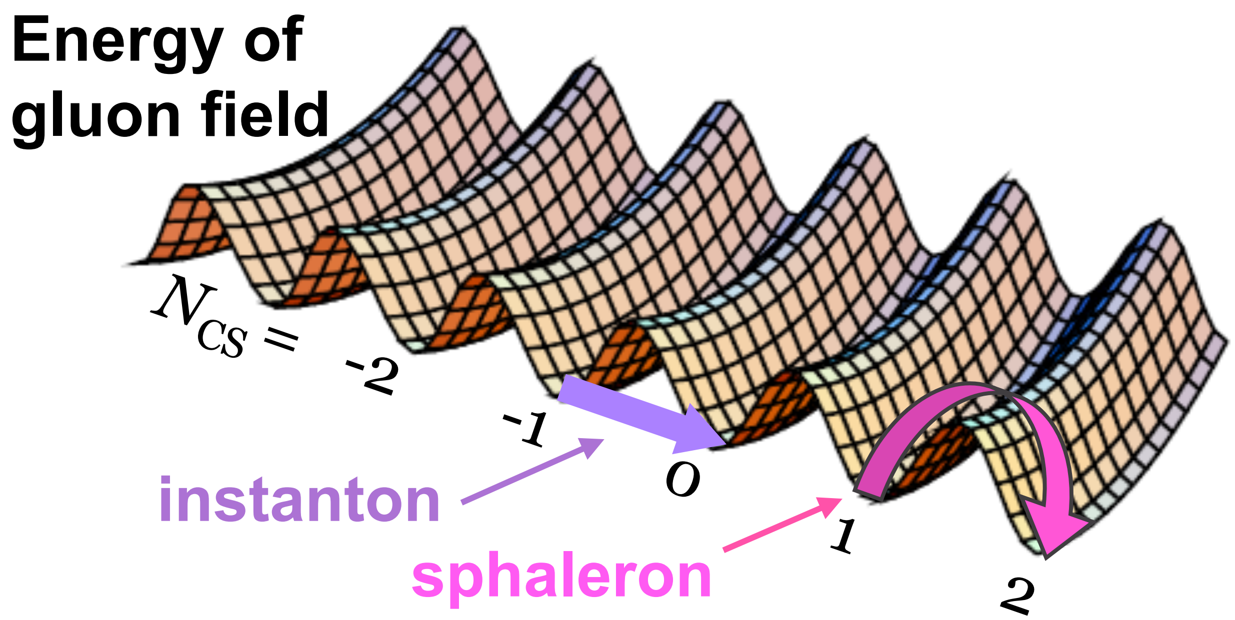
TOPOLOGY OF THE QCD VACUUM

QCD vacuum is a superposition of states with different topology, characterized by the topological charge density

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}(x)$$



Transitions between such states creates local imbalance of chirality.

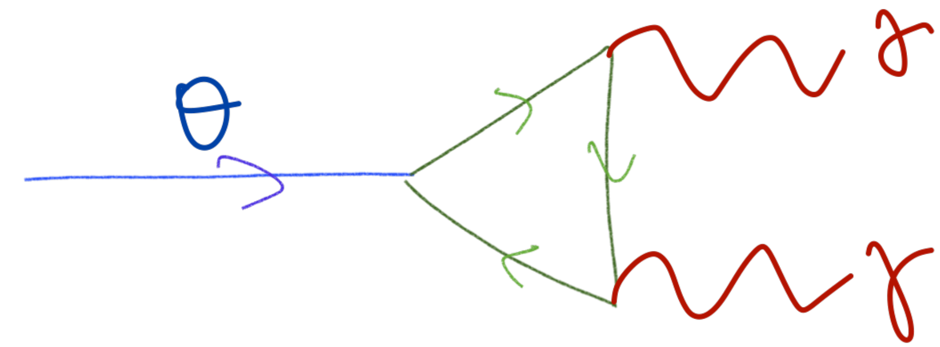
The transition rate per unit volume is exponentially suppressed at low temperatures, but increases at high temperatures as

$$\Gamma_{\rm sph} \sim (\alpha_s N_c)^5 T^4$$

→ Important phenomenological implications for QGP.

MAXWELL-CHERN-SIMONS THEORY

Electromagnetic field in chiral matter $\Delta\mathcal{L} = -\frac{c_A}{4}\theta F\tilde{F}$



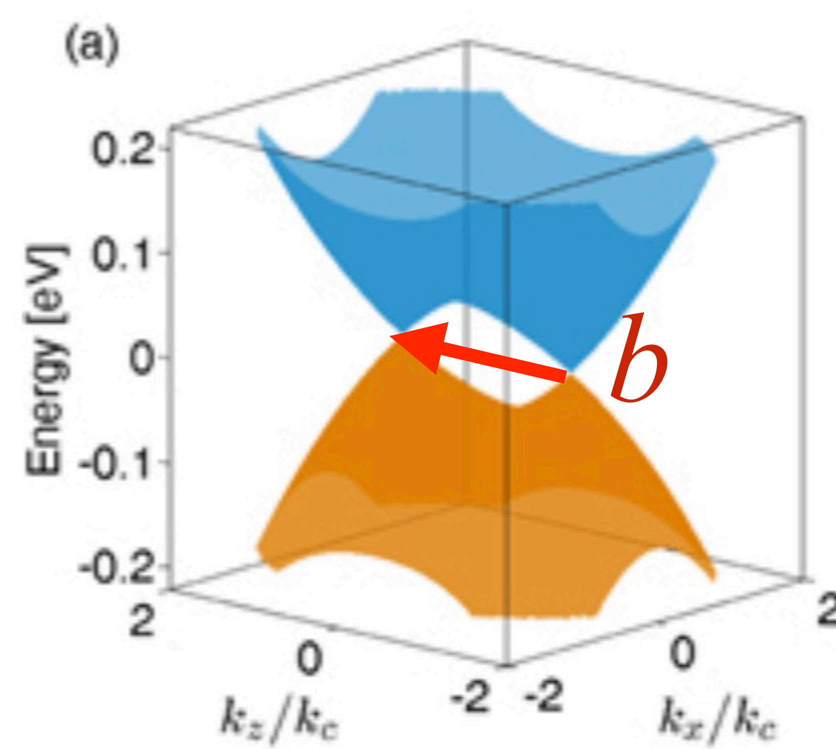
$$\partial_\mu F^{\mu\nu} = j^\nu - c_A \tilde{F}^{\mu\nu}\partial_\mu\theta\,,$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0\,.$$

In perturbation theory
$\theta = \text{const}$

Often used notations: $\sigma_\chi = c_A\dot{\theta}$ $\quad$ $\boldsymbol{b} = c_A \boldsymbol{\nabla}\theta.$

"chiral conductivity" $\quad$ axial chemical potential



Weyl semimetals

ANOMALOUS CURRENTS IN QGP

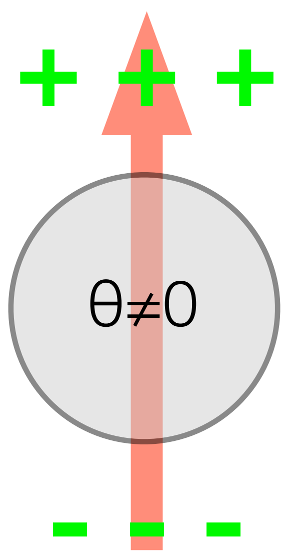
$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + c_A(\boxed{\partial_t\theta\, \boldsymbol{B}} + \boxed{\boldsymbol{\nabla}\theta \times \boldsymbol{E}})$$

Anomalous currents: $\boldsymbol{j}_{\rm CME} = \sigma_\chi \boldsymbol{B}$ $\qquad$ $\boldsymbol{j}_{\rm AHE} = \boldsymbol{b} \times \boldsymbol{E}$

P-odd,
T-odd

P-even,
T-odd

Anomalous Hall Effect



External magnetic field drives the charge separation.

Breaks Parity!

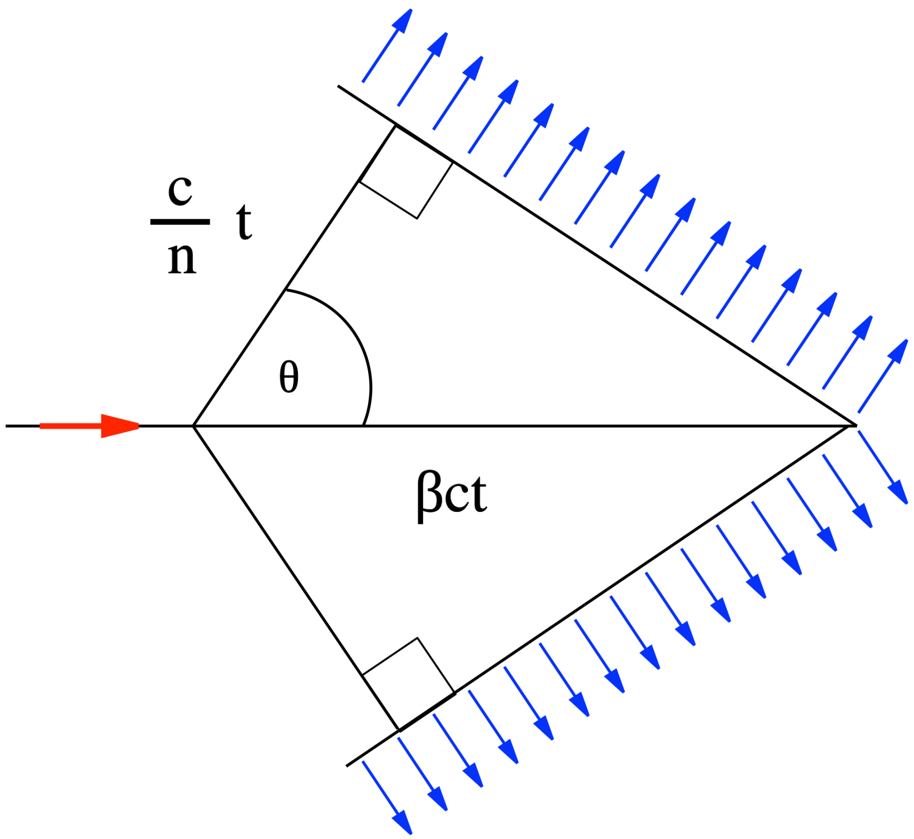
In HIC ∇θ is small

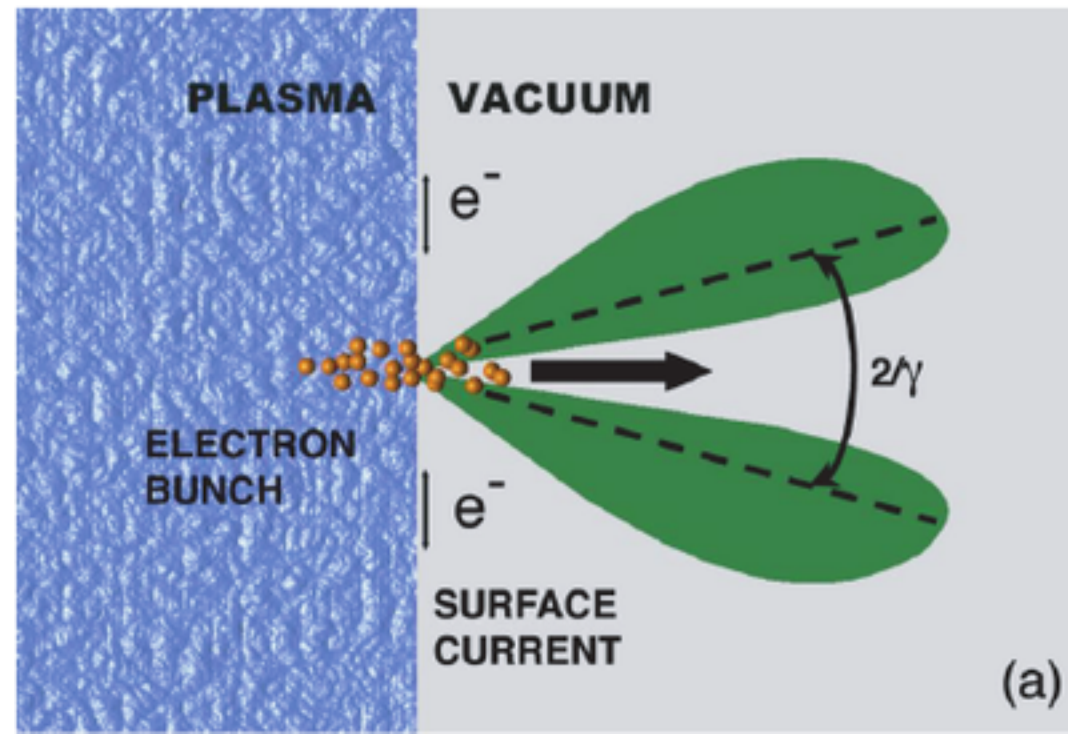
Zhitnitsky 2013

Chiral magnetic effect

Kharzeev, McLerran, Warringa (2008)

PARTICLE RADIATION IN MATTER: CHERENKOV AND TRANSITION RADIATION





Classical Cherenkov radiation is emitted by a charged particle that moves faster than the phase velocity of light: *vn>1*

$$\cos\theta = \frac{1}{\beta\sqrt{\epsilon}} = \frac{1}{\beta n}$$

Classical transition radiation is emitted by a charged particle that moves through inhomogeneous matter.

33.7. Cherenkov and transition radiation [33,77,78]

A charged particle radiates if its velocity is greater than the local phase velocity of light (Cherenkov radiation) or if it crosses suddenly from one medium to another with different optical properties (transition radiation). Neither process is important for energy loss, but both are used in high-energy and cosmic-ray physics detectors.

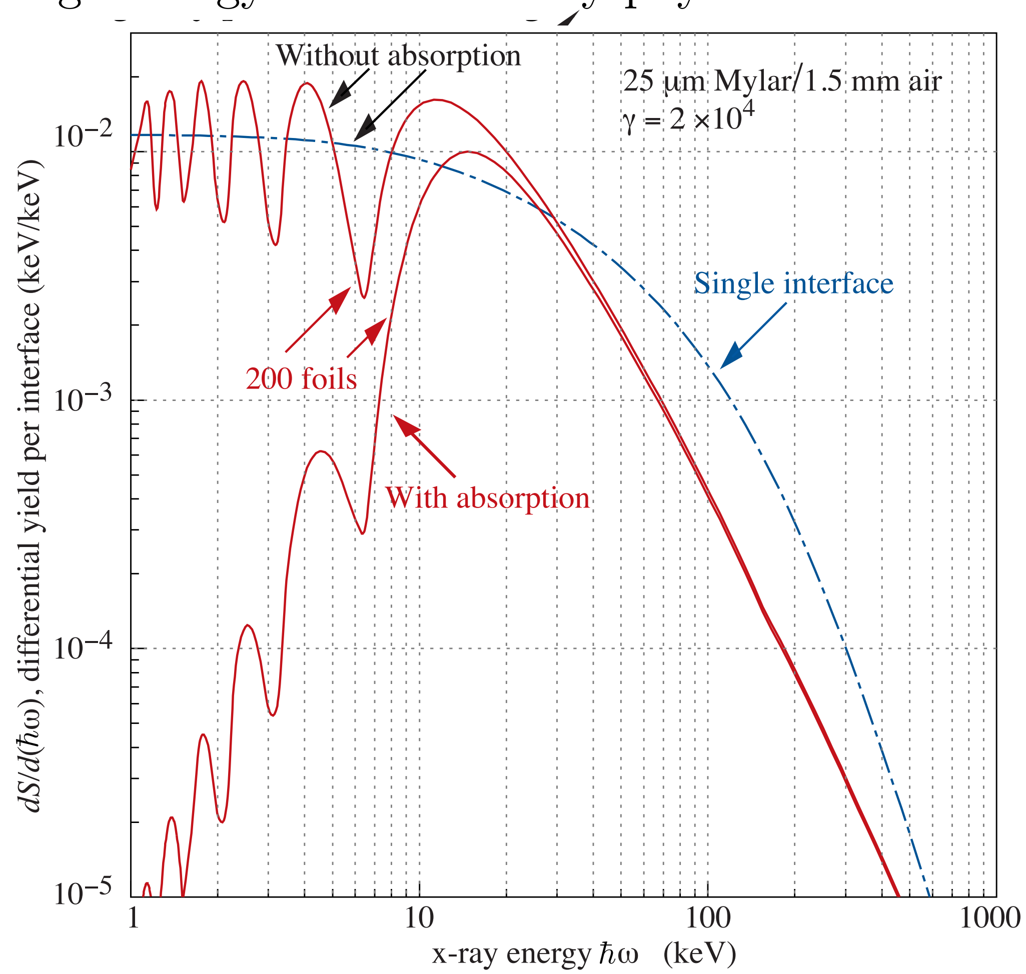


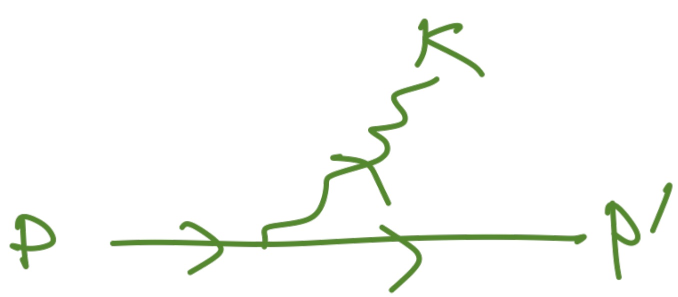
Figure 33.27: X-ray photon energy spectra for a radiator consisting of 200 25 μm thick foils of Mylar with 1.5 mm spacing in air (solid lines) and for a single surface (dashed line). Curves are shown with and without absorption. Adapted from Ref. 88.

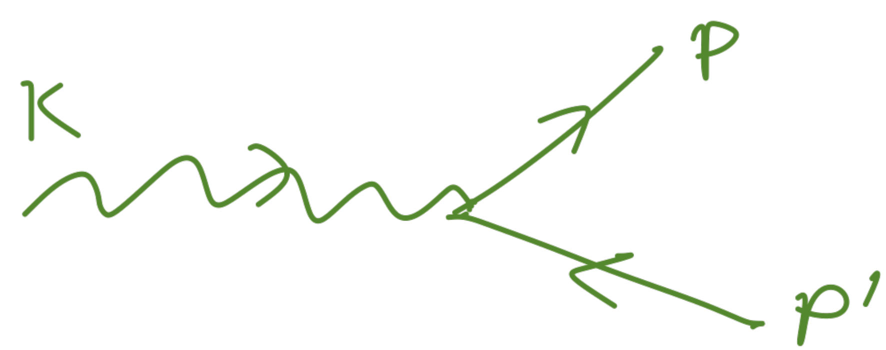
1→2 PROCESSES IN CHIRAL MATTER

Let field θ be homogenous and weekly time-dependent $\dot{\theta} = \mathrm{const}$

In radiation gauge: $\nabla^2 \boldsymbol{A} = \partial_t^2 \boldsymbol{A} - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{A}$

The dispersion relation $k^2 = -\lambda \sigma_\chi |\boldsymbol{k}|$ → photon becomes space- or timelike





$k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$ forbidden in vacuum, but allowed in chiral medium

Pair production: $k^2 > 0 \Rightarrow \lambda \sigma_\chi < 0$

Photon radiation: $k^2 < 0 \Rightarrow \lambda \sigma_\chi > 0$

THE WAVE FUNCTIONS

Scattering matrix $S = -ieQ\int \bar{\psi}\gamma^\mu\psi A_\mu d^4x$ $\Rightarrow$ need photon wave function.

Use the ultra-relativistic approximation:

1. no need to worry about the instability of EM field in the infrared
2. no reflection off domain walls boundaries

$$-\nabla^2 \boldsymbol{A} = -\partial_t^2 \boldsymbol{A} + \sigma_\chi(\boldsymbol{\nabla}\times\boldsymbol{A})$$

$$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}}\boldsymbol{\epsilon}_\lambda\, e^{i\omega z + i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp - i\omega t}\exp\left\{-i\frac{1}{2\omega}\int_0^z \left[k_\perp^2 - \underbrace{\sigma_\chi(z')\omega\lambda}_{\text{"}m_\gamma^2\text{"}}\right]dz'\right\}$$

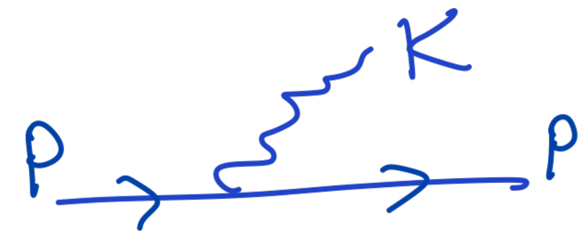
$$\boldsymbol{\epsilon}_\lambda = \boldsymbol{e}_\lambda - \frac{\boldsymbol{e}_\lambda\cdot\boldsymbol{k}_\perp}{\omega}\hat{\boldsymbol{z}}$$

$$\boldsymbol{e}_\lambda = (\hat{\boldsymbol{x}} + i\lambda\hat{\boldsymbol{y}})/\sqrt{2}.$$

Fermion wave function $\psi = \frac{1}{\sqrt{2\varepsilon V}}u(p)e^{i\varepsilon z - i\varepsilon t}\exp\left\{i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp - iz\frac{\boldsymbol{p}_\perp^2 + m^2}{2\varepsilon}\right\}$

SCATTERING MATRIX FOR RADIATION

Photon spectrum:



$$dN = \frac{1}{(2\pi)^3}\frac{1}{8x(1-x)\varepsilon^2}\frac{1}{2}\sum_{\lambda,\sigma,\sigma'}|\mathcal{M}|^2 d^2q_\perp dx$$

$$\mathcal{M} = \underbrace{-\,eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu}_{-\frac{eQ}{\sqrt{2(1-x)}}\left[xm(\sigma+\lambda)\delta_{\sigma',-\sigma} - \frac{1}{x}(2-x+x\lambda\sigma)(q_x - i\lambda q_y)\delta_{\sigma',\sigma}\right]} \int_{-\infty}^{\infty} dz\, \exp\left\{i\int_0^z \frac{q_\perp^2 + \kappa_\lambda(z')}{2\varepsilon x(1-x)}dz'\right\}$$

$x = \omega/\varepsilon$ $\quad\quad$ $\boldsymbol{q}_\perp = x\boldsymbol{p'} - (1-x)\boldsymbol{k}_\perp$ $\quad\quad$ $\kappa_\lambda(z) = x^2m^2 - (1-x)x\lambda\sigma_\chi\varepsilon$

A SINGLE UNIFORM INFINITE DOMAIN ($\kappa_\lambda = \text{const}$)

$$\mathcal{M} = -\,eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu \underbrace{\int_{-\infty}^{\infty} dz \exp\left\{ i\int_0^z \frac{q_\perp^2 + \kappa_\lambda(z')}{2\varepsilon x(1-x)}dz' \right\}}_{2\varepsilon x(1-x)\,2\pi\,\delta(q_\perp^2+\kappa_\lambda)}$$

$$\boxed{\kappa_\lambda(z) = x^2m^2 - (1-x)x\lambda\sigma_\chi\varepsilon}$$

can become negative!

“chiral Cherenkov effect” $\vartheta \sim \sqrt{|\sigma_\chi|/\omega}$

Photon radiation rate $$\frac{dW}{dx} = \frac{1}{16\pi\varepsilon}\frac{1}{2}\sum_{\lambda,\sigma,\sigma'}|\mathcal{M}_0|^2\,\theta(-\kappa_\lambda)$$

Kappa is negative if $\lambda\sigma_\chi > 0$ and $x < x_0 = \dfrac{1}{1+m^2/(\lambda\sigma_\chi\varepsilon)}$

$$\frac{dW_+}{dx} = \frac{\alpha Q^2}{2\varepsilon x}\left\{\sigma_\chi\varepsilon\left(\frac{x^2}{2} - x + 1\right) - m^2x\right\}\theta(x_0 - x)$$

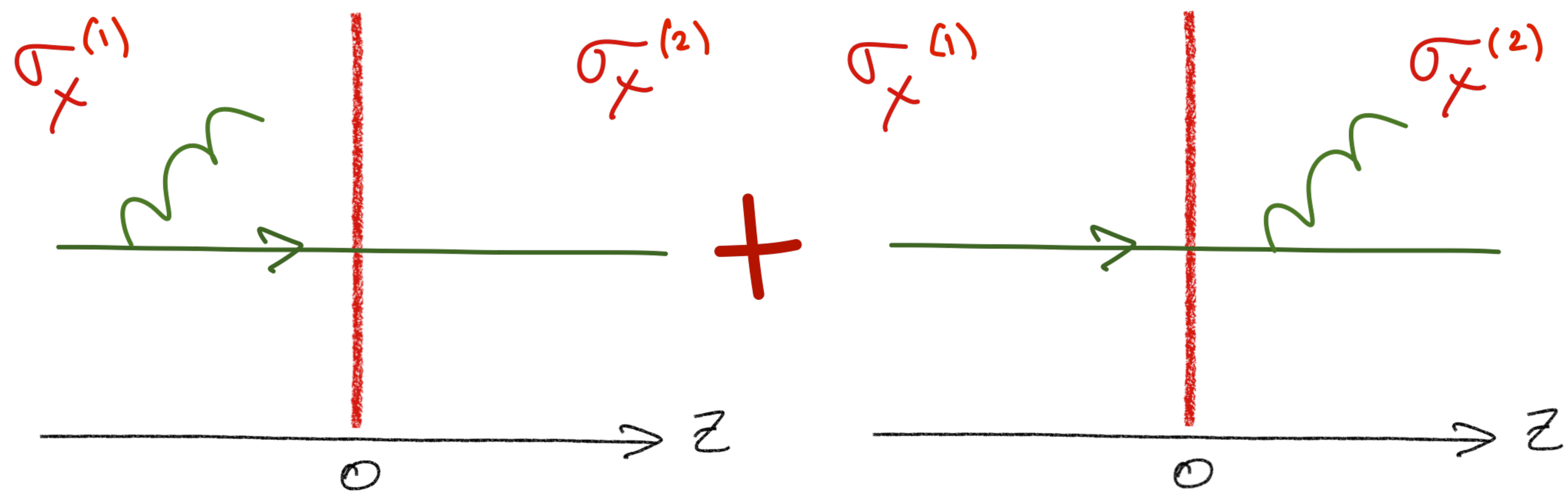
$$\frac{dW_-}{dx} = 0\,.$$

Vanishes as $\hbar \to 0$

Quantum anomaly!

Total rate of energy loss $$\frac{\Delta\varepsilon}{T} = \int_0^1 \frac{dW_+}{dx}x\varepsilon dx = \frac{1}{3}\alpha Q^2\sigma_\chi\varepsilon$$

TWO SEMI-INFINITE DOMAINS



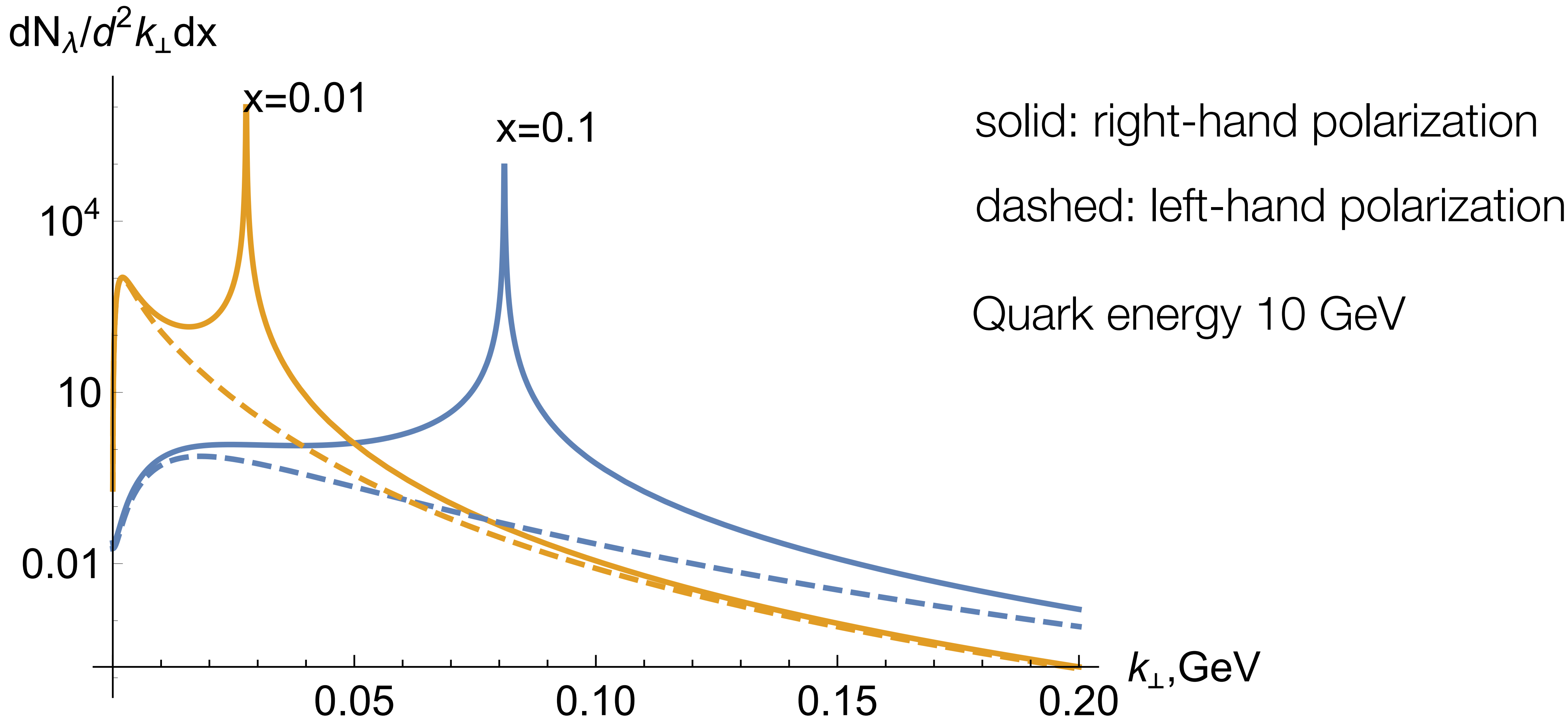
$$\frac{dN}{d^2q_\perp dx} = \frac{\alpha Q^2}{2\pi^2 x}\left\{\left(\frac{x^2}{2} - x + 1\right) q_\perp^2 + \frac{x^4 m^2}{2}\right\}\sum_\lambda \left|\frac{1}{q_\perp^2 + \kappa_\lambda^{(1)} - i\delta} - \frac{1}{q_\perp^2 + \kappa_\lambda^{(2)} + i\delta}\right|^2$$

(Transition radiation in ordinary materials corresponds to $\kappa_{\rm tr} = m^2x^2 + m_\gamma^2(1-x)$ finite at $\hbar \to 0$)

Contribution of the pole at $q_\perp^2 + \kappa_\lambda = 0$ is the chiral Cherenkov radiation.

The rest is the "chiral transition radiation"

CHERENKOV + TRANSITION RADIATION IN QGP



CONCLUSION 1

1. Charged particles traveling through the chiral medium emit electromagnetic radiation sensitive to the chiral anomaly.

2. It is

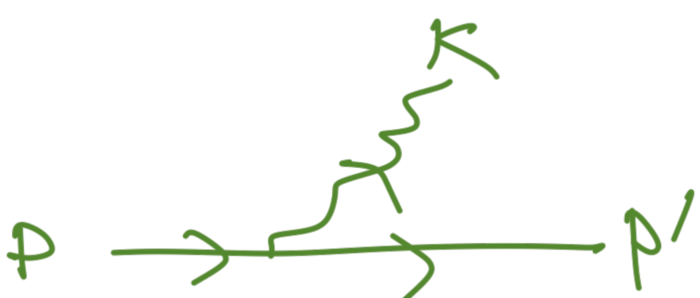
    a. circularly polarized,

    b. has resonant peaks at angles proportional to the anomaly,

    c. has characteristic angular distribution.

3. It can be used to investigate the chiral anomaly in QGP, Weyl semimetals, axionic dark matter.

PHOTON PRODUCTION BY QGP VIA THE CHIRAL ANOMALY (W/O EXTERNAL MAGNETIC FIELD)



Thermal q
and anti-q

The photon energy produced by thermal quarks is controlled by the plasma temperature
→ must take into account the plasma frequency

$$\omega_{\rm pl}^2 = \frac{m_D^2}{2} = \frac{e^2}{2}\left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)$$

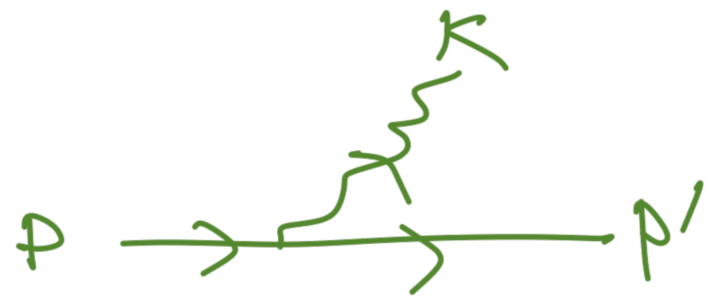
Photon mass gets two contributions: $\omega^2 - k^2 = \omega_{\rm pl}^2 + m_A^2 + \mathcal{O}(\omega - k)$

$$m_A^2 = -\lambda\sigma_\chi\omega\,, \qquad \text{or} \qquad m_A^2 = -\lambda\boldsymbol{k}\cdot\boldsymbol{b}\,,$$

Due to the topological number fluctuations $m_A \sim \sqrt{\langle\theta^2\rangle} \sim \Gamma_{\rm sp} \sim T^4$

Thus, at high enough T $m_A \gg \omega_{\rm pl}$ → Cherenkov radiation is possible

THE DECAY CHANNEL



$$x = k^0/p^0 = \omega/\varepsilon$$

$$S_D = (2\pi)^4 \delta^{(4)}(p' + k - p) i\mathcal{M}_D \qquad\qquad i\mathcal{M}_D = -ieQ \frac{\bar{u}_{\boldsymbol{p}'s'} \not{\epsilon}^*_{\boldsymbol{k}\lambda} u_{\boldsymbol{p}s}}{\sqrt{8\varepsilon\varepsilon'\omega V^3}}$$

The decay probability $dw_D = 2N_c \frac{1}{2} \sum_{\lambda s s'} |S_D|^2 f(\varepsilon)[1 - f(\varepsilon')] \frac{V d^3p'}{(2\pi)^3} \frac{V d^3k}{(2\pi)^3} \frac{V d^3p}{(2\pi)^3}$

$$\omega \frac{d\Gamma_D}{d^3k} = 2N_c \frac{e^2 Q^2 \pi}{4(2\pi)^5} \int_0^1 \frac{dx}{x^4} f\left(\frac{\omega}{x}\right) \left[1 - f\left(\frac{\omega(1-x)}{x}\right)\right] \sum_\lambda \left\{-m_A^2[(1-x)^2 + 1] - 2m^2x^2\right\} \theta(-\kappa_\lambda)$$

$$\kappa_\lambda = m_A^2(1-x) + m^2x^2$$

κ_λ is negative only if $|m_A^2|(1-x) > x^2m^2$ and $m_A^2 < 0$

$$👉 \quad 0 \le x < \frac{|m_A^2|}{2m^2} \left(\sqrt{1 + \frac{4m^2}{|m_A^2|}} - 1\right)$$

Quark thermal mass $m = gT/\sqrt{3}$

HIGH-T LIMIT

- In the limit $m \ll |m_A|$ (hight T) $\quad 0 \leq x \leq 1 \qquad \xi = 1/x - 1$

$$\omega \frac{d\Gamma_D}{d^3k} = 2N_c \frac{e^2Q^2}{8(2\pi)^4} |m_A^2| \int_0^\infty d\xi \left[\xi^2 + (1-\xi)^2\right] f(\omega(1+\xi)) \left[1 - f(\omega\xi)\right] .$$

$$\approx 0.73 \cdot 2N_c \frac{e^2Q^2}{8(2\pi)^4} |m_A^2| \left[\frac{\ln(1+e^{-\beta\omega})}{\beta\omega} + \frac{2\mathrm{Li}_2(-e^{-\beta\omega})}{(\beta\omega)^2} - \frac{4\mathrm{Li}_3(-e^{-\beta\omega})}{(\beta\omega)^3}\right] .$$

Asymptotically:

$$\omega \frac{d\Gamma_D}{d^3k} = 0.73 \cdot 2N_c \frac{e^2Q^2}{8(2\pi)^4} |m_A^2| \begin{cases} \frac{3\zeta(3)}{(\beta\omega)^3} , & \omega \ll T \\ \frac{1}{\beta\omega} e^{-\beta\omega} , & \omega \gg T . \end{cases}$$

Since $|m_A|^2 \sim \omega$, the total rate Γ_D is dominated by soft photons yielding log(T/m)

THE ANNIHILATION CHANNEL

$$y = p^0/k^0 = \varepsilon/\omega$$

$$\omega \frac{d\Gamma_A}{d^3k} = N_c \frac{e^2 Q^2 \pi}{8(2\pi)^5} \int_0^1 dy f(y\omega) f((1-y)\omega) \sum_\lambda \left[m_A^2 (2y^2 - 2y + 1) + 2m^2 \right] \theta(-\varkappa_\lambda) \,,$$

$$\varkappa_\lambda = -m_A^2 y(1-y) + m^2$$

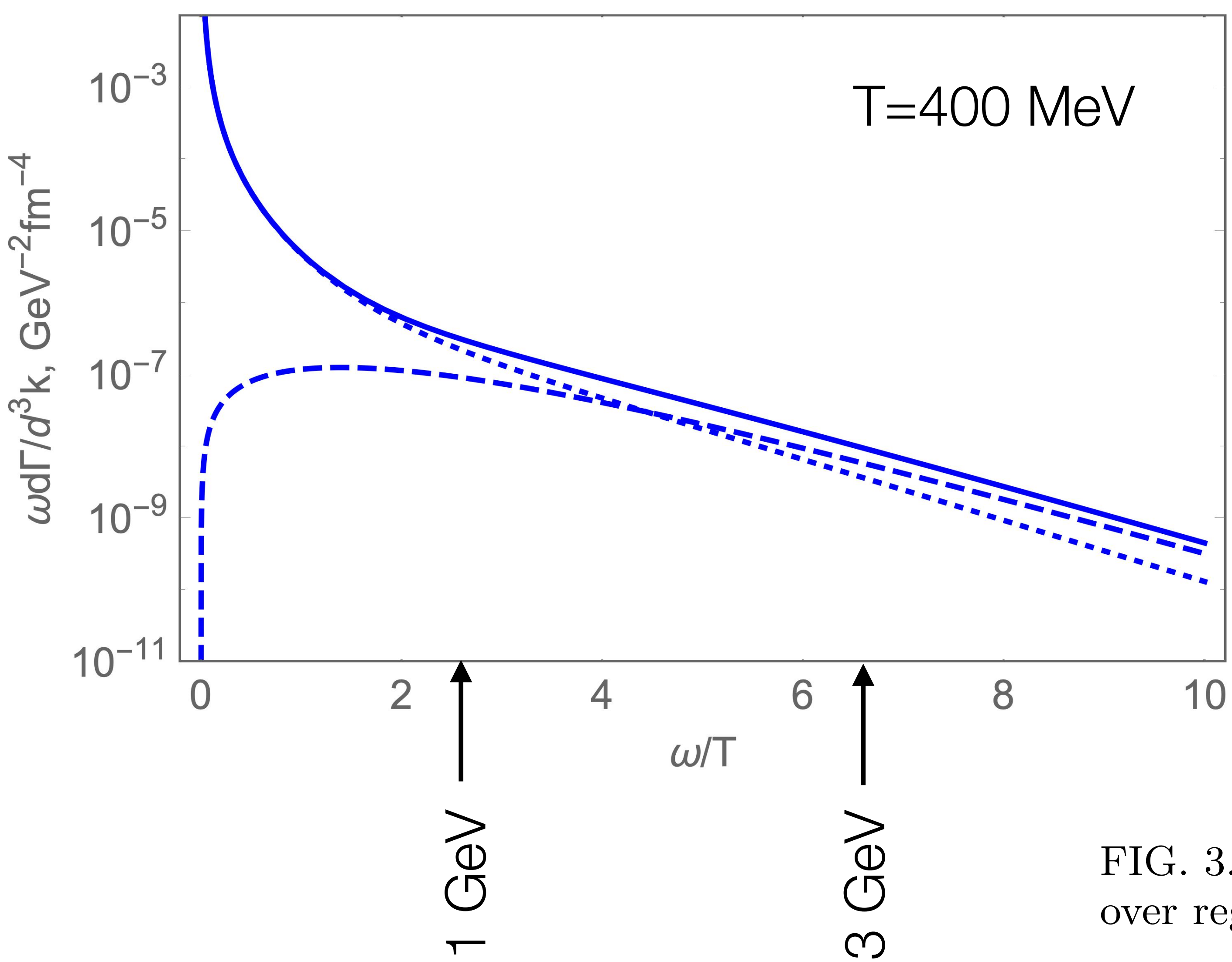
The allowed values of y: $\frac{1}{2}\left(1 - \sqrt{1 - \frac{4m^2}{|m_A^2|}}\right) < y < \frac{1}{2}\left(1 + \sqrt{1 - \frac{4m^2}{|m_A^2|}}\right)$

In the limit $m \ll |m_A|$

$$\omega \frac{d\Gamma_A}{d^3k} = N_c \frac{e^2 Q^2}{16(2\pi)^4} |m_A^2| \frac{1}{e^{\beta\omega} - 1} \left[-\frac{8\mathrm{Li}_3(-e^{\beta\omega}) + 6\zeta(3)}{(\beta\omega)^3} + \frac{4\mathrm{Li}_2(-e^{\beta\omega}) - \pi^2/3}{(\beta\omega)^2} + \frac{2\ln(1 + e^{\beta\omega}) - \ln 4}{\beta\omega} - \frac{2}{3} \right]$$

$$\omega \frac{d\Gamma_A}{d^3k} = N_c \frac{e^2 Q^2}{16(2\pi)^4} |m_A^2| \begin{cases} \frac{1}{6}\,, & \omega \ll T \\ \frac{2}{3} e^{-\beta\omega}\,, & \omega \gg T\,. \end{cases}$$

TOTAL CHIRAL CHERENKOV RADIATION



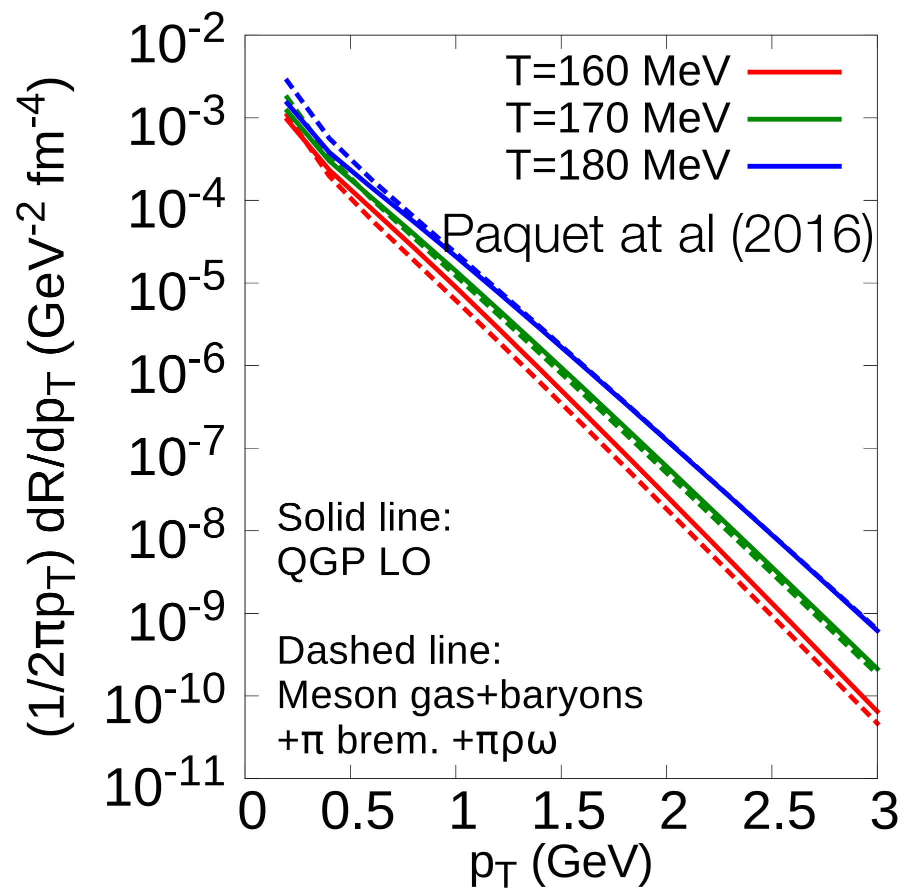


FIG. 3. Ideal QGP and hadronic photon rate near the cross-over region.

The contributions are comparable. Worth doing a more accurate estimate.

CONCLUSION 2

At high temperature QGP emits photons via the chiral Cherenkov radiation (without any magnetic field).

This is a missing contribution at low p_T