PHOTON RADIATION IN HOT NUCLEAR MATTER BY MEANS OF CHIRAL ANOMALIES

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Hard Probes, June 4, 2020
**TOPOLOGY OF THE QCD VACUUM**

QCD vacuum is a superposition of states with different topology, characterized by the topological charge density

\[ q(x) = \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}(x) \]

Transitions between such states creates local imbalance of chirality.

The transition rate per unit volume is exponentially suppressed at low temperatures, but increases at high temperatures as

\[ \Gamma_{\text{sph}} \sim (\alpha_s N_c)^5 T^4. \]

→ Important phenomenological implications for QGP.
MAXWELL–CHERN–SIMONS THEORY

Electromagnetic field in chiral matter

\[ \Delta \mathcal{L} = -\frac{c_A}{4} \theta F \tilde{F} \]

\[ \partial_\mu F^{\mu \nu} = j^\nu - c_A \tilde{F}^{\mu \nu} \partial_\mu \theta, \]

\[ \partial_\mu \tilde{F}^{\mu \nu} = 0. \]

Often used notations: \( \sigma_\chi = c_A \dot{\theta} \)

\( b = c_A \nabla \theta. \)

“chiral conductivity” axial chemical potential

In perturbation theory \( \theta = \text{const} \)

Weyl semimetals
ANOMALOUS CURRENTS IN QGP

\[ \nabla \times B = \partial_t E + j + c_A (\partial_t \theta B + \nabla \theta \times E) \]

Anomalous currents:

\[ j_{\text{CME}} = \sigma \chi B \quad \text{P-odd, T-odd} \]

\[ j_{\text{AHE}} = b \times E \quad \text{P-even, T-odd} \]

External magnetic field drives the charge separation.

\[ \theta \neq 0 \]

Breaks Parity!

Chiral magnetic effect

Kharzeev, McLerran, Warringa (2008)

In HIC \( \nabla \theta \) is small

Zhitnitsky 2013
PARTICLE RADIATION IN MATTER: CHERENKOV AND TRANSITION RADIATION

Classical Cherenkov radiation is emitted by a charged particle that moves faster than the phase velocity of light: \( vn > 1 \)

\[
\cos \theta = \frac{1}{\beta \sqrt{\epsilon}} = \frac{1}{\beta n}
\]

Classical transition radiation is emitted by a charged particle that moves through inhomogeneous matter.
33.7. Cherenkov and transition radiation [33,77,78]

A charged particle radiates if its velocity is greater than the local phase velocity of light (Cherenkov radiation) or if it crosses suddenly from one medium to another with different optical properties (transition radiation). Neither process is important for energy loss, but both are used in high-energy and cosmic-ray physics detectors.

![Figure 33.27](image)

**Figure 33.27:** X-ray photon energy spectra for a radiator consisting of 200 25 µm thick foils of Mylar with 1.5 mm spacing in air (solid lines) and for a single surface (dashed line). Curves are shown with and without absorption. Adapted from Ref. 88.
1→2 PROCESSES IN CHIRAL MATTER

Let field \( \theta \) be homogenous and weekly time-dependent \( \dot{\theta} = \text{const} \)

In radiation gauge: \( \nabla^2 A = \partial_t^2 A - \sigma_\chi \nabla \times A \)

The dispersion relation \( k^2 = -\lambda \sigma_\chi |k| \rightarrow \) photon becomes space- or timelike

\[ k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon) \] forbidden in vacuum, but allowed in chiral medium

Pair production: \( k^2 > 0 \Rightarrow \lambda \sigma_\chi < 0 \)

Photon radiation: \( k^2 < 0 \Rightarrow \lambda \sigma_\chi > 0 \)
THE WAVE FUNCTIONS

Scattering matrix \( S = -ieQ \int \bar{\psi} \gamma^\mu \psi A_\mu d^4x \) \( \Rightarrow \) need photon wave function.

Use the ultra-relativistic approximation:

1. no need to worry about the instability of EM field in the infrared
2. no reflection off domain walls boundaries

\[
-\nabla^2 A = -\partial_t^2 A + \sigma_\chi (\nabla \times A)
\]

\[
A = \frac{1}{\sqrt{2\omega V}} \varepsilon_\lambda e^{i\omega z + i k_\perp \cdot x_\perp - i\omega t} \exp \left\{ -i \frac{1}{2\omega} \int_0^z \left[ k^2_\perp - \sigma_\chi (z') \omega \right] dz' \right\}
\]

\[
\varepsilon_\lambda = e_\lambda - \frac{e_\lambda \cdot k_\perp}{\omega} \hat{z}
\]

\[
e_\lambda = (\hat{x} + i\lambda \hat{y})/\sqrt{2}.
\]

Fermion wave function

\[
\psi = \frac{1}{\sqrt{2\varepsilon V}} u(p) e^{i\varepsilon z - i\varepsilon t} \exp \left\{ i p_\perp \cdot x_\perp - iz \frac{p^2_\perp + m^2}{2\varepsilon} \right\}
\]
Photon spectrum:

\[ dN = \frac{1}{(2\pi)^3} \frac{1}{8x(1-x)\varepsilon^2} \frac{1}{2} \sum_{\lambda,\sigma,\sigma'} |M|^2 d^2q_\perp dx \]

\[ M = -eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu \int_{-\infty}^\infty dz \exp \left\{ i \int_0^z \frac{q_\perp^2 + \kappa_\lambda(z')}{2\varepsilon x(1-x)} dz' \right\} \]

\[ \kappa_\lambda(z) = x^2m^2 - (1-x)x\lambda\sigma \chi \varepsilon \]

\[ x = \omega/\varepsilon \quad q_\perp = xp' - (1-x)k_\perp \]
\[ M = -eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu \int_{-\infty}^{\infty} dz \exp \left\{ i \int_0^z \frac{q^2_1 + \kappa_\lambda(z')}{2\varepsilon x(1-x)} dz' \right\} \]

\[ \kappa_\lambda(z) = x^2m^2 - (1-x)x\lambda\sigma\chi\varepsilon \]

can become negative!

\[ \text{Photon radiation rate} \quad \frac{dW}{dx} = \frac{1}{16\pi\varepsilon} \frac{1}{2} \sum_{\lambda,\sigma,\sigma'} |M_0|^2 \theta(-\kappa_\lambda) \]

Kappa is negative if \( \lambda\sigma\chi > 0 \)  and \( x < x_0 = \frac{1}{1 + m^2/(\lambda\sigma\chi\varepsilon)} \)

\[ \frac{dW_+}{dx} = \frac{\alpha Q^2}{2\varepsilon x} \left\{ \sigma\chi\varepsilon \left( \frac{x^2}{2} - x + 1 \right) - m^2 x \right\} \theta(x_0 - x) \]

\[ \frac{dW_-}{dx} = 0 \,. \]

Total rate of energy loss \[ \frac{\Delta\varepsilon}{T} = \int_0^1 \frac{dW_+}{dx} x\varepsilon dx = \frac{1}{3} \alpha Q^2 \sigma\chi\varepsilon \]

\[ \Psi = \omega_{\nu}(T) \]

“chiral Cherenkov effect” \( \theta \sim \sqrt{|\sigma\chi|/\omega} \)

Vanishes as \hbar \to 0

Quantum anomaly!
TWO SEMI-INFINITE DOMAINS

\[
\frac{dN}{d^2 q_\perp dx} = \frac{\alpha Q^2}{2\pi^2 x} \left\{ \left( \frac{x^2}{2} - x + 1 \right) q_\perp^2 + \frac{x^4 m^2}{2} \right\} \sum_\lambda \left| \frac{1}{q_\perp^2 + \kappa_\lambda^0 - i\delta} - \frac{1}{q_\perp^2 + \kappa_\lambda^0 + i\delta} \right|^2
\]

(Transition radiation in ordinary materials corresponds to \( \kappa_{tr} = m^2 x^2 + m_\gamma^2 (1-x) \) finite at \( \hbar \to 0 \))

Contribution of the pole at \( q_\perp^2 + \kappa_\lambda = 0 \) is the chiral Cherenkov radiation.

The rest is the “chiral transition radiation”
CHERENKOV + TRANSITION RADIATION IN QGP

\[
dN_\lambda / d^2 k_\perp dx = \begin{cases} 10^4 & \text{for } x=0.01 \\ 10^4 & \text{for } x=0.1 \end{cases}
\]

solid: right-hand polarization
dashed: left-hand polarization

Quark energy 10 GeV
CONCLUSION 1

1. Charged particles traveling through the chiral medium emit electromagnetic radiation sensitive to the chiral anomaly.

2. It is
   a. circularly polarized,
   b. has resonant peaks at angles proportional to the anomaly,
   c. has characteristic angular distribution.

3. It can be used to investigate the chiral anomaly in QGP, Weyl semimetals, axionic dark matter.
The photon energy produced by thermal quarks is controlled by the plasma temperature → must take into account the plasma frequency

\[ \omega_{pl}^2 = \frac{m_D^2}{2} = \frac{e^2}{2} \left( \frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right) \]

Photon mass gets two contributions:

\[ \omega^2 - k^2 = \omega_{pl}^2 + m_A^2 + \mathcal{O}(\omega - k) \]

\[ m_A^2 = -\lambda \sigma \chi \omega, \quad \text{or} \quad m_A^2 = -\lambda \kappa \cdot b, \]

Due to the topological number fluctuations

\[ m_A \sim \sqrt{\langle \theta^2 \rangle} \sim \Gamma_{sp} \sim T^4 \]

Thus, at high enough T \( m_A \gg \omega_{pl} \) → Cherenkov radiation is possible
THE DECAY CHANNEL

\[ x = k^0/p^0 = \omega/\varepsilon \]

\[ S_D = (2\pi)^4 \delta^{(4)}(p' + k - p)iM_D \quad iM_D = -ieQ \frac{\bar{u}_{p's'} \gamma^*_{k\lambda} u_{ps}}{\sqrt{8\varepsilon\varepsilon'\omega V^3}} \]

The decay probability \( dw_D = 2N_c \frac{1}{2} \sum_{\lambda ss'} |S_D|^2 f(\varepsilon)[1 - f(\varepsilon')] \frac{V d^3p' V d^3k' V d^3p}{(2\pi)^3 (2\pi)^3 (2\pi)^3} \)

\[ \omega \frac{d\Gamma_D}{d^3k} = 2N_c \frac{e^2Q^2\pi}{4(2\pi)^5} \int_0^1 \frac{dx}{x^4} f \left( \frac{\omega}{x} \right) \left[ 1 - f \left( \frac{\omega(1-x)}{x} \right) \right] \sum_{\lambda} \left\{ -m_A^2[(1-x)^2 + 1] - 2m^2x^2 \right\} \theta(-\kappa_{\lambda}) \]

\[ \kappa_{\lambda} = m_A^2(1 - x) + m^2x^2. \]

\( \kappa_{\lambda} \) is negative only if \( |m_A^2|(1 - x) > x^2m^2 \) and \( m_A^2 < 0 \)

\[ 0 \leq x < \frac{|m_A^2|}{2m^2} \left( \sqrt{1 + \frac{4m^2}{|m_A^2|}} - 1 \right) \]

Quark thermal mass \( m = gT/\sqrt{3} \)
**HIGH-T LIMIT**

- In the limit $m \ll |m_A|$ (high $T$) \hspace{1cm} 0 \leq x \leq 1 \hspace{1cm} \xi = 1/x - 1

\[
\omega \frac{d\Gamma_D}{d^3 k} = 2N_c \frac{e^2 Q^2}{8(2\pi)^4} |m_A|^2 \int_0^\infty d\xi \left( \xi^2 + (1 - \xi)^2 \right) f(\omega(1 + \xi)) [1 - f(\omega \xi)].
\]

\[
\approx 0.73 \cdot 2N_c \frac{e^2 Q^2}{8(2\pi)^4} |m_A|^2 \left[ \ln(1 + e^{-\beta \omega}) - \frac{2\text{Li}_2(-e^{-\beta \omega})}{(\beta \omega)^2} - \frac{4\text{Li}_3(-e^{-\beta \omega})}{(\beta \omega)^3} \right].
\]

Asymptotically:

\[
\omega \frac{d\Gamma_D}{d^3 k} = 0.73 \cdot 2N_c \frac{e^2 Q^2}{8(2\pi)^4} |m_A|^2 \begin{cases} 
\frac{3\zeta(3)}{(\beta \omega)^3}, & \omega \ll T \\
\frac{1}{\beta \omega} e^{-\beta \omega}, & \omega \gg T.
\end{cases}
\]

Since $|m_A|^2 \sim \omega$, the total rate $\Gamma_D$ is dominated by soft photons yielding $\log(T/m)$
THE ANNIHILATION CHANNEL

\[ y = p^0/k^0 = \varepsilon/\omega \]

\[ \omega \frac{d\Gamma_A}{d^3k} = N_c \frac{e^2 Q^2}{8(2\pi)^5} \int_0^1 dy f(y\omega) f((1-y)\omega) \sum_\lambda [m_A^2(2y^2 - 2y + 1) + 2m^2] \theta(-\kappa_\lambda), \]

\[ \kappa_\lambda = -m_A^2 y(1 - y) + m^2 \]

The allowed values of y:

\[ \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4m^2}{m_A^2}} \right) < y < \frac{1}{2} \left( 1 + \sqrt{1 - \frac{4m^2}{m_A^2}} \right) \]

In the limit \( m \ll |m_A| \)

\[ \omega \frac{d\Gamma_A}{d^3k} = N_c \frac{e^2 Q^2}{16(2\pi)^4} |m_A^2| \cdot \frac{1}{e^{\beta\omega} - 1} \left[ -\frac{8\text{Li}_3(-e^{\beta\omega}) + 6\zeta(3)}{(\beta\omega)^3} + \frac{4\text{Li}_2(-e^{\beta\omega}) - \pi^2/3}{(\beta\omega)^2} + \frac{2\ln(1 + e^{\beta\omega}) - \ln 4}{\beta\omega} - \frac{2}{3} \right] \]

\[ \omega \frac{d\Gamma_A}{d^3k} = N_c \frac{e^2 Q^2}{16(2\pi)^4} |m_A^2| \left\{ \begin{array}{ll} \frac{1}{6}, & \omega \ll T \\ \frac{2}{3} e^{-\beta\omega}, & \omega \gg T. \end{array} \right. \]
The contributions are comparable. Worth doing a more accurate estimate.

**FIG. 3.** Ideal QGP and hadronic photon rate near the crossover region.
CONCLUSION 2

At high temperature QGP emits photons via the chiral Cherenkov radiation (without any magnetic field).

This is a missing contribution at low $p_T$. 