10th international conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions (online)

Transport coefficient $\hat{q}$ in 2+1 flavor QCD plasma and pure gluon plasma

AMIT KUMAR
Wayne State University

Collaborators: Prof. Abhijit Majumder (WSU)
Dr. Johannes Weber (MSU)

WAYNE STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY
Office of Science

Outline

- Phenomenology based extraction of transport coefficient $\hat{q}$ in heavy-ion collisions

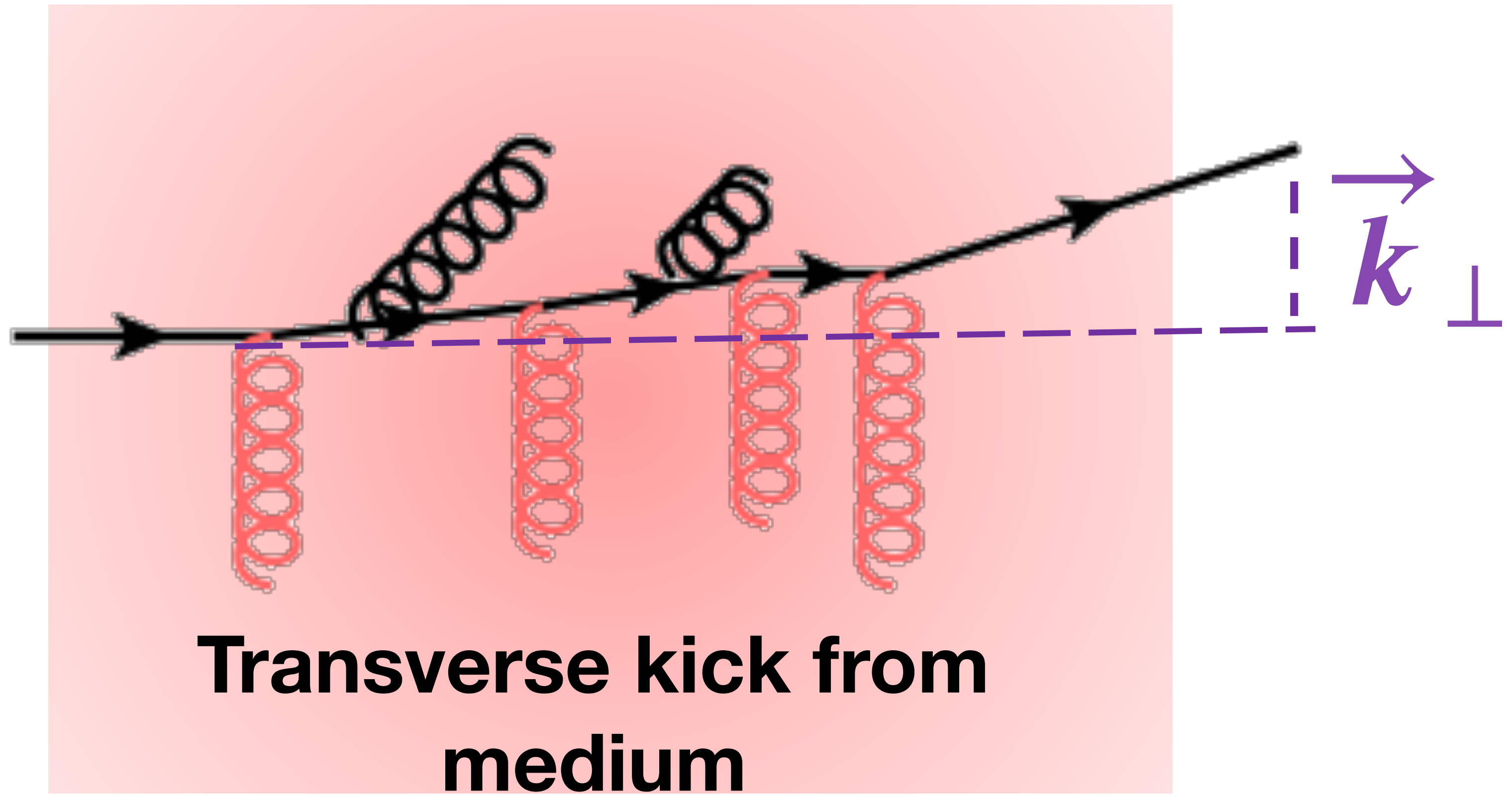
- Formulating $\hat{q}$ for hot QGP using Lattice gauge theory

1) Express $\hat{q}$ as a series of local operators using dispersion relation
2) Computing operators on quenched SU(3) lattices
3) Computing operators on unquenched SU(3) lattices

- Estimates of $\hat{q}$ for pure gluon plasma and 2+1 flavor QGP

First systematic extraction of $\hat{q}$ based on phenomenology

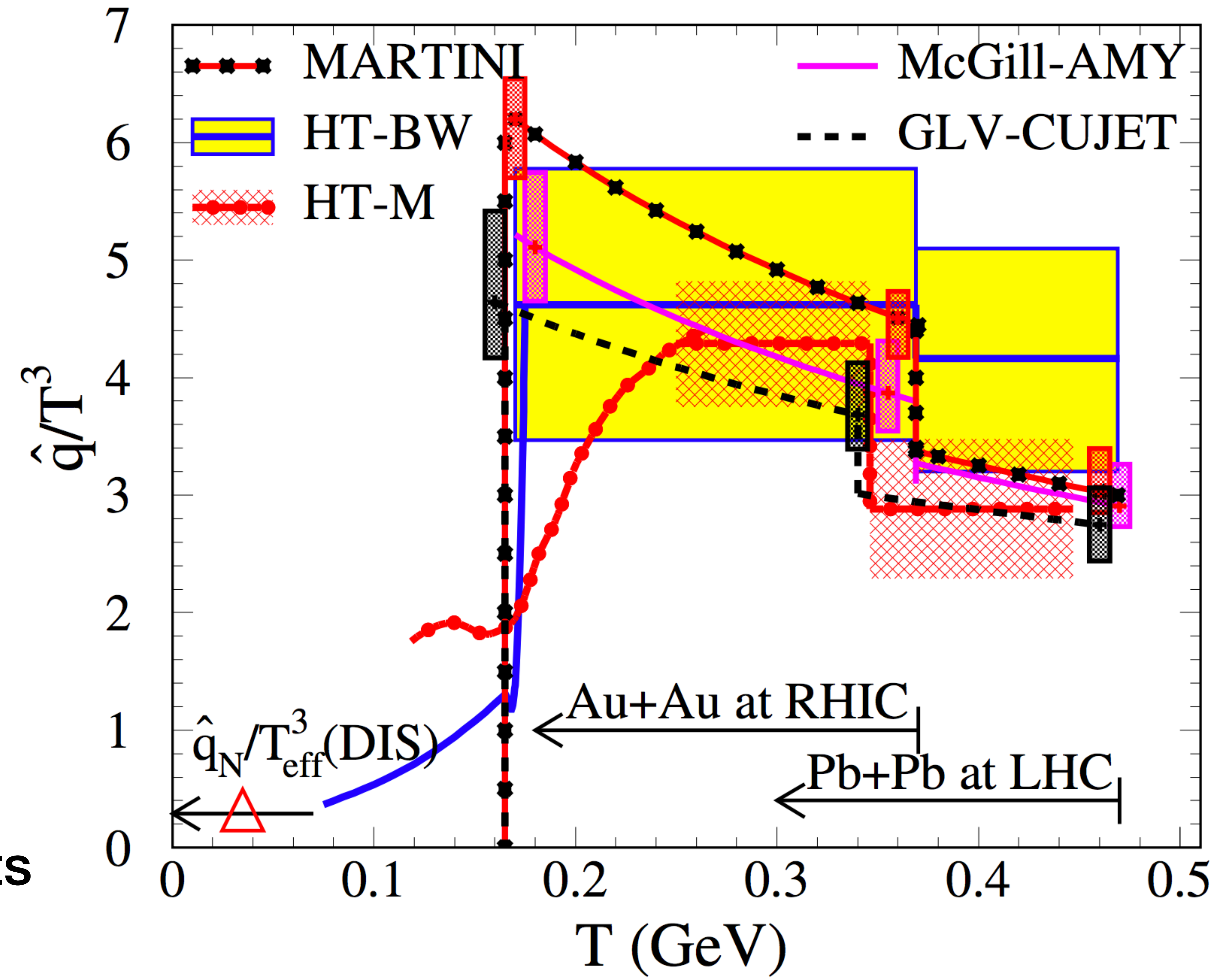
Leading parton going through medium



$$\hat{q}(\vec{r},t) = \frac{< \vec{k}_\perp^2 >}{L}$$

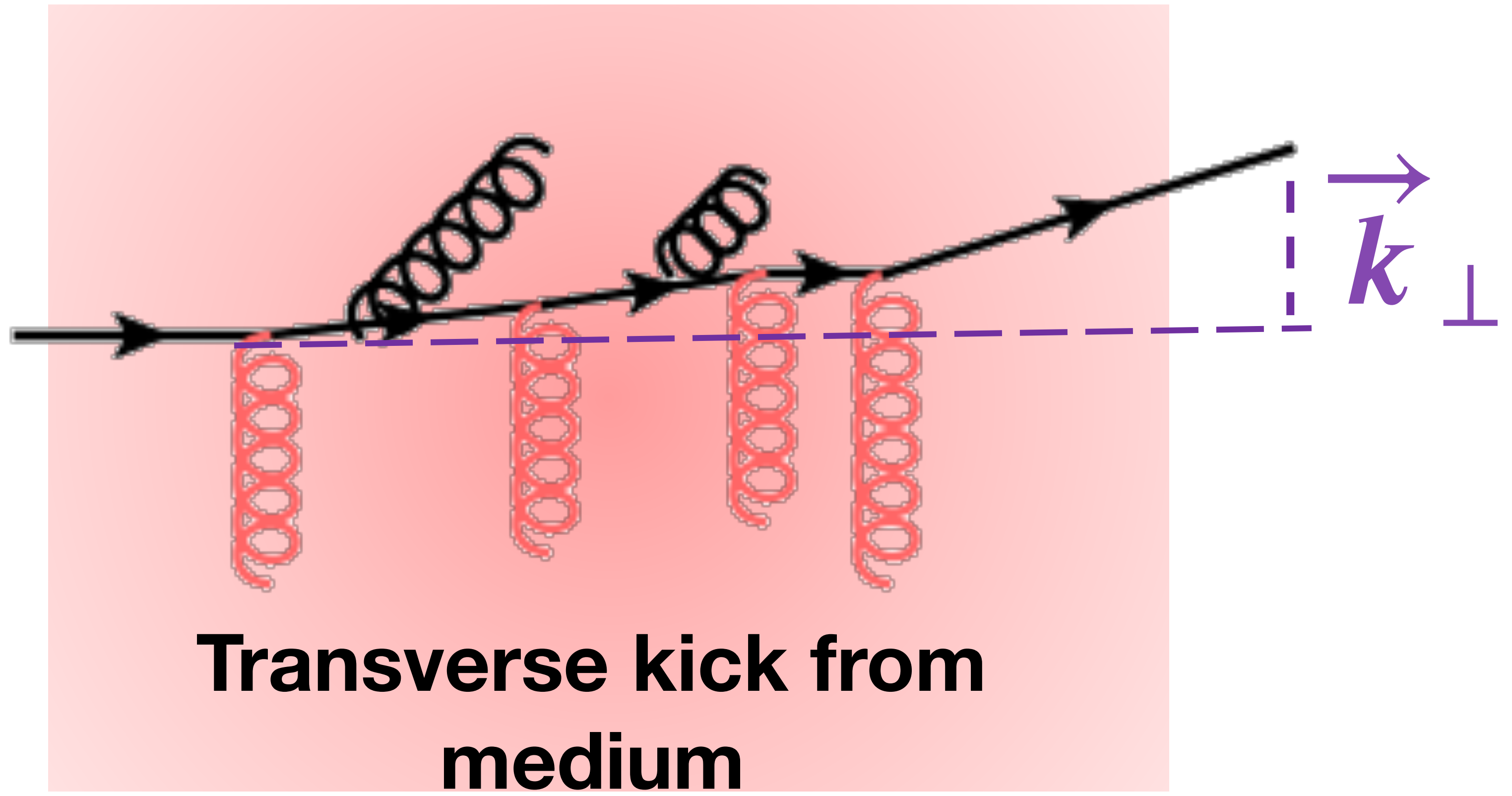
- ❑ $\frac{\hat{q}}{T^3}$ ~ 3-6 , Decreases as T increases
- ❑ Based on fit to hadron-R_{AA} measurements
- ❑ Based on single energy-loss scheme

JET Collaboration(Burke et al. 2014)



First systematic extraction of $\hat{q}$ based on phenomenology

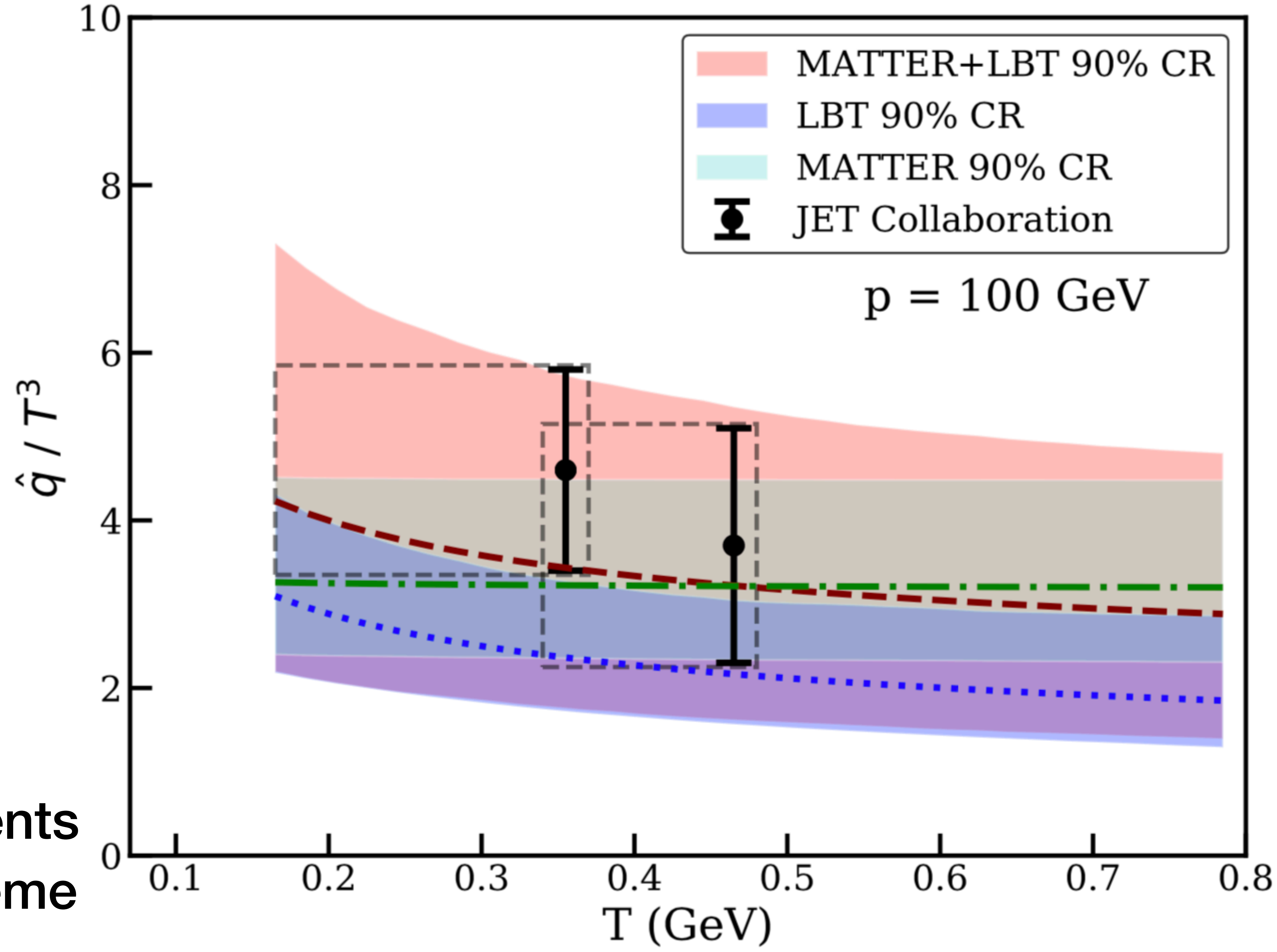
Leading parton going through medium



$$\hat{q}(\vec{r},t) = \frac{<\vec{k}_\perp^2>}{L}$$

- $\frac{\hat{q}}{T^3} \sim$ 3-6 , Decreases as T increases
- Based on fit to hadron-R_{AA} measurements
- Based on Multi-stage energy-loss scheme

Using JETSCAPE framework



(Taken from Ron Soltz's slides (HardProbes 2018))

Lattice formulation of $\hat{q}$

A. Majumder, PRC 87, 034905 (2013)

- **Leading order (LO) process:** A high energy quark propagating (along -ve z-dir) through plasma

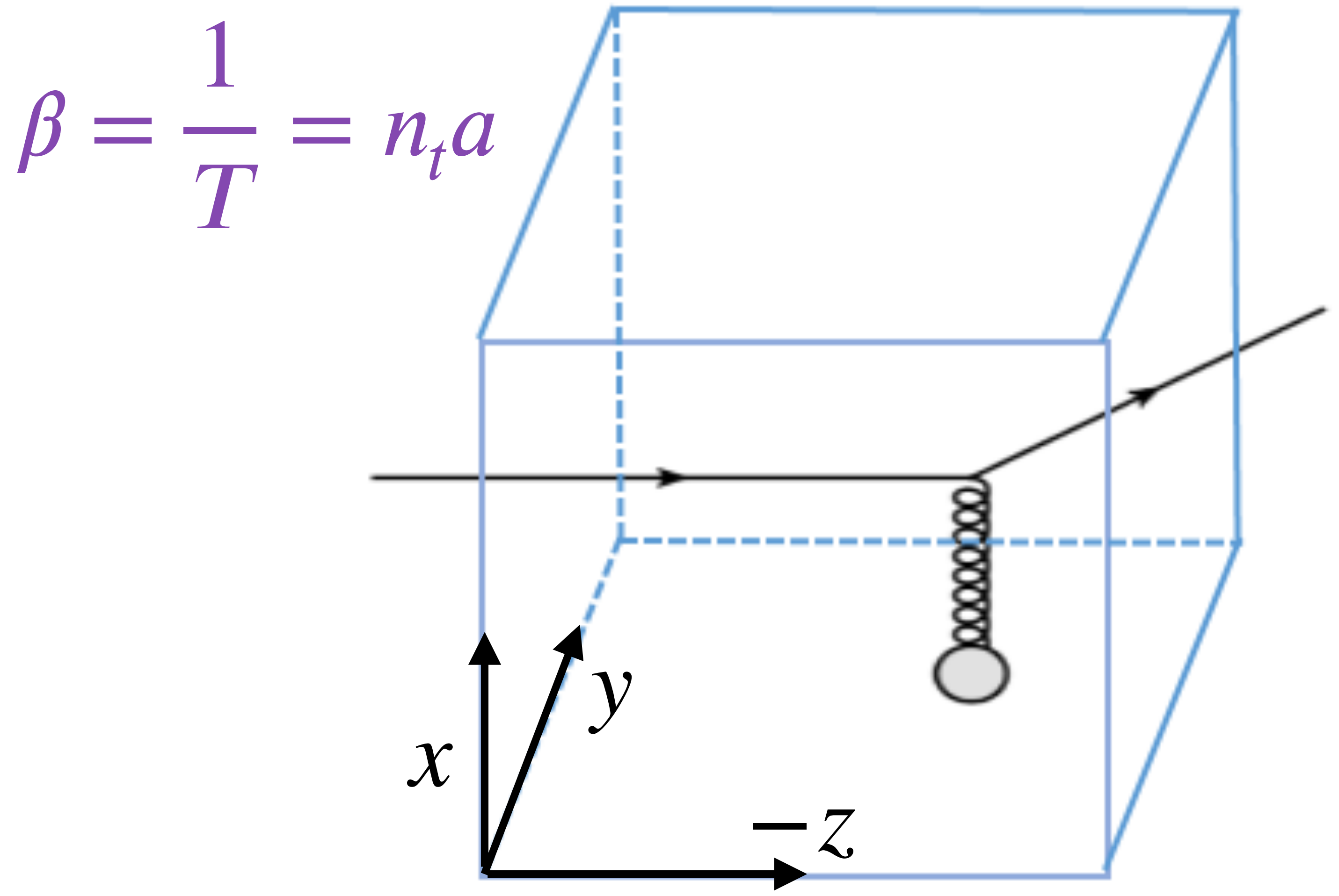
Quark momentum: $q = (\mu^2/2q^-, q^-, 0) \sim (\lambda^2, 1, 0)q^-;$

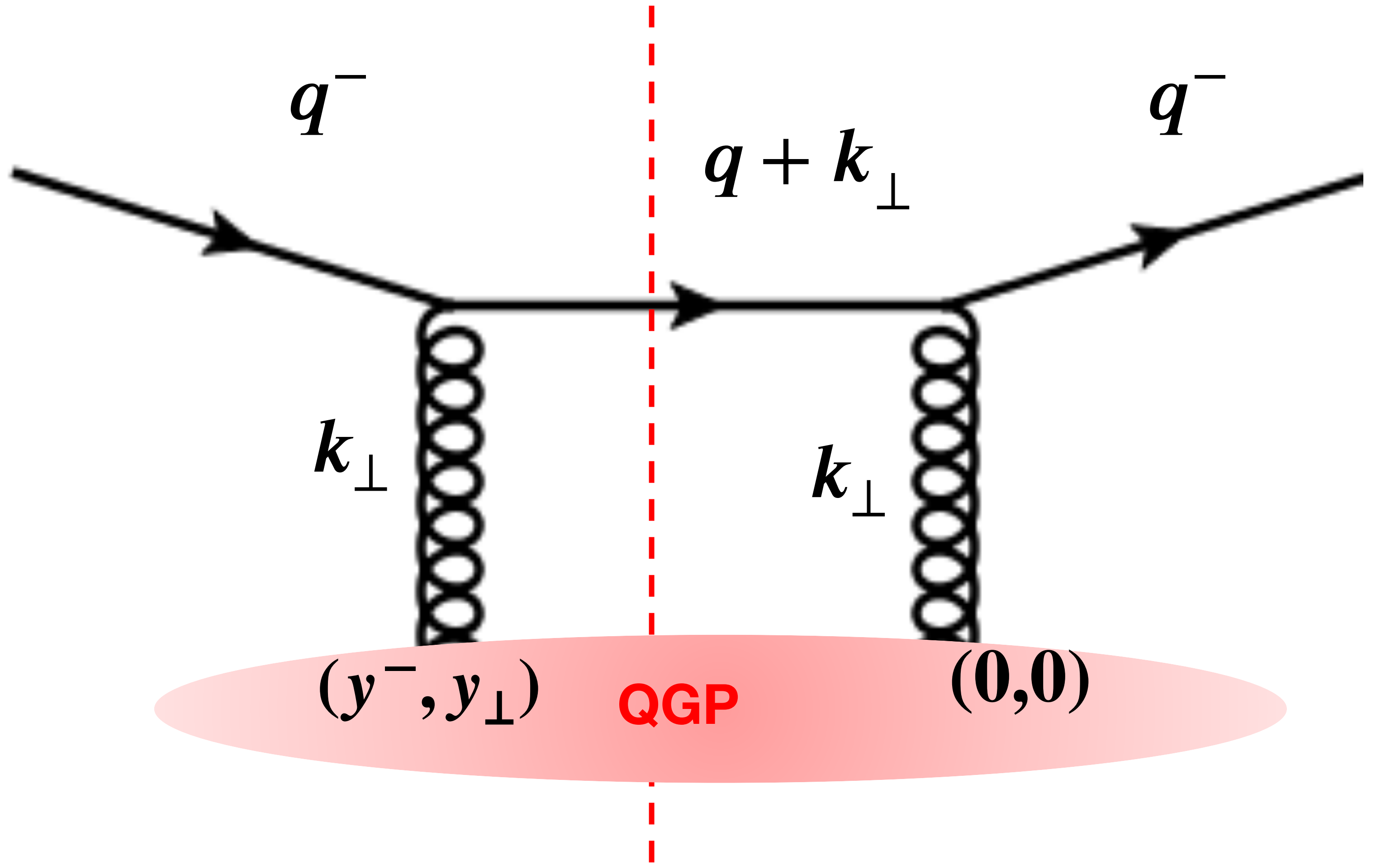
where $\lambda << 1;$ $\quad q^- = (q^0 - q^3)/\sqrt{2} \equiv$ hard scale

Transverse gluon: $k = (k^+, k^-, k_\perp) \sim (\lambda^2, \lambda^2, \lambda)q^-$

$$\hat{q} = \sum_k k_\perp^2 \frac{Disc\left[\mathscr{W}(k)\right]}{L^0};$$

$\mathscr{W}(k)$: **Transition probability**





$$\hat{q} = \frac{16\sqrt{2}\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp}{(2\pi)^3} d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp \vec{y}_\perp}$$
$$\times \langle M | \mathrm{Tr}[F^{+\perp_\mu}(y^-, y_\perp) F^+_{\perp_\mu}(0)] | M \rangle$$

(Lattice QCD)

Non-perturbative part

Constructing a more general expression as $\hat{Q}$

❖**Generalized object** $\hat{Q}$ **:** with q^- fixed and q^+ is variable

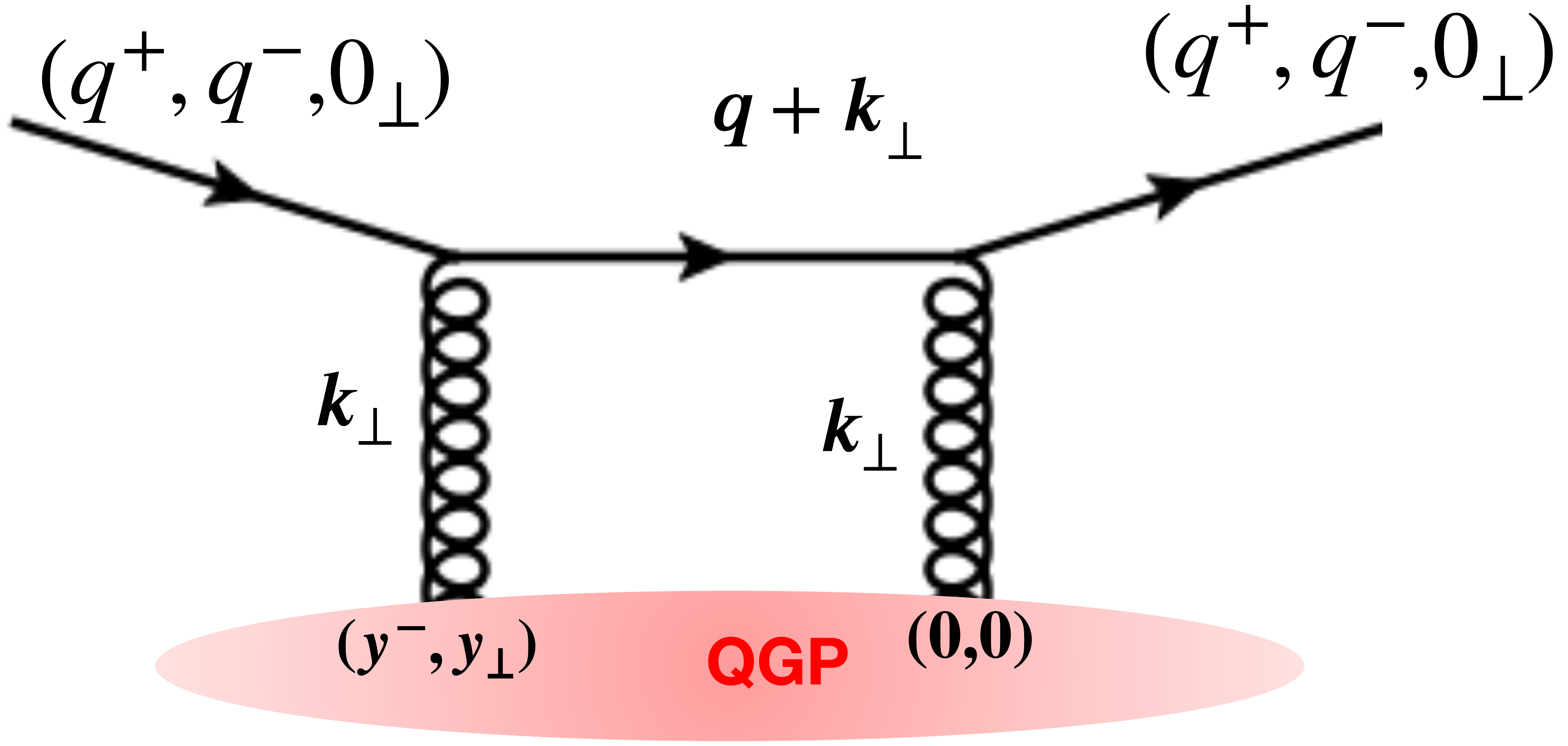
$$\hat{q} = \sum_k k_\perp^2 \frac{Disc\left[\mathscr{W}(k)\right]}{L^0};$$

$$\hat{Q} = \sum_k k_\perp^2 \frac{\mathscr{W}(k)}{L^0};$$

$$\hat{Q}(q^+) = \frac{16\sqrt{2}\pi\alpha_s}{N_c} \int \frac{d^4y d^4k}{(2\pi)^4} e^{iky} q^- \frac{\langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) F^+_{\perp_\mu}(y^-, y_\perp)] | M \rangle}{(q+k)^2 + i\epsilon}$$

▸ **When** $q^+ \sim 0 << q^-$ **:**

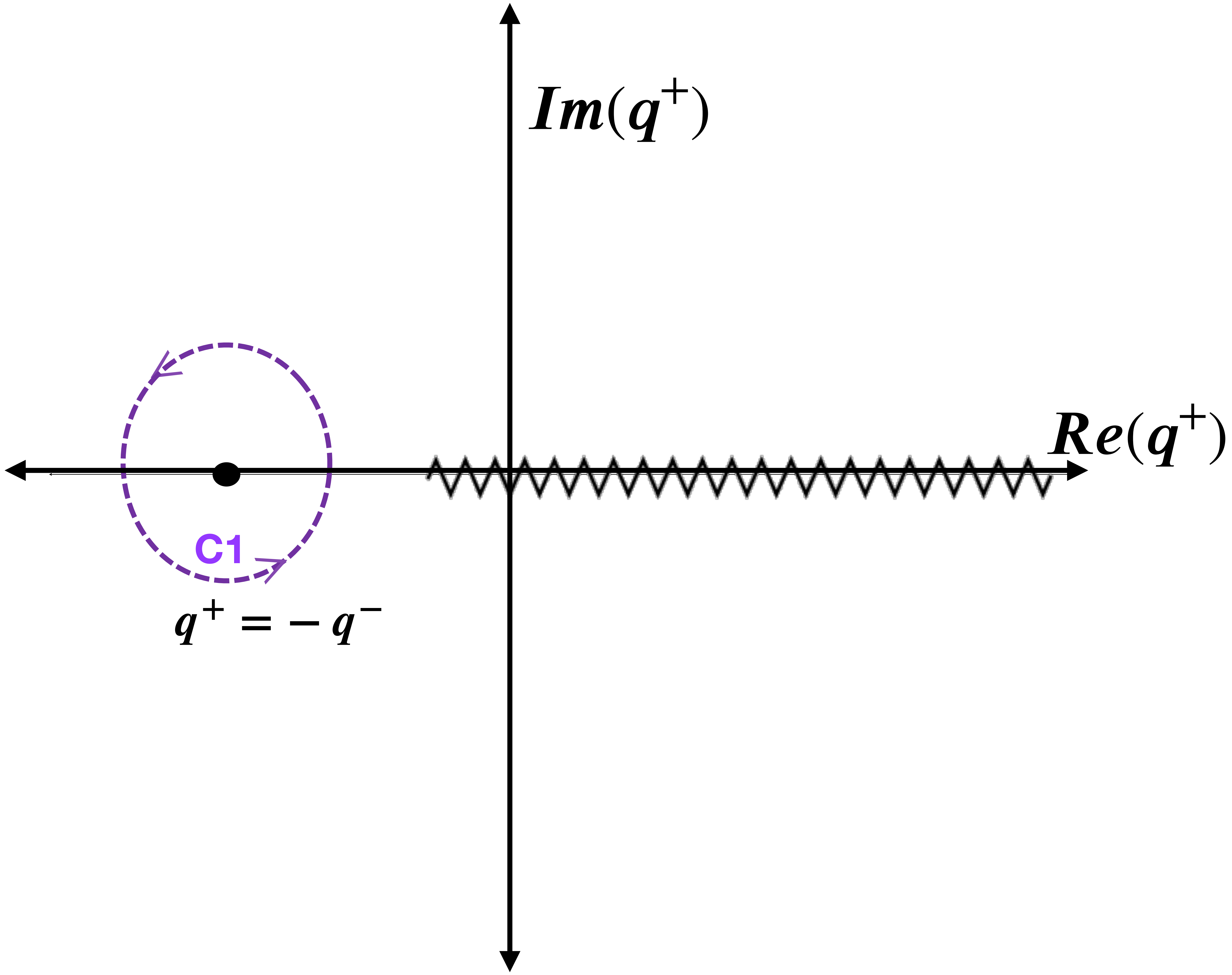
$$\mathrm{Disc}\left[\hat{Q}(q^+)\right]\Big|_{q^+\sim 0} = \hat{q}$$



▸ **When** $q^+ = -q^-$ **:** Perform d^4y and d^4k integration

$$\hat{Q}(q^+ = -q^-) = \frac{8\sqrt{2}\pi\alpha_s}{N_c q^-} \langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F^+_{\perp_\mu}(0)] | M \rangle$$

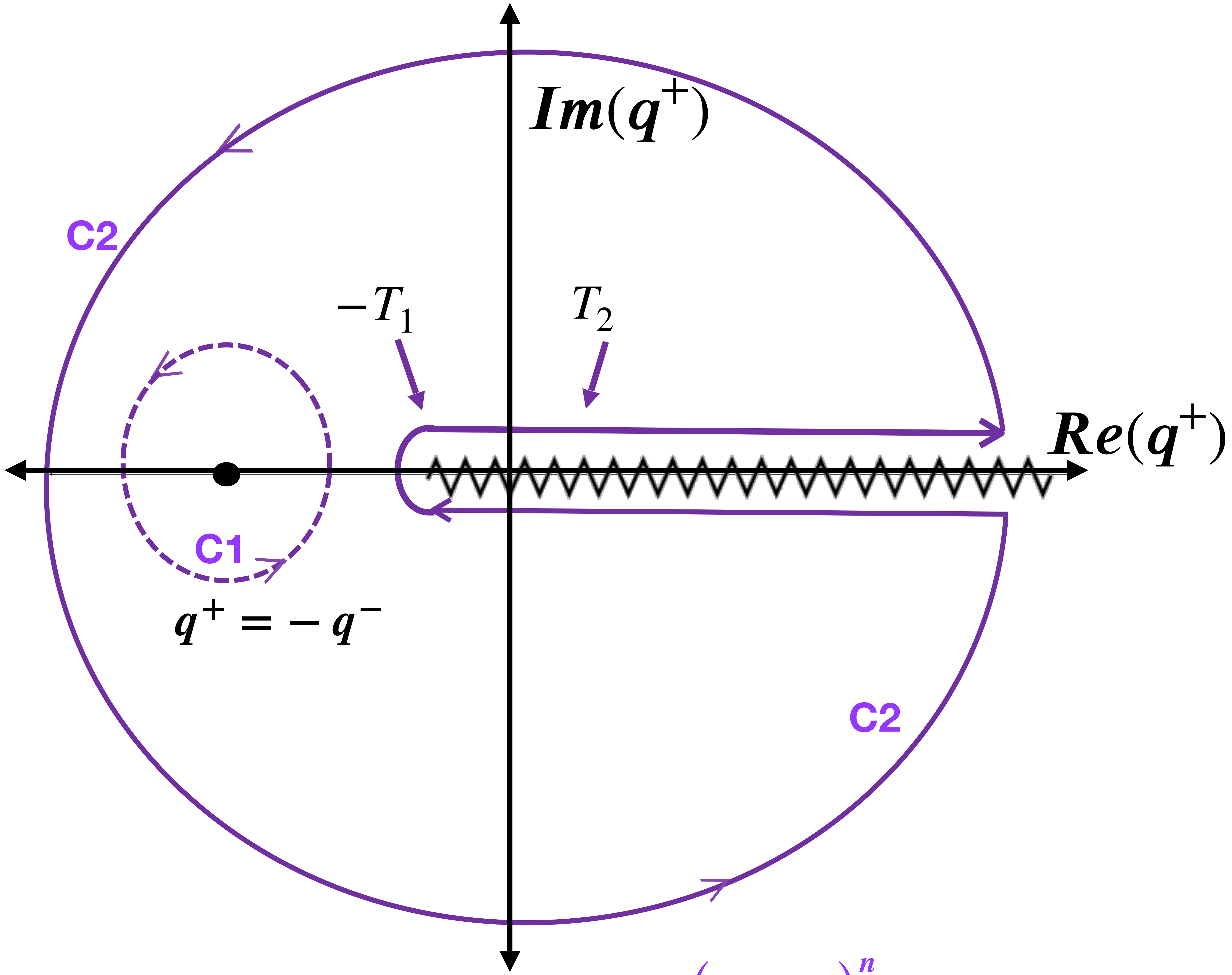
Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



Contour C1:

$$I = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



Contour C1:

$$I = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Contour C2: On extending it to infinity

$$I = \int_{-T_1}^{T_2} dq^+ \frac{\hat{q}(q^+)}{(q^+ + q^-)} + \int_0^\infty \frac{dq^+}{2\pi i} \frac{Disc[\hat{Q}(q^+)]}{(q^+ + q^-)}$$

Pure thermal part   **Pure Vacuum part**

$$\hat{q} = \frac{8\sqrt{2}\pi\alpha_s}{N_c(T_1+T_2)} \langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F^{+}_{\perp_\mu}(0)] | M \rangle_{(\mathrm{Thermal-Vacuum})}$$

Width of thermal discontinuity $2T$ or $4T$ (HTL analysis)

$\Longrightarrow$ Source of systematic error

$\hat{q}$ as a series of local operators

X. Ji, PRL 110, 262002 (2013)
Parton PDF: Operator product expansion with D_z derivatives

❖ **Physical form of $\hat{q}$ at LO:**

$$\hat{q} = \frac{8\sqrt{2}\pi\alpha_s}{N_c(T_1+T_2)} \langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F^{+}_{\perp_\mu}(0)] | M \rangle_{(\mathrm{Thermal-Vacuum})}$$

Rotating to Euclidean space: $x^0 \to -ix^4;\ A^0 \to iA^4$

$$\implies F^{0i} \to iF^{4i}$$

❖ **Non-zero operators:**

LO operators at n=0 : $\mathrm{Tr}\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right] \implies E_i^2 + B_i^2$

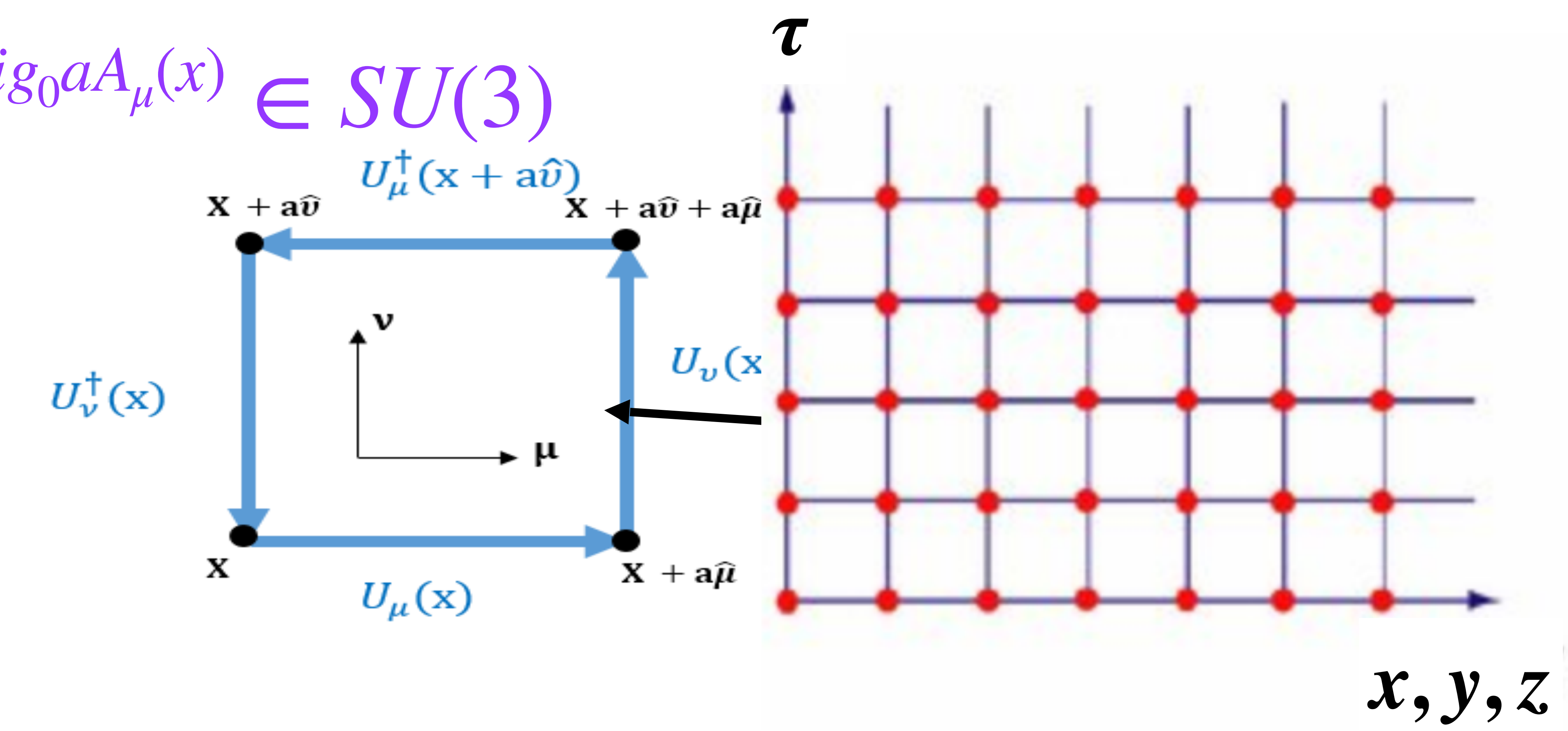
LO operators with D_z^2 derivative at n=2 : $\mathrm{Tr}\left[F^{3i}D_z^2F^{3i} - F^{4i}D_z^2F^{4i}\right]$

LO operators with D_z^4 derivative at n=4 : $\mathrm{Tr}\left[F^{3i}D_z^4F^{3i} - F^{4i}D_z^4F^{4i}\right]$

where, D_z is covariant derivative along leading parton direction (z-dir)

Basic elements of lattice QCD

- Periodic in imaginary time direction
- **Fields:** Fermions (on sites) and Gauge field (on Link), $U_\mu(x) = e^{ig_0 a A_\mu(x)} \in SU(3)$
- **Finite temperature lattice:** $n_\tau a \ll n_s a$, where $T = 1/(n_\tau a)$
- **Vacuum lattice:** $n_\tau a \geq n_s a$



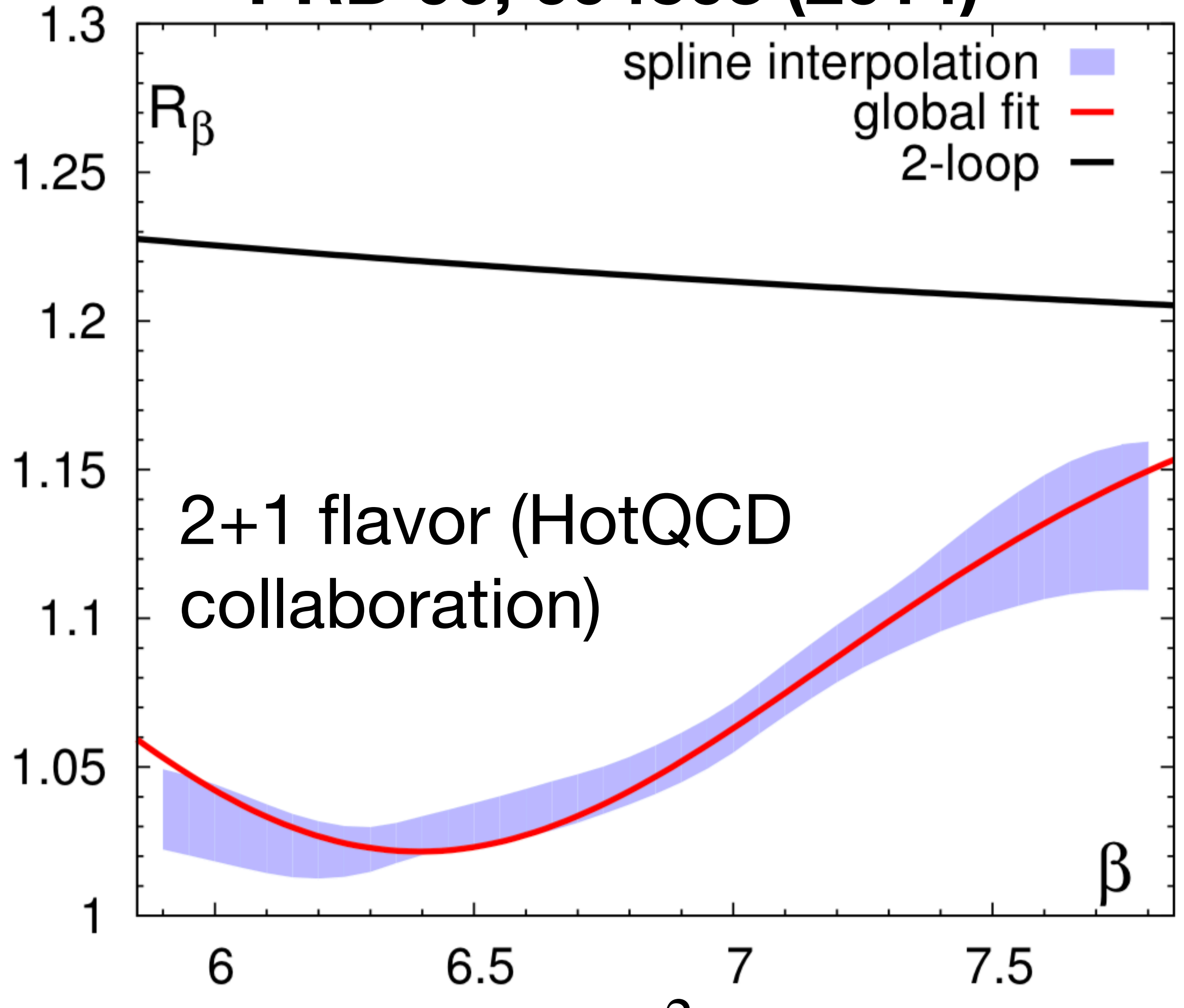
$$< O > = \frac{\int DU\, D\bar{\psi} D\psi\, O\, e^{-S_f - S_g}}{\int DU\, D\bar{\psi} D\psi\, e^{-S_f - S_g}} = \frac{\int DU\, det(\not{D}[U] + m_f)\, O\, e^{-S_g}}{\int DU\, det(\not{D}[U] + m_f)\, e^{-S_g}}$$

- **Full QCD action:** Highly-improved Staggered quark (HISQ) action
  + Tree-level Symanzik improved gauge action $\mathcal{O}(a^4) + \mathcal{O}(g_0^2 a^2)$,
  - Strange quark mass: fixed to physical value
  - Light quark mass: fixed to $m_{u,d} = m_s/20 \longrightarrow m_\pi \approx 160\text{MeV}$ (in continuum),
  - Configurations are generated using public version of MILC code
- **Pure SU(3) gauge action:** $< O > = \frac{\int DU\, O\, e^{-S_g}}{\int DU\, e^{-S_g}}$, Wilson's gauge action $\mathcal{O}(a^2) + \mathcal{O}(g_0^2 a^2)$, bare coupling $\beta = 6/g_0^2$

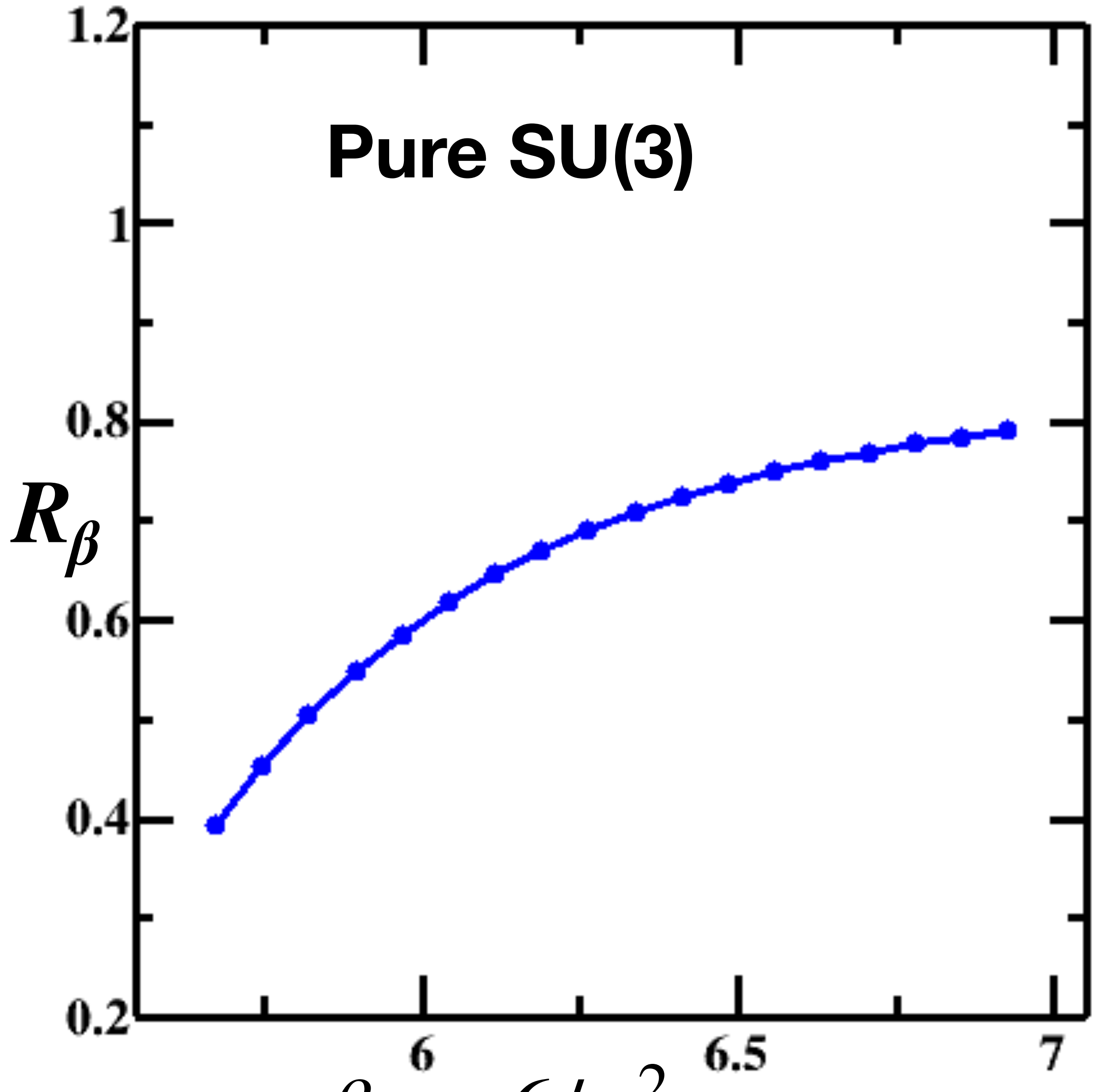
Renormalization factors for FF correlators

- **Lattice beta-function:** $R_\beta = -a\frac{d\beta_0}{da}$

PRD 90, 094503 (2014)



$\beta_0 = 10/g_0^2$



$\beta_0 = 6/g_0^2$

- **For pure SU(3) gauge:**

$$a_L = \frac{f}{\Lambda_L}\left(\frac{11}{16\pi^2 g_0^2}\right)^{\frac{-51}{121}} exp\left(-\frac{8\pi^2}{11 g_0^2}\right)$$

$$T_c = 265\text{MeV}, \Lambda_L = 5.5\text{MeV}$$

Tune $f(g_0^2)$ such that T_c/Λ_L is independent of g_0^2

- **Tadpole improvements for Covariant derivative terms:** Minimize Discretization effects

Tadpole factor, $u_0 = \langle \text{Average Plaquette} \rangle^{1/4} \equiv$ **(mean link)**

Covariant derivative terms:

$$D_z^2 F_{3i}(z) = \frac{1}{a^2}\left[\frac{U_z(z)F_{3i}(z+a)U_z^\dagger(z) + U_z^\dagger(z-a)F_{3i}(z-a)U_z(z-a)}{u_0^2} - 2F_{3i}(z)\right]$$

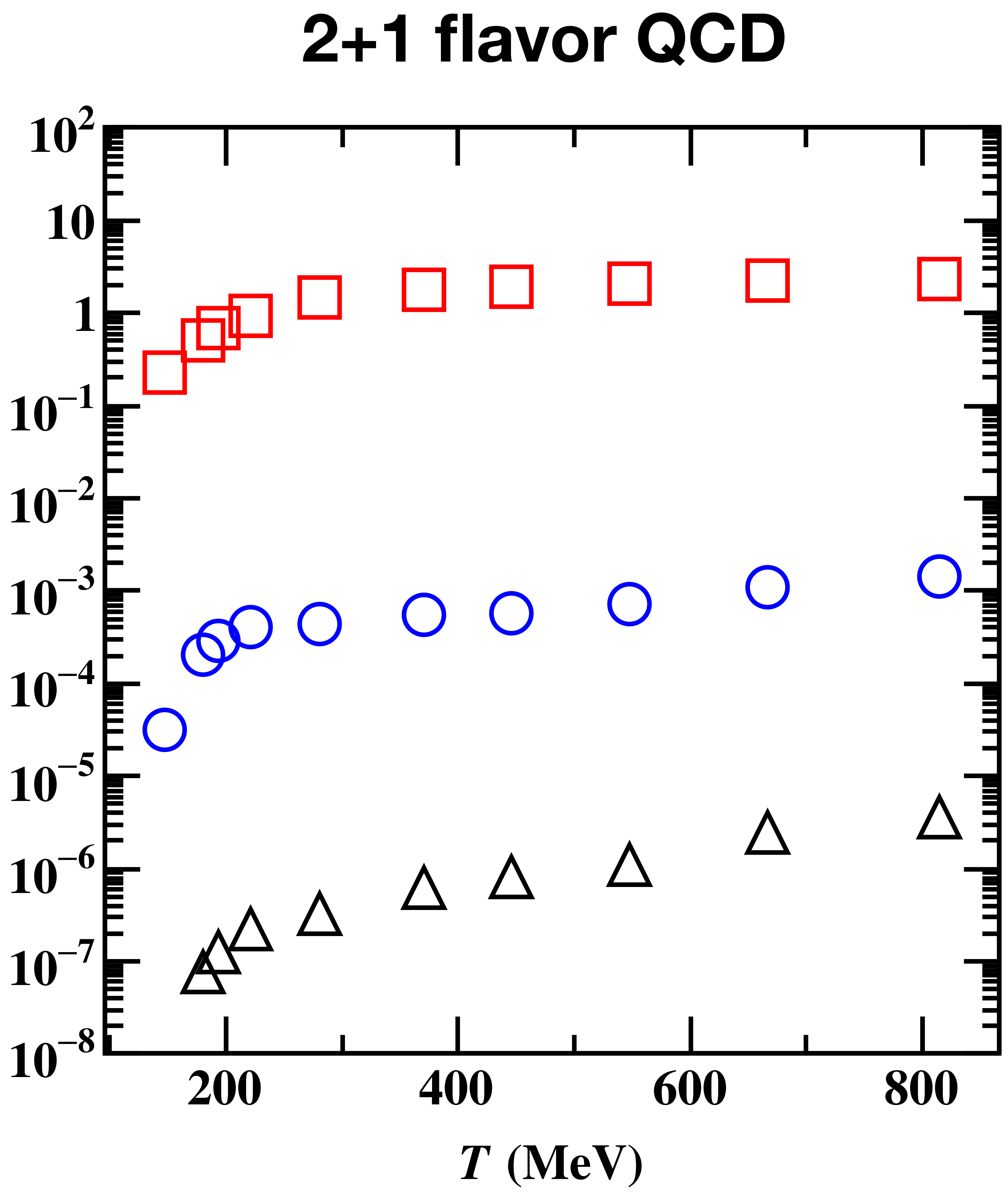
FF correlator in unquenched and quenched SU(3) lattices

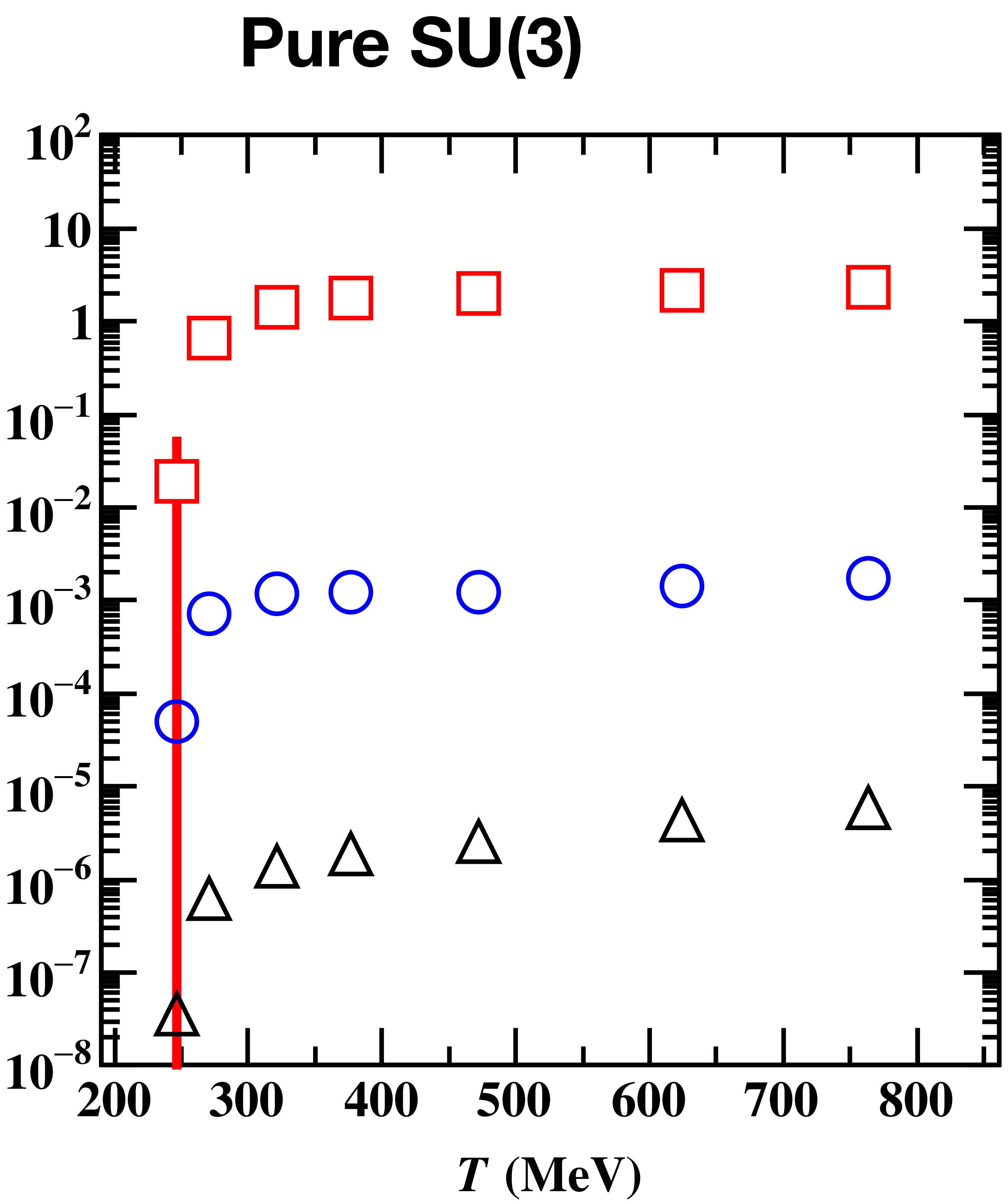
$$n_\tau = 6, n_s = 4n_\tau$$

$$\frac{1}{T^4}\mathrm{Tr}\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right]$$

$$\frac{-1}{T^4(q^-)^2}\mathrm{Tr}\left[F^{3i}D_z^2F^{3i} - F^{4i}D_z^2F^{4i}\right]$$

$$\frac{1}{T^4(q^-)^4}\mathrm{Tr}\left[F^{3i}D_z^4F^{3i} - F^{4i}D_z^4F^{4i}\right]$$





- LO FF correlator is dominant
- FF correlator with D_z^2 derivative are suppressed by a factor of 10^3 compared LO operator
- FF correlator with D_z^4 derivative are suppressed by a factor of 10^6 compared LO operator

$\hat{q}$ for 2+1 flavor QCD and pure gluon plasma

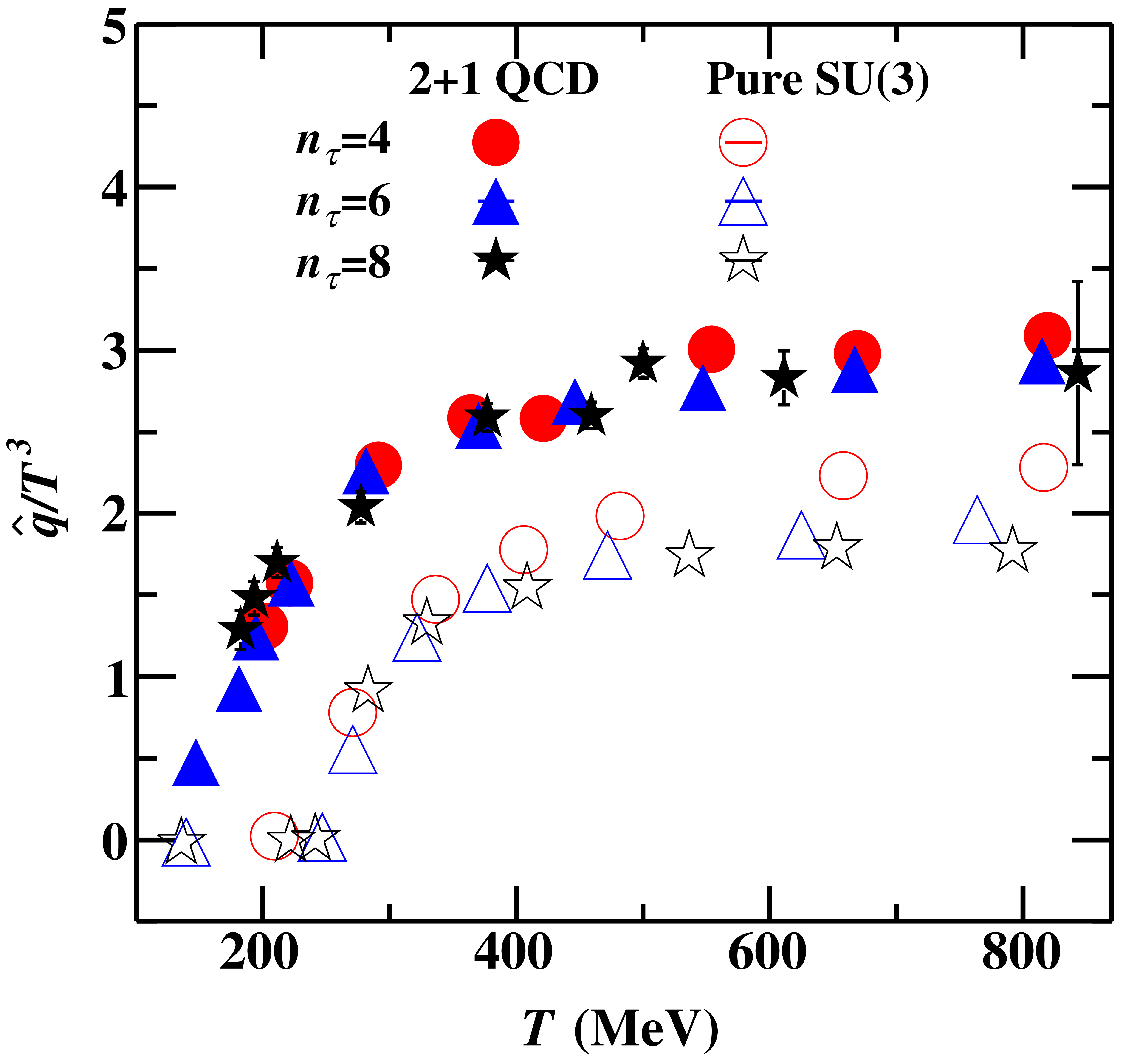
- **At high temperature:** $\hat{q} \propto T^3$
  - $\dfrac{\hat{q}}{T^3} \sim$ 2.5-3.5 (2+1 flavor QCD plasma)
  - $\dfrac{\hat{q}}{T^3} \sim$ 1.5-2.5 (pure gluon plasma)

- **At low temperature:**

  $\hat{q}/T^3$ becomes smaller and No signature of log-dependence as in HTL form

  $$(\text{HTL formula}) \equiv \hat{q} \propto T^3 \ln(E/T)$$

- **The qualitative behavior is similar to entropy density**

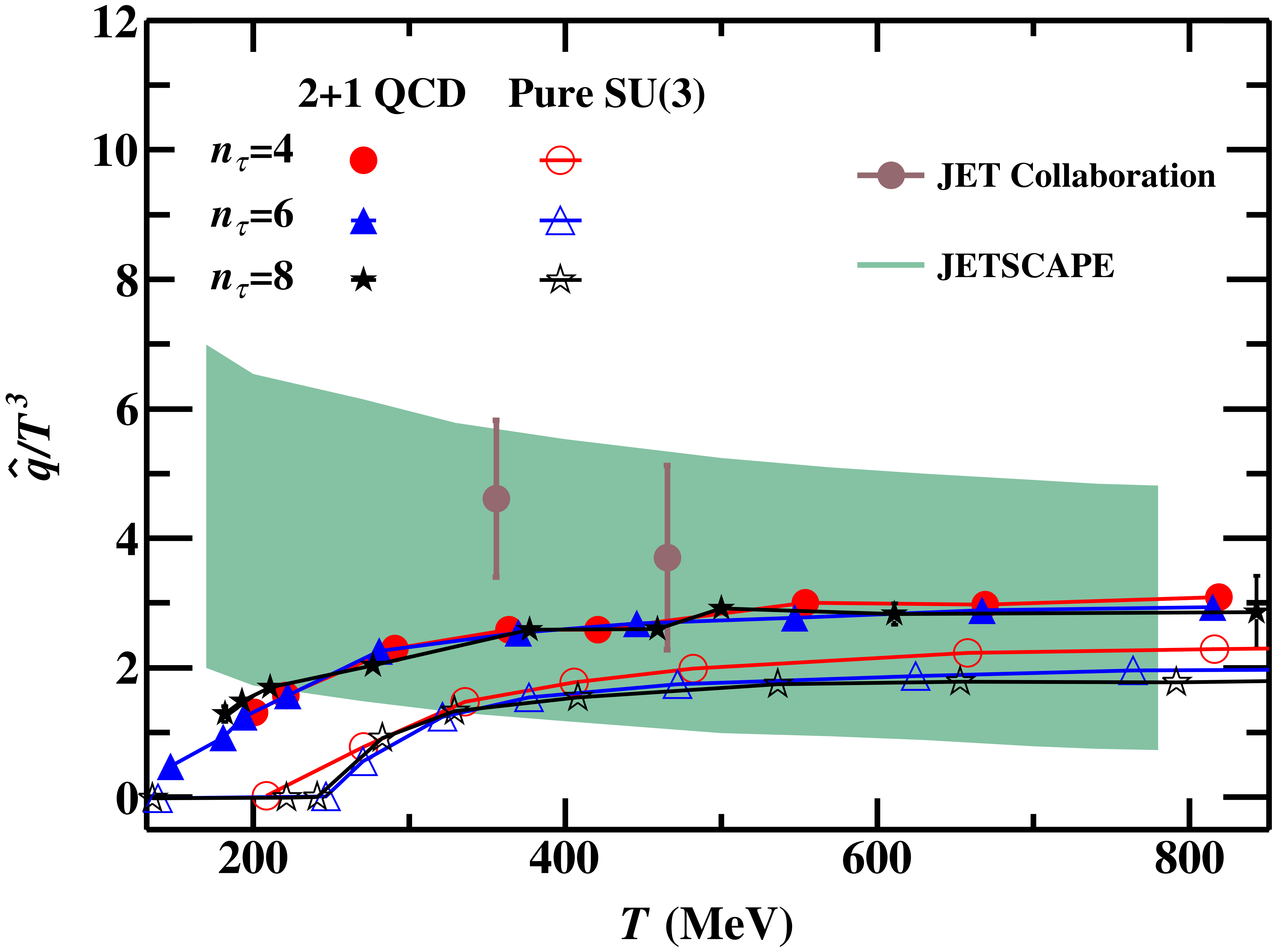


Comparison with extractions based on heavy-ions experimental data

- Lattice extractions are consistent to JET and JETSCAPE collaboration extraction

- Lattice extracted $\hat{q}$ does not show a log-like behavior

$$\hat{q} \propto T^3 \ln\left(\frac{E}{T}\right) \text{ (HTL formula)}$$



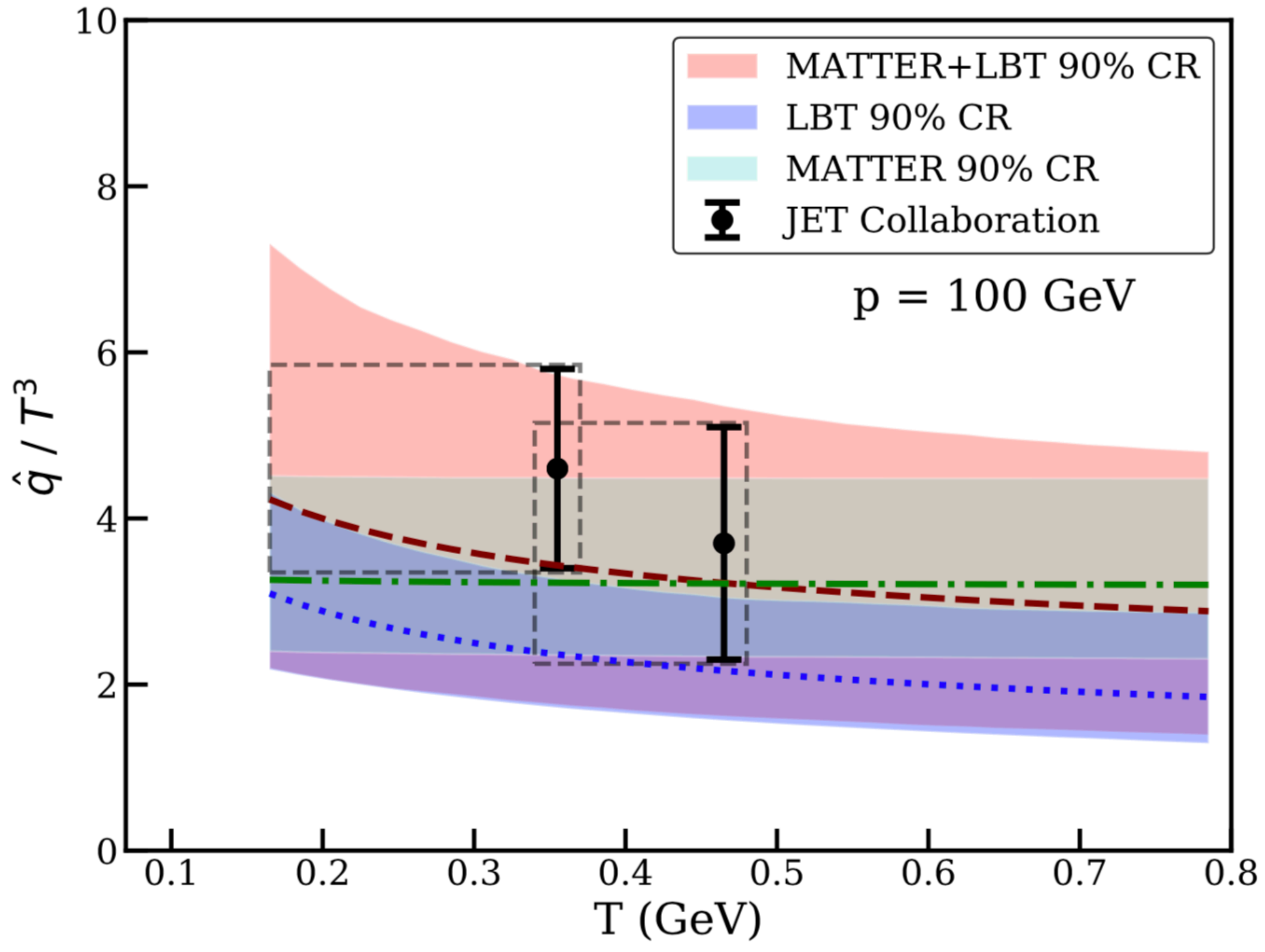
Summary and Future work

- **First lattice calculation of $\hat{q}$ on 2+1 flavor QCD plasma**
  - $\hat{q}/T^3 \sim$ 2.5-3.5 (QCD plasma) and 1.5-2.5 (pure gluon plasma)
  - Consistent with JET and JETSCAPE collaboration results within their uncertainty band
  - Analytic continuation to deep Euclidean space and expressed as local operators
  - $\hat{q}$ does not show a log-dependence as in the HTL formula

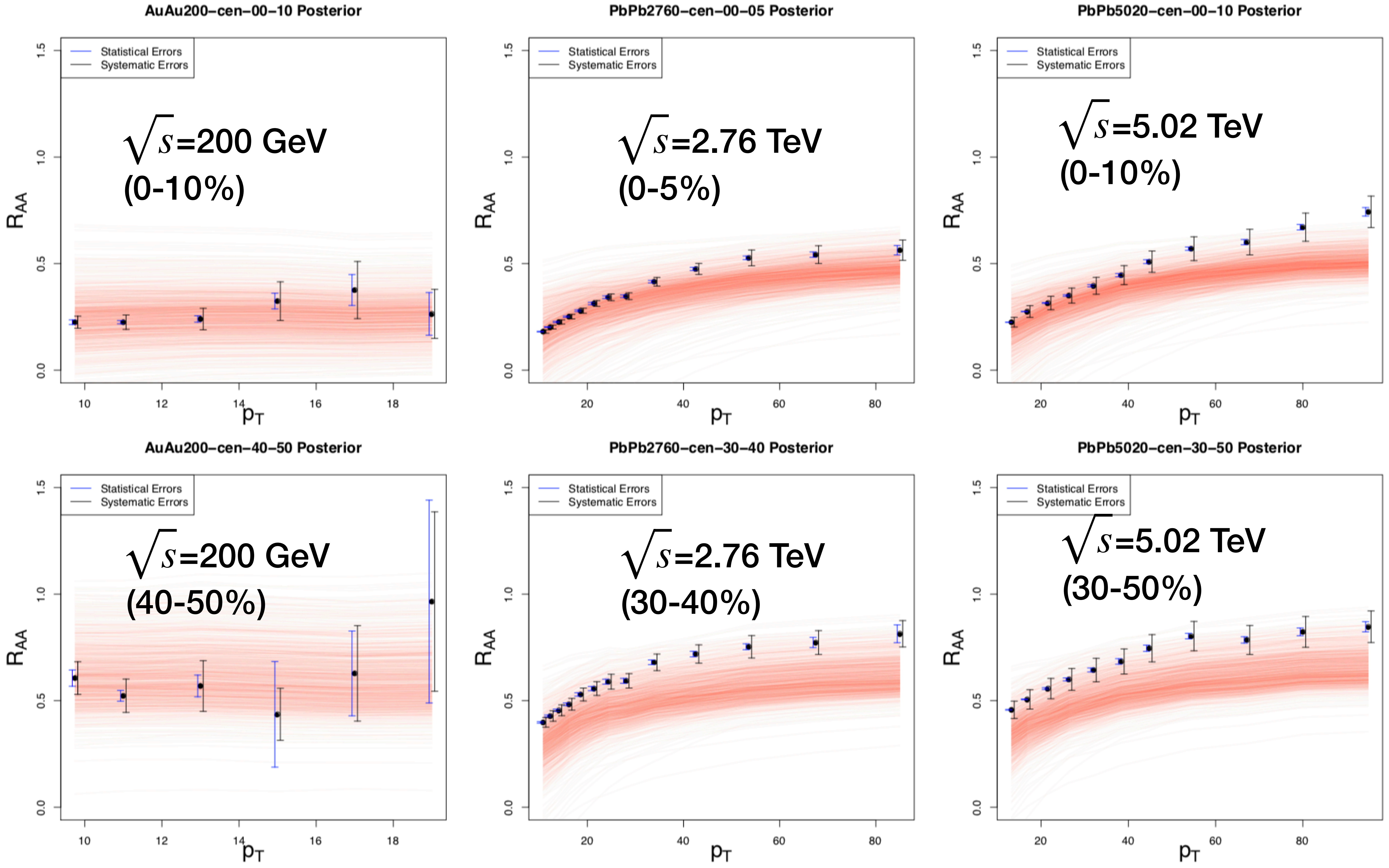
- **Future work**
  - Improve the statistics for $n_\tau = 8$ (Full QCD case)
  - Extend calculation to finer lattices $n_\tau = 10, 12$ and perform continuum extrapolation
  - Include medium induced radiative corrections

First Bayesian and multi-stage extraction of $\hat{q}$

Using JETSCAPE framework



(Taken from Ron Soltz's slides (HP 2018))



- Multi-stage energy-loss approach within JETSCAPE
- Jet energy loss model: MATTER+LBT with switching virtuality Q0 ~ 1-2 GeV
- Based on fit to experimental data

HISQ action

- **Staggered fermion action** $$S_{KS} = \sum_x \bar{\chi}(x)\left\{\sum_\mu \eta_\mu(x)\nabla_\mu\chi(x) + m\chi(x)\right\}$$

$$\psi(x) = \Gamma_x\chi(x), \quad \bar{\psi}(x) = \bar{\chi}(x)\Gamma_x^\dagger, \qquad \Gamma_x = \gamma_1^{(x_1/a)}\gamma_2^{(x_2/a)}\gamma_3^{(x_3/a)}\gamma_4^{(x_4/a)}$$

$$(-1)^{(x_1+\cdots+x_{\mu-1})/a} \equiv \eta_\mu(x)$$

- **Replace the covariant derivative with the Naik term with smearing and reunitarization**

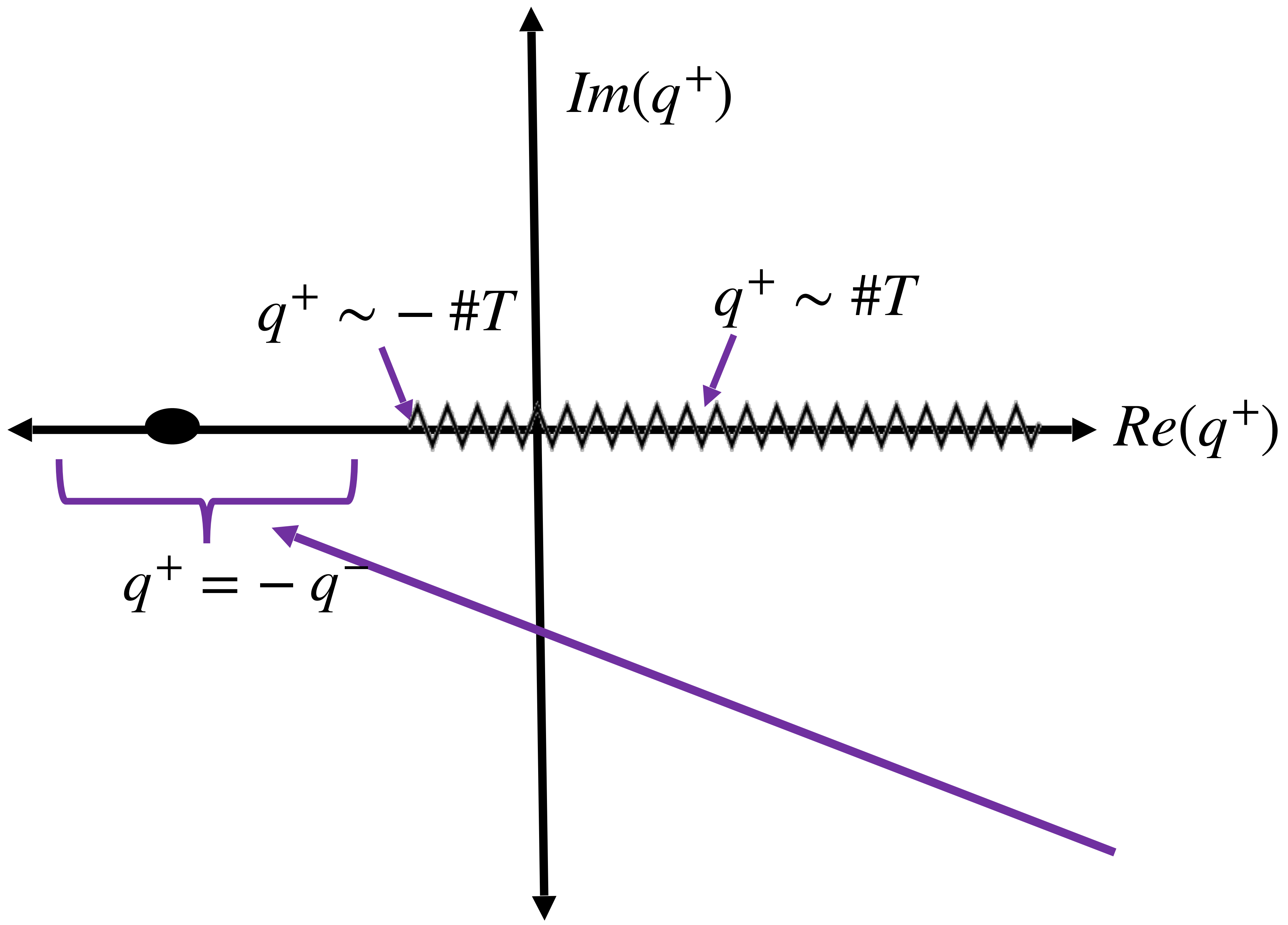
$$\nabla_\mu[U]\chi(x) \to \nabla_\mu(x)[X]\chi(x) - \frac{a^2}{6}(1+\varepsilon)(\nabla_\mu)^3[W]\chi(x).$$ **, where**

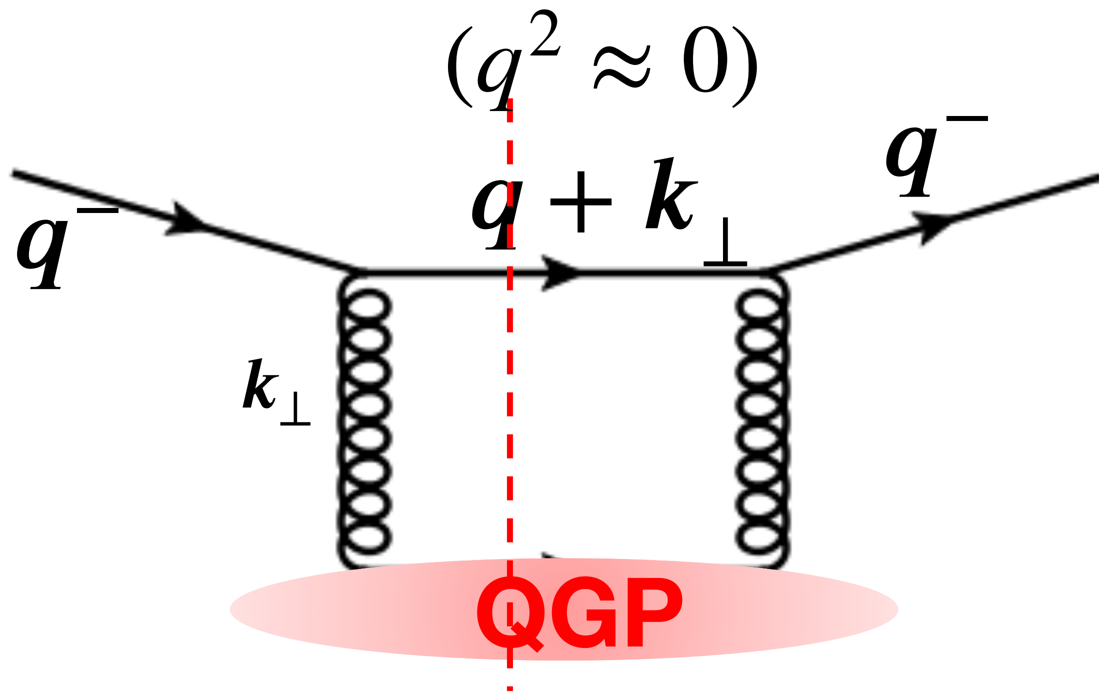
$$V_\mu(x) = \mathcal{F}^{f7}U_\mu(x),$$

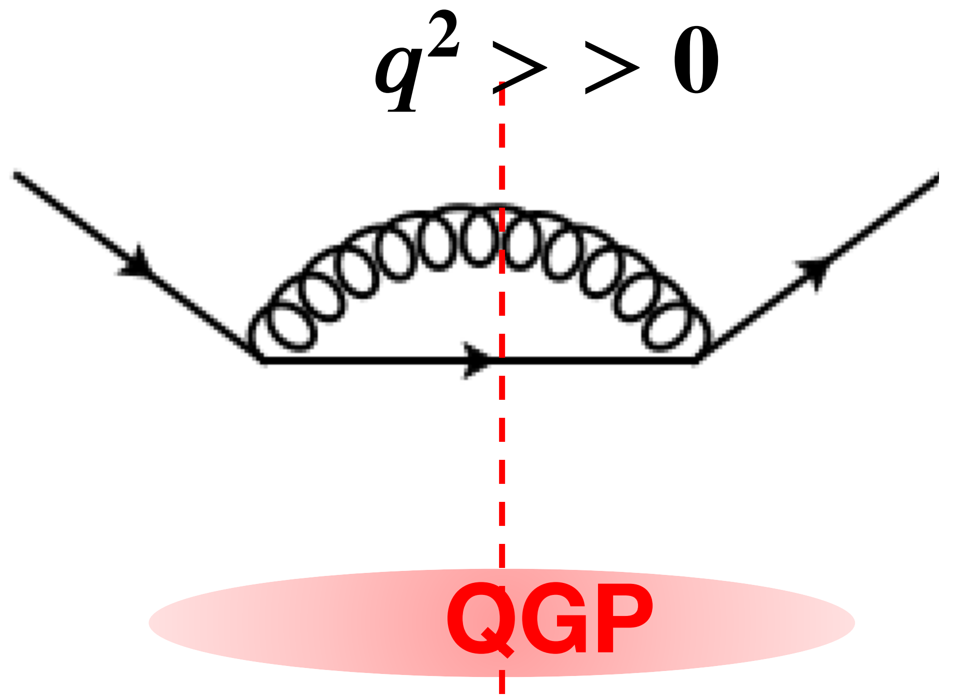
$$W_\mu(x) = \mathcal{U}V_\mu(x) = \mathcal{U}\mathcal{F}^{f7}U_\mu(x),$$

$$X_\mu(x) = \mathcal{F}^{f7L}W_\mu(x) = \mathcal{F}^{HISQ}U_\mu(x),$$

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$







$$\hat{Q}(q^+) = \frac{16\sqrt{2}\pi\alpha_s}{N_c} \int \frac{d^4y\, d^4k}{(2\pi)^4} e^{iky} q^- \frac{\langle M | F^{+\perp_\mu}(0) F^+_{\perp_\mu}(y^-, y_\perp) | M \rangle}{(q+k)^2 + i\epsilon}$$

1)When $q^+ \in \left[-\#T, \#T\right]$

$q^2 \approx 0$ **(in-medium scattering) ;**

2) When $q^+ \in [\#T, +\infty)$

$q^2 \gg 0$ **(Bremsstrahlung radiation) ;**

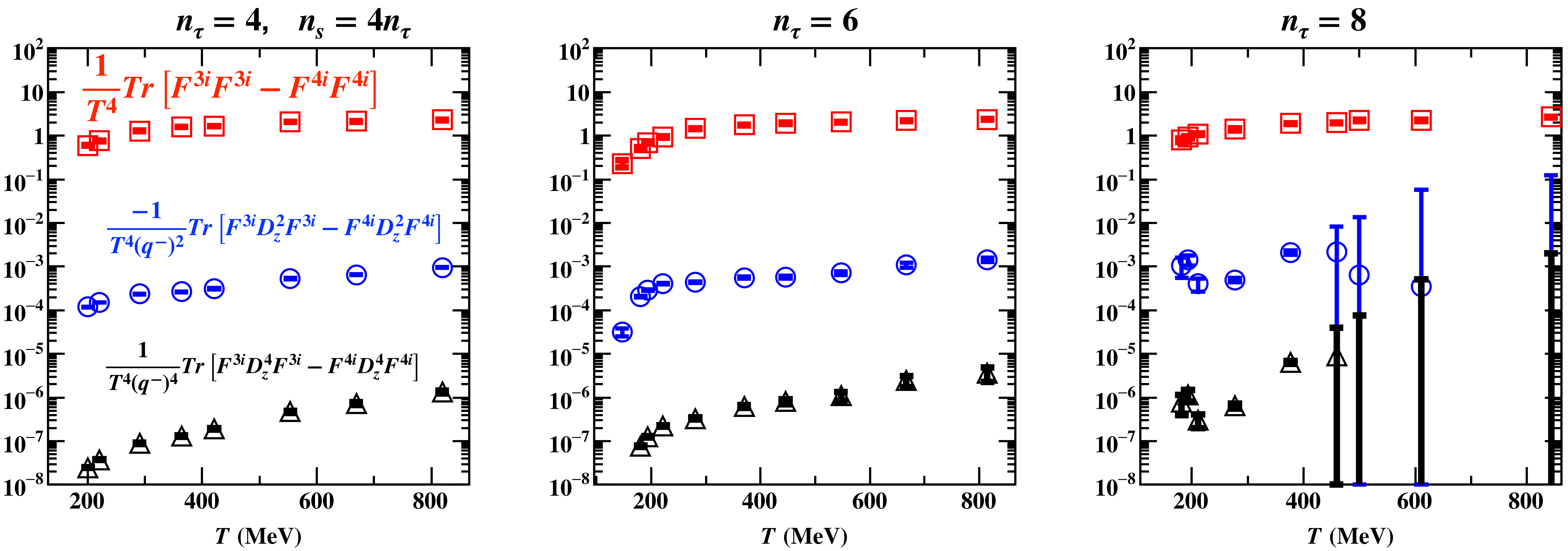
3) When $q^+ \in (-\infty, -\#T]$

$q^2 \ll 0$ **(Space-like) ;**

$$\lim_{q^+ \to -q^-} \mathrm{Disc}\left[\hat{Q}(q^+)\right] = 0$$

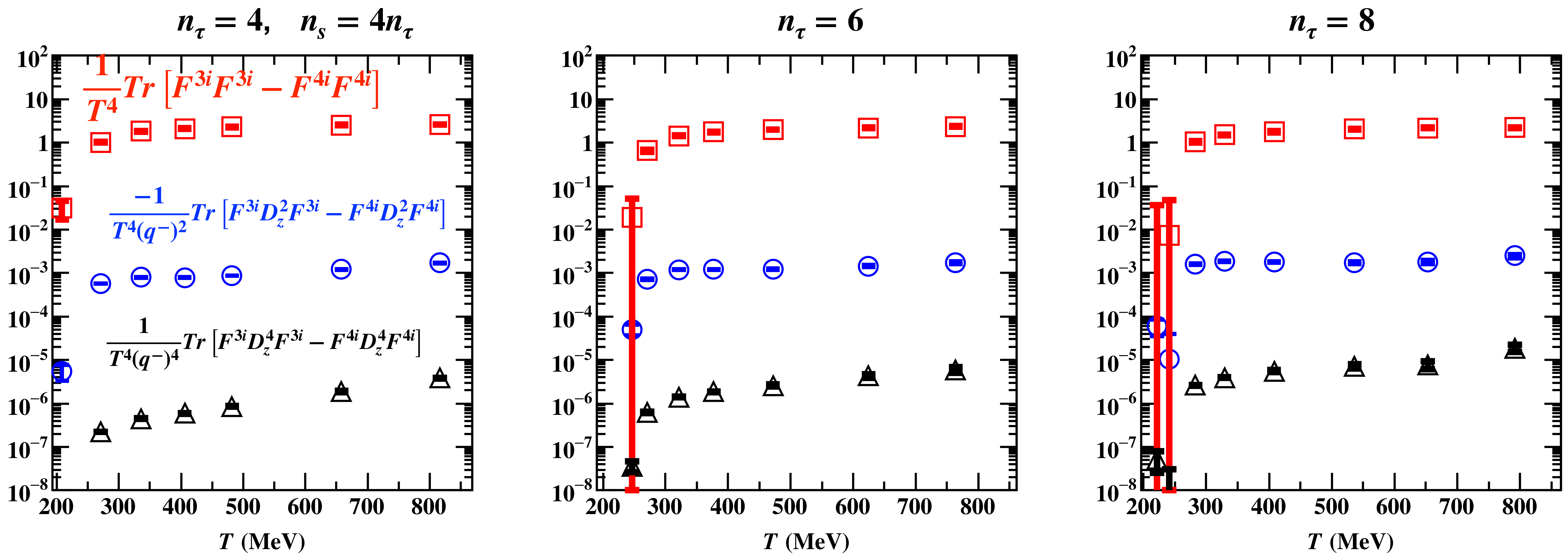
$$\hat{Q}(q^+ = -q^-) = \frac{8\sqrt{2}\pi\alpha_s}{N_c q^-} \langle M | F^{+\perp_\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F^+_{\perp_\mu}(0) | M \rangle$$

FF correlator in unquenched SU(3) lattices



- LO FF correlator is dominant among all operators
- FF correlator with D_z^2 derivative are suppressed by a factor of 10^3 compared LO operator
- FF correlates with D_z^4 derivative are suppressed by a factor of 10^6 compared LO operator

FF correlator in quenched SU(3) lattices



- LO FF correlator is dominant among all operators
- FF correlator with D_z^2 derivative are suppressed by a factor of 10^3 compared LO operator
- FF correlates with D_z^4 derivative are suppressed by a factor of 10^6 compared LO operator

Operators in quenched SU(2) plasma

A. Majumder, PRC 87, 034905 (2013)