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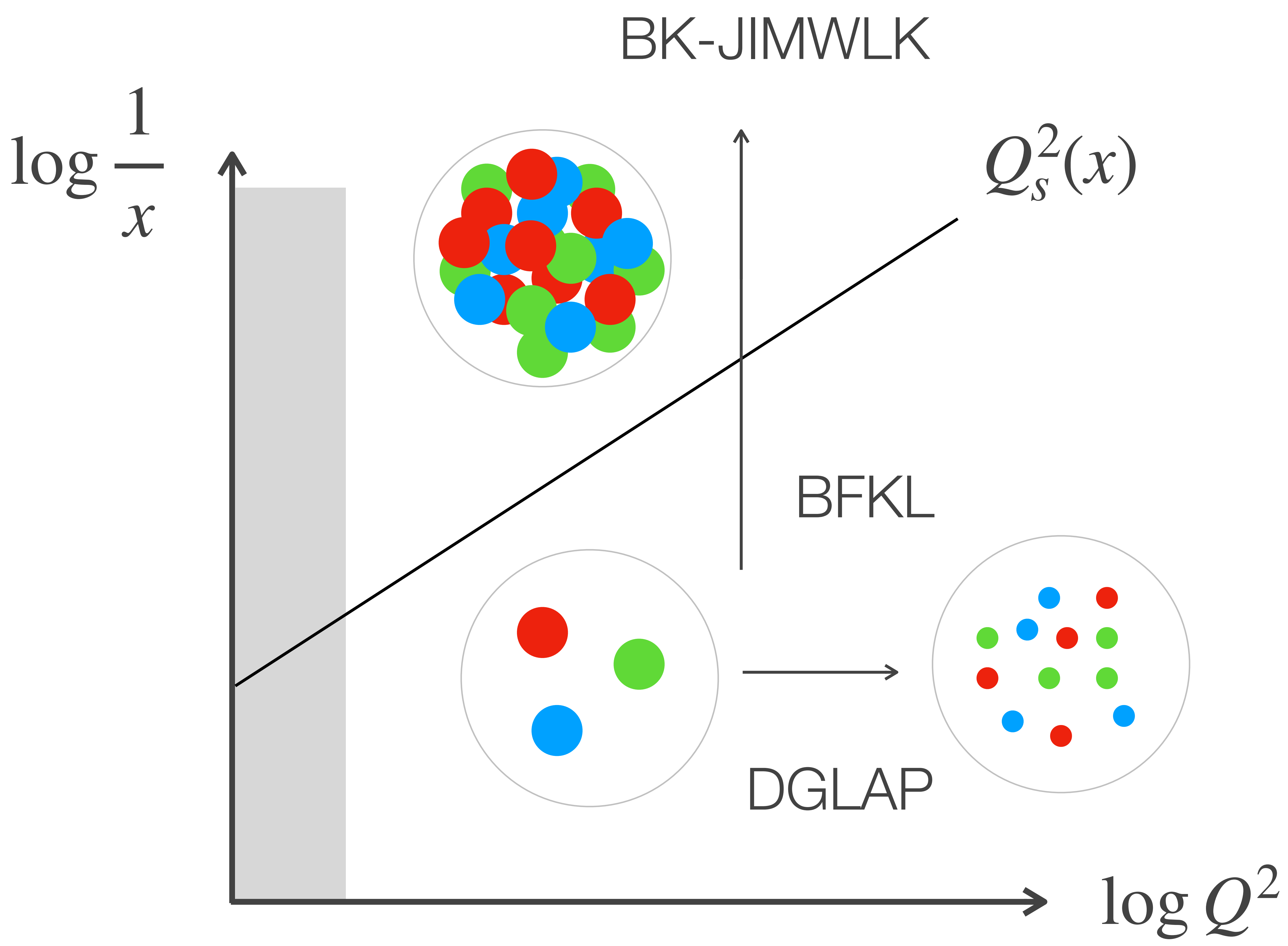
On gauge invariance of TMD distributions at small x

Yacine Mehtar-Tani (BNL/RBRC)

@Hard Probes 2020, June 3

2001.06449 [hep-ph] in collaboration with Renaud Boussarie

Saturation and Wilson lines at small x

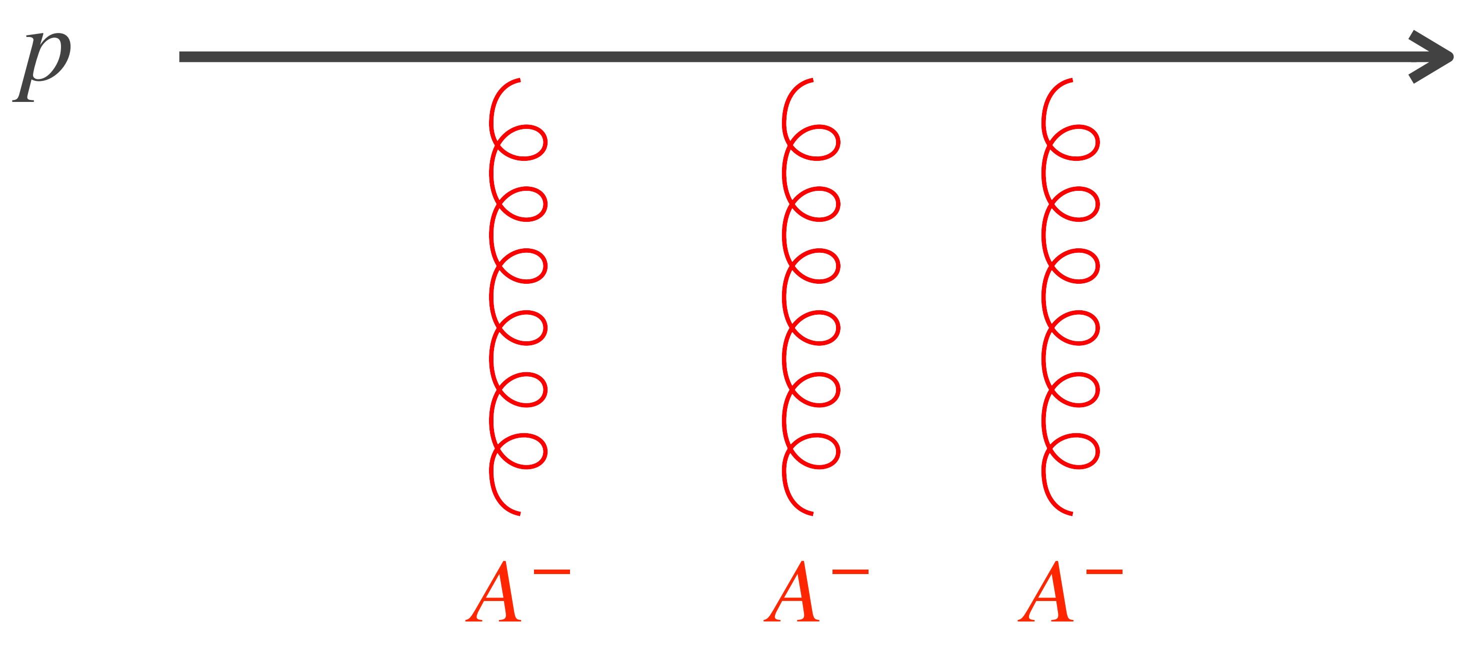


Plenary by M. Sievert

- at large Q^2 and moderate $x \sim Q^2/s$ the proton can be described as a collection of weakly interacting partons
- partonic interpretation breaks down beyond leading twist
- at small x gluon occupation number increases and eventually saturates due to non-linear effects (gluon recombination)

[Venugopalan, McLerran (MV), Balitsky, Kovchegov (BK) Jallilian-Marian, Iancu, Weigert, Leonidov, Kovner (JIMWLK) (1993-2001)]

Saturation and Wilson lines at small x



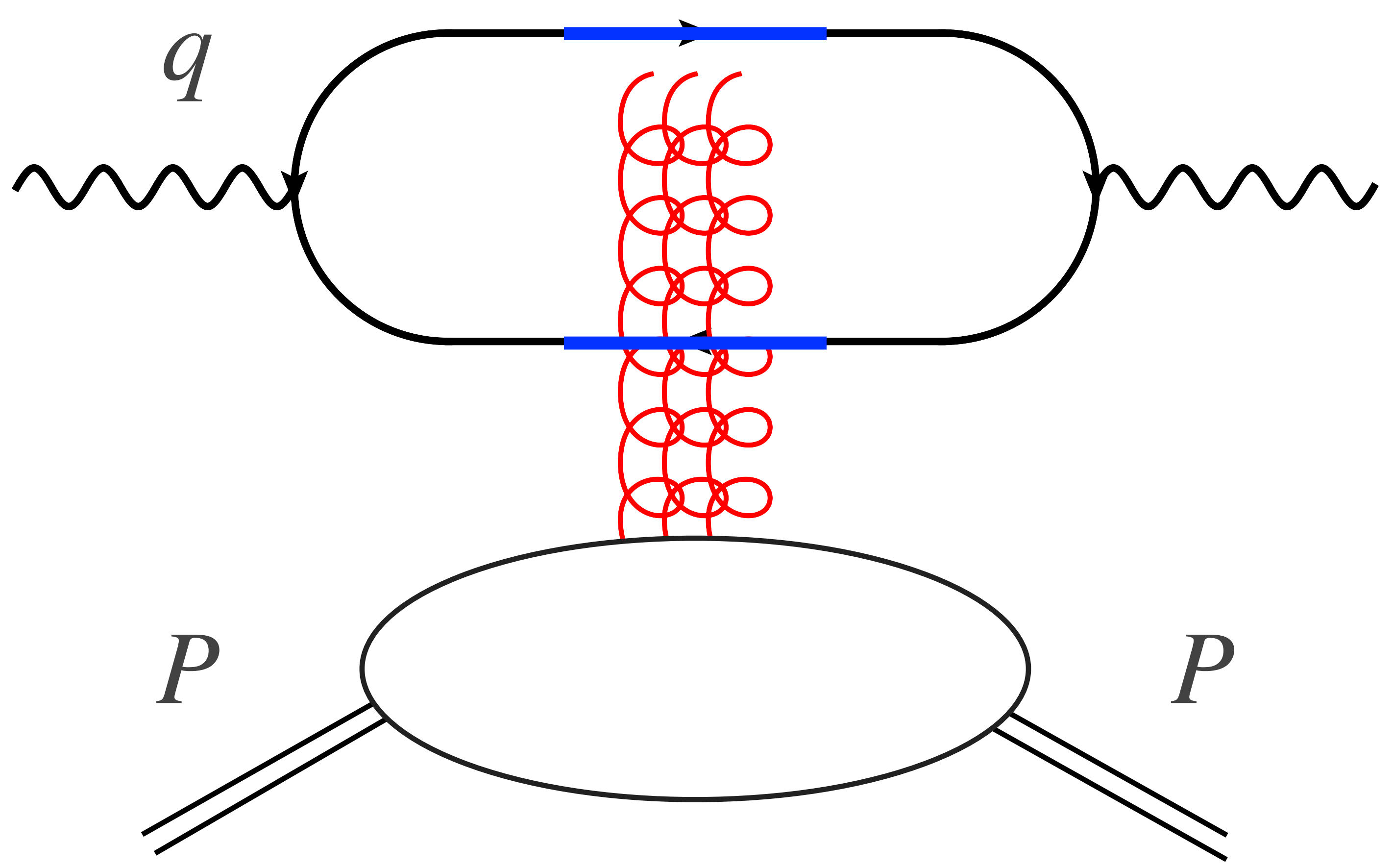
- the relevant d.o.f. in the saturation regime are strong classical fields $gA^- \sim 1$
- eikonal lines: $p^+ \gg k^+$ and $k^- \ll P^-$

- path ordered Wilson line

$$U_{\boldsymbol{x}} \equiv [+\infty, -\infty]_{\boldsymbol{x}} = P\exp\left[ig\int_{-\infty}^{+\infty} \mathrm{d}x^+ A^-(x^+, \boldsymbol{x})\right]$$

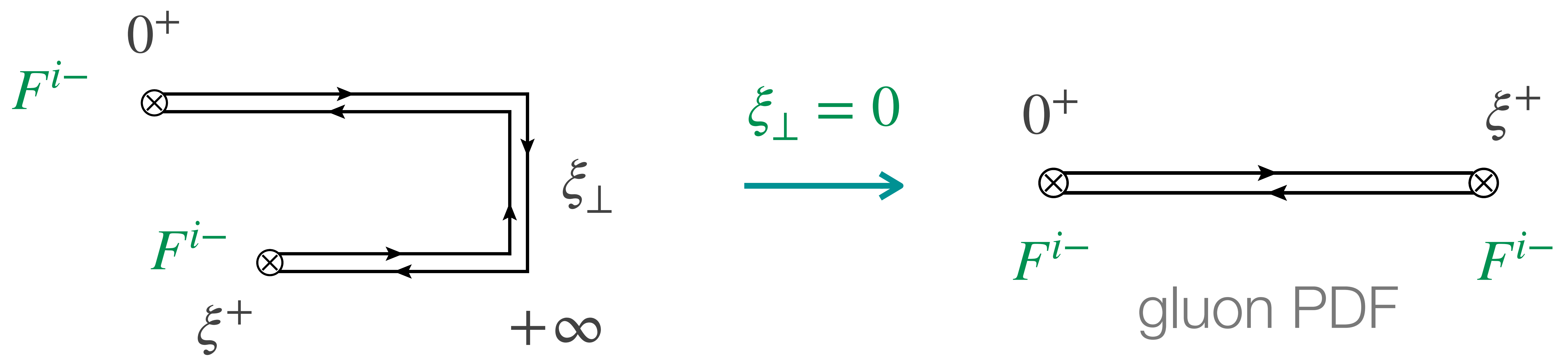
- dominant contribution in DIS: dipole scattering

$$\langle P\,|\,\mathrm{Tr}\; U_{\boldsymbol{x}_1}\; U^\dagger_{\boldsymbol{x}_2}\,|\,P\rangle$$



shock wave limit: $t_f \equiv Q^2/q^+ \gg 1/P^- \;\rightarrow x = Q^2/s \ll 1$

Parton distributions at small x



Weizsacker-Williams (WW) gluon Transverse Momentum Dependent (TMD) distribution

$$xG_{WW}(x,k_\perp) = \int_\xi \delta(\xi^-)\, e^{ixP^-\xi^+ - i\xi_\perp\cdot k_\perp} \;\langle P\,|\,\mathrm{Tr}\; F^{i-}(0)\;\; W(0,\xi), F^{i-}(\xi)\;\; W(\xi,0)\,|\,P\rangle$$

$\updownarrow$

- distributions of two kinds:
  1. PDF, GPD, TMD at moderate x
  2. dipole at small x (Wilson line correlators)

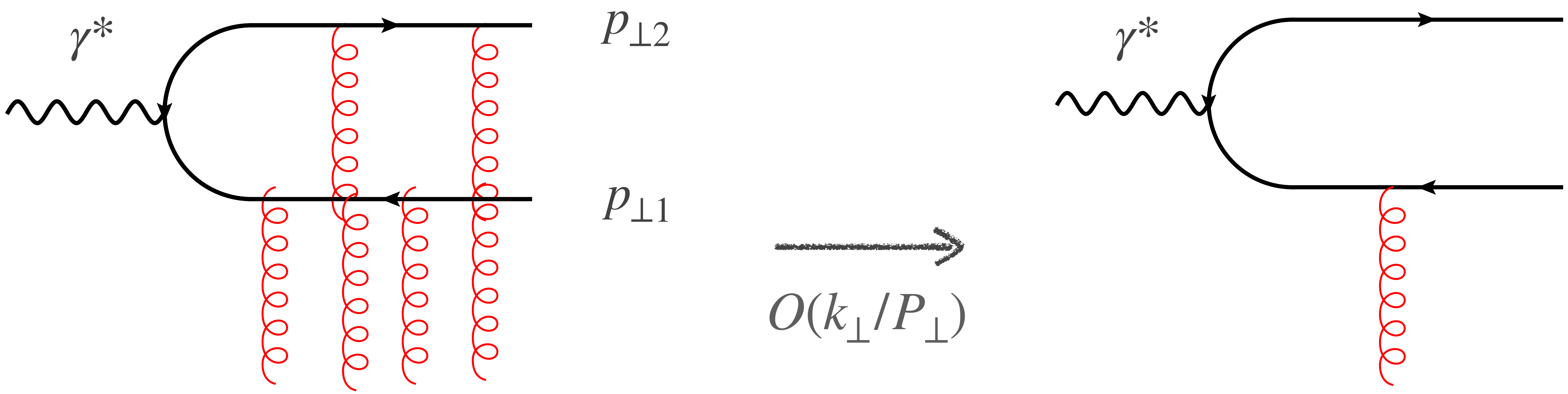
$$\langle P\,|\,\mathrm{Tr}\; U_{x_1}\;\; U^\dagger_{x_2}\,|\,P\rangle$$

dipole scattering amplitude

How can we reconcile/unify the two pictures?

Dijet production and TMD's at small x

- A first step toward connecting small x operators to large x was done at leading twist [Dominguez, Marquet, Xiao, Yuan (2011)]



$k_\perp \sim p_{\perp 1} \sim p_{\perp 2}$

$k_\perp \ll p_{\perp 1} \sim p_{\perp 2}$

back-to-back limit

- dijet production amplitude:

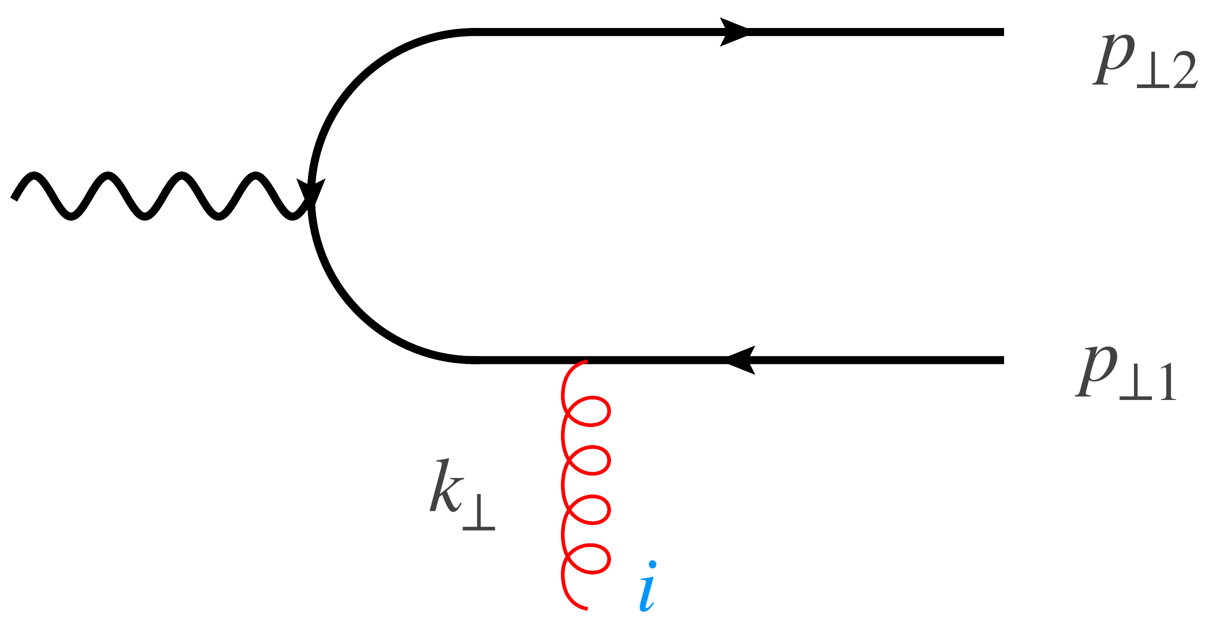
$$M \equiv H(k_{\perp 1}, k_{\perp 2}) \otimes \; U(k_{\perp 1}) U^\dagger(k_{\perp 2})$$

$$H(k_\perp = 0) \;\; [+\infty, x^+]_{\boldsymbol{x}} F^{i-}(x^+, \boldsymbol{x}) \; [x^+, +\infty]_{\boldsymbol{x}}$$

- power expansion generates derivatives of Wilson lines $\partial^i U \sim [+\infty, x^+] \; F^{i-}(x^+) \; [x^+, +\infty]$

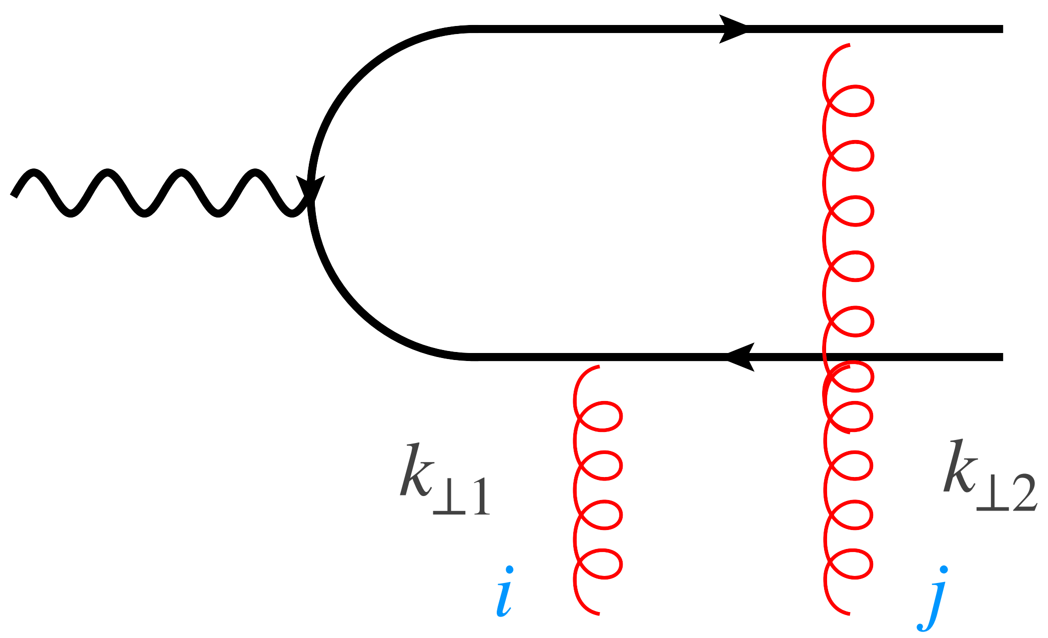
Dijet production and TMD's at small x

[Kokto, Kutak, Marquet, Petreska, Sapeta, van Hameren (2015)]



$$M_{1-\text{body}} \equiv H_1(k_\perp) \int_b e^{-i\boldsymbol{k}\cdot\boldsymbol{b}} \left([+\infty, b^+]F^{i-}(b^+)[b^+, +\infty]\right)_{\boldsymbol{b}}$$

[Altinoluk, Boussarie, Kokto (2019)]



$$M_{2-\text{body}} \equiv \int_{k_{\perp 1}, k_{\perp 1}} H_2(k_{\perp 1}, k_{\perp 2}) \int_{b_1, b_2} e^{-i\boldsymbol{k}_1\cdot\boldsymbol{b}_1 - i\boldsymbol{k}_2\cdot\boldsymbol{b}_2}\, \delta(\boldsymbol{k} - \boldsymbol{k}_1 - \boldsymbol{k}_2)$$

$$\left([+\infty, x_1^+]F^{i-}(x_1^+)[x^+, -\infty]\right)_{\boldsymbol{b}_2} \left([+\infty, x^+]F^{j-}(x_2^+)[x^+, -\infty]\right)_{\boldsymbol{b}_1}$$

Dijet production amplitude at small x to all orders in $k_\perp/Q$ is the sum of a 1-body contribution (leading genuine twist) and 2-body amplitude

Dijet production and TMD's at small x

[Altinoluk, Boussarie, Kokto (2019)]

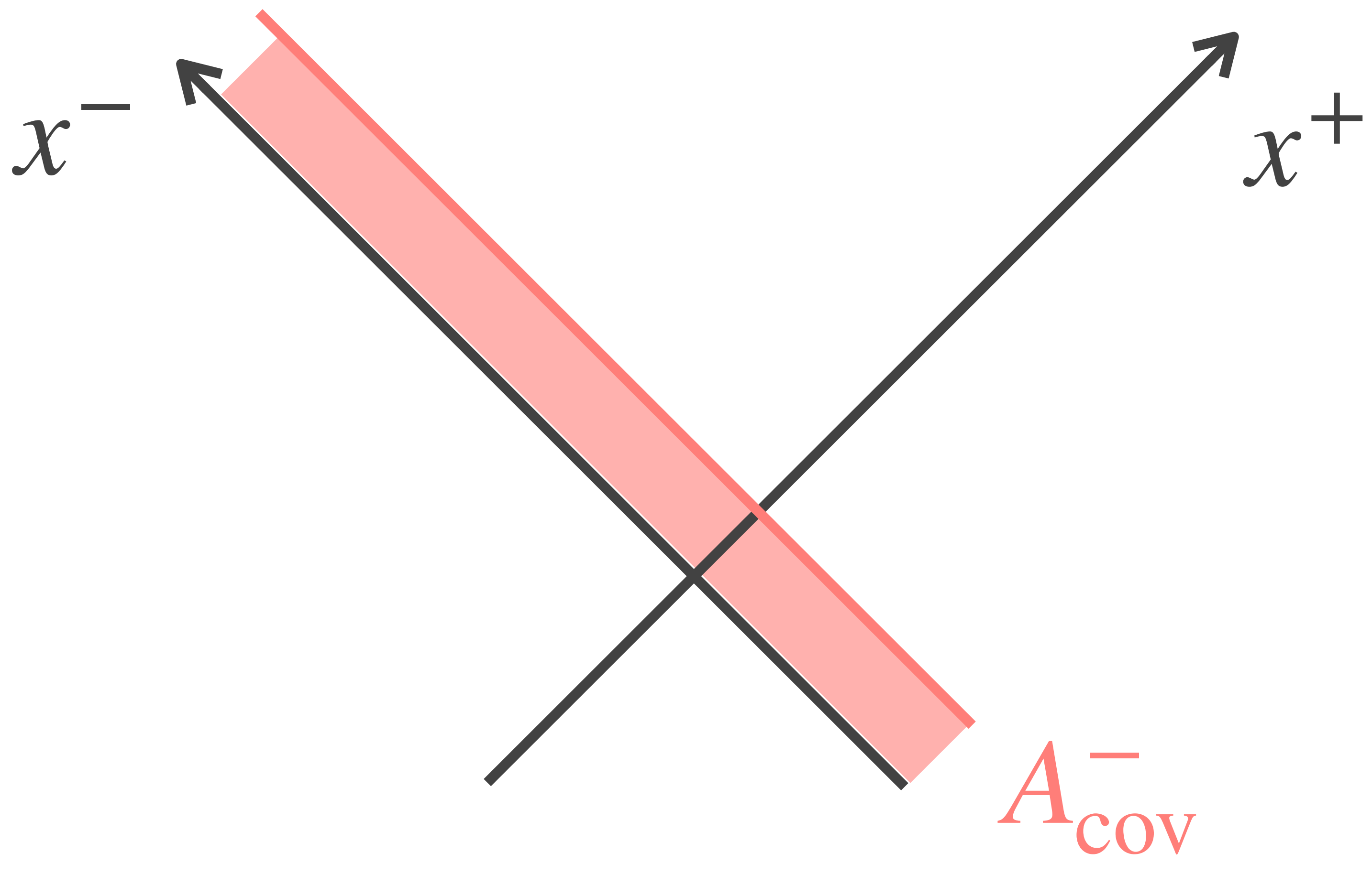
- resummation of powers of $k_\perp / Q$
- separation of kinematic twists (expansion of the hard matrix element) and genuine twists: expansion of the hadronic operator in $Q_s/k_\perp$ and $M/k_\perp$

This talk:

[MT, Boussarie, (2020))]

- geometric interpretation
- gauge invariant formulation of small x operators
- generalization to other observables

The target background field



target sources

$$J^\nu(x) \;\rightarrow\; J^-(x^+, x_\perp)$$

and $\quad J^+ = J_\perp = 0$

- consider a target boosted along the $-z$ direction close to the light cone. Due to time dilation the target color sources are "frozen" in the - direction
- Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ can be solved exactly (together with the continuity equation $[D_\mu, J^\mu] = 0$) in covariant gauge $\partial \cdot A = 0$ (or light-cone gauge $A^+ = 0$)

$$A^-_{cov} = -\frac{1}{\partial_\perp^2} J^- \quad \text{and} \quad A^+ = A_\perp = 0$$

The target background field

- under an arbitrary gauge rotation $\Omega(x^+, x_\perp)$ the target field transforms as

$$A^- \; \rightarrow \; \Omega_{\boldsymbol{x}}(x^+) \; A^-_{\rm cov} \; (x^+, \boldsymbol{x}) \; \Omega^{-1}_{\boldsymbol{x}}(x^+) \; - \; \frac{1}{ig} \Omega_{\boldsymbol{x}}(x^+) \; \partial^- \; \Omega^{-1}_{\boldsymbol{x}}(x^+)$$

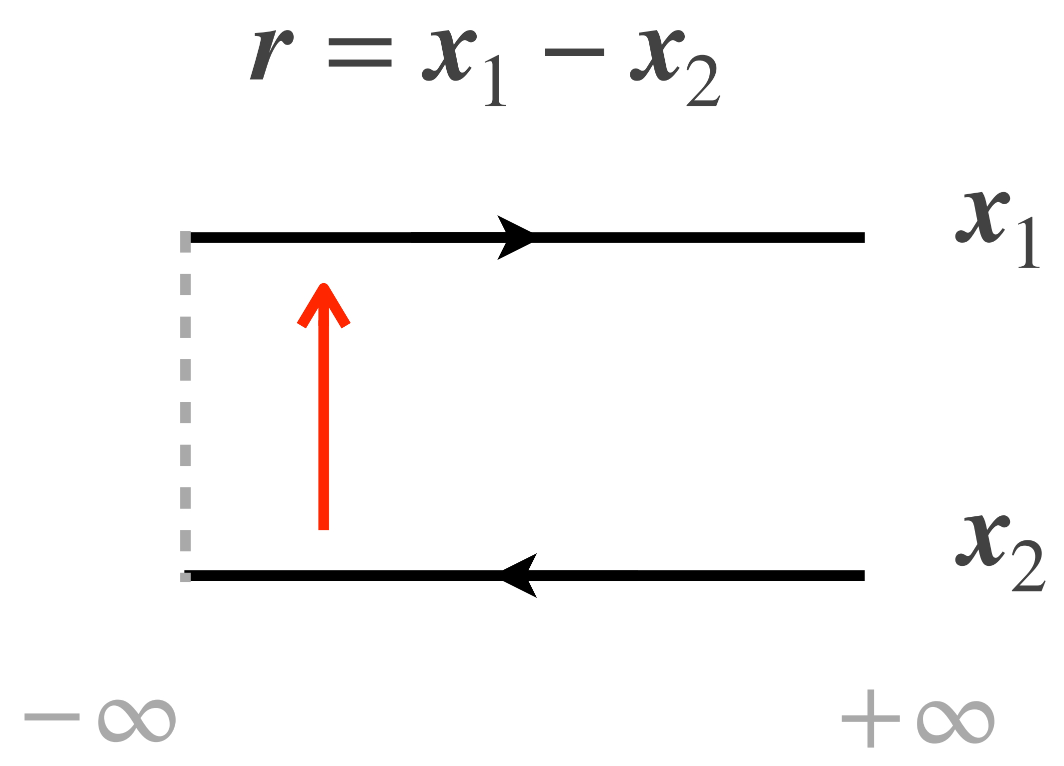
$$\boldsymbol{A}^i \; \rightarrow -\frac{1}{ig} \; \Omega_{\boldsymbol{x}}(x^+) \; \partial^i \; \Omega^{-1}_{\boldsymbol{x}}(x^+)$$

- exploiting the residual gauge freedom we can generate a transverse pure gauge
- N.B.: the partonic picture is manifest in the LC-gauge $A^- = 0$ (with $A_\perp \neq 0$)
- small x observables are (in the dilute/dense limit) more naturally expressed in the wrong LC-gauge $A^- \neq 0$ (with $A_\perp = 0$).
- in order to connect to the partonic interpretation one needs to deal with transverse fields

Parallel transport and transverse link operator

- geometric interpretation of the all twist resummation

$$U_{x_1} = U_{x_2} - r^i \int_0^1 \mathrm{d}s \, (\partial^i U_{x_2+sr})$$

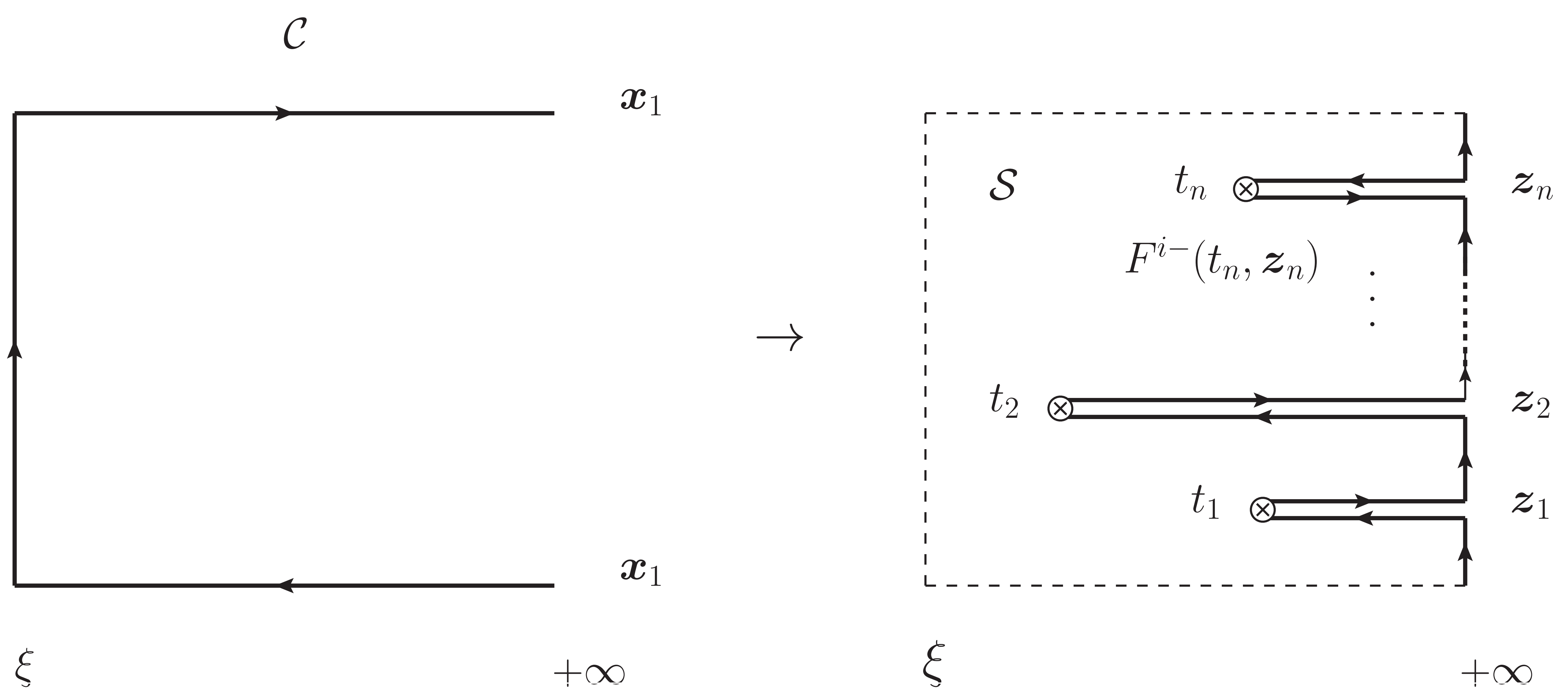


- and noticing that $\frac{1}{ig}(\partial^i U_x) U_x \equiv A^i(x)$, one can express the dipole operator (in the background field A^-) as a transverse gauge link:

$$U_{x_1} U^\dagger_{x_2} = [x_1, x_2] = 1 - ig \int_{x_2}^{x_1} \mathrm{d}z \, A^i(z) \, [z, x_2]$$

Wilson lines and the partonic interpretation

- non-Abelian Stokes theorem: more generally, the dipole operator can be written as a path ordered tower of "twisted" field strength tensor (i.e. dressed with future pointing Wilson lines)



[Fishbane, Gasiorowicz, Kaus (1981) Wiedemann (2000) YMT, Boussarie (2020)]

$$U_{x_2} U^\dagger_{x_1} \equiv P \exp\left[-ig \int_S \mathrm{d}t\mathrm{d}z\, [+\infty, x^+]_x\, F^{i-}(x^+, x)\, [x^+, +\infty]_x\right]$$

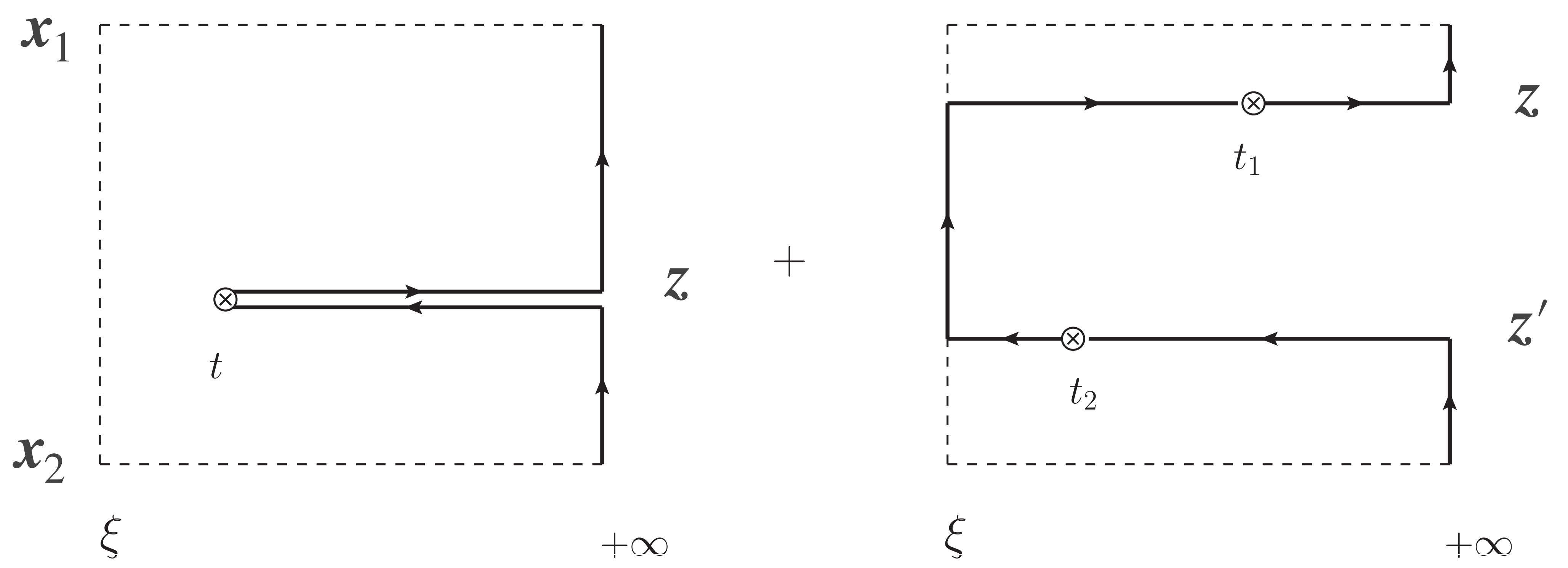
Wilson lines and the partonic interpretation

arXiv:2001.06449

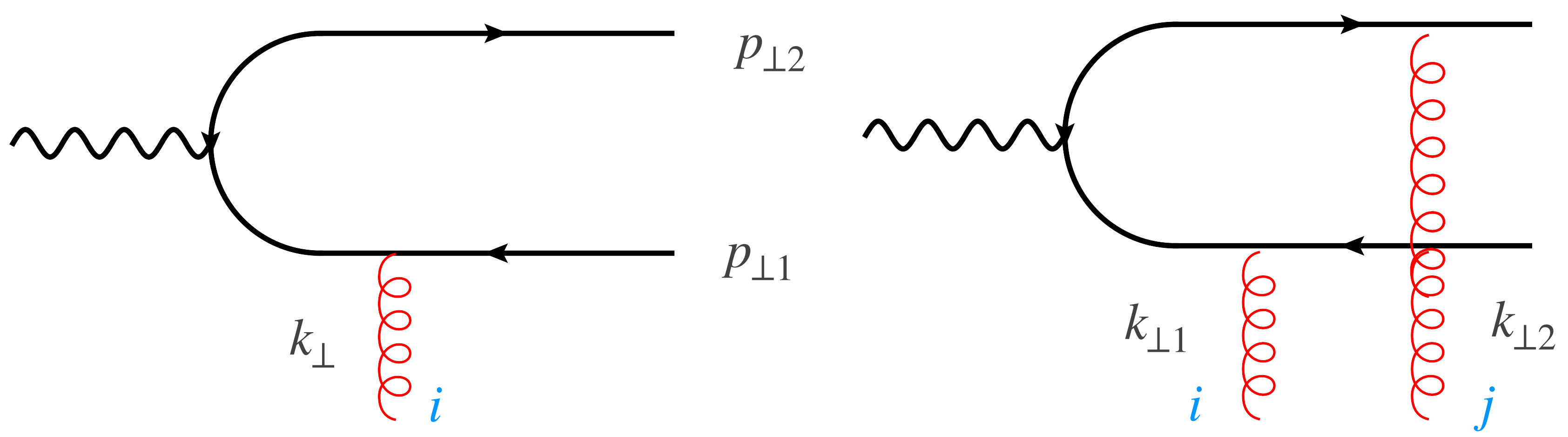
- the dipole operator appear in the dijet amplitude takes the compact form

$$U_{x_1}U^\dagger_{x_2} \to 1 - ig\int_{x_2}^{x_1} dz^i\, \hat{A}^i(z) + (ig)^2 \int_{x_2}^{x_1} dz \int_{x_2}^{z} dz'\, \hat{A}^i(z)\,[\hat{z},\hat{z}']\,\hat{A}^j(z')$$

$$\hat{A}^i \sim U A^i U^\dagger - \frac{1}{ig} U\partial^i U^\dagger$$



$\otimes$

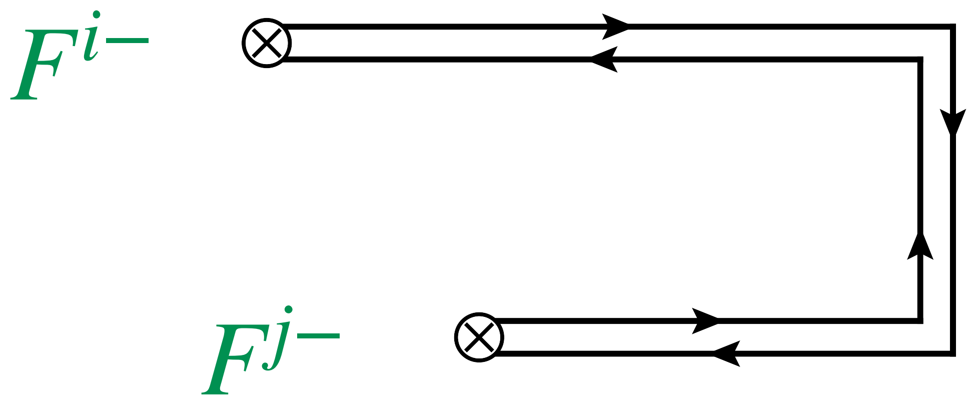


hadronic operator

hard matrix element

- the first non-trivial term generates the Weizsacker-Williams gluon TMD

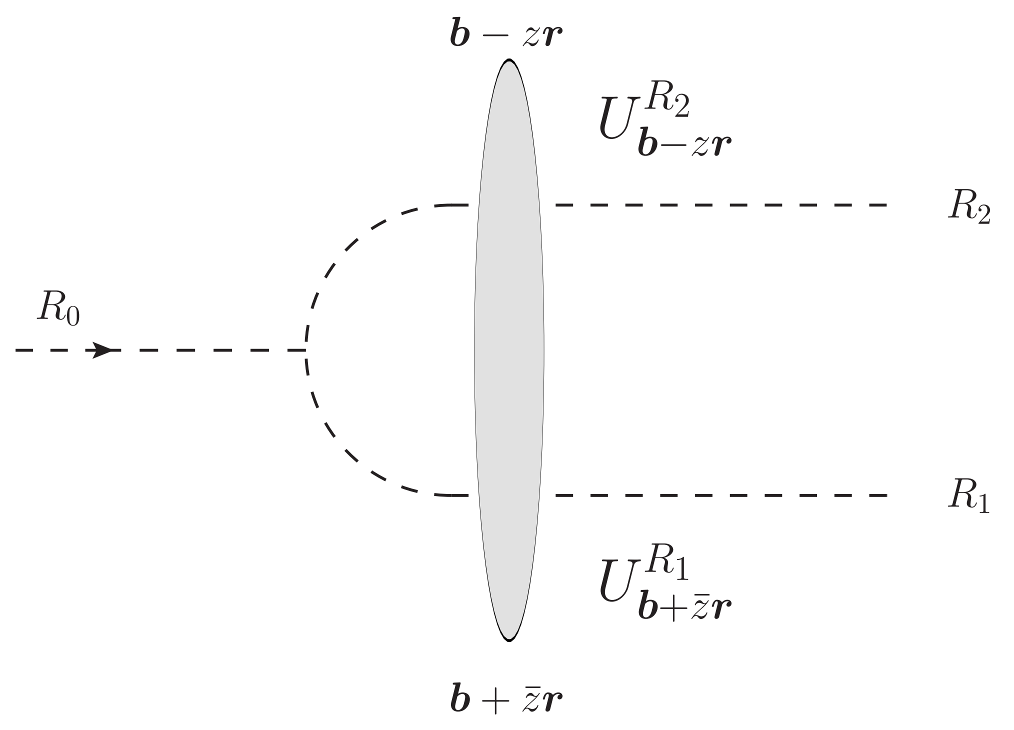
$$\hat{A}^i\hat{A}^j \sim \langle P\,|\,\mathrm{Tr}\; F^{i-}(0)\; W(0,\xi), F^{j-}(\xi)\; W(\xi,0)\,|\,P\rangle$$

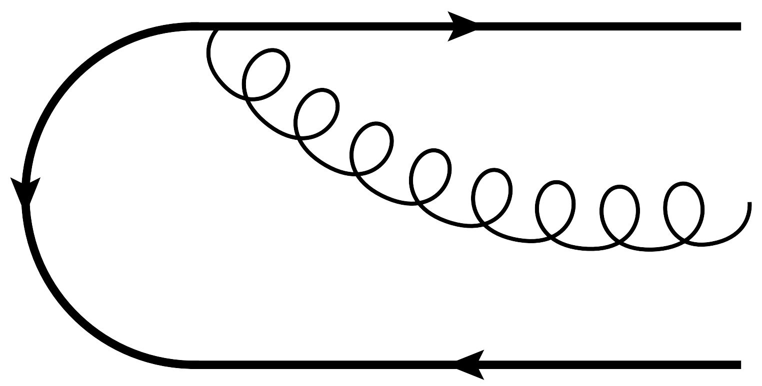


Wilson lines and the partonic interpretation

arXiv:2001.06449

- Straightforward generalization to:
    - arbitrary color representation (application to pA collisions for instance)
    - 3-Wilson-line operator (3-jet events, BK evolution, etc)





- Local formulation: systematic power expansion

$$\mathrm{Tr}\, U_b U^\dagger_{b-r} \equiv \mathrm{Tr} \exp\left[-\boldsymbol{r}\cdot \hat{\boldsymbol{D}}_\perp(\boldsymbol{b})\right] \qquad \hat{D}_\perp(\boldsymbol{b}) = \partial - ig\hat{A}_\perp(\boldsymbol{b})$$

$$\sim r^i r^j \;\mathrm{Tr}\,(F^{i-} U F^{j-} U^\dagger)_b + O(r^3)$$

Summary and outlook

- Gauge invariant formulation of TMD distributions at small x allow a direct connection to parton distribution functions at moderate x
- Dipole operators at small x involving Wilson lines along the - direction can be rewritten as transverse link operators
- More generally, owing to the Stokes' theorem one can express the dipole operator as an infinite tower of "twisted" field strength tensors, i.e, $[+\infty, x^+]_{\boldsymbol{x}}\, F^{i-}(x^+, \boldsymbol{x})\, [x^+, +\infty]_{\boldsymbol{x}}$ with a more transparent partonic interpretation
- Systematic generalization to arbitrary color structure and n-point correlation functions
- **Open questions:** including quantum evolution (BK equation) ? sub-eikonal corrections? exclusive observables, etc