

# Revisiting heavy quark radiative energy loss in nuclei within the high-twist approach

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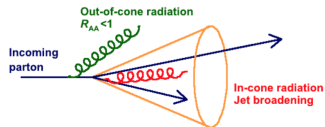
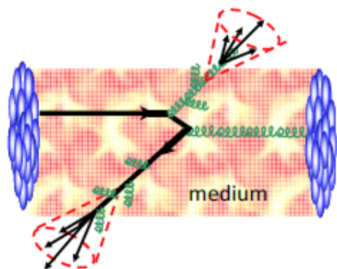
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# Outline

- Introduction
- SIDIS at Next-to-Leading Order & Twist 4
- Improved high-twist factorization formalism: gauge invariance
- Summary & Outlook

# Parton energy loss in medium and jet quenching

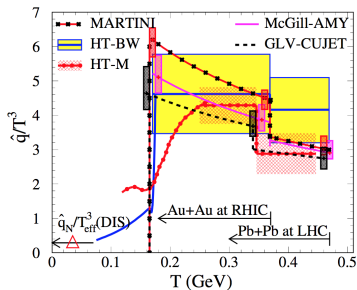


- Energetic partons will lose energy when traveling through hot QGP or large cold nuclei (eA DIS process)
- Medium-induced gluon radiations give rise to jet energy loss

# Jet transport parameter $\hat{q}$

$$\hat{q} = \frac{4\pi C_F \alpha_s}{N_c^2 - 1} \int dy^- \langle F^{ai+}(0) F_i^{a+}(y^-) \rangle e^{i\xi p^+ y^-},$$

- Average squared transverse momentum transfer per unit distance in the medium, reflecting the ability of the medium to “quench” jets.
- Proportional to the **gluon density** of the medium.
- Medium property can be probed by parton energy loss mechanisms



$\hat{q} = 1.3 \pm 0.3 \text{ GeV}^2/\text{fm}$  Au+Au at  
RHIC  $\sqrt{s} = 0.2 \text{ TeV}/n$ ;

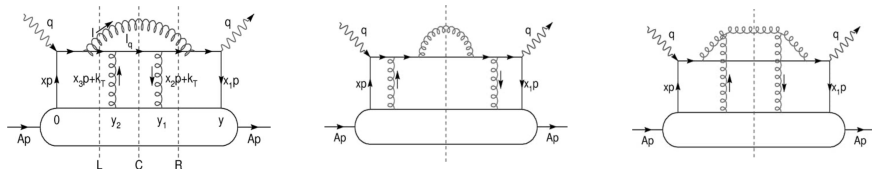
$\hat{q} = 2.2 \pm 0.5 \text{ GeV}^2/\text{fm}$  Pb+Pb at  
LHC  $\sqrt{s} = 2.76 \text{ TeV}/n$ ;

Extracted from **HT-BW** model via  
 $R_{AA}$ . PRC 90, 014909

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- Introduction
- **SIDIS at Next-to-Leading Order & Twist 4**
- Improved high-twist factorization formalism: gauge invariance
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# SIDIS at NLO & Twist 4



Identify one hadron in final state

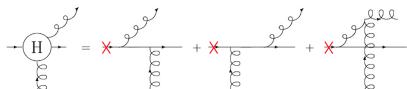
- \* Light quark:  $l(L_1) + A(p) \rightarrow l(L_2) + h(l_h) + X$ , via  $\gamma^*$  exchange. Nucl. Phys. A696, 788 (2001)
- \* Heavy quark:  $l(L_1) + A(p) \rightarrow \nu_l(L_2) + H(l_H) + X$ , via  $W^\pm$  exchange. Nucl. Phys. A757, 493 (2005)
- Single rescattering approximation: dilute medium,  $l_{mfp} \gg L$
- Dynamic scattering center: momentum and energy ( $\sim q$ ) transfer
- Hard radiated gluon: finite  $z_g$
- Typical transverse momentum transfer from the medium is much smaller than radiated gluon transverse momentum:  $k_T \ll l_T$

# Collinear Expansion and Factorization

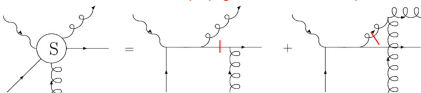
- $W_{\mu\nu} \propto \text{Tr}[\hat{H}_{\mu\nu}^{\alpha\beta}(x_1 p^+, k_2, k_3) \langle A | \bar{\psi}(0) A_\beta(y_2) A_\alpha(y_1) \psi(y) | A \rangle]$
- $\hat{H}_{\mu\nu}^{\alpha\beta}(x_1 p^+, k_2, k_3) = \hat{H}_{\mu\nu}^{\alpha\beta}(k_\perp = 0) + \frac{\partial \hat{H}_{\mu\nu}^{\alpha\beta}}{\partial k_{\perp\rho}} \Big|_{k_\perp=0_\perp} k_{\perp\rho}$   
 $+ \frac{1}{2} \frac{\partial^2 \hat{H}_{\mu\nu}^{\alpha\beta}}{\partial k_{\perp\rho} \partial k_{\perp\sigma}} \Big|_{k_\perp=0_\perp} k_{\perp\rho} k_{\perp\sigma} + \dots$
- $k_\perp = 0$  term contributes to gauge link of initial quark PDF without  $L$  enhancement.
- $k_{\perp\rho}$  term contributes zero for unpolarized beam, vanishing on taking a spin average.
- $W_{\mu\nu}^D \propto \int dz_H \int_{z_H}^1 \frac{dz}{z} D_{Q \rightarrow H}(\frac{z_H}{z}) \int \frac{dy^-}{2\pi} dy_1^- dy_2^-$   
 $\left( -\frac{1}{2} g^{\rho\sigma} \right) \left[ \frac{\partial^2}{\partial k_{\perp}^\rho \partial k_{\perp}^\sigma} \bar{H}_{\mu\nu}^D \right] \Big|_{k_\perp=0}$   
 $\times \frac{1}{2} \langle A | \bar{\psi}(0) \gamma^+ F_\sigma^+(y_2^-) F^{+\sigma}(y_1^-) \psi(y^-) | A \rangle$

# SIDIS at NLO & Twist 4 with central-cut

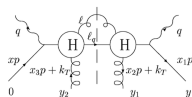
Twist: Dimension-Spin+Momentum in operator definition



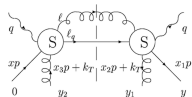
The cross marks indicate the propagators where the hard poles arise



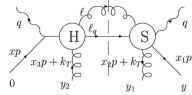
The short bars indicate the propagators where the soft poles arise



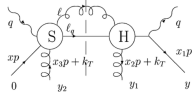
(a)



(b)



(c)

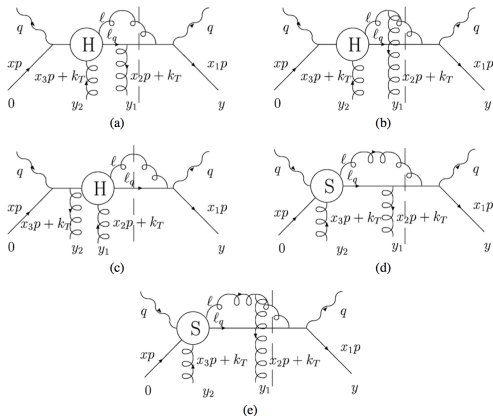


(d)

- Power suppressed compared with twist-2 contribution
- Nuclear size enhancement  $\propto L$
- LPM interference effect  $\propto L^2$



# SIDIS at NLO & Twist 4 with asymmetric-cut



Interference between vacuum and medium induced radiation

**Note:** Virtual corrections and gluon fragmentation are also needed

# Nucleus modified Splitting Functions & FFs

- Splitting Functions

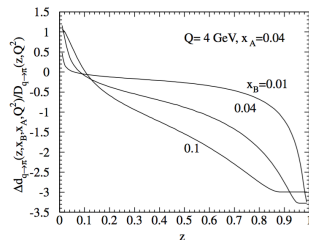
$$\Delta\gamma_{q\rightarrow qg}(z, x, x_L, \ell_T^2) = \left[ \frac{1+z^2}{(1-z)_+} T_{qg}^A(x, x_L) + \delta(1-z) \Delta T_{qg}^A(x, \ell_T^2) \right] \frac{C_A 2\pi\alpha_s}{(\ell_T^2 + \langle k_T^2 \rangle) N_c \tilde{f}_q^A(x, \mu_1^2)}$$

$$\Delta\gamma_{q\rightarrow gq}(z, x, x_L, \ell_T^2) = \Delta\gamma_{q\rightarrow qg}(1-z, x, x_L, \ell_T^2).$$

- Quark-gluon correlation function at Twist-4, analogous to PDFs at Twist-2

$$T_{qg}^A(x, x_L) = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{i(x+x_L)p^+ y^-} (1 - e^{-ix_L p^+ y_2^-}) (1 - e^{-ix_L p^+ (y^- - y_1^-)}) \\ \times \frac{1}{2} \langle A | \bar{\psi}_q(0) \gamma^+ F_{\sigma^+}(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | A \rangle \theta(-y_2^-) \theta(y_2^- - y_1^-)$$

$$\tilde{D}_{q\rightarrow h}(z_h, \mu^2) \equiv D_{q\rightarrow h}(z_h, \mu^2) + \int_0^{\mu^2} \frac{dl_T^2}{l_T^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[ \Delta\gamma_{q\rightarrow qg}(z, x, x_L, l_T^2) D_{q\rightarrow h}(z_h/z, \mu^2) + \Delta\gamma_{q\rightarrow gq}(z, x, x_L, l_T^2) D_{g\rightarrow h}(z_h/z, \mu^2) \right].$$



# Jet transport parameter $\hat{q}$

Parton energy loss and jet transport coefficient  $\hat{q}$  are both related to the quark-gluon correlation functions.

- Parton energy loss: carried away by the radiated gluons

$$\begin{aligned}\frac{\Delta E}{E} &= \langle \Delta z_g \rangle = \frac{\alpha_s}{2\pi} \int \frac{dl_T^2}{l_T^2} \int dz z \Delta \gamma_{q \rightarrow gq}(z, l_T^2) \\ &= \alpha_s^2 \int dl_T^2 \int_0^1 dz \frac{C_A}{N_c} \frac{1 + (1-z)^2}{l_T^4} \frac{T_{qg}^A(x_B, x_L)}{f_q^A(x_B)}.\end{aligned}$$

- Decomposition:

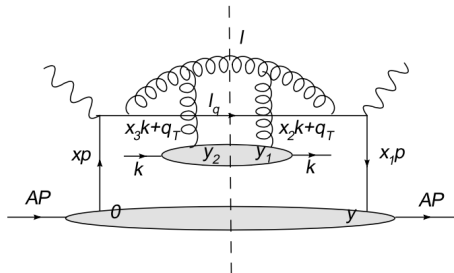
$$\begin{aligned}\frac{2\pi\alpha_s T_{qg}^A(x_B, x_L)}{N_c f_q^A(x_B)} &= \frac{2\pi\alpha_s}{N_c} \pi \int dy^- \rho_N^A(y) [1 - \cos(x_L p^+ y^-)] \\ &\quad \times [(x_L G_N(x_L) + c(x_L)[x G_N(x)]_{x \approx 0}) \\ &= \int dy^- [1 - \cos(x_L p^+ y^-)] [\hat{q}_F(x_L, y) + c(x_L) \hat{q}_F(0, y)]\end{aligned}$$

- Jet transport parameter:

$$\hat{q}_R(x_L, y) = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \rho_N^A(y) x_L G_N(x_L),$$

W.-T Deng and X.-N. Wang, Phys. Rev. C 81, 024902

# Applied in hot medium



X-F Chen et al, Phys. Rev. C 81, 064908

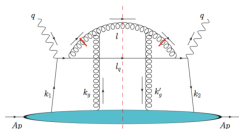
- Apply the modified splitting function in the hot medium.
- Coupled to the dynamical evolution models of the bulk matter.

# Outline

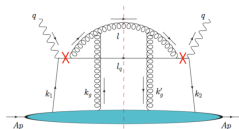
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# SIDIS at NLO & Twist 4 ensuring gauge invariance

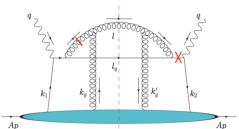
Z-B Kang, E-k Wang, X-N Wang, and H-x Xing, PRL 112,102001 &  
Z-B Kang, J-W Qiu, X-N Wang, and H-x Xing, PRD 94, 074038



(a) The hard-soft process. We assign the four initial partons' momenta as  $k_1 = xp$ ,  $k_2 = x_3p + k_T$  and  $k'_g = x_2p + k_T$ , respectively.

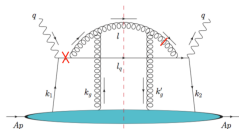


(b) The double hard process. We assign the four initial partons' momenta as  $k_1 = xp$ ,  $k_2 = x_1p + k_{3T} - k_{2T}$ ,  $k_g = x_3p + k_{3T}$  and  $k'_g = x_2p + k_{2T}$ , respectively.



(c) The interference between hard-soft and double hard process.

We assign the four initial partons' momenta as  $k_1 = xp$ ,  $k_2 = x_1p + k_{3T} - k_{2T}$ ,  $k_g = x_3p + k_{3T}$  and  $k'_g = x_2p + k_{2T}$ , respectively.

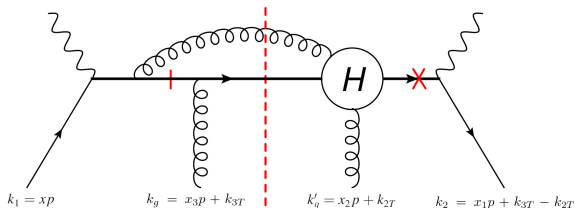


(d) The interference between double hard and soft-hard process.

We assign the four initial partons' momenta as  $k_1 = xp + k_{2T} - k_{3T}$ ,  $k_2 = x_1p$ ,  $k_g = x_3p + k_{3T}$  and  $k'_g = x_2p + k_{2T}$ , respectively.

$$\frac{1}{2} \frac{\partial^2}{\partial k_T^\alpha \partial k_T^\beta} \bar{H}_{\mu\nu}^D(k_T) \Big|_{k_T=0} \rightarrow \frac{\partial^2}{\partial k_{2T}^\alpha \partial k_{3T}^\beta} \bar{H}_{\mu\nu}^D(k_{2T}, k_{3T}) \Big|_{\substack{k_{2T}=0 \\ k_{3T}=0}}$$

# Example: soft-hard double scatterings



Two factorized scatterings at amplitude level

- Left: first hard scattering  $W + q \rightarrow Q + g$  (NLO,  $\alpha\alpha_s$ ) and then soft rescattering  $Q + g \rightarrow Q$  (LO QCD,  $\alpha_s$ ) by exchanging a soft gluon: on-shell initial quark  $k_1^2 = 0$  for the first scattering  $\rightarrow k_1 = xp$ .
- Right: first hard scattering  $W + q \rightarrow Q$  (LO EW,  $\alpha$ ) and then hard rescattering  $Q + g \rightarrow Q + g$  (NLO,  $\alpha_s^2$ ): on shell initial gluon  $k_g'^2 = 0$  for the second scattering, up to the order in which we perform collinear expansion,  $\mathcal{O}(k_{2T})$  and  $\mathcal{O}(k_{3T})$  if  $k_{2T} \neq k_{3T}$ .
- Momentum conservation:  $k_2 = x_1 p + k_{3T} - k_{2T}$ .

# Results and corrections

$$T_{qg}^{A,C}(x, x_L, M^2) \approx \frac{\tilde{C}}{x_A} f_q^A(x) (1 - e^{-\tilde{x}_L^2/x_A^2}) a(z, \frac{M^2}{\tilde{\ell}_T^2})$$

$$a(z, \frac{M^2}{\tilde{\ell}_T^2}) = \frac{(1+z)}{2} + [\dots] \frac{(1-z)^2}{1+z^2} \frac{M^2}{\tilde{\ell}_T^2} + [\dots] \frac{(1-z)^4}{1+z^2} \frac{M^4}{\tilde{\ell}_T^4}$$

$$\Delta a = \left[ z - \frac{1}{2} + \frac{C_F}{C_A} (1-z)^2 \right] \left[ (1+z)^2 + (1-z)^4 \frac{M^2}{\tilde{\ell}_T^2} \right] \frac{(1-z)^2}{1+z^2} \frac{M^2}{\tilde{\ell}_T^2}.$$

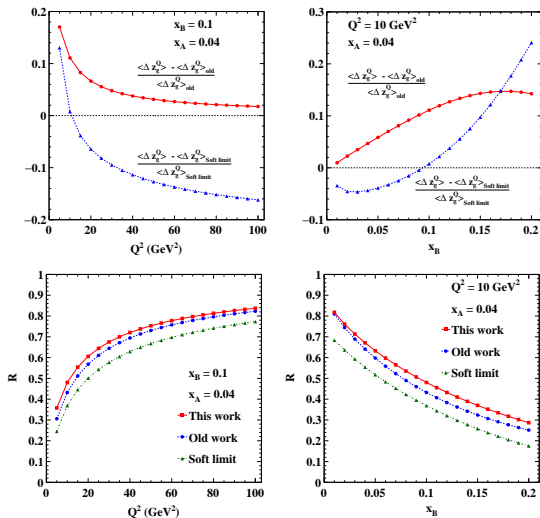
$$\langle \Delta z_g^Q \rangle = \frac{\tilde{C} C_A \alpha_s^2 x_B}{N_c Q^2 x_A} \int_0^1 dz \frac{1+(1-z)^2}{z(1-z)} \int_{\tilde{x}_M}^{\tilde{x}_L} d\tilde{x}_L \frac{(\tilde{x}_L - \tilde{x}_M)^2}{\tilde{x}_L^4} (1 - e^{-\frac{\tilde{x}_L^2}{x_A^2}}) a,$$

- Light quark energy loss and mDGLAP Eqs for FF remains the same
- New correction terms in the heavy quark energy loss, which vanishes in the soft gluon radiation limit  $z \rightarrow 1$ .

Y.-L Du, Y.-Y He, X.-N Wang, H.-X Xing, H.-S Zong, PRD 98, 054015 (2018)



# Relative correction of energy loss and Ratio $R$



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# Summary & Outlook

- **Gauge invariance** is ensured by a delicate setup of the initial partons' transverse momenta in SIDIS at twist 4.
- New correction terms **only in the heavy quark energy loss**, which vanishes in the soft gluon radiation limit.
- Significant correction in the **small  $Q^2$  and large  $x_B$  (small heavy quark energy) regions** compared with the old result and that with soft gluon approximation.
- Necessity to go beyond the soft gluon limit: a **global fitting** on light and heavy quark energy loss.

Future work:

- Numerical simulation on heavy quark energy loss in transport models
- A complete & gauge invariant calculation of parton energy loss at NLO & twist 4.

