Heavy Flavor Hadronization and Hadron Chemistry in Heavy-ion Collisions

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Hadronization is an important but difficult topic

Soft: NCQ scaling of hadron $\nu_2$

- Coalescence of quarks into hadron
- Quark degree of freedom inside the hot nuclear matter in heavy-ion collisions

Hard: cone size dependence of $\sigma$(jet)

- Similar contributions from hadronization and NLO effects
- No state-of-the-art hadronization model for hard probes yet
Charmed hadron chemistry

- Heavy quarks: early production in collisions, interact with QGP with flavor conservation
- Ideal probe of the in-medium hadronization mechanism of hard partons
- Few precise model descriptions of data, puzzling smaller $\Lambda_c/D^0$ at LHC than at RHIC
- Goal of this work: develop a comprehensive hadronization model and understand the heavy flavor hadron chemistry (arXiv:1911.00456)
Two major hadronization mechanisms

**Fragmentation:**
High momentum heavy quarks are more likely to fragment into hadrons
[ Peterson, FONLL, Pythia, etc. ]

**Coalescence (recombination):**
Low momentum heavy quarks are more likely to combine with thermal partons into hadrons
Oh, Ko, Lee and Yasui, PRC 79 (2019)
Plumari, Minissale, Das, Coci and Greco, EPJC 98 (2018)
Cho, Sun, Ko, Lee and Oh, PRC 101 (2020)
SC, Sun, Li, Liu, Xing, Qin and Ko, arXiv:1911.00456
Coalescence model

• Sudden approximation: $|q, g\rangle \rightarrow |h\rangle$ as $T$ drops across $T_c$

• Probability for quarks to combine into hadrons: Wigner function (wave function overlap)

• Example: 2-body system for meson formation

$$W(\vec{r}, \vec{k}) \equiv |\langle M | q_1, q_2 \rangle|^2 = g_M \int d^3r' e^{-i\vec{k} \cdot \vec{r}'} \phi_M(\vec{r} + \vec{r}'/2)\phi_M^*(\vec{r} - \vec{r}'/2)$$

$g_M$: ratio of spin-color degeneracy between meson and quark states

$\phi_M$: meson wavefunction (S.H.O. approximation with a frequency parameter $\omega$)

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \quad \vec{k} = \frac{1}{E_1' + E_2'}(E_2'\vec{p}_1' - E_1'\vec{p}_2') \quad (r' \text{ and } p' \text{ defined in the meson rest frame})$$

• Momentum space Wigner function (after averaging over position space) for $s$ and $p$ wave $\phi_M$:

$$W_s = g_M \frac{(2\sqrt{\pi\sigma})^3}{V} e^{-\sigma^2k^2} \quad W_p = g_M \frac{(2\sqrt{\pi\sigma})^3}{V} 2 \frac{2}{3} \sigma^2k^2 e^{-\sigma^2k^2} \quad (\sigma = 1/\sqrt{\mu\omega}, \mu: \text{reduced mass})$$
Coalescence model

• Hadron spectrum from coalescence

\[
f_M(\vec{p}'_M) = \int d^3p_1 d^3p_2 f_1(\vec{p}_1)f_2(\vec{p}_2)W(\vec{p}_1, \vec{p}_2)\delta(\vec{p}'_M - \vec{p}_1 - \vec{p}_2)\]

\[ f_i(\vec{p}_i)\): distribution of constituent quarks

*Light quarks*: thermal distribution in the local rest frame of the QGP (gluons are converted to light quark pairs by \(gg \rightarrow q\bar{q}\))

*Heavy quarks*: from a Langevin-hydrodynamics simulation (discuss later)

• Straightforward to extend to a 3-body system for baryon formation

• Coalescence probability for a single charm quark with a given \(p_c\) into a particular hadron species

\[
P_{\text{coal}}(p_c) = \int d^3p'_M f_M(\vec{p}'_M) \quad \text{with} \quad f_c(\vec{p}) = \delta(\vec{p} - \vec{p}_c)
\]
Coalescence probability

- Include both $s$ and $p$-wave states in a full 3-D calculation
  
  e.g. $D^0$ ($c\bar{u}$) meson formation with $S = 0, 1$

  $s$ wave ($L = 0$): $S = 0 \rightarrow J = 0$ ($D^0$); $S = 1 \rightarrow J = 1$ ($D^{*0}$)

  $p$ wave ($L = 1$): $S = 0 \rightarrow J = 1$ ($D^0_1$); 

  \[ S = 1 \rightarrow J = 0 \ (D^0_0), \quad J = 1 \ (D^{*0}_1), \quad J = 2 \ (D^{*0}_2) \]

- Cover nearly all charmed hadrons in PDG

- Enhance the total $P_{\text{coal}}$
Coalescence probability

- Include both s and p-wave states in a full 3-D calculation, e.g. \( D^0 (c\bar{u}) \) meson formation with \( S = 0, 1 \)
- s wave (\( L = 0 \)): \( S = 0 \rightarrow J = 0 (D^0); \ S = 1 \rightarrow J = 1 (D^{*0}) \)
- p wave (\( L = 1 \)): \( S = 0 \rightarrow J = 1 (D^0_1); \)
  \[
  S = 1 \rightarrow J = 0 (D^{*0}_0), \ J = 1 (D^{*0}_1), \ J = 2 (D^{*0}_2)
  \]
- Cover nearly all charmed hadrons in PDG
- Enhance the total \( P_{\text{coal}} \)
- Allow normalizing \( P_{\text{coal}}(p_c = 0) = 1 \) with a proper \( \omega = 0.24 \) GeV, abandoning arbitrary normalization factors in literature
- Predict larger in-medium hadron size (\( r_{D^0} = \sqrt{3/(2\mu\omega)} = 0.97 \) fm) than in vacuum (0.83 fm), consistent with relativistic potential model prediction (Shi, Zhao, Zhuang, arXiv:1905.10627)
- Coalescence-fragmentation model: use Pythia to fragment heavy quarks that do not coalesce
Energy conservation and thermal limit

Recall: \( f_M(p'_M) = \int d^3p_1d^3p_2f_1(p_1)f_2(p_2)W(p_1, p_2)\delta(p'_M - p_1 - p_2) \)

- Energy is not conserved if \( p'_M \) is directly put on-shell with the hadron mass
- 3-\( p \rightarrow 4-\)\( p \) conservation: coalesce to an off-shell c-hadron \( (E'_M, p'_M) \) and then decay it to an on-shell c-hadron with a pion \( (E_M, p_M) + (E_\pi, p_\pi) \)
Energy conservation and thermal limit

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- 3-\( p \rightarrow 4-\)\( p \) conservation: coalesce to an off-shell \( c \)-hadron \((E'_M, p'_M)\) and then decay it to an on-shell \( c \)-hadron with a pion \((E_M, p_M) + (E_\pi, p_\pi)\)

- Guarantee boost invariance

- Respect the thermal equilibrium limit of \( c \)-hadrons: thermal \( c \) + thermal \( q \rightarrow \) thermal \( D^0 \)

- Sudden approximation \(|q, g\rangle \rightarrow |h\rangle\) (no inverse process) does not require the chemical equilibrium
Heavy quark evolution in heavy-ion collisions

- **Initial production**: MC-Glauber for $x$ space, FONLL with CT14NLO (+EPPS16) for $p$ space
- **Interaction with QGP**: Langevin-hydrodynamics model [SC, Qin and Bass, PRC 88 (2013)]

  \[
  \text{Langevin: } \frac{d \vec{p}}{dt} = -\eta_D(p) \vec{p} + \vec{\xi} + \vec{f}_g, \text{ with gluon radiation } \vec{f}_g = -\frac{d \vec{p}_g}{dt}
  \]

  The medium-induced gluon momentum $\vec{p}_g$ follows the spectra from the higher-twist formalism

- **Hydrodynamics**: VISHNEW [Qiu, Shen, Heinz, PLB 707 (2012)]
- **Hadronization**: Coalescence-Fragmentation at the $T_c = 160$ MeV hypersurface

- Model parameter: heavy quark diffusion coefficient $D_s(2\pi T)$ — 3.5 at RHIC and 4 at LHC
Examples of heavy flavor $R_{AA}$ and $v_2$

$D$ meson $R_{AA}$

- Coalescence enhances the $D^0$ $R_{AA}$ at medium $p_T$, generates its bump structure

$c$ and $b$ decay electron $R_{AA}$ and $v_2$

(taken from STAR presentation at QM2019 by M. Kelsey)
Charmed hadron spectra: QGP flow effect

- Coalescence dominates $\Lambda_c$ production over a wider $p_T$ region than $D^0$
- The QGP radial flow significantly enhances the coalescence contribution
- The inaccuracy of default Pythia fragmentation in pp should have minor effects on AA results, could be improved later (color reconnection [Velasquez et al., PRL 111 (2013)], or coalescence in pp [Song, Li, Shao, EPJC 78 (2018)])
Charmed hadron chemistry at RHIC

- (a) Stronger QGP flow boost on heavier hadrons => increasing $\Lambda_c/D^0$ with $N_{\text{part}}$

- (b) Coalescence significantly increases $\Lambda_c/D^0$, larger value in more central collisions (stronger QGP flow)

- (c) Enhanced $D_s/D^0$ due to strangeness enhancement in QGP and larger $D_s$ mass than $D^0$
IF charm quarks have the same initial spectrum at RHIC and LHC, $\Lambda_c/D^0$ would be larger at LHC than RHIC due to the flow effect.

The harder initial charm quark spectra at LHC reduces $\Lambda_c/D^0$.

Similar theoretical prediction on $D_s/D^0$. 
Prediction on beauty hadron chemistry

- More constraints on the mass (velocity/momentum) dependence of hadronization models
- Assume same diffusion coefficient $D_s$ between $c$ and $b$ quarks
- Only difference: $\omega_c = 0.24$ GeV $\rightarrow$ $\omega_b = 0.14$ GeV so that $P_{coal}(p_b = 0) = 1$ for $b$ quarks

taken from CMS presentation at HP2020 by Z. Shi
Summary

- Developed a comprehensive hadronization model for heavy quarks
- Included a complete set of $s$ and $p$-wave hadron states in coalescence, allowing to normalize the heavy quark coalescence probability at $p = 0$ with proper $\omega$
- Introduced 4-$p$ conservation to respect boost invariance and thermal equilibrium limit
- Revealed the strong QGP flow effect on the heavy flavor hadron chemistry
- Provided a good prediction on $\Lambda_c/D^0$, $D_s/D^0$ and $B_s/B^+$ at RHIC and LHC
- Found the competing effects of QGP flow and charm quark spectra yield the different observations at RHIC vs. LHC