Effect of strong magnetic field on the interplay of momentum, heat and charge transports in a hot QCD matter

Binoy Krishna Patra and Shubhalaxmi Rath

Department of Physics, Indian Institute of Technology Roorkee

binoy.purva@gmail.com, shubhalaxmirath@gmail.com

Introduction

- The peripheral events of ultrarelativistic heavy ion collisions produce extremely strong magnetic fields.
- High energetic areas where strong magnetic fields might exist :

(1) Peripheral heavy ion collisions (10^{18} G to 10^{20} G)
($1\ m_\pi^2 \sim 0.02\ \mathrm{GeV}^2 \sim 10^{18}$ G)
(2) Cores of compact stars (10^{13} G to 10^{16} G)
(3) The early universe (much higher than the above cases)

- Depending on the electrical conductivity, the magnetic field may remain strong during the lifetime of the partonic medium [1, 2].
- Thus, it is interesting to explore the effect of strong magnetic field on the interplay of momentum, heat and charge transports in a hot QCD matter.
- Strong magnetic field (SMF) limit : $|q_iB| \gg T^2$, $|q_iB| \gg m_i^2$.
- Magnetic field breaks the rotational symmetry. So, momentum gets separated into transverse (p_T) and longitudinal (p_L) components and in SMF limit, $p_L \gg p_T$. Thus, an anisotropy is created.
- Another anisotropy is also created at the early stage of heavy ion collision due to the preferential expansion of the matter, where $p_T \gg p_L$.

Different anisotropies

- Anisotropic parameter -: $\xi = \frac{\langle \mathbf{p}_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$.
- Isotropic distribution functions :

$$f_i^{\rm iso}(\mathbf{p};T) = \frac{1}{e^{\beta\omega_i}+1}\,,$$

where $T = 1/\beta$ and $\omega_i = \sqrt{\mathrm{p}^2 + \mathrm{m}_i^2}$,

$$f_g^{\rm iso}(\mathbf{p};T) = \frac{1}{e^{\beta\omega_g}-1}\,.$$

- Expansion-driven anisotropic distribution functions :

$$f_{{\rm ex},i}^{\rm aniso}(\mathbf{p};T) = f_i^{\rm iso} - \frac{\xi\beta(\mathbf{p}\cdot\mathbf{n})^2}{2\omega_i} f_i^{\rm iso}(1-f_i^{\rm iso})\,,$$

$$f_{{\rm ex},g}^{\rm aniso}(\mathbf{p};T) = f_g^{\rm iso} - \frac{\xi\beta(\mathbf{p}\cdot\mathbf{n})^2}{2\omega_g} f_g^{\rm iso}(1+f_g^{\rm iso})\,.$$

- Magnetic field-driven anisotropic distribution function :

$$f_{{\rm B},i}^{\rm aniso}(\mathbf{p}';T) = f_i - \frac{\xi\beta p_3^2}{2\omega_i} f_i(1-f_i)\,,$$

where $\mathbf{p}' = (0,0,p_3)$ and $f_i = \frac{1}{e^{\beta\sqrt{p_3^2+m_i^2}}+1}$.

- Quasiparticle masses :

Pure thermal medium : $m_{qT}^2 = \frac{g^2T^2}{6}$, $m_{gT}^2 = \frac{g^2T^2}{6}\left(N_c + \frac{N_i}{2}\right)$.

Thermal medium in strong magnetic field :
$m_{iT,B}^2 = \frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_{i0}} - \ln(2)\right]$, $m_{gT,B}^2 = \frac{g^2T^2N_c}{6} + \frac{g^2}{8\pi^2}\sum_i |q_iB|$.

- Effective quasiparticle masses in the presence of strong magnetic field get enhanced as compared to the masses at zero magnetic field.

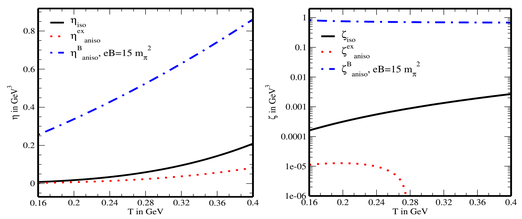
Shear and bulk viscosities

- For a nonequilibrium system, $T^{\mu\nu} = T_{(0)}^{\mu\nu} + \Delta T^{\mu\nu}$.
- The traceless and trace parts of ΔT^{ij} give the shear and bulk viscosities, respectively,

$$\Delta T^{ij} = -\eta\left(\partial^i u^j + \partial^j u^i - \frac{2}{3}\delta^{ij}\partial_l u^l\right) - \zeta\delta^{ij}\partial_l u^l.$$

- In the strong magnetic field regime, components of velocity transverse to the magnetic field vanish. Thus, only $\Delta\tilde{T}^{ij}$ (ΔT^{33}) component survives [3, 4],

$$\Delta\tilde{T}^{ij} = -\eta^B\left(\partial^i u^j + \partial^j u^i - \frac{2}{3}\delta^{ij}\partial_l u^l\right) - \zeta^B\delta^{ij}\partial_l u^l.$$



Electrical and thermal conductivities

- Infinitesimal disturbance by an electric field induces an electric current as

$$J_\mu = \sum_i 2q_ig_i\int\frac{d^3\mathrm{p}}{(2\pi)^3\omega_i}p_\mu\delta f_i\,.$$

- Ohm's law -: $\mathbf{J} = \sigma_{\rm el}\mathbf{E}$.
- The relativistic Boltzmann transport (RBT) equation in relaxation time approximation (RTA) [5] :

$$p^\mu\frac{\partial f_i}{\partial x^\mu} + q_iF^{\rho\sigma}p_\sigma\frac{\partial f_i}{\partial p^\rho} = -\frac{p_\nu u^\nu}{\tau_i}\delta f_i\,,$$

$$\tau_i = 1/\left[5.1T\alpha_s^2\log(1/\alpha_s)(1+0.12(2N_i+1))\right].$$

- Ohm's law in the presence of strong magnetic field -: $J_3 = \sigma_{\rm el}E_3$.
- The RBT equation in RTA, in a strong magnetic field [6] :

$$p^0\frac{\partial f_i}{\partial x^0} + p^3\frac{\partial f_i}{\partial x^3} + q_iF^{03}p_3\frac{\partial f_i}{\partial p^0} + q_iF^{30}p_0\frac{\partial f_i}{\partial p^3} = -\frac{p_0}{\tau_i^B}\delta f_i\,,$$

$$\tau_i^B = \frac{\omega_i\left(e^{\beta\omega_i}-1\right)}{\alpha_sC_2m_i^2\left(e^{\beta\omega_i}+1\right)}\frac{1}{\int dp_3'\frac{1}{\omega_i'\left(e^{\beta\omega_i'}+1\right)}}\,.$$

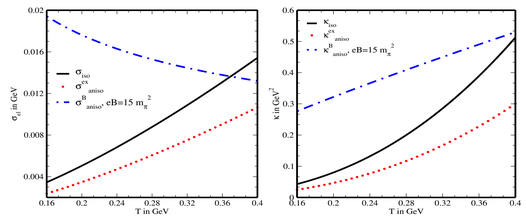
- Spatial component of heat flow due to the external disturbance :

$$\mathbf{Q} = \sum_i 2g_i\int\frac{d^3\mathrm{p}}{(2\pi)^3}\frac{\mathbf{p}}{\omega_i}(\omega_i - h_i)\delta f_i\,.$$

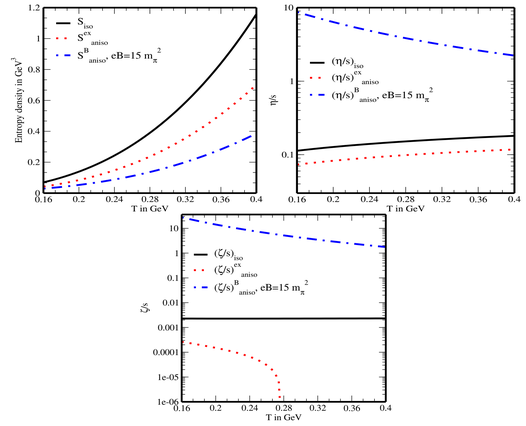
- The flow of heat in a medium : $\mathbf{Q} = -\kappa\frac{\partial T}{\partial \mathbf{x}}$.
- In strong magnetic field case :

$$Q_3 = \sum_i\frac{g_i|q_iB|}{2\pi^2}\int dp_3\,\frac{p_3}{\omega_i}(\omega_i - h_i^B)\delta f_i\,,$$

$$Q_3 = -\kappa\frac{\partial T}{\partial x_3}\,.$$

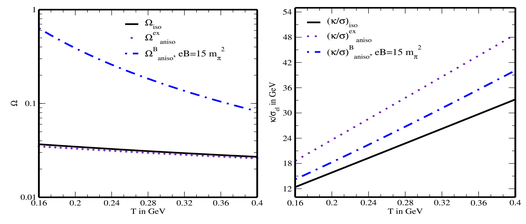


Entropy density, η/s and ζ/s



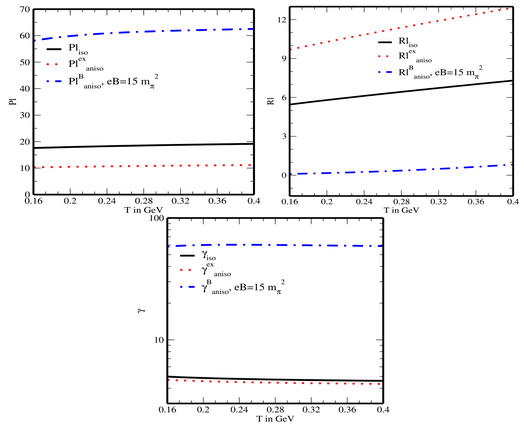
- In B-driven anisotropy, η/s and ζ/s get larger than their respective isotropic counterparts. So, the medium slightly deviates from the perfect fluid characteristic.

Knudsen number: $\Omega = \frac{\lambda}{L} = \frac{3\kappa}{LvC_V}$ and Wiedemann-Franz law: $\frac{\kappa}{\sigma_{\rm el}} = LT$



- In the presence of strong magnetic field, since Ω becomes nearer to 1, the medium may move slightly away from the equilibrium state.
- The Wiedemann-Franz law gets violated for a hot QCD matter in the presence of strong magnetic field.

Prandtl number: $\mathrm{Pl} = \frac{\eta/\rho}{\kappa/C_p}$, Reynolds number: $\mathrm{Rl} = \frac{Lv}{\eta/\rho}$ and $\gamma = \frac{\eta/s}{\sigma_{el}/T}$



- Since $\mathrm{Pl} > 1$, the sound attenuation in a hot QCD matter is mostly governed by the momentum diffusion.
- In B-driven anisotropy, the kinematic viscosity is larger than the length scale of the hot QCD matter (as $\mathrm{Rl} < 1$), whereas the opposite is true (as $\mathrm{Rl} > 1$) in expansion-driven anisotropy.
- The dominance of momentum diffusion over charge diffusion is more pronounced in the presence of strong magnetic field than in the absence of magnetic field.

Conclusions

- The B-driven and expansion-driven anisotropies affect η, ζ, $\sigma_{\rm el}$ and κ differently with respect to the isotropic medium.
- Thus, the viscosities and the conductivities can distinguish the abovementioned anisotropies.
- In addition, the transport coefficients derived from momentum, heat and charge transports, *viz.* η/s, ζ/s, Ω, L, Pl, Rl and $(\eta/s)/(\sigma_{el}/T)$ also get significantly affected by the strong magnetic field.

References

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