Heavy Quarkonia in a Magnetic Field

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Abstract

It is predicted that for the noncentral events in ultrarelativistic heavy-ion collisions (URHICs), a strong magnetic field is generated at the very early stages of the collisions. However, as we know the quarkonia, the physical resonances of QQ states, are formed in the plasma frame at a time, $t_F(=\gamma\tau_F)$, which is order of 1-2 fm, depending on the resonances and their momenta. By the time elapsed, the magnetic field may become weak. This motivates us to explore the effects of both the weak magnetic field ($T^2 > |q_f B|$, $T^2 > m_f^2$, where $|q_f|$ (m_f) is the absolute electric charge (mass) of the f-th quark flavour) and strong magnetic field ($|q_f B| > T^2$ and $|q_f B| > m_f^2$) on the properties of heavy quarkonia immersed in a thermal medium of quarks and gluons and then studied how the magnetic field affects the quasi-free dissociation in the aforesaid medium.

Methodology

1. First we have revisited the structure of gluon self-energy tensor in the presence of both weak and strong magnetic fields in thermal QCD and obtained the relevant form factors, that in turn computes the real and imaginary parts of the resummed gluon propagator.
2. Then the linear response theory yields the real and imaginary parts of the dielectric permittivity from the respective resummed propagators.
3. Finally, the inverse Fourier transform of the permittivities of the above propagators in the static limit obtains the complex heavy quark potential.
4. Further, the real-part of the potential is used in the Schrödinger equation to obtain the binding energy, whereas the imaginary part is used to calculate the thermal decay width of heavy quarkonia.
5. Finally we have studied the quasi free dissociation of quarkonia and obtained the dissociation temperatures of heavy quarkonia.

Heavy Quark Potential

The mass of heavy quark (m_Q) is very large, so the requirements - $m_Q \gg T \gg \Lambda_{QCD}$ and $m_Q \gg \sqrt{eB}$ are satisfied for the description of the interactions between a pair of heavy quark and anti-quark at finite temperature in a magnetic field in terms of quantum mechanical potential, that leads to the validity of taking the static heavy quark potential

$$V(r;T,B) = \int \frac{d^3k}{(2\pi)^{3/2}}(e^{ik.r}-1)\frac{V(k)}{\epsilon(k)}, \quad (1)$$

$\Rightarrow$ Dielectric permittivity: $\frac{1}{\epsilon(k)} = -\lim_{k_0\to 0} k^2 D^{00}(k_0,k)$ and $V(k)$ is the Fourier transform of the Cornell potential $V(r) = -\frac{\alpha}{r} + \sigma r$

Resummed Gluon Propagator

Real and imaginary parts of the resummed gluon propagator

In strong magnetic field

$$\mathrm{Re}\, D^{00}(k_0=0,k) = -\frac{1}{k^2+m_D^2}, \quad (2)$$

$$\mathrm{Im}\, D^{00}(k_0=0,k) = \frac{g^2}{4\pi k_z^2}\frac{\sum_f |q_f B| m_f^2}{(k^2+m_D^2)^2} + \frac{\pi g'^2 T^3}{k(k^2+m_D^2)^2}, \quad (3)$$

$\Rightarrow$ Debye mass

$$m_D^2 = g'^2T^2 + \frac{g^2}{4\pi^2 T}\sum_f |q_f B|\int_0^\infty dp_z \frac{e^{\beta\sqrt{p_z^2+m_f^2}}}{\left(1+e^{\beta\sqrt{p_z^2+m_f^2}}\right)^2}. \quad (4)$$

In weak magnetic field

$$\mathrm{Re}\, D^{00}(k_0=0,k) = -\frac{1}{k^2+M_D^2} - \frac{m_G^2}{(k^2+M_D^2)^2}, \quad (5)$$

$$\mathrm{Im}\, D^{00}(k_0=0,k) = \frac{\pi T M_{(T,B)}^2}{k(k^2+M_D^2)^2} + \frac{2\pi T M_{(T,B)}^2 m_G^2}{k(k^2+M_D^2)^3}, \quad (6)$$

$\Rightarrow$ m_G^2 is a dimensional constant, related to the string tension through the relation $\sigma = \alpha m_G^2/2$

$$M_{(T,B)}^2 = g^2T^2\left(\frac{N_c}{3}+\frac{N_f}{6}\right) + \left[\sum_f \frac{g^2(q_fB)^2}{8\pi^2T^2}\sum_{l=1}^{\infty}(-1)^{l+1}l^2K_0\left(\frac{m_f l}{T}\right) - \sum_f \frac{g^2(q_fB)^2}{48\pi^2T^2}\sum_{l=1}^{\infty}(-1)^{l+1}l^2K_2\left(\frac{m_f l}{T}\right) + \sum_f \frac{g^2(q_fB)^2}{384\pi^2}\frac{(8T-7\pi m_f)}{m_f^2 T}\right], \quad (7)$$

$\Rightarrow$ Debye mass

$$M_D^2 = g^2T^2\left(\frac{N_c}{3}+\frac{N_f}{6}\right) + \sum_f \frac{g^2}{12\pi^2T^2}(q_fB)^2\sum_{l=1}^{\infty}(-1)^{l+1}l^2K_0\left(\frac{m_f l}{T}\right) \quad (8)$$

Real and Imaginary Parts of the $Q\bar{Q}$ Potential

In presence of strong magnetic field (with $\hat{r} = rm_D$) [1, 2]

$$\mathrm{ReV(r;T,B)} = -\frac{4}{3}\alpha_s\left(\frac{e^{-\hat{r}}}{r}+m_D\right) + \frac{2\sigma}{m_D}\left[\frac{(e^{-\hat{r}}-1)}{\hat{r}}+1\right] \quad (9)$$

$$\mathrm{ImV_C(r;T,B)} = \frac{\alpha_s g^2}{3\pi^2}\sum_f |q_fB| m_f^2\left(\frac{\pi}{2m_D^3} - \left[\frac{\pi e^{-\hat{r}}}{2m_D^3} + \frac{\hat{r}\pi e^{-\hat{r}}}{2m_D^3}\right] - 2\frac{\hat{r}}{m_D}\int_0^\infty \frac{dk\, k}{(\mathbf{k}^2+m_D^2)^2}\int_0^{kr} dx\frac{\sin x}{x}\right) - \frac{8\alpha_s' T}{3}\int_0^\infty \frac{dz\, z}{(z^2+1)^2}\left(1-\frac{\sin z\hat{r}}{z\hat{r}}\right) \quad (10)$$

$$\mathrm{ImV_S^q(r;T,B)} = \frac{\sigma g^2}{2\pi^2}\sum_f |q_fB| m_f^2\left(\frac{\pi}{2m_D^5}\left[\hat{r}e^{-\hat{r}} - 3(1-e^{-\hat{r}}) + 2\hat{r}\right] - 2\frac{\hat{r}}{m_D}\int_0^\infty \frac{dk}{k(\mathbf{k}^2+m_D^2)^2}\int_0^{kr}\frac{\sin x}{x}dx\right) - \frac{4\sigma T}{m_D^2}\int_0^\infty \frac{dz}{z(z^2+1)^2}\left(1-\frac{\sin z\hat{r}}{z\hat{r}}\right) \quad (11)$$

In presence of weak magnetic field (with $\hat{r} = rM_D$) [3]

$$\mathrm{ReV(r;T,B)} = -\frac{4}{3}\alpha_s\left(\frac{e^{-\hat{r}}}{r}+M_D\right) + \frac{4}{3}\frac{\sigma}{M_D}\left(1-e^{-\hat{r}}\right) \quad (12)$$

$$\mathrm{ImV_C(r;T,B)} = -\frac{8}{3}\frac{\alpha_s T M_{(T,B)}^2}{M_D^2}\int_0^\infty \frac{zdz}{(z^2+1)^2}\left[1-\frac{\sin z\hat{r}}{z\hat{r}}\right] \quad (13)$$

$$\mathrm{ImV_S(r;T,B)} = -\frac{8\sigma T M_{(T,B)}^2}{M_D^4}\int_0^\infty \frac{zdz}{(z^2+1)^3}\left[1-\frac{\sin z\hat{r}}{z\hat{r}}\right] \quad (14)$$

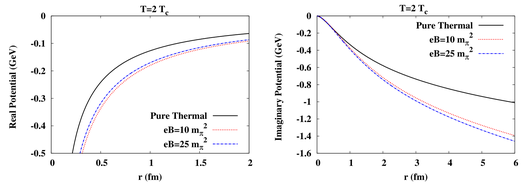

Figure 1: Real and imagniary parts of potential in presence of strong magnetic field

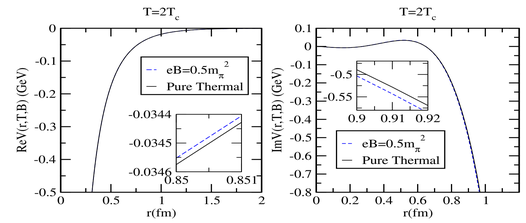

Figure 2: Real and imagniary parts of potential in presence of weak magnetic field

Binding Energy:

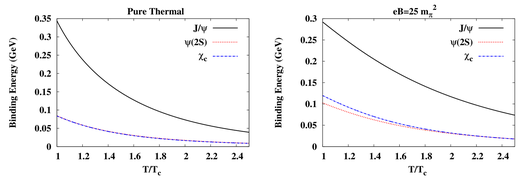

Figure 3: Binding Energy in presence of strong magnetic field

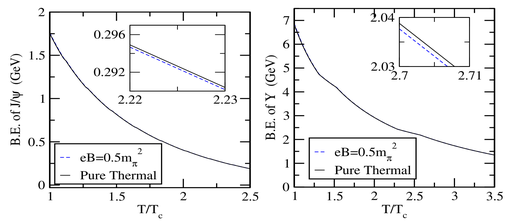

Figure 4: Binding Energy in presence of weak magnetic field

Decay Width ($\Gamma(\mathrm{T,B}) = -2\int_0^\infty \mathrm{Im\ V(r;T,B)}|\Phi(\mathrm{r})|^2 d\tau$)

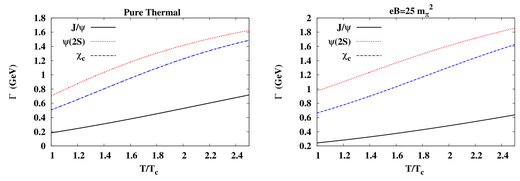

Figure 5: Deacay width in presence of strong magnetic field

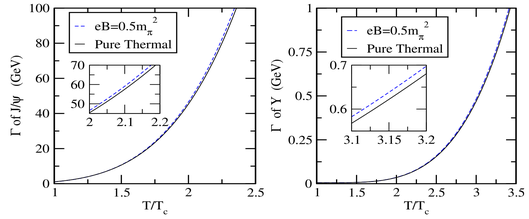

Figure 6: Deacay width in presence of weak magnetic field

Dissociation Temperatures (Criteria: $\Gamma_i \geq 2\,\mathrm{BE_i}$)

State	T_D (in T_c)[eB in m_π^2]	T_D (in T_c)[$eB=0$]
J/ψ	2.0 [6.50]	1.60
χ_c	1.1 [3.7]	0.80
$\psi(2S)$	$< 1 (< m_\pi^2)$	0.70

Table 1: Dissociation temperatures for strong magnetic field

State	T_D (in T_c)[eB in m_π^2]	T_D (in T_c)[$eB=0$]
J/ψ	1.13[0.5]	1.14
Υ	3.94 [0.5]	3.98

Table 2: Dissociation temperatures for weak magnetic field

Conclusions

- The real-part gets screened more in the presence of weak magnetic field, whereas it becomes less screened in the strong magnetic field compared to their counterparts in the absence of magnetic field. On the other hand, the magnitude of the imaginary-part becomes larger both in weak and strong magnetic field as compared to that in the absence of magnetic field.
- With the weak and strong magnetic field both the observed screening in the real-part of the potential can be attributedin terms of the decrease in the binding energy, whereas the increase in the magnitude of the imaginary-part of the potential will leads to the enhancement of decay width of quarkonia.
- The dissociation temperature in the presence of weak magnetic field becomes slightly lower, whereas in the presence of strong magnetic field it becomes higher compared to the one in absence of magnetic field.

References

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