Validation and Improvement of the ZPC Parton Cascade inside a Box

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1. Motivation

- Ensure that the parton cascade solution is accurate in solving the corresponding Boltzmann equation is important to understand the origin of collectivity and the difference between kinetic theory and hydrodynamics.
- Causality violation [1,2] is inherent in cascade simulations due to the geometrical interpretation of cross section. It leads to inaccurate numerical results at large opacities and the different choices of doing collisions and/or the reference frame can lead to different numerical results. Parton subdivision [3-5] can solve this problem but it is much more computationally expensive.
- The goal of this work [6] is to find a parton cascade algorithm that is accurate enough without using parton subdivision.

2. ZPC parton cascade

- The default differential cross section in ZPC for two-parton scatterings is [7,8]

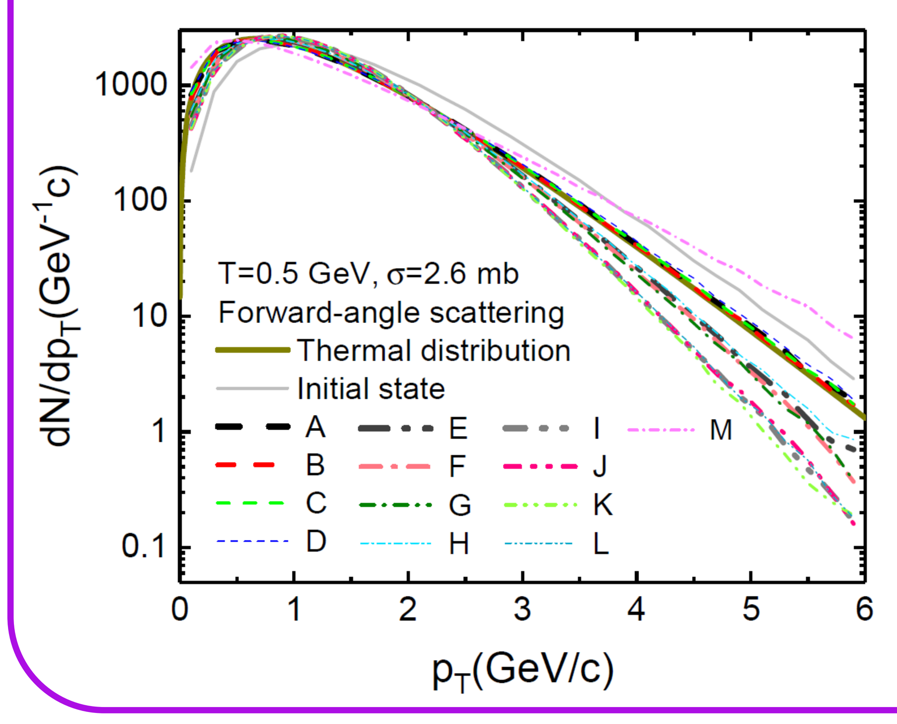
$$\frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{2}\left(1+\frac{\mu^2}{s}\right)\frac{1}{(t-\mu^2)^2} \qquad (1)$$

This equation represents forward-angle scatterings but $d\sigma/dt$ is independent of the scattering angle for isotropic scatterings. The total cross section is $\sigma = 9\pi\alpha_s^2/(2\mu^2)$.

- Different collision schemes for ZPC. ct_1 and ct_2 represent the collision times [6].

Ordering time \ Collision time	ct_1 & ct_2	$min(ct_1, ct_2)$	$(ct_1+ct_2)/2$	$max(ct_1, ct_2)$
$min(ct_1, ct_2)$	A	B (new scheme)	C	D
$(ct_1+ct_2)/2$	E	F	G (default ZPC scheme)	H
$max(ct_1, ct_2)$	I	J	K	L

The ordering time is the time of a collision in the ordering frame that we use for ordering the collisions.



The final p_T distributions

- They are different from these schemes.
- The default ZPC scheme deviates from the expected thermal distribution, and so do the most collision schemes.
- The collision scheme B (new scheme) is the closest to the expected thermal distribution.

3. Parton subdivision

- The opacity of the parton system is represented as [5]

$$\chi = \sqrt{\frac{\sigma}{\pi}}/\lambda = n\sqrt{\frac{\sigma^3}{\pi}}, \qquad (2)$$

- The transformation for parton subdivision is

$$f(x,p,t) \to l \times f(x,p,t), \quad \frac{d\sigma}{dt} \to \frac{d\sigma}{dt}/l \qquad (3)$$

- The transformation for the standard subdivision method is:

$$N \to l \times N, V \text{ unchanged}, \qquad (4)$$

- The transformation for our novel subdivision method can be represented by

$$N \text{ unchanged}, V \to V/l \qquad (5)$$

Because N does not change, this subdivision method is much more efficient than the standard subdivision method, therefore we can afford a huge subdivision factor such as 10^6 (instead of the usual value of up to a few hundreds).

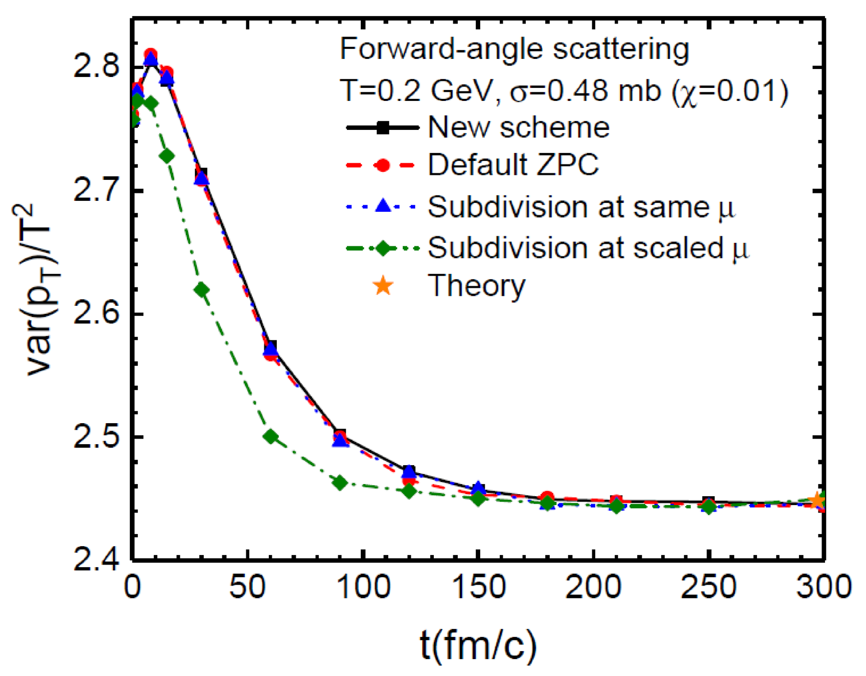
- $\text{var}(p_T) = \langle p_T^2\rangle - \langle p_T\rangle^2$; (6)

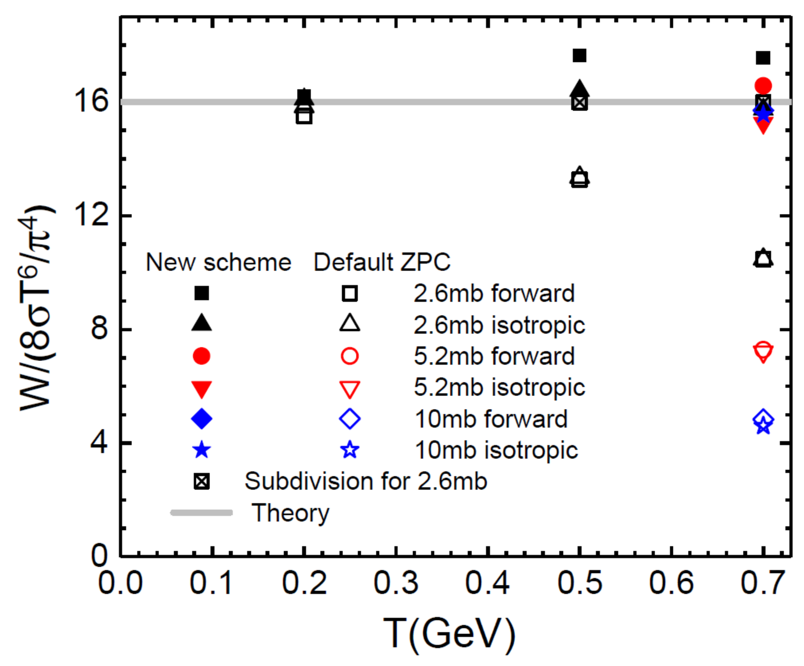
$\text{var}(p_T) = (8 - \frac{9\pi^2}{16})T^2$ in thermal equilibrium.

- For forward angle scatterings, the parton subdivision method needs to use the same μ value so that the scattering angular distribution is unchanged [6]. And the scaled μ means that increase μ to decrease σ.

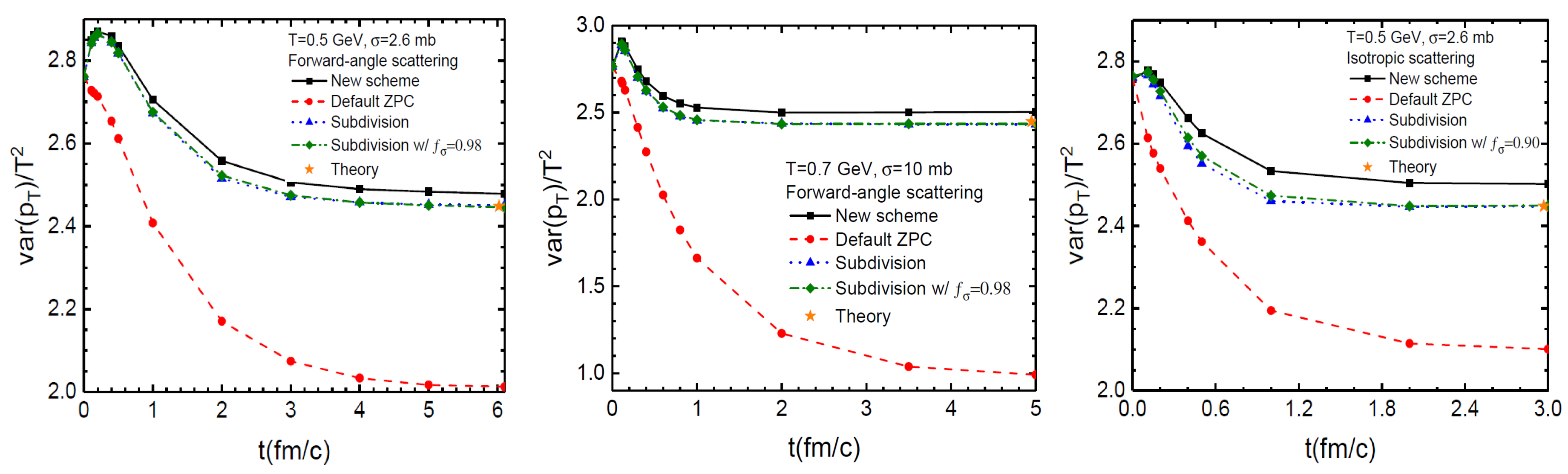
The collision rates per volume

- The default ZPC scheme gets much lower than the theoretical expectation for large cross sections.
- The new scheme are much closer to the theoretical value, even at large opacities.

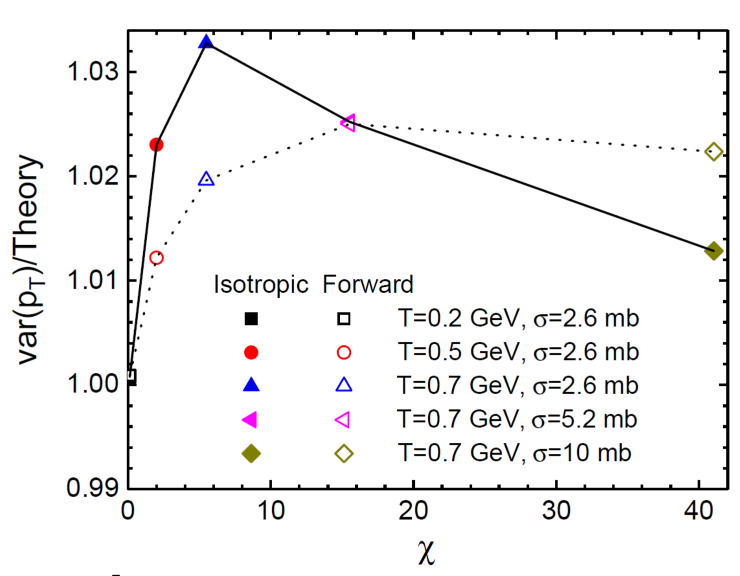




4. Results



- From low opacity to extreme opacities, the new scheme results are very close to the parton subdivision results, which agree with theoretical expectations at late times.
- The default ZPC scheme results are much further away from the parton subdivision results.
- Same conclusions for isotropic scatterings.



- At low opacities, the new scheme is very accurate as expected. At moderate to high opacities, the deviations of the variance of the p_T distribution are quite small.
- For isotropic scatterings, the maximum deviation does not occur at the highest opacity shown but at a moderate opacity.

5. Conclusions

- We have evaluated and then improved the accuracy of the ZPC parton cascade for elastic scatterings inside a box.
- A new collision scheme that is accurate enough without parton subdivision and much better than the default ZPC collision scheme has been found.
- A novel parton subdivision method is used to obtain the "exact" time evolution of the momentum distribution towards equilibrium. This subdivision method is valid for such box calculations.
- The novel subdivision method is much more efficient than the traditional subdivision method that we typically use a subdivision factor of 10^6.
- This work is the first step towards the validation and improvement of the ZPC parton cascade for scatterings in 3-dimensional expansion cases.

References

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