Validation and Improvement of the ZPC Parton Cascade inside a Box

Xin-Li Zhao, Guo-Liang Ma, Yu-Gang Ma, and Zi-Wei Lin

1 Fudan University, China
2 East Carolina University, USA
3 Shanghai Institute of Applied Physics, China

zhaoxilin@sinap.ac.cn  linz@ecu.edu

1. Motivation

- Ensure that the parton cascade solution is accurate in solving the corresponding Boltzmann equation is important to understand the origin of collectivity and the difference between kinetic theory and hydrodynamics.
- Causality violation [1,2] is inherent in cascade simulations due to the geometrical interpretation of cross section. It leads to inaccurate numerical results at large opacities and the different choices of doing collisions and/or the reference frame can lead to different numerical results. Parton subdivision [3-5] can solve this problem but it is much more computationally expensive.
- The goal of this work [6] is to find a parton cascade algorithm that is accurate enough without using parton subdivision.

2. ZPC parton cascade

- The default differential cross section in ZPC for two-parton scatterings is [7,8]
  \[ \frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{2} \left(1 + \mu^2\right) \frac{1}{(t - \mu^2)^2} \]
  This equation represents forward-angle scatterings but \(dt/dt\) is independent of the scattering angle for isotropic scatterings. The total cross section is \(\sigma = 9\pi\alpha_s^2/(2\mu^2)\).
- Different collision schemes for ZPC: \(c_t, c_t\) and \(c_t\) represent the collision times [6].

The ordering time is the time of a collision in the ordering frame that we use for ordering the collisions.

3. Parton subdivision

- The opacity of the parton system is represented as [5]
  \[ \chi = \frac{\sigma}{\pi \lambda} = \pi \frac{\sigma}{\pi \lambda}. \]
- The transformation for parton subdivision is
  \[ f(x, p_t, t) \rightarrow f(x, p_t, t), \quad \frac{dt}{dt'} = \frac{dt}{dt'}/l \]
- The transformation for the standard subdivision method is:
  \[ N \rightarrow t \times N/V \text{ unchanged}, \]
- The transformation for our novel subdivision method can be represented by
  \[ \frac{N}{N} \text{ unchanged}, V \rightarrow V/l \]
  Because \(N\) does not change, this subdivision method is much more efficient than the standard subdivision method, therefore we can afford a huge subdivision factor such as \(10^6\) (instead of the usual value of up to a few hundreds).
- \[ \var(p_T) = \left(p_T^2 \right) - \left(p_T^2\right)^2; \]
- \[ \var(p_T) = (8 - 9\pi^2/16)^2 \text{ in thermal equilibrium.} \]
- For forward angle scatterings, the parton subdivision method needs to use the same \(\mu\) value so that the scattering angular distribution is unchanged [6]. And the scaled \(\mu\) means that increase \(\mu\) to decrease \(\chi\).

The collision rates per volume

- The default ZPC scheme gets much lower than the theoretical expectation for large cross sections.
- The new scheme are much closer to the theoretical value, even at large opacities.

4. Results

- From low opacity to extreme opacities, the new scheme results are very close to the parton subdivision results, which agree with theoretical expectations at late times.
- The default ZPC scheme results are much further away from the parton subdivision results.
- Same conclusions for isotropic scatterings.

5. Conclusions

- We have evaluated and then improved the accuracy of the ZPC parton cascade for elastic scatterings inside a box.
- A new collision scheme that is accurate enough without parton subdivision and much better than the default ZPC collision scheme has been found.
- A novel parton subdivision method is used to obtain the “exact” time evolution of the momentum distribution towards equilibrium. This subdivision method is valid for such box calculations.
- The novel subdivision method is much more efficient than the traditional subdivision method that we typically use a subdivision factor of \(10^6\).
- This work is the first step towards the validation and improvement of the ZPC parton cascade for scatterings in 3-dimensional expansion cases.

References